THE POLY-REFERENCE LEAST-SQUARE COMPLEX FREQUENCY IDENTIFICATION REVISED TO IMPROVE DAMPING ESTIMATION

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ABSTRACT
The Poly-Reference Least-Square Complex Frequency (p-LSCF) identification method is a well-established method currently available in commercial software. It is known to deliver clear stabilization diagrams in which spurious poles can hopefully be separated from physical ones. It is shown in the literature [1] that this property is achieved thanks to an appropriate choice of constraints for the least-square (LS) problem to be solved. It is also demonstrated that, in case of low signal to noise ratio, the extracted damping ratio value is usually underestimated, the discrepancy being worse in situations in which the damping level is high. Solutions have been proposed to improve the damping estimation such as the frequency-domain maximum likelihood estimation. However, this type of identification method is iterative and particularly time consuming and therefore not suitable in many industrial cases.
In this paper, a solution is proposed to tackle damping underestimation and to improve pole identification. The developed methodology solves two subsequent least-square problems: one for each of the two opposite choices of constraints. The stabilization diagram is then built according to the p-LSCF method, but the damping ratios are computed solving the second LS problem on the paired poles.
The first part of the paper describes the method and the implementation in an in-house modal analysis software.
In the second part, the methodology is applied to two meaningful test cases to assess the improvement achieved. First, a finite element model of a clamped beam is considered to explore the sensitivity of the proposed damping estimation with respect to the noise level. If the noise level becomes too high, the proposed method behaves similarly to the LSCE method but with a cleaned stabilization diagram which improves identification of the poles. The relative error on the damping ratio estimation remains low while it becomes significant if the p-LSCF method is used. The method is then applied to an industrial test case and the results are compared and discussed.

KEYWORDS: EMA, p-LSCF, damping.

NOTATION

ARX	AutoRegressive model with eXogenous input
EMA	Experimental Modal Analysis
FRF	Frequency Response Function
p-LSCF	Polyreference Least-Square Complex Frequency
LS	Least-Square
LSCE	Least-Square Complex Exponential
MIMO	Multi-Input Multi-Output
SNR	Signal-to-Noise Ratio

1. INTRODUCTION

Experimental modal analysis has been one of the core topics of the engineering research in the last years. Several methodologies have been proposed to be able to correctly identify the modal parameters in different measurements conditions ([1] [2] [3] [4]). While the frequency estimation is straightforward, in the larger majority of the cases, the estimation of the damping still suffers from high uncertainties. When the measurement conditions are not ideal (in situ measurements) or when the design of the structure is such to induce highly damped modes, the available commercial solutions struggle to perform at their best. The numerical implementation of some of the existent well-established methodologies, such as p-LSCF and LSCE, is created to facilitate the structural poles identification by means of a stabilization diagram. Unfortunately, as the damping increases, or as the Signal-to-Noise ratio of the measurements worsens, the stabilization diagram starts to become chaotic and it becomes more difficult to clearly identify the physical poles and the relative modal parameters. Some algorithms have been recently implemented to overcome this bottleneck, but the required computational time and cost makes the available solutions not appealing for industrial applications [5]. In the current paper, a modified p-LSCF method is presented which allows to improve the damping estimation in non-ideal measurement cases providing a modal parameter estimation algorithm which is relatively non-cumbersome in terms of computational cost. The first part of the paper goes through the numerical implementation of the proposed solution highlighting the differences with respect to the state-of-the-art methods. The second part of the paper presents first a simple numerical validation case which is followed by a real industrial case in order to emphasize the potentialities of the modified methodologies.

2. DEFINITION OF THE PARAMETER ESTIMATION MATHEMATICAL PROBLEM

Most of the modal parameter estimation algorithms (LSCE [1], LSCF [3], subspace method [4]) require solving an over-determined system of equations, by means of a least-square approach (1):

$$\boldsymbol{A}x = b \tag{1}$$

An appropriate number of constraints has to be imposed on the parameters *x* to reduce the redundancy in the least-square problem.
The solution of (1) will return a certain number of mathematical poles in addition to the physical poles of the system, which could affect the parameters estimation accuracy under certain noise conditions. While the presence of mathematical poles is unavoidable, a proper choice of constraint facilitates discerning the physical poles from the mathematical ones by acting on their stability.

2.1. Constraint choice and mathematical pole stability

Considering a MIMO problem and a classical AR model (2):

$$A_0\, x[t] + A_1 x[t-1] + \cdots + A_r\, x[t-r] = e[t\,] \tag{2}$$

two typical constraint choices are available:

a) Constraining the first block: $A_0 = I_{N_i}$ with N_i being the number of inputs
b) Constraining the last block: $A_r = I_{N_i}$ with r being the maximum pole order

It is demonstrated in [6] that these particular constraint choices play an important role for the stability of the mathematical poles (also called stochastic poles or spurious poles):

- For case (a), the mathematical poles are stable (negative damping). The physical (a.k.a. deterministic) and mathematical poles are all within the unit circle. Deterministic and mathematical poles are differentiated thanks to their stability over increasing pole order. In practice, considering measurements with non-negligible noise component on a structure having rich modal content, the stabilization diagram becomes difficult to interpret due to the presence of a high number of stabilized poles. This approach is commonly used by LSCE and it can be used by one-step parameter estimation methods, such as least squares or subspace methods, both in the discrete time or frequency domain.
- For case (b), the mathematical poles are pushed outside the unit circle and are, therefore, unstable. The stabilization diagram is very clear, as only the deterministic poles are present. This approach is commonly used by LSCF algorithm family and it can be used by one-step parameter estimation methods, such as least squares or subspace methods, both in the discrete time or frequency domain.

In practice, it is possible to pass from one type of constraint to the other by performing either a transformation of the input data (time or frequency measurement) or a modification of the basis function considered in the problem solving. The LSCE method, for instance, can be applied as it is classically implemented but it is possible to reverse the time domain data; the resulting effect is similar to the one of constraining on the last block of the least-square equations. The stability of the mathematical poles is reversed, passing from stable to unstable. Equivalently, taking the complex conjugate of the frequency response functions as input of the parameter estimation method allows to inverse the stability behavior of the mathematical poles. Section 3 will present one practical implemented solution to tweak the stability of LSCF by modifying the basis function.

2.2. Impact of the stochastic part the parameter estimation

The LSCF identification methods fall under the category of autoregressive models with exogenous input (ARX) which are described in the frequency domain as the sum of a deterministic part and a stochastic part (3):

$$H_k = \frac{N_k}{D_k} + \frac{1}{D_k} E_k \tag{3}$$

With:

- k equal to the number of inputs times the number of outputs $N_i \times N_o$
- D_k, the common-denominator polynomial
- E_k, circular complex independent and identically distributed noise

In case of low noise level (or equivalently for high SNR), the contribution of the stochastic part is marginal, and it does not influence the parameter estimation accuracy. When very high level of noise is present, the stochastic part becomes more important and impacts the parameter estimation. While the frequency estimation is not affected, the stochastic part, which is forced to be unstable, pushes the physical poles from the stable side to the unstable region. This results into an underestimation of the damping ratio. This situation can be also encountered for highly damped systems with a relatively high noise level inducing a low SNR.

3. THE REVISED POLY-REFERENCE LEAST-SQUARE COMPLEX FREQUENCY IDENTIFICATION

A revised version of the p-LSCF algorithm is proposed in this paper. The method aims to keep the advantages of the classical method, i.e. based on a clear stabilization diagram, but improving the damping estimation.

The proposed methodology combines the two constraints typologies described in 2.1, resolving two different least square problems for each considered order. In a p-LSCF framework, it translates into having two different basis functions which derive from the clockwise and counterclockwise direction of the unit circle. As the estimated frequencies are proportional to the poles amplitude and they are not impacted by the constraint choice, it is possible to correlate the two sets of poles which are computed at each order. In particular, based on the pole amplitude, for each counterclockwise direction poles (for which the mathematical poles are unstable), the correlated pole is found in the set of clockwise direction poles (for which the mathematical and physical poles are stable). The stabilization diagram is then built with the counterclockwise direction poles and the frequency and damping are estimated with the clockwise direction poles. The flow diagram of the proposed algorithm is shown in Figure 1.

The main drawback of the proposed approach is the doubling of the computational time. However, the resolution of the two least square problems (positive and negative) can be easily parallelized and the numerical solver can be optimized with a relatively low effort.

The revised LSCF algorithm is included in a home-made identification software developed at V2i. The software (Figure 2) links the data acquisition, data treatment and correlation tools functionalities incorporating also a data converter from and towards commercial software formats.

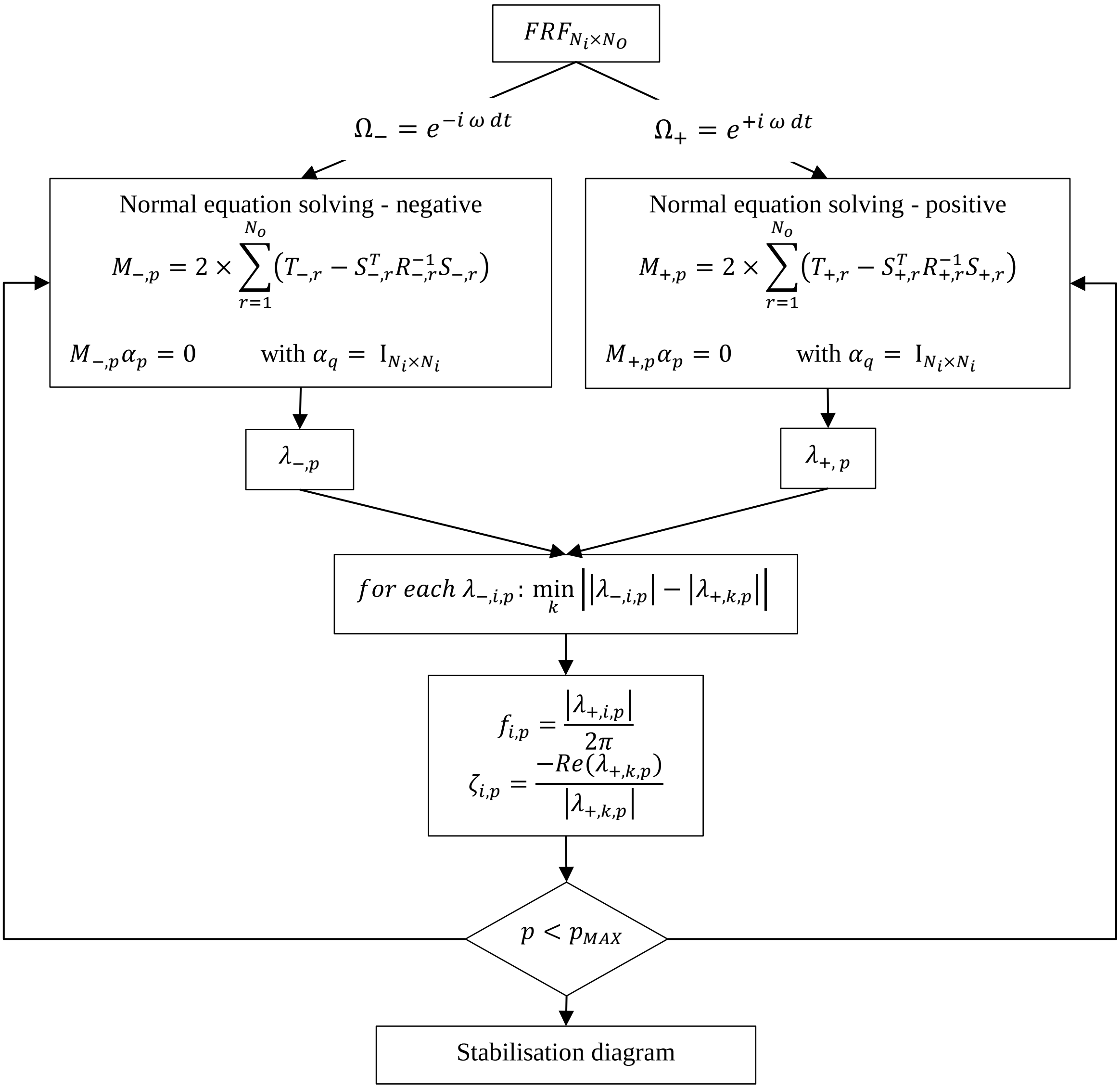


Figure 1 – Revised p-LSCF algorithm flow diagram

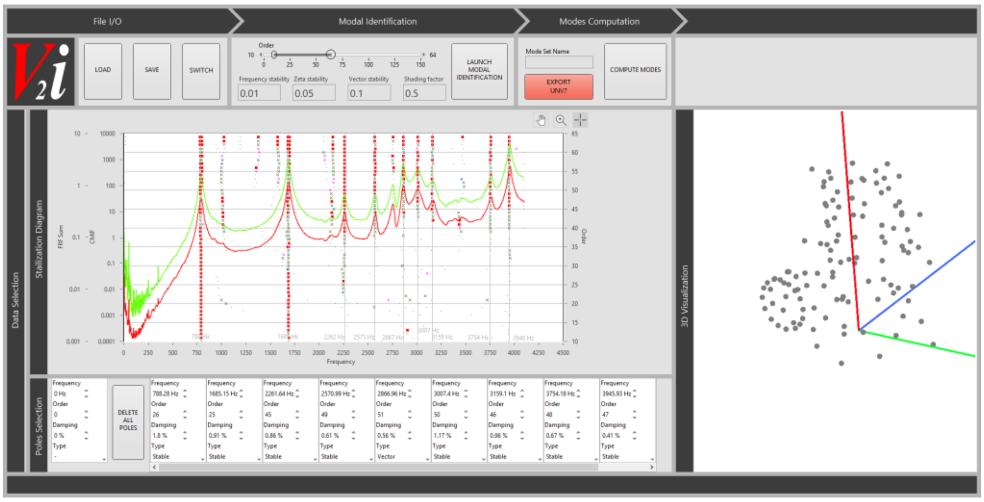
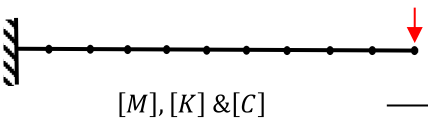


Figure 2 – V2i modal analysis software user interface illustration

4. NUMERICAL EXAMPLE

In order to illustrate the previous explanation, a numerical case is considered. A clamped beam finite element model has been implemented imposing a 1% damping ratio for each mode. 10 FRFs are computed at equally spaced points, considering a single point excitation applied at the beam tip. Artificial uncorrelated noise (uniformly distributed random noise) is added to the real (ϑ_1) and the imaginary parts (ϑ_2) of the FRFs independently (Figure 3). 7 different noise levels with increasing noise amplitude are considered (from 0.01% to 7.5%). The different noise percentages are computed with respect to the maximal absolute value of the FRFs, one value for the real part and one for the imaginary part.



$$H(\omega) = \frac{F}{-\omega^2 M + i\omega C + K}$$

$$H_{noise}(\omega) = \left[Re\big(H(\omega)\big) + \vartheta_1\right] + i\left[Im\big(H(\omega)\big) + \vartheta_2\right]$$

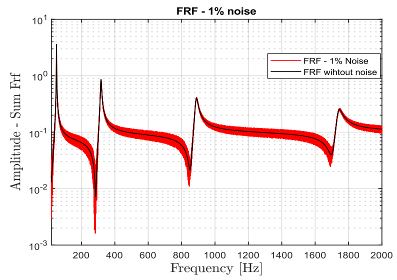


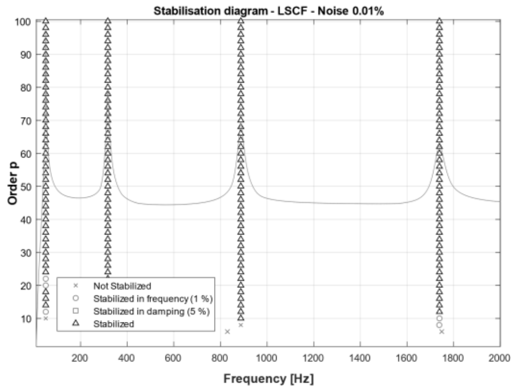
Figure 3 – Beam Model – FRF computation and noise addition

Four different modal identification methods (p-LSCF, LSCE, counterclockwise p-LSCF and a modified version of LSCE) are used to assess the sensitivity of the accuracy of the pole estimation with respect to the noise level. The two adapted algorithms, based on the well-known LSCF and LSCE, consider the alternative constraint for the least-square problem resolution, with respect to their typical implementation.
For the LSCF, it results in either inverting the sense of rotation on the unit circle ($\Omega = e^{+i\omega t}$ in place of $\Omega = e^{-i\omega t}$) or in taking the conjugate of the frequency response function matrix. For the LSCE, it is simply done by inverting the time axis.
Considering the lowest noise level, it can be observed that (Figure 4):

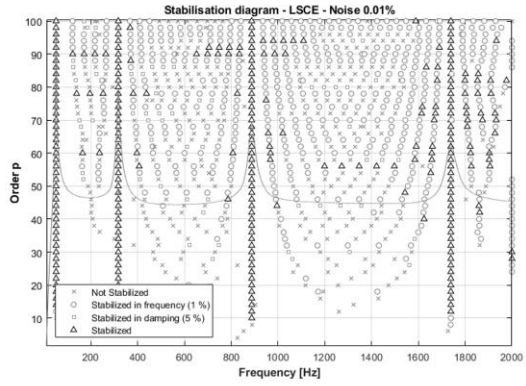
- clear stabilization diagrams are obtained for the original LSCF and the adapted LSCE algorithms;
- stabilization diagrams are polluted by the mathematical poles for the original LSCE and the revised LSCF algorithms;
- the stabilization is achieved for low pole order for all the algorithms.

The increment of noise level on the FRFs induces some trouble in the detection of stabilized poles. However, the LSCF algorithms seem to be less sensitive to the noise level, with respect to the other algorithms. Even with high noise level (5%), all the 4 resonances are identified with stabilized poles. Conversely, the revised LSCE is not capable of identifying the fourth pole. The original LSCE is able to locate the pole but the physical stable poles are lost among a dense forest of mathematical poles and it is difficult to distinguish a physical pole from a stochastic one.

The highest level of noise considered is this example may seem unrealistic. However, this level of low signal-to-noise ratio are typically encountered when the measurements are not taken in clean laboratory environment but in situ where several external uncontrollable noise sources could perturb the measurements.



(a)



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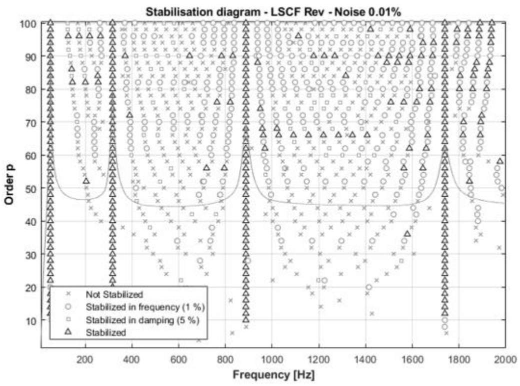

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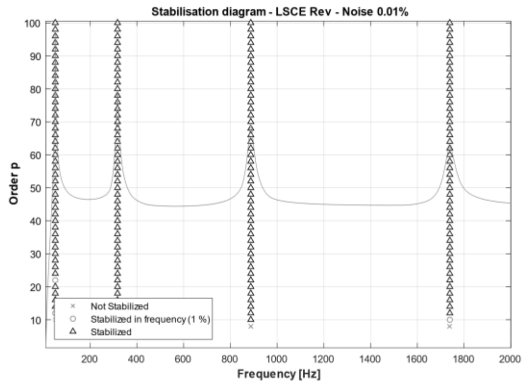
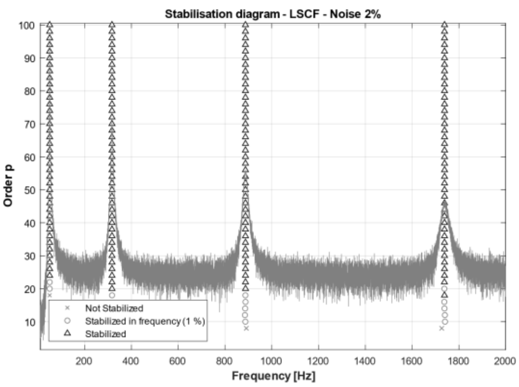
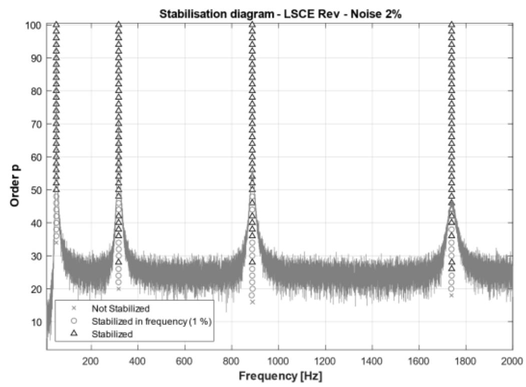


(d)

Figure 4 – Stabilization diagram – Low level of noise: (a) LSCF – (b) LSCE – (c) LSCF with alternative constraint – (d) LSCE with alternative constraint



(a)



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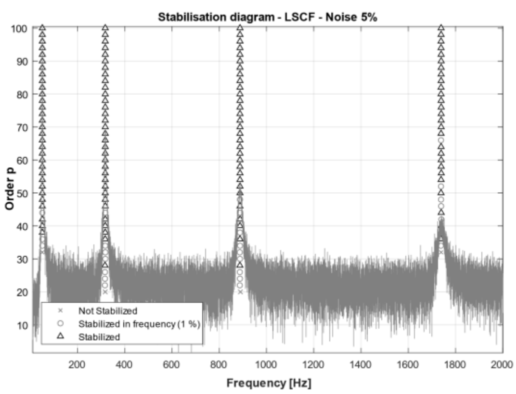

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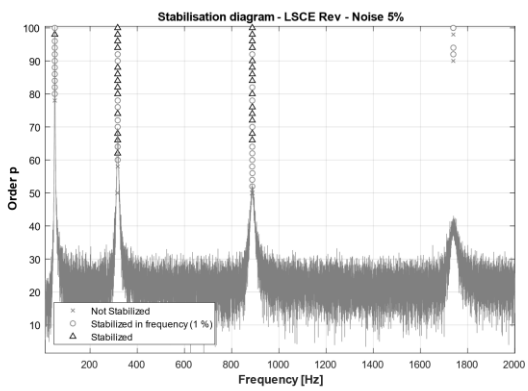
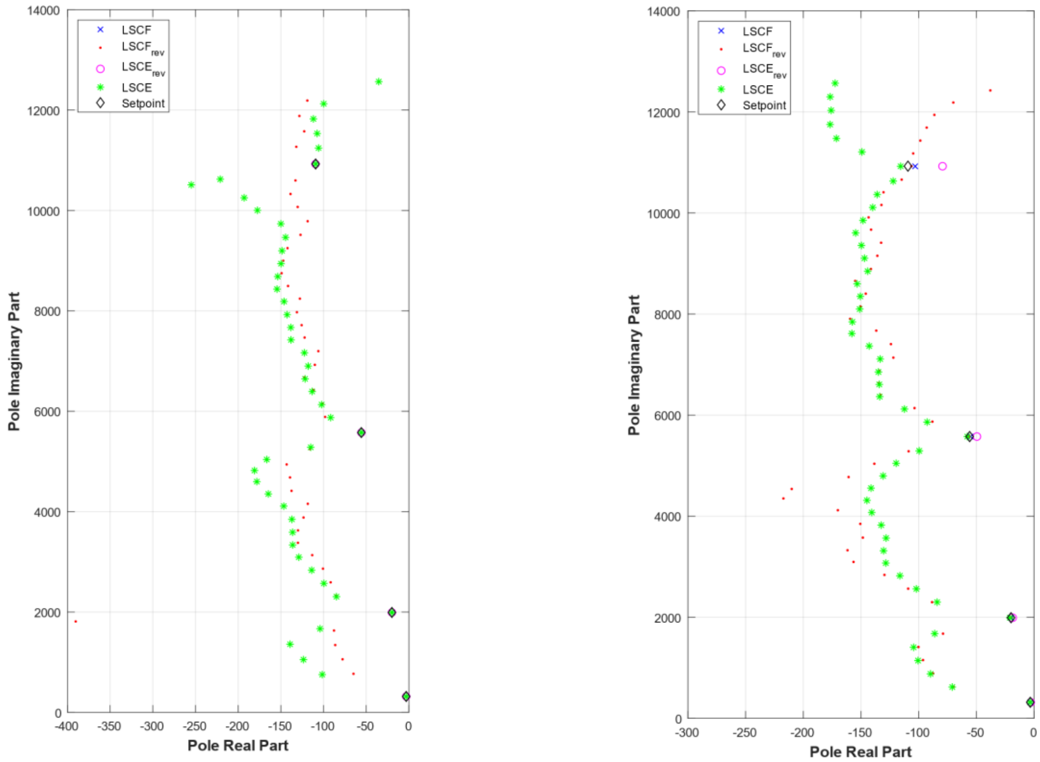


(d)

Figure 5 – Stabilization diagram: (a) LSCF with 2% of noise – (b) LSCE revised with 2% of noise – (c) LSCF with 5% of noise and (d) LSCE revised with 5% of noise

An analysis of the poles in their complex representation gives more insight of the impact of the constraint choice on the pole's values – see Figure 6 :

- for the low noise level, all the methods identify nearly identical poles;
- when the noise level increases, the behaviour described in §2.2 becomes more evident. The methods which confine the mathematical poles in the positive part (unstable) push the poles towards the unstable zone (underestimating the damping) while the methods for which the mathematical pole are stables tend to push the poles toward the left inducing a damping overestimation.
- In the analysed case, the shifting is lower in amplitude for the LSCF than for the LSCE.
- For the revised LSCE with unstable mathematical poles, when the noise level is higher than 2%, the poles are so much shifted towards the unstable part that also the physical poles become unstable.
- For the 4th mode, the shift direction is inverted for the counterclockwise LSCF, which is not in line with the behaviour of the other poles. The classical LSCF pole has always lower real value than the counterclockwise LSCF but side effects seem to affect the identification.



(a) (b)

Figure 6 – Complex representation of poles: (a) low noise level and (b) high noise level (2%)

Figure 7 and Figure 8 show the relative estimation error for the frequency and damping of the 4 first modes. The error on the frequency remains very low even for high noise level (below 0.1% of error for 7.5% of noise). Concerning the frequency estimation, no method appears to outclass the others for the investigated case. The differences are more significant if the damping estimation error is analyzed. The high sensitivity to noise of the LSCE methods, when compared to the LSCF methods, becomes evident again. The revised p-LSCF with stable mathematical poles is less sensitive to the noise and shows lower relative error in the damping compared to the LSCF with unstable mathematical poles. For the sake of comparison, for the mode 3 with 5% of noise, the relative error is about 6% for the p-LSCF with stable mathematical poles and is about 12% for the original LSCF.

In conclusion of this numerical example, the LSCE algorithm shows the highest sensitivity with respect to the noise level and the original LSCF exhibits higher error in the damping estimation than the version with stable mathematical poles.

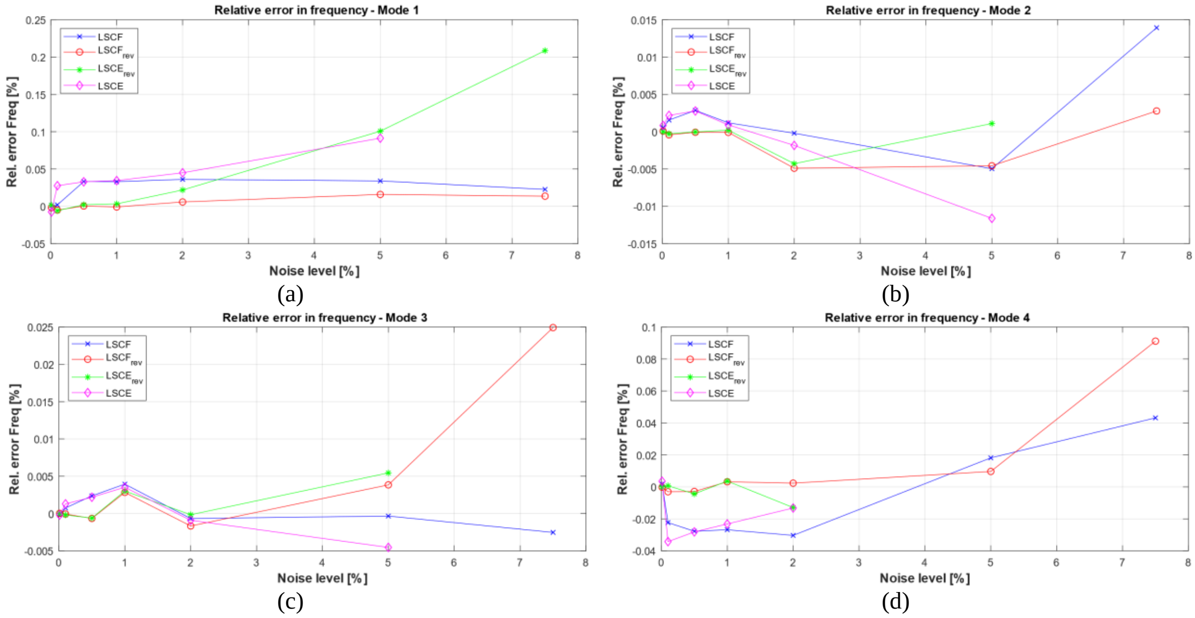


Figure 7 – Relative error in frequency: (a) Mode 1 – (b) Mode 2 – (c) Mode 3 – (d) Mode 4

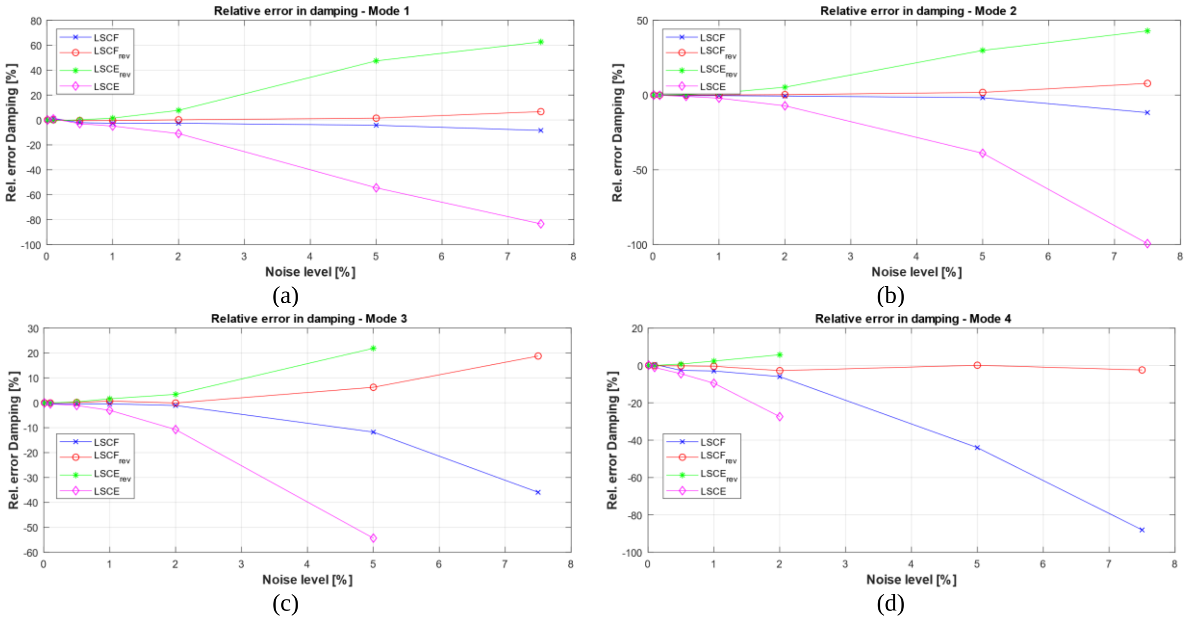


Figure 8 – Relative error in damping : (a) Mode 1 – (b) Mode 2 – (c) Mode 3 – (d) Mode 4

5. INDUSTRIAL TEST CASE

In order to illustrate a practical application of the proposed method, a modal analysis on an industrial case is presented in this section. The test case has been already described in [7], in which the other V2i modal analysis tools are described (optimal sensor placement, geometry validation, expansion and reduction). The present section describes only the modal identification algorithm.

The test object is a pyramidal structure which can be used as tooling for shaker based vibration tests. The accurate determination of the tooling modal behavior is extremely important in order to guarantee a correct interpretation of the vibration tests.
The structure is made of aluminum and weights approximately 220 kg. The pyramid base is clamped to the shaker table for the whole modal analysis campaign. In order to reproduce the boundary conditions that are encountered during a standard vibration test as closest as possible, the shaker is switched on, inducing a non-negligible noise on the measurements.

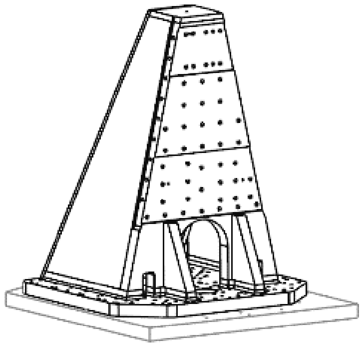


Figure 9 – Geometry of the considered structure

The structure is excited by an impact hammer on the top part of the tooling while the response points are the points determined as being optimal. The FRFs are measured thanks to LMS Test.Lab software [8]. Conversion tools have been developed to facilitate the transfer of data between commercial software and the in-house identification tools therefore allowing the loading of the measured FRFs into the modal identification algorithm

Among the modes computed between 50 and 2000 Hz, 8 modes have been identified as extremely relevant for the modal analysis. As an example, two modes extracted from the corresponding numerical model are shown in Figure 10.

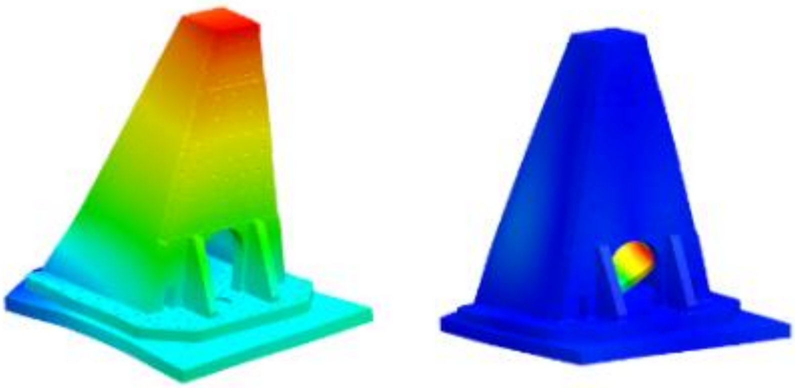


Figure 10 – Example of a global and local mode

In this paper, the results obtained with the revised version of the p-LSCF are compared with the results of the Polymax and the LSCE algorithm which are available in commercial software.
Table 1 compares the results (frequencies and damping ratio) identified with the three different algorithms. Figure 11 to Figure 13 show the stabilization diagrams obtained for the three methods (the revised p-LSCF is built with V2i software and the two others with LMS Test.Lab). The poles which are not clearly stabilized, which mostly correspond to not enough excited modes (orthogonal to the excitation), are not considered. If these modes would be of interest, a MIMO analysis would be needed to identify properly these poles.
Based on the following table and figures, the following conclusions can be drawn:

- There are no significant differences in terms of identified frequencies;
- The damping ratio determined with the revised p-LSCF and the LSCE algorithm are similar and are higher than the ones determined with Polymax. The available data do not allow to determine which one is the correct estimation, anyway the considered testcase proves that a special attention has to be taken when dealing with highly damped modes (or with low signal to noise ratio).
- The stabilization diagrams for the two versions of the p-LSCF give clear stabilization diagram with the non-physical poles eliminated. The LSCE diagram is really complicated to interpret. Due to noise level and the considered frequency bandwidth, a high order is needed to stabilize on all the modes.

Frequency [Hz]			
#	Revised p-LSCF	Polymax	LSCE
1	130.00	129.75	130.25
2	287.25	291.25	/
3	400.00	399.75	397.50
4	543.75	539.50	/
5	628.25	628.25	628.25
6	750.50	751.75	/
7	813.75	814.25	813.50
8	833.25	833.00	833.25

Damping ratio [%]			
#	Revised p-LSCF	Polymax	LSCE
1	3.50	2.90	3.50
2	2.90	0.43	/
3	2.20	2.00	2.20
4	4.10	1.68	/
5	0.28	0.26	0.29
6	1.71	1.96	/
7	2.26	1.62	2.20
8	0.62	0.59	0.61

Table 1 – Comparison of identified frequencies and damping ratio for: a) revised p-LSCF – b) Polymax – c) LSCE

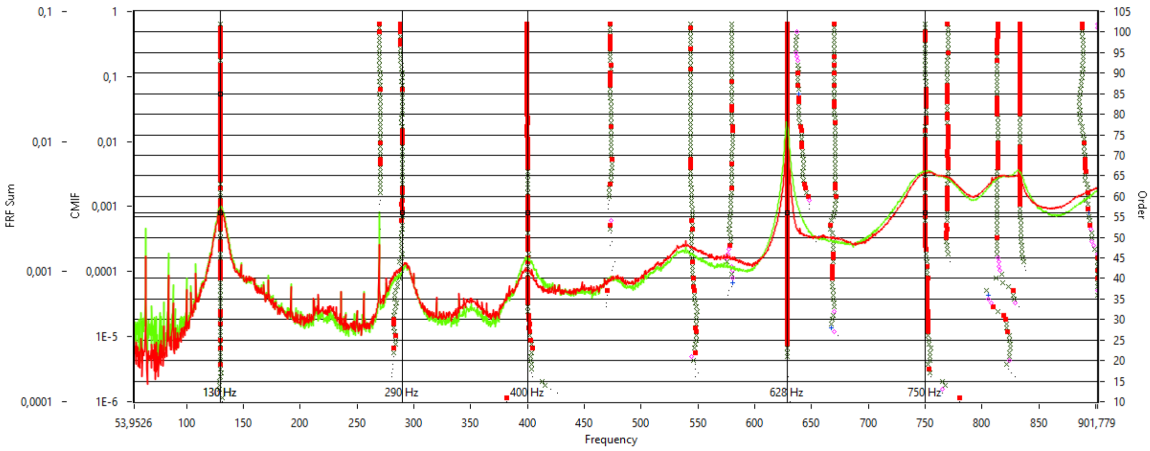


Figure 11 – Revised p-LSCF stabilization diagram

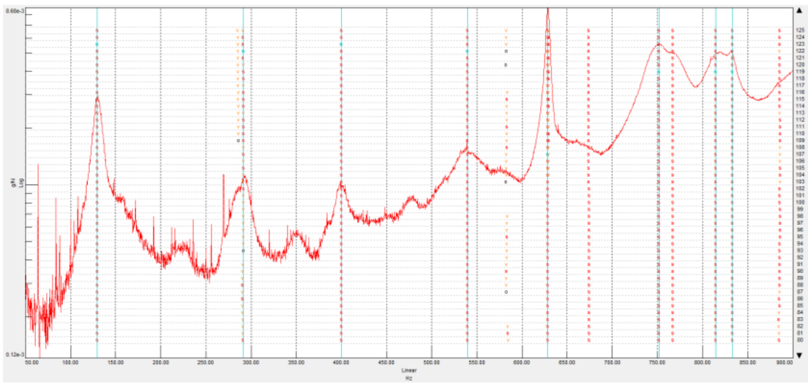


Figure 12 – Polymax stabilization diagram

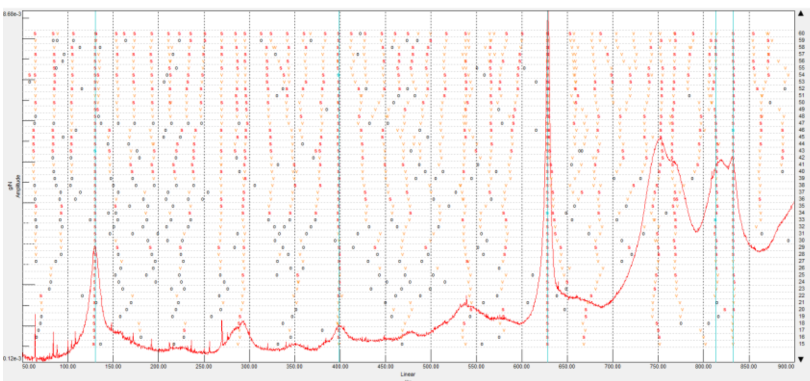


Figure 13 – LSCE stabilization diagram

The test case shows that the modifications of the p-LSCF algorithm allow to obtain a clear stabilization diagram while improving at the same time the damping ratio estimation, especially in case of low Signal-to-Noise ratios. The observation of the evolution of the frequency and damping parameters according to the pole order gives an insight into the stabilization of the parameters. The Figure 14 and Figure 15 show how the frequency and the damping estimation evolves for an increasing pole order considering a resonance at about 120 Hz and one at about 400 Hz. The frequency values are stable after a pole order of 20. The damping estimation is more variable, but it becomes stable as the order increases. Sudden jumps in the evolution of the damping are observed in Figure 15 that correspond to a bad correlation between the clockwise and counterclockwise pole. The analysis of the estimation trends toward the pole orders can help to judge the stability of the estimated parameter and it provides information about the margin of error.

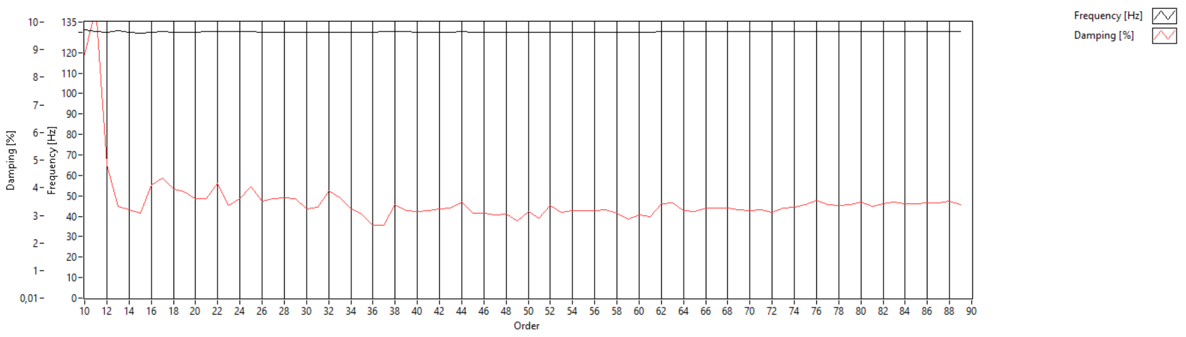


Figure 14 – Frequency and damping evolution vs pole order – 130 Hz

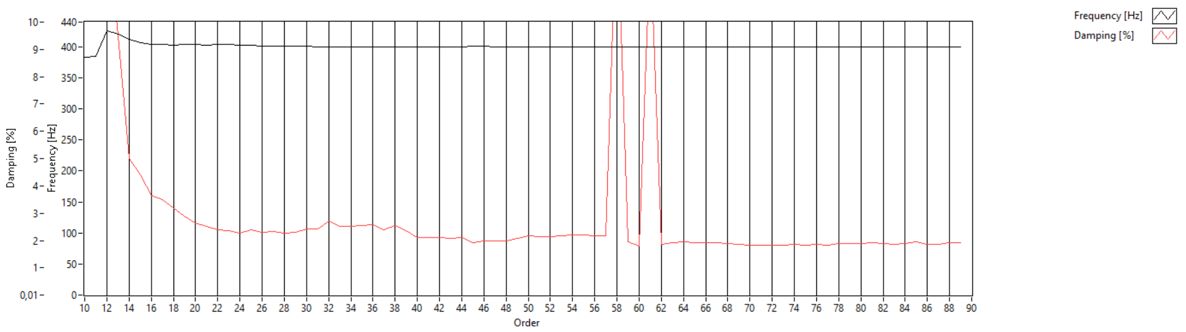


Figure 15 – Frequency and damping evolution vs pole order – 400 Hz

6. CONCLUSIONS

A revised version of the well-known p-LSCF identification algorithm is presented.
The influence of the constraint choice on the damping estimation accuracy is addressed thanks to mathematical formulation of the least-square problem. The constraint choice determines the stability of the mathematical poles. This behavior explains why the stabilization diagram of the p-LSCF identification are so clear by a rejection of the stochastic poles having negative damping. It explains also why the classical implementation of the LSCE can give a stabilization diagram which is polluted by a lot of non-physical poles. The error made on the damping estimation in presence of low Signal-to-Noise ratio signal is shown on a finite element model analysis of a clamped beam. The observed error is lower when the stability of the mathematical poles is not forced to be unstable.
The proposed algorithm aims at merging in a single algorithm the advantages of a clear stabilization diagram and a diminished pollution of the unstable poles on the stable ones. The solving at each pole order requests the consideration of two bases (clockwise and counterclockwise) and a pairing between the two solutions with a criterion on the pole amplitude. In the future, other correlation criterion will be explored. Moreover, some other rules to compute the frequency and damping estimation such as a combination of the two poles (positive and negative) will be considered and compared to the proposed methodology.
The algorithm is included in the V2i identification software and its application on an industrial test case is presented.

ACKNOWLEDGEMENTS

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