MODELLING OF BEAM-TO-COLUMN JOINTS FOR THE DESIGN OF STEEL BUILDING FRAMES

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ABSTRACT

Present paper is aimed at presenting and classifying well known second-order inelastic computer programs for the analysis of building frames with semi-rigid beam-to-column joints according to their type of numerical joint modelling and at highlighting and discussing the influence of this modelling on the structural frame response.

INTRODUCTION

For sake of economy, beam-to-column bolted joints without any column web stiffener become a common practice (joints between H or I sections). Such a joint has a non-linear behaviour: when the beam is subject to bending, the axes of the connected members do not rotate a same angle, what results in a relative rotation that is not proportional to the beam bending moment.

In a strong axis beam-to-column joint, two main sources of deformability are identified (fig. 1):

a) The deformation of the connection associated to the deformation of the connection elements (end plate, angles, bolts,...), to that of the column flange and to the load-introduction deformability of the column web;

b) The shear deformation of the column web associated mostly to the pair of forces $F_b$ carried over by the beam(s) and acting on the column web at the level of the joint; these forces are statically equivalent to the beam moment $M_b$.

These components are illustrated in figure 2 for the particular case of a joint between a column and a one-sided beam. The flexural deformability of the connection elements is concentrated at the end of the beam (fig. 2.a). The associated behaviour is expressed in the format of a $M_b - \phi$ curve.

The deformation of the ABCD column web panel is divided into:

- The load-introduction deformability which consists in the local deformation of the column web in both tension and compression zones of the joint (respectively a lengthening and a
shortening) and which results in a relative rotation $\phi$ between the beam and column axes; this rotation concentrates mainly along edge BC (fig. 2.b) and provides also a deformability curve $M_b - \phi$.

The shear effect - due to shear force $V_s$ - which results in a relative rotation $\gamma$ between the beam and column axes (fig. 2.c); this rotation makes it possible to establish a second deformability curve $V_s - \gamma$.

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constraint of the column normally a lengthening and a

Figure 1 - Deformation of a strong axis joint

Figure 2 - Joint deformability components

It is important to stress that the deformability of the connection (connection elements + load-introduction) is only due to the forces carried over by the flanges of the beam (equivalent to the beam moment $M_b$), while the shear in a column web panel is the result of the combined action of these equal but opposite forces and of the shear forces in the column at the level of the beam flanges. In fact, the actual value of the shear force $V_s$ may be obtained from the equilibrium equations of the web panel [1]; it is given by the following formula (figure 3):

$$V_s = \frac{M_{bl} + M_{b2}}{d_p} - \frac{V_{cl} + V_{c2}}{2}$$

(1)

As highlighted herein under, most of the researchers refer to another formula:

$$V_s = \frac{M_{bl} + M_{b2}}{d_p}$$

(2)

which is nothing but a rough approximation of the actual one (formula 1). The validity of formula (1) has been clearly demonstrated in [1].

The difference between the loading of the connection and that of the column web in a specified joint requires, at a theoretical point of view, that account be taken separately of both deformability sources when designing a building frame (figure 4.a). However doing so is only practicable when the frame is analysed by means of a sophisticated computer program allowing for the separate modeling of both deformability sources. In all other cases, the
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*: stiffnesses of the connection under normal and shear forces may also be simulated by the spring block

**: interaction between normal force and bending moment in the connection may be taken into account.
actual behaviour of the joints must be simplified by concentrating the whole deformability at the beam end (figure 4.b). Reference [2] gives guidelines on how to "concentrate" the joint deformability in an accurate and safe manner for design practice.

Figure 3 - Loading of an interior joint

Figure 4 - Actual and simplified joint modelling
(a) Actual one
(b) Simplified one (after concentration [2])

JOINT REPRESENTATIONS IN COMPUTER PROGRAMS

Several joint representations have been adopted in the last years by different authors. They have to be classified in two categories (table 1):
- representations allowing for the separate modelling of the connection and column web panel behaviours;
- representations allowing for the modelling of the connection behaviour only (or of the joint behaviour by means of the "concentration" concept).

Table 1 presents an outline of the main representations proposed as well as of the computer programs in which they have been implemented. For each of these programs, informations relative to:
- the definition of the shear force $V_a$ to which it is referred when simulating the behaviour of the sheared column web panel or when concentrating the joint deformability (formula 1 or 2);
- the way in which the spread of plasticity in the connected beam and column members is taken into account (plastic zone or plastic hinge theory);
are also reported in table 1. People interested in is begged to consult the references given in table 1 as well as reference [3] where a detailed description of the programs and joint representations is given. The sake of consideration for the real dimensions of the joint in the representations presented in table 1 has to be pointed out.
FINELG FINITE ELEMENT PROGRAM

The non-linear finite element program FINELG developed jointly at the University of Liège, Belgium, and at the Polytechnic Federal School of Lausanne, Switzerland, has been recently implemented in Liège to simulate accurately the non-linear behaviour of connections and sheared column web panels [8]. The flexural behaviour of the connection and of the adjacent beam as well as that of the column web panel are gathered into a single 'plane beam + connection + sheared panel' finite element. Any type of non-linear response may be associated to the behaviour of the beam, of the connection and of the web panel respectively. Aforementioned element can be used according anyone of the three different manners sketched in figure 5.

![Figure 5 - Use of the finite element](image)

(a) "Beam" element
(b) "Beam+connection" element
(c) "Beam+connection+sheared web panel" element

It exhibits several superiorities:
- in contrast to approaches [4], [5] and [6], it fulfills the equilibrium equations of the web panel;
- it makes possible an accurate and realistic picture of the actual macroscopic behaviour of the column web panel;
- it does not need a more refined discretization than that just required when rigid joints (in contrast to [4], [5], [6] and [7]);
- the number of equations which have to be solved at each step of the structural analysis is not increased due to the non-linear semi-rigidity of the joints.

NUMERICAL APPLICATIONS

In a recent document issued by ECCS TWG 8.1/8.2 ([3]), most of the numerical procedures described in the previous section (except [4], [7] and [8]) have been applied to the study of two unbraced and sway plane building frames with semi-rigid joints (figure 6). The comparison diagrams, on which the curves resulting from the application of procedures [7]...
and [8] have been superimposed, are presented in figure 7.

It is worthwhile stressing that: i) the deformability curves which characterize the web panel, connection and joint semi-rigid response (figure 6.b) have been analytically obtained by means of a mechanical prediction model proposed by TSCHEMMERNEGG in Innsbruck and, ii) it is explicitly referred, in this model, to formula (2) for the determination of the shear force $V_n$. That means, in practice, that the $V_n - \gamma$ curve which has to be introduced as a data in all the computer programs allowing for the separate modelling of connections and sheared column web panels has to be simply derived from the shear curves given in figure 6.b by dividing the values of the bending moment $M_b$ by the beam height $d_b$ (figure 1); this $V_n - \gamma$ curve must thus be considered as the actual one.

At each step in the non-linear numerical computation, the loading of the column web panel (figure 3) is assessed and the shear force $V_n$ is evaluated by means of formula 1 (for procedures [7] and [8]) or of approximate formula 2 (for procedures [5] and [6]); the relative rotation $\gamma$ is then deduced from the $V_n - \gamma$ data curve.

These informations are useful when interpreting the comparison diagrams (figure 7) which clearly show that (figure 6):

- The $\lambda - V_n$ curves obtained by means of procedures [9], [10], [11] and [12] (concentration of the joint deformability) are in close agreement;
- Procedures [5] and [6], which adopt more refined models but with a shear force $V_n$ evaluated by means of the approximate formula 2, provide quite similar results;
- Procedures [7] and [8], which are based on a separate modelling of the joint deformability components and on an actual definition of $V_n$ (formula 1), lead to higher values of the ultimate load factor $\lambda$ and of the frame stiffness at each step of loading.

**CONCLUSIONS**

From this study, it may be concluded that:

- The concentration of the joint deformability into rotational elements at beam end provides generally a safe appraisal of the frame response (overestimation of the transverse displacement of the beams and of all the storeys under service loads/underestimation of the ultimate resistance under factored loads). This safe character results mainly from the overestimation of the shear force resulting from the evaluation of $V_n$ by means of formula 2, when "concentrating" the joint deformability.

This conclusion is supported by an extensive parametric study recently performed at the University of Liège [2]. This study shows that the concentration provides an accurate prediction of the actual response of braced and unbraced frames, except when the beam-to-column connections have a stiffness and a strength similar to (or higher than) those of the web panel in shear, as for frames A and B (see figure 6.b). In the latter case, the actual frame collapse is somewhat underestimated. The concentration is however sufficiently accurate to be recommended for practical applications.

- The separate modelling of the joint deformability components provides only a better assessment of the actual frame response if reference is made to the exact definition of the shear force $V_n$ (formula 1). The use of procedures [5] and [6], which, in contrast to procedures [9], [10], [11], and [12], require special 'sheared panel' elements (what increases, often substantially [8], the size of the numerical system which has to be solved
at each loading step in the computation) becomes consequently somewhat questionable.

These conclusions have to be stressed just as databases for semi-rigid joints are developed in different countries with the view to validate existing analytical models for the prediction of the joint response. As a matter of fact, due to the dimensions of the connected members commonly used for joint laboratory tests, the influence of the $V_a$ definition on the experimentally obtained $V_a - \gamma$ curve may be sometimes especially significant (26% and 37% for instance for the two Austrian tests on welded joints reported and discussed in [1] and [4]), which reduces considerably, when use is made of formula 2, the interest of such a validation process.

![Diagram](image_url)

**Figure 6 - Frames A and B**
(a) Configurations and loadings
(b) Deformability curves
(c) Initial deflected shape
Figure 7 - Load-drift diagrams of frames A and B
REFERENCES


