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"SEMI-RIGID BEHAVIOUR"

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TITLE: Survey of existing types of joint modelling

POINTS OF REFERENCE: The present paper is a survey of the existing types of joint modellings which have been proposed during these last years by plenty of different authors. It is more aimed at describing the philosophies used than at presenting separately all the modellings. However the interested reader will find information relative to the different modellings in the original papers, a list of which is reported in the references [1,2] of the present paper.

KEY WORDS: Semi-rigid joints, modelling of M-phi curves

ABSTRACTS: The design of semi-continuous building frames requires a sufficient knowledge of the semi-rigid and partial-strength behaviour of the beam-to-column joints, beam-to-beam connections and column bases. The present paper is aimed at presenting and comparing the different existing types of modelling which have been proposed till now in view of the mathematical prediction of the deformability and resistance characteristics of joints and connections in building frames.

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SURVEY OF THE EXISTING TYPES OF JOINT MODELLING

The design and the analysis of semi-continuous frames require a sufficient knowledge of the semi-rigid and/or partial strength behaviour of the beam-to-column joints, beam-to-beam connection and column bases. In reality, the level of information required for each joint is dependent on the type of global frame analysis which is planned by the designer.

The more accurate but also expensive way to characterize the deformability and the resistance of the joints is the experimentation in laboratory. The use of this technique - which requires much money and much time - is basically limited to research activities and can consequently not be recommended for daily practice.

The existence of numerous test results for a large variety of joint configurations and connection types led progressively some researchers to develop computerized data banks. The low probability for the designer to find informations relative to the specific joint he is studying and the risk to misinterpretate the results listed in the databank - no standardized procedure for the testing of joint exits at present - limit considerably the practical interest of these tools. On the other hand, it appears now clearly that the databanks have to be considered as quite convenient tools for the validation of mathematical models aimed at predicting the semi-rigid joint response from the knowledge of the geometrical and mechanical joint properties.

Most of the well-known mathematical models available in the literature are described in [1] by NETHERCOT and ZANDONINI and in [2] by BURSI. They can be classified in four main categories:

A - Curve fitting
B - Simplified analytical models
C - Mechanical models
D - Finite element analysis.

The objective of the present paper is to highlight the differences and similarities between these categories through four tables focusing for each category on:
- the definition;
- the advantages and disadvantages (practical interest, field of application, ...)

The description of a specific model as well as a list of available models - without any reference/see [1] and [2] if needed - complete also the information given in each table.

Such a synthetic presentation is aimed at helping the researchers who intend to develop mathematical predicting tools for joints - whatever be the material used - to choose on good grounds the type of modelling to which he will refer.
CURVE FITTING

**Definition**

- Attempt to fit a mathematical representation to characteristic \( M \cdot \phi \) curves obtained by means of:
  * experimental tests in laboratory;
  * numerical simulations.

- Possible attempt to link the coefficients of the mathematical representation with physical parameters of the connection.

**Example**

**FRYE and MORRIS formula:**

\[
\phi = C_1 (kM) + C_2 (kM)^3 + C_3 (kM)^5
\]

in which: \( k \) is depending on the main parameters of the considered connection;

\( C \) coefficients are curve fitting constants.
### List of existing models

<table>
<thead>
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<th>Mathematical representations</th>
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<tr>
<td>Polynomial</td>
<td>Sommer 1969</td>
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<td></td>
<td>Kennedy 1969</td>
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<tr>
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<td>Richard formula</td>
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<td>Ramberg-Osgood</td>
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<td>Exponential</td>
<td>Lui and Chen 1986</td>
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<tr>
<td>Power</td>
<td>Krishnamurthy and al 1979</td>
</tr>
<tr>
<td></td>
<td>Murray and al 1987</td>
</tr>
</tbody>
</table>

### General advantages and/or disadvantages

- Capacity of representing with extreme accuracy any shape of M-\(\phi\) curve

- Purely empirical
  - range of application limited to joints, the geometrical and mechanical properties of which are similar to those considered when calibrating the formula

- Inability to recognize that, depending on the geometrical and mechanical parameters, the type of connection behaviour as well as the contribution of each component to the overall joint response may change significantly.
SIMPLIFIED ANALYTICAL MODELS

Definition

- Simplified analytical methods to predict the main characteristic values of the M-\(\phi\) curves (initial stiffness, design moment resistance,...); this step requires the knowledge of the mechanical and geometrical properties of the joints.

- Verification of these analytical methods by comparison with test data or results of numerical simulations.

- Description of M-\(\phi\) behaviour by curve fitting using the calculated initial stiffness, plastic and ultimate moment, ... in suitable mathematical expressions.

Example

JASPART's model:

\[
M_b = \frac{(K_i - K_u) \phi}{\{1 + \left[\frac{(K_s - K_u)\phi}{M_v}\right]^{1/C}\}} + K_s \phi + M_u
\]

with:  
- \(K_i\) = initial stiffness;  
- \(K_{st}\) = strain-hardening (and possible membranar) stiffness;  
- \(M_v\) = pseudo-plastic resistance;  
- \(M_u\) = ultimate resistance;  
- \(C\) = curve fitting constant.
List of existing models

- Lothers 1951  
  (double web cleat)

- Johnson and Law 1981  
  (flush endplate)

- Chen and al (1987/88)  
  (flange and/or web cleat(s))

- Yee and Melchers 1986  
  (end plate)

- Jaspart 1991  
  (welded - end plate - flange cleats)

General advantages and/or disadvantages

- Allow to approximate the form of the M-\(\phi\) curves without resort to testing

- Still require empirical curve fitting to generate the full curve (but limited !)

Special remark

EC3 method (Annex J for end plate and welded joints) refers basically to this section but provides an original way to generate the M-\(\phi\) curves:

\[ M = f(\phi) \]

\(f\) lies on the study of simplified analytical models
Definition

- Set of rigid and deformable elements representing each the behaviour of specific parts of the joint.

- Non-linearity of the joint response is then accounted for by inelastic constitutive laws adopted for the deformable elements.

- These constitutive laws are obtained from test data, numerical simulations or analytical models.

Example

TSCHERMERNEGG's model:
List of existing model

- Kennedy and Hafez 1984  
  (Header plate connections)

- Wales and Rossow 1983  
  (Double web cleat connections)

- Richard and al 1987  
  (Cleated connections)

- Tschemmernegg and al 1988  
  (Fully welded and endplate connections)

- Jaspart 1990  
  (Composite connections with endplate and cleated connections -  
  also applicable to similar steel connections)

General advantages and/or disadvantages

- Really suitable for modelling provided that a knowledge of the load  
  deformation curve of the key components is available.

- May be easily extended to different types of joint configurations and of  
  connections provided the knowledge of the key components is available.

- May require the use of computer programs to generate the curve.
### Definition

- Prediction of $M-\phi$ curves by means of a finite element analysis.

### Example

**KRISHNAMURTHY’s analysis**

![Diagram with labeled components: Bolt, End plate, Roller, Beam]
### List of existing models

<table>
<thead>
<tr>
<th>Model Description</th>
<th>References</th>
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<tr>
<td><strong>Welded connections</strong></td>
<td></td>
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<tr>
<td></td>
<td>Bose and al 1972</td>
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<tr>
<td></td>
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<td>Atarnaz Sibai, Jaspart, Frey 1988</td>
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<td><strong>Bolted connections</strong></td>
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<td>Lipson and Hague 1978 (single angle bolted-welded connections)</td>
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<tr>
<td></td>
<td>Patel and Chen 1985 (fully bolted connections)</td>
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<tr>
<td></td>
<td>Richard and al 1983 (double web cleat connections)</td>
</tr>
<tr>
<td></td>
<td>Krishnamurty 1980 (end plate connections)</td>
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<tr>
<td></td>
<td>Murray and al 1987 (flush end plate connections)</td>
</tr>
</tbody>
</table>

### General advantages and/or disadvantages

- Suitable to predict the response of welded connections
- Sufficient ability to model the non linear 3-D response of bolted connections and in particular:
  - the actual bolt action
  - the contact phenomena
  - the slips
  - ...
  
  not yet attained
From the four tables, it may be concluded that:

1. The finite element analysis is not yet likely to be used to predict the semi-rigid response for any type of connection detailing and that its use is reserved to research activities.
2. The simplified analytical models and the mechanical models are characterized by a wider field of application than curve fitting; this results from the theoretical background of these two kinds of modelling. They have, however, to be extended to other types of connections in order to cover the main needs of the designers.
3. The simplified analytical models are the only ones - with the formulae resulting from curve fitting - to be suitable for hand calculations.

REFERENCES


[2] BURSI, O.: "Behaviour and modelling of semi-rigid beam-to-column steel joints", SPRINT Contract RA351, University of Trento (I), University of Liège (B), CRIF (B), CTICM (F), ENSAIS Strasbourg (F), LABELIN (E).