Bayesian Inference of Visco-Elastic Visco-Plastic Material Model Parameters for SLS-printed polyamide lattices

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MOAMMM

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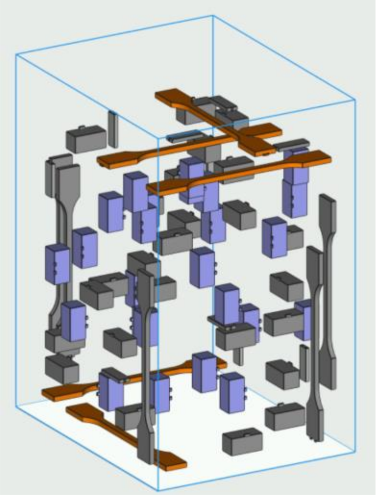
- Selective Laser Sintering (SLS) Polyamide 12 (PA12)
  - SLS-printed polyamide lattices
  - Material constitutive for numerical modelling of structure
  - Bulk sample preparation



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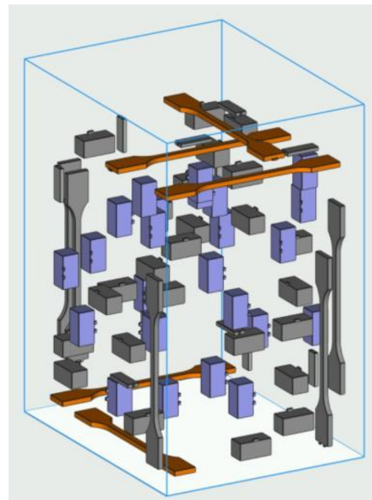

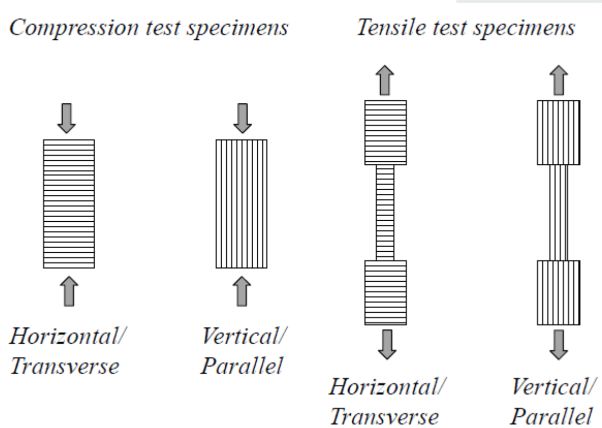

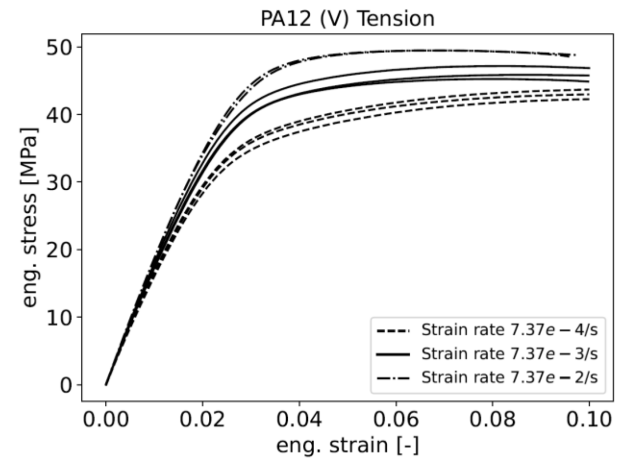

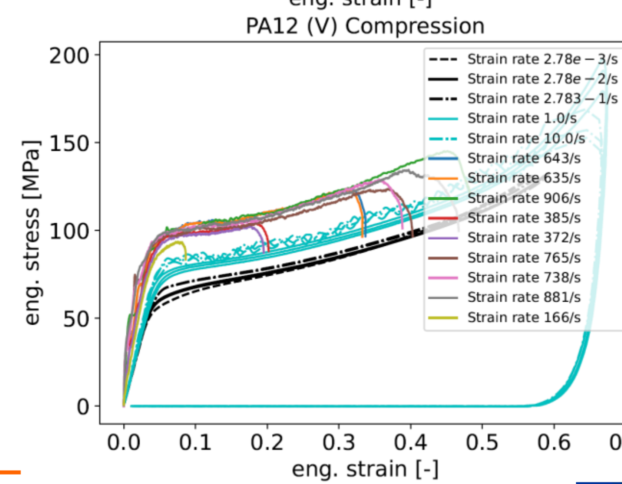

- Model: Hyperelastic visco-elastic-visco-plastic material model
  - Strain measure:
    - $\mathbf{F} = \mathbf{F}^{\mathrm{ve}} \cdot \mathbf{F}^{\mathrm{vp}}, \ \mathbf{C}^{\mathrm{ve}} = \mathbf{F}^{\mathrm{veT}} \cdot \mathbf{F}^{\mathrm{ve}}, \ \mathbf{E}^{\mathrm{ve}} = \frac{1}{2}\ln(\mathbf{C}^{\mathrm{ve}})$
    - $\mathbf{L}^{\mathrm{vp}} = \dot{\mathbf{F}}^{\mathrm{vp}} \cdot \mathbf{F}^{\mathrm{vp}-1}, \ \mathbf{D}^{\mathrm{vp}} = \mathbf{L}^{\mathrm{vp}}$
  - Visco-elastic part

$$\left\{ \begin{aligned} \boldsymbol{\tau}^{\mathrm{dev}} &= \int_{-\infty}^{t} G(t-s)\frac{\mathrm{d}}{\mathrm{d}s}\mathrm{dev}(\mathbf{E}^{\mathrm{ve}}(s))\mathrm{d}s \\ p = \frac{1}{3}\boldsymbol{\tau}^{\mathrm{tr}} &= \int_{-\infty}^{t} K(t-s)\frac{\mathrm{d}}{\mathrm{d}s}\mathrm{tr}(\mathbf{E}^{\mathrm{ve}}(s))\mathrm{d}s \end{aligned} \right.$$

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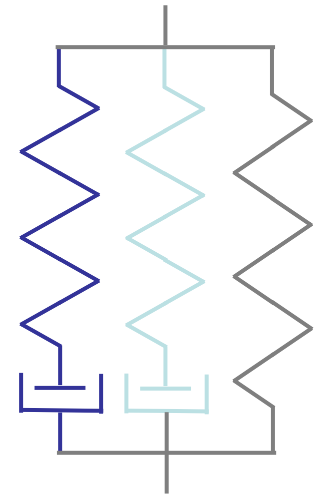


  - Maxwell model
    - $G(t) = G_\infty + \sum_{i=1}^{N} G_i \exp(-\frac{t}{g_i})$
    - $K(t) = K_\infty + \sum_{i=1}^{N} K_i \exp(-\frac{t}{k_i})$
  - Parameters:

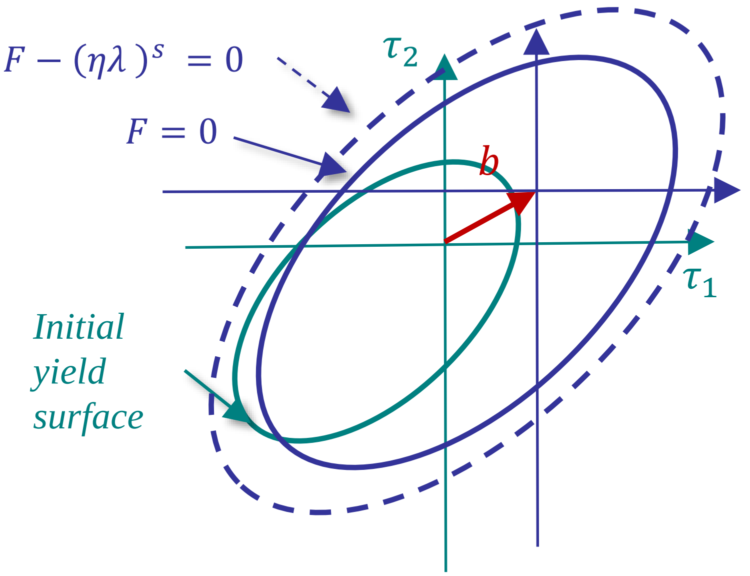
K_∞ , G_∞ , G_i, g_i K_i, k_i , $i = 1, \ldots, N$

- Model: Hyperelastic visco-elastic-visco-plastic material model
  - Visco-plastic part
    - back-stress

$$\boldsymbol{\varphi} = \boldsymbol{\tau} - \boldsymbol{b}$$

    - Perzina plastic flow rule

$$\mathbf{D}^{\mathrm{vp}} = \dot{\mathbf{F}}^{\mathbf{vp}} \cdot \mathbf{F}^{\mathbf{vp-1}} = \frac{1}{\eta}\langle F\rangle^{\frac{1}{s}} \frac{\partial P}{\partial \boldsymbol{\tau}} = \lambda \frac{\partial P}{\partial \boldsymbol{\tau}}$$



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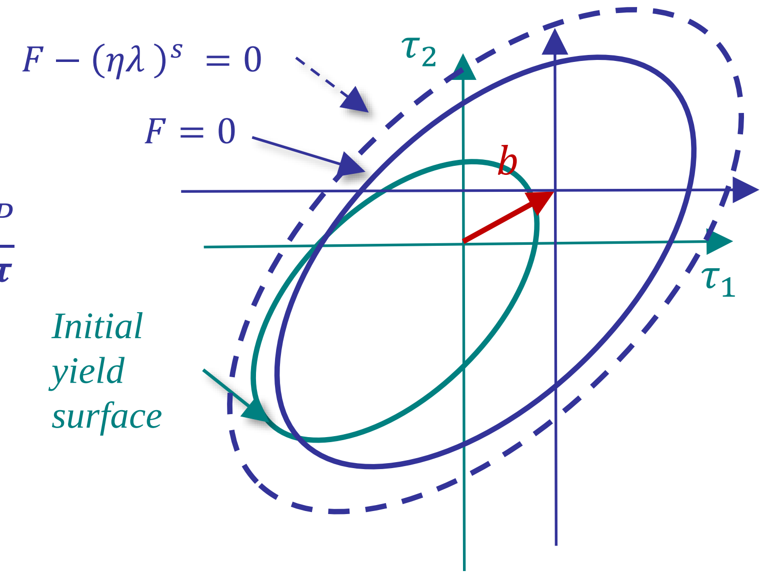
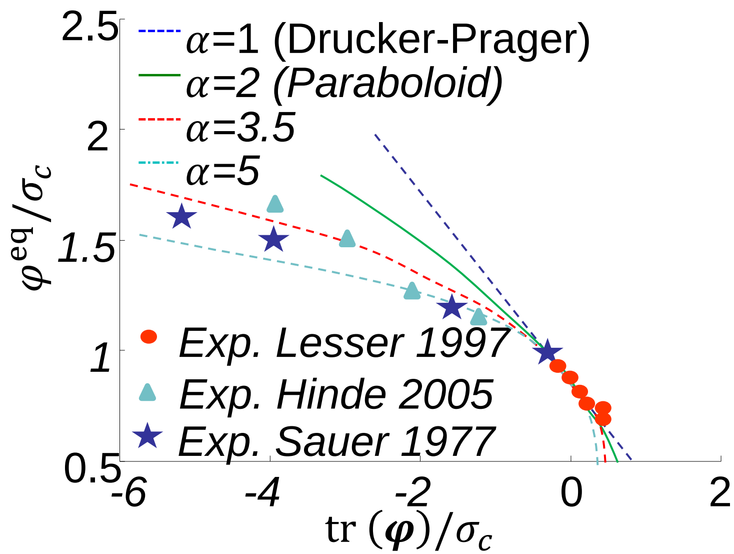
    - Yield surface, flow potential

$$\begin{cases} F = \left(\frac{\varphi^{\mathrm{eq}}}{Y_c}\right)^{\alpha} - \frac{m^{\alpha}-1}{m+1}\frac{\mathrm{tr}\boldsymbol{\varphi}}{Y_c} - \frac{m^{\alpha}+m}{m+1} \\ m = \frac{\sigma_t}{\sigma_c} \\ P = (\varphi^{\mathrm{eq}})^2 + \beta\left(\frac{\mathrm{tr}\boldsymbol{\varphi}}{3}\right)^2 \end{cases}$$



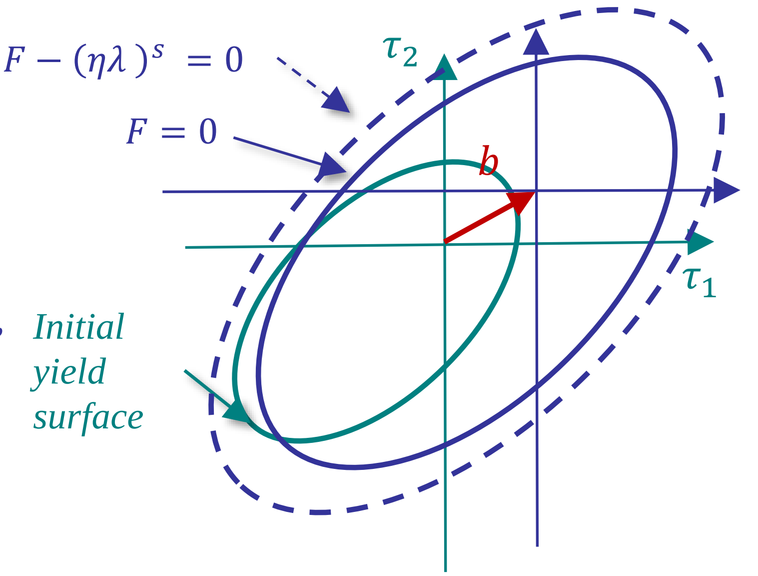
    - Equivalent plastic strain rate:

$$\begin{cases} \dot{\gamma} = \frac{\sqrt{\mathbf{D}^{\mathrm{vp}}:\mathbf{D}^{\mathrm{vp}}}}{\sqrt{\mathbf{1}+\mathbf{2}v_p^2}} \\ v_p = \frac{9-2\beta}{18+2\beta} \end{cases}$$



- Model: Hyperelastic visco-elastic-visco-plastic material model
  - Visco-plastic part
    - Hardening

$$\begin{cases} \dot{Y}_c = H_c(\gamma)\dot{\gamma} \text{ with } H_c(\gamma) = c_0 + h_c\ H_c^1 \exp(-h_c\ \gamma), \\ \dot{Y}_t = H_t(\gamma)\dot{\gamma} \text{ with } H_t(\gamma) = t_0 + h_t\ H_t^1 \exp(-h_t\ \gamma), \\ \dot{\mathbf{b}} = \frac{H_b(\gamma)}{\sqrt{1+2\nu_p^2}} \mathbf{D}^{vp} \text{ with } H_b(\gamma) = b_0 + h_b\ H_b^0 \exp(-h_b\ \gamma), \end{cases}$$



- Model: Hyperelastic visco-elastic-visco-plastic material model
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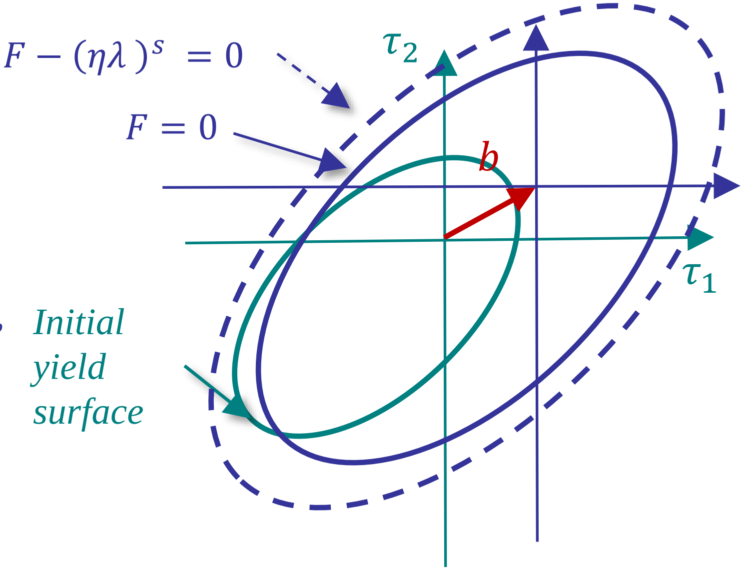
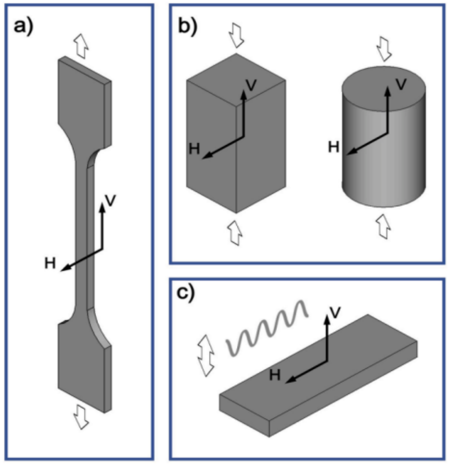
\dot{\mathbf{b}} = \frac{H_b(\gamma)}{\sqrt{1+2\nu_p^2}} \mathbf{D}^{vp} \ \text{with } \ H_b(\gamma) = b_0 + h_b \ H_b^0 \exp(-h_b \ \gamma),
\end{cases}
$$



  - Parameters: ~~(back stress)~~
    - $\sigma_{c0}, \ c_0, \ h_c \ , H_c^0$
    - σ_{t0} or $m_0, \ t_0, \ h_t \ , H_t^0$
    - $\eta, \ s, \ \alpha, \ \beta$

$$
\begin{cases}
F = \left(\frac{\varphi^{\text{eq}}}{Y_c}\right)^{\alpha} - \frac{m^{\alpha} - 1}{m + 1} \frac{\text{tr}\boldsymbol{\varphi}}{Y_c} - \frac{m^{\alpha} + m}{m + 1} \\
m = \frac{\sigma_t}{\sigma_c} \\
\nu_p = \frac{9 - 2\beta}{18 + 2\beta}
\end{cases}
$$

- VE-Identification (PA12-V):
  - Since, Poisson's ratio, ν, of polymer material is also a function of time
  - We consider



$$\begin{cases} G(t) = G_\infty + \sum_{i=1}^{N} G_i \exp(-\frac{t}{g_i}) \\ K(t) = K_\infty + \sum_{i=1}^{N} K_i \exp(-\frac{t}{k_i}) \\ G_\infty = \frac{E_\infty}{2(1+\nu_\infty)} \qquad K_\infty = \frac{E_\infty}{3(1-2\nu_\infty)} \end{cases}$$

- The testing strain rates vary from 10^{-4}/s to 10^{3}/s, and the relaxation test lasts 1800 s
- In the relaxation test, no obvious stress decrease has been seen after 1000 s

N = 8 branches are considered

to have the characteristic relaxation times of $\propto 10^{-5}$, $\propto 10^{-4}$, $\propto 10^{-3}$, $\propto 10^{-2}$, $\propto 10^{-1}$, $\propto 10^{0}$, $\propto 10^{1}$ and $\propto 10^{2}$ s.

- Bayesian-Identification (PA12-V):
  - Viscoelastic parameters $\boldsymbol{\vartheta}^{\mathrm{ve}}$:

    $E_\infty,\ \nu_\infty,\ G_i,\ K_i,\ g_1,\ k_1,$

    $g_i = \frac{g_1}{10^{i-1}},\ k_i = \frac{k_1}{10^{i-1}}$ with i =1,…,8

  - Viscoplastic parameters $\boldsymbol{\vartheta}^{\mathrm{vp}}$:

    $\eta,\ s,\ \alpha,\ \beta,\ \sigma_c^0,\ c_0, H_c^0, h_c\ ,\ m^0,\ t_0, H_t^0, h_t$

- Bayes theorem

$$\pi_{\mathrm{post}}(\boldsymbol{\vartheta}|\boldsymbol{\varepsilon},\boldsymbol{\sigma}_{\mathrm{exp}}) \propto \pi_{\mathrm{likelihood}}(\boldsymbol{\sigma}_{\mathrm{exp}}|\boldsymbol{\vartheta},\boldsymbol{\varepsilon})\,\pi_{\mathrm{prior}}(\boldsymbol{\vartheta})$$

What we want (π_{post})

To be evaluated from $\boldsymbol{\sigma}_{\mathrm{model}}^{(i)}$ *and* $\mathrm{noise}^{(i,j)}$ ($\pi_{\mathrm{likelihood}}$)

What we know (physical bounds) (π_{prior})

- Bayesian-Identification (PA12-V):
  - Viscoelastic parameters $\boldsymbol{\vartheta}^{\mathrm{ve}}$:

    $E_\infty,\, \nu_\infty,\, G_i,\ K_i,\, g_1,\, k_1,$

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  - Viscoplastic parameters $\boldsymbol{\vartheta}^{\mathrm{vp}}$:

    $\eta,\, s,\, \alpha,\, \beta,\, \sigma_c^0,\, c_0, H_c^0, h_c\ ,\, m^0,\, t_0, H_t^0, h_t$

- Bayes theorem

$$\pi_{\mathrm{post}}(\boldsymbol{\vartheta}_1, \boldsymbol{\vartheta}_2 \,|\, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2) \propto \pi_{\mathrm{likelihood}}(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2 | \boldsymbol{\vartheta}_1, \boldsymbol{\vartheta}_2)\, \pi_{\mathrm{prior}}(\boldsymbol{\vartheta}_1, \boldsymbol{\vartheta}_2)$$

$\boldsymbol{\beta}_1, \boldsymbol{\beta}_2$
independent observations

$$\propto \pi_{\mathrm{likelihood}}(\boldsymbol{\beta}_2 | \boldsymbol{\vartheta}_1, \boldsymbol{\vartheta}_2)\, \pi_{\mathrm{likelihood}}(\boldsymbol{\beta}_1 | \boldsymbol{\vartheta}_1, \boldsymbol{\vartheta}_2) \pi_{\mathrm{prior}}(\boldsymbol{\vartheta}_1, \boldsymbol{\vartheta}_2)$$

$\boldsymbol{\vartheta}_1, \boldsymbol{\vartheta}_2$
independent prior distribution

$$\propto \pi_{\mathrm{likelihood}}(\boldsymbol{\beta}_2 | \boldsymbol{\vartheta}_1, \boldsymbol{\vartheta}_2) \pi_{\mathrm{prior}}(\boldsymbol{\vartheta}_2) \pi_{\mathrm{likelihood}}(\boldsymbol{\beta}_1 | \boldsymbol{\vartheta}_1, \boldsymbol{\vartheta}_2) \pi_{\mathrm{prior}}(\boldsymbol{\vartheta}_1)$$

- Bayesian-Identification (PA12-V):
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$E_\infty,\ \nu_\infty,\ G_i,\ K_i,\ g_1,\ k_1,$

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  - Viscoplastic parameters $\boldsymbol{\vartheta}^{\mathrm{vp}}$:

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$\boldsymbol{\beta}_1, \boldsymbol{\beta}_2$ independent observations

$$\propto \pi_{\text{likelihood}}(\boldsymbol{\beta}_2 | \boldsymbol{\vartheta}_1, \boldsymbol{\vartheta}_2)\, \pi_{\text{likelihood}}(\boldsymbol{\beta}_1 | \boldsymbol{\vartheta}_1, \boldsymbol{\vartheta}_2) \pi_{\text{prior}}(\boldsymbol{\vartheta}_1, \boldsymbol{\vartheta}_2)$$

$\boldsymbol{\vartheta}_1, \boldsymbol{\vartheta}_2$ independent prior distribution

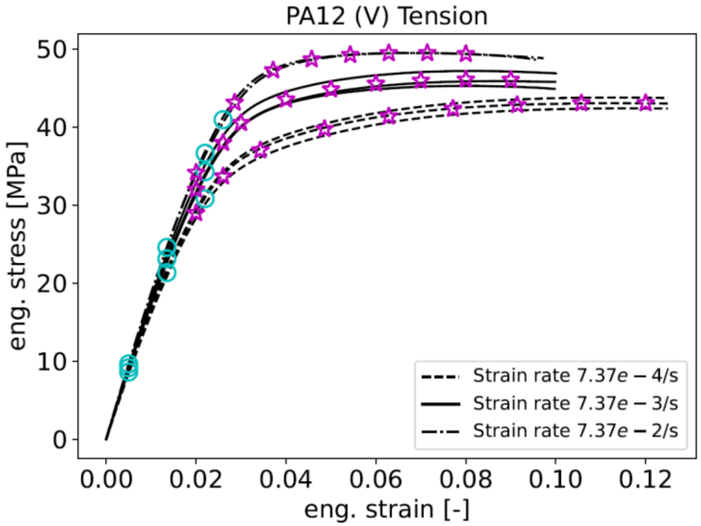
$$\propto \pi_{\text{likelihood}}(\boldsymbol{\beta}_2 | \boldsymbol{\vartheta}_1, \boldsymbol{\vartheta}_2) \pi_{\text{prior}}(\boldsymbol{\vartheta}_2) \pi_{\text{likelihood}}(\boldsymbol{\beta}_1 | \boldsymbol{\vartheta}_1, \boldsymbol{\vartheta}_2) \pi_{\text{prior}}(\boldsymbol{\vartheta}_1)$$

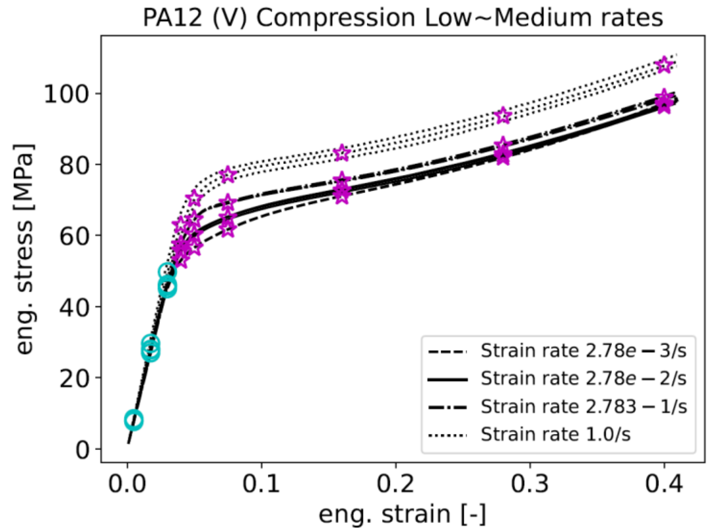
$\boldsymbol{\beta}_1, \boldsymbol{\vartheta}_2$ independent distribution

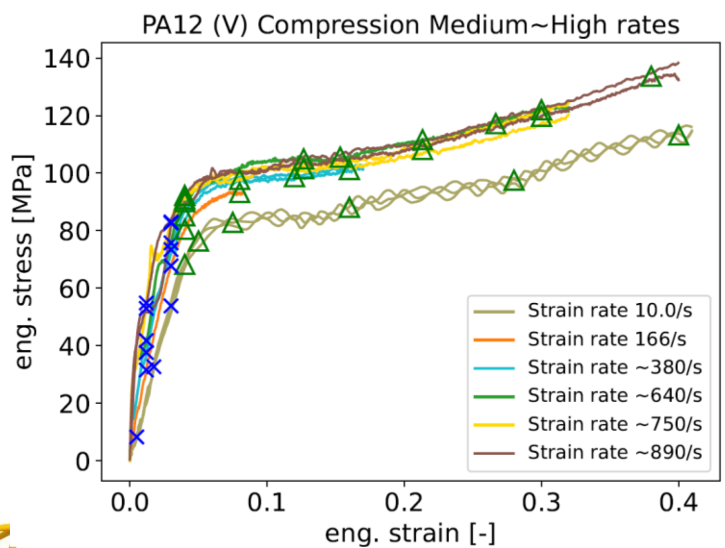
$$\propto \pi_{\text{likelihood}}(\boldsymbol{\beta}_2 | \boldsymbol{\vartheta}_1, \boldsymbol{\vartheta}_2) \pi_{\text{prior}}(\boldsymbol{\vartheta}_2) \pi_{\text{likelihood}}(\boldsymbol{\beta}_1 | \boldsymbol{\vartheta}_1) \pi_{\text{prior}}(\boldsymbol{\vartheta}_1)$$

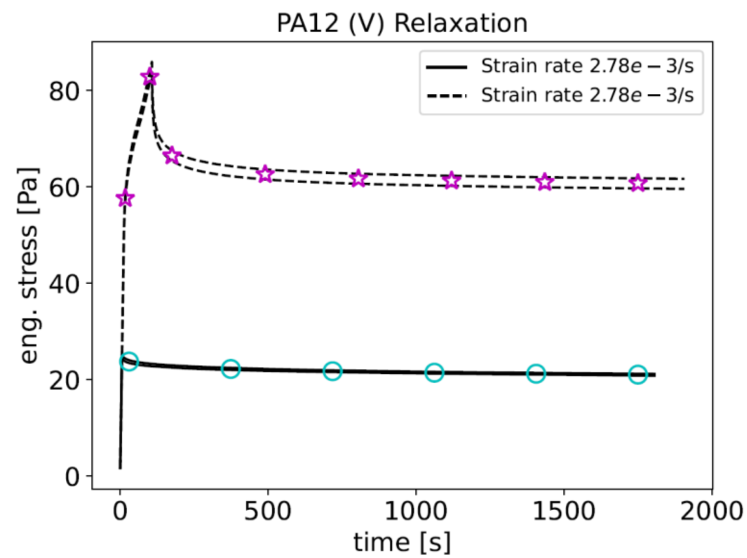
$$\pi_{\text{post}}(\boldsymbol{\vartheta}_1 \,|\, \boldsymbol{\beta}_1) \propto$$

- Observations $\boldsymbol{\beta} = (\boldsymbol{\varepsilon}, \boldsymbol{\sigma}_{\mathrm{exp}})$









- Bayesian inference in sequence
  - $\boldsymbol{\beta} = [\boldsymbol{\beta}^{\mathrm{ve}}, \boldsymbol{\beta}^{\mathrm{vp}}]$, $\boldsymbol{\beta}^{\mathrm{ve}} = [\boldsymbol{\beta}_1^{\mathrm{ve}}, \boldsymbol{\beta}_2^{\mathrm{ve}}]$, $\boldsymbol{\beta}^{\mathrm{vp}} = [\boldsymbol{\beta}_1^{\mathrm{vp}}, \boldsymbol{\beta}_2^{\mathrm{vp}}]$
  - Observations $\boldsymbol{\beta}_1^{\mathrm{ve}}$, $\boldsymbol{\beta}_2^{\mathrm{ve}}$, $\boldsymbol{\beta}_1^{\mathrm{vp}}$, $\boldsymbol{\beta}_2^{\mathrm{vp}}$ are used in sequence

Viscoelastic parameters $\boldsymbol{\vartheta}^{\mathrm{ve}}$		
BI step	I	II
observation	$\boldsymbol{\beta}_1^{\mathrm{ve}}$, cyan circle	$\boldsymbol{\beta}_2^{\mathrm{ve}}$, blue "x"
likelihood	$\pi(\boldsymbol{\beta}_1^{\mathrm{ve}} \mid \boldsymbol{\vartheta}^{\mathrm{ve}})$	$\pi(\boldsymbol{\beta}_2^{\mathrm{ve}} \mid \boldsymbol{\vartheta}^{\mathrm{ve}})$
prior	$\pi_{\mathrm{prior}}(\boldsymbol{\vartheta}^{\mathrm{ve}})$	$\pi_{\mathrm{post}}(\boldsymbol{\vartheta}^{\mathrm{ve}} \mid \boldsymbol{\beta}_1^{\mathrm{ve}})$
posterior	$\pi_{\mathrm{post}}(\boldsymbol{\vartheta}^{\mathrm{ve}} \mid \boldsymbol{\beta}_1^{\mathrm{ve}})$	$\pi_{\mathrm{post}}(\boldsymbol{\vartheta}^{\mathrm{ve}} \mid \boldsymbol{\beta}^{\mathrm{ve}})$
Viscoplastic parameters $\boldsymbol{\vartheta}^{\mathrm{vp}}$		
BI step	III	IV
observation	$\boldsymbol{\beta}_1^{\mathrm{vp}}$, magenta star	$\boldsymbol{\beta}_2^{\mathrm{vp}}$, green triangles
likelihood	$\pi(\boldsymbol{\beta}_1^{\mathrm{vp}} \mid \boldsymbol{\vartheta}^{\mathrm{ve}}, \boldsymbol{\vartheta}^{\mathrm{vp}})$	$\pi(\boldsymbol{\beta}_2^{\mathrm{vp}} \mid \boldsymbol{\vartheta}^{\mathrm{ve}}, \boldsymbol{\vartheta}^{\mathrm{vp}})$
prior	$\pi_{\mathrm{prior}}(\boldsymbol{\vartheta}^{\mathrm{vp}})\pi_{\mathrm{post}}(\boldsymbol{\vartheta}^{\mathrm{ve}} \mid \boldsymbol{\beta}^{\mathrm{ve}})$	$\pi_{\mathrm{post}}(\boldsymbol{\vartheta}^{\mathrm{ve}}, \boldsymbol{\vartheta}^{\mathrm{vp}} \mid \boldsymbol{\beta}^{\mathrm{ve}}, \boldsymbol{\beta}_1^{\mathrm{vp}})$
posterior	$\pi_{\mathrm{post}}(\boldsymbol{\vartheta}^{\mathrm{ve}}, \boldsymbol{\vartheta}^{\mathrm{vp}} \mid \boldsymbol{\beta}^{\mathrm{ve}}, \boldsymbol{\beta}_1^{\mathrm{vp}})$	$\pi_{\mathrm{post}}(\boldsymbol{\vartheta}^{\mathrm{ve}}, \boldsymbol{\vartheta}^{\mathrm{vp}} \mid \boldsymbol{\beta}^{\mathrm{ve}}, \boldsymbol{\beta}^{\mathrm{vp}})$

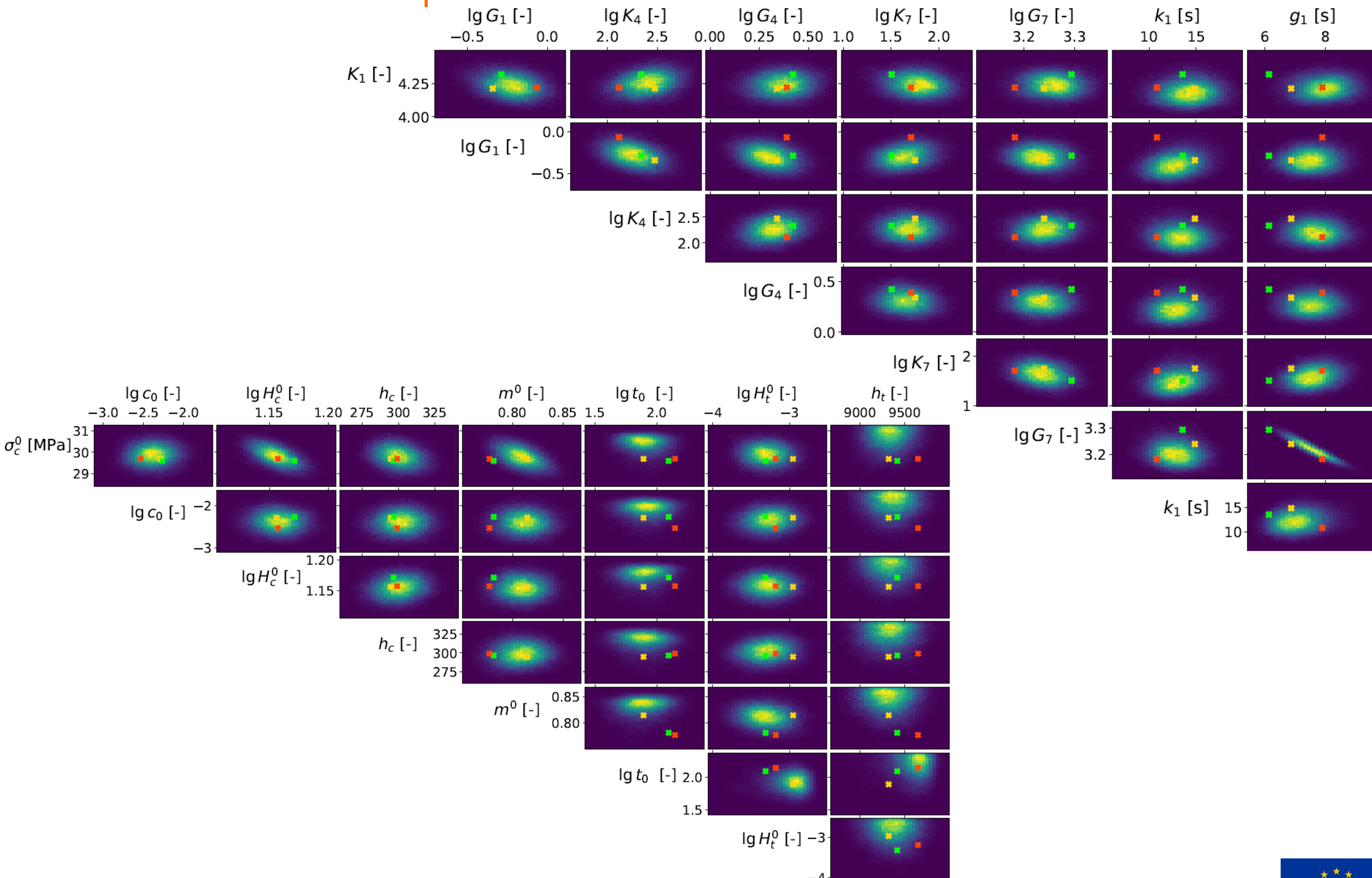
- ## Bayesian inference
  - The maximum a posteriori (MAP) estimate of viscoelastic material parameters

$\boldsymbol{\vartheta}^{\mathrm{ve}}$	E_∞ [MPa]	ν_∞ [-]	g_1 [s]	k_1 [s]				
Value	1366.7	0.278	7.36	9.12				
G_i [MPa]	G_1	G_2	G_3	G_4	G_5	G_6	G_7	G_8
	0.33	27.23	18.55	1.62	21.66	9.91	1722.24	6.13
K_i [MPa]	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8
	28281.49	0.30	747.17	467.82	0.51	7874.44	39.13	2422.40

  - The maximum a posteriori (MAP) estimate of viscoelastic material parameters

$\boldsymbol{\vartheta}^{\mathrm{vp}}$	η [MPa $\cdot$ s]	s [-]	α [-]	β [-]
Value	1.33×10^5	0.18	3.63	0.18
$\boldsymbol{\vartheta}^{\mathrm{vp}}$	σ_{c}^0 [MPa]	c_0 [MPa]	H_{c}^0 [MPa]	h_{c} [-]
Value	30.04	7.85×10^{-3}	14.41	296.38
$\boldsymbol{\vartheta}^{\mathrm{vp}}$	m^0 [-]	t_0 [MPa]	H_{t}^0 [MPa]	h_{t} [-]
Value	0.80	146.34	5.24×10^5	9955.79

- Run a Markov Chain process



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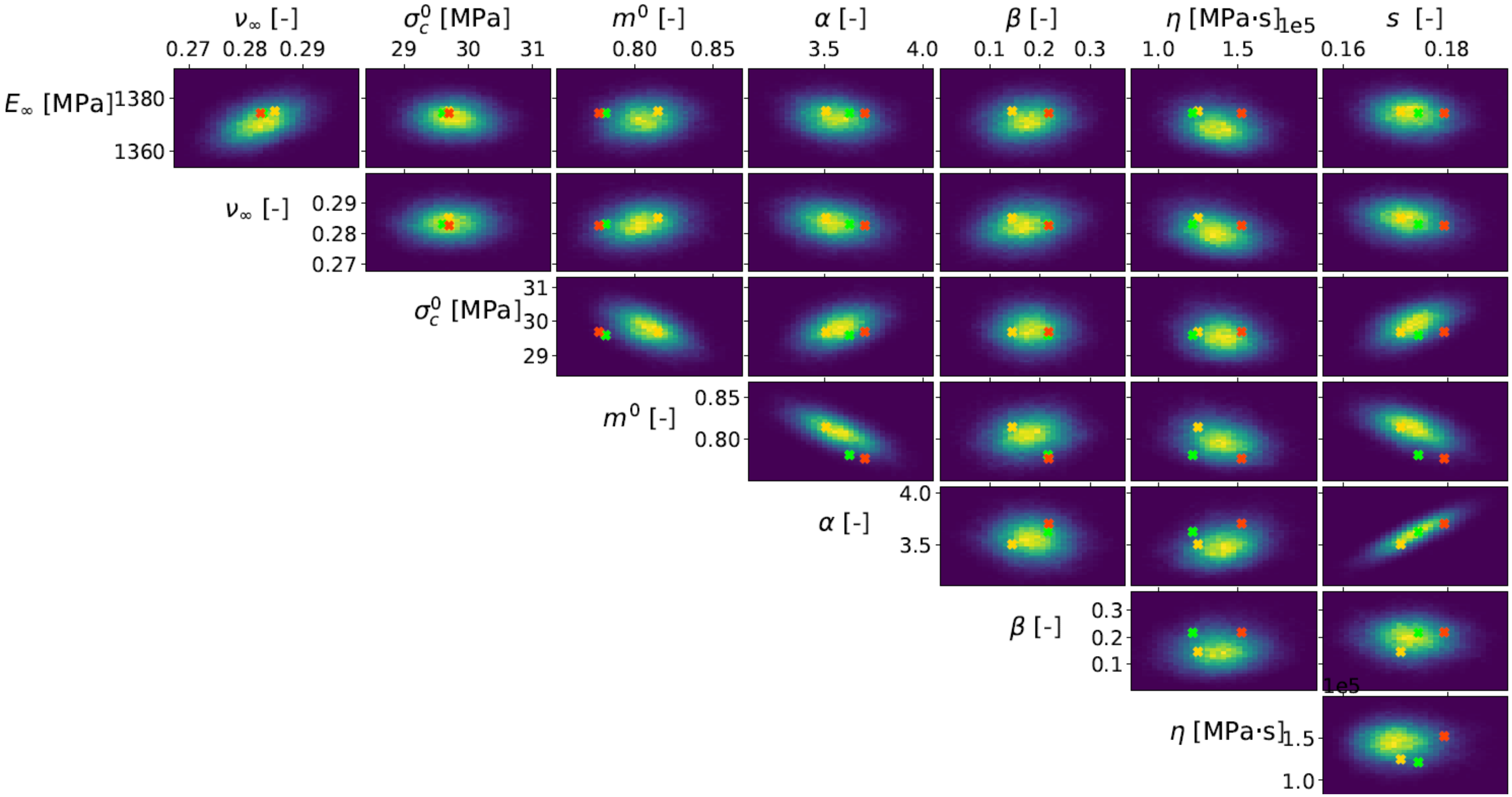
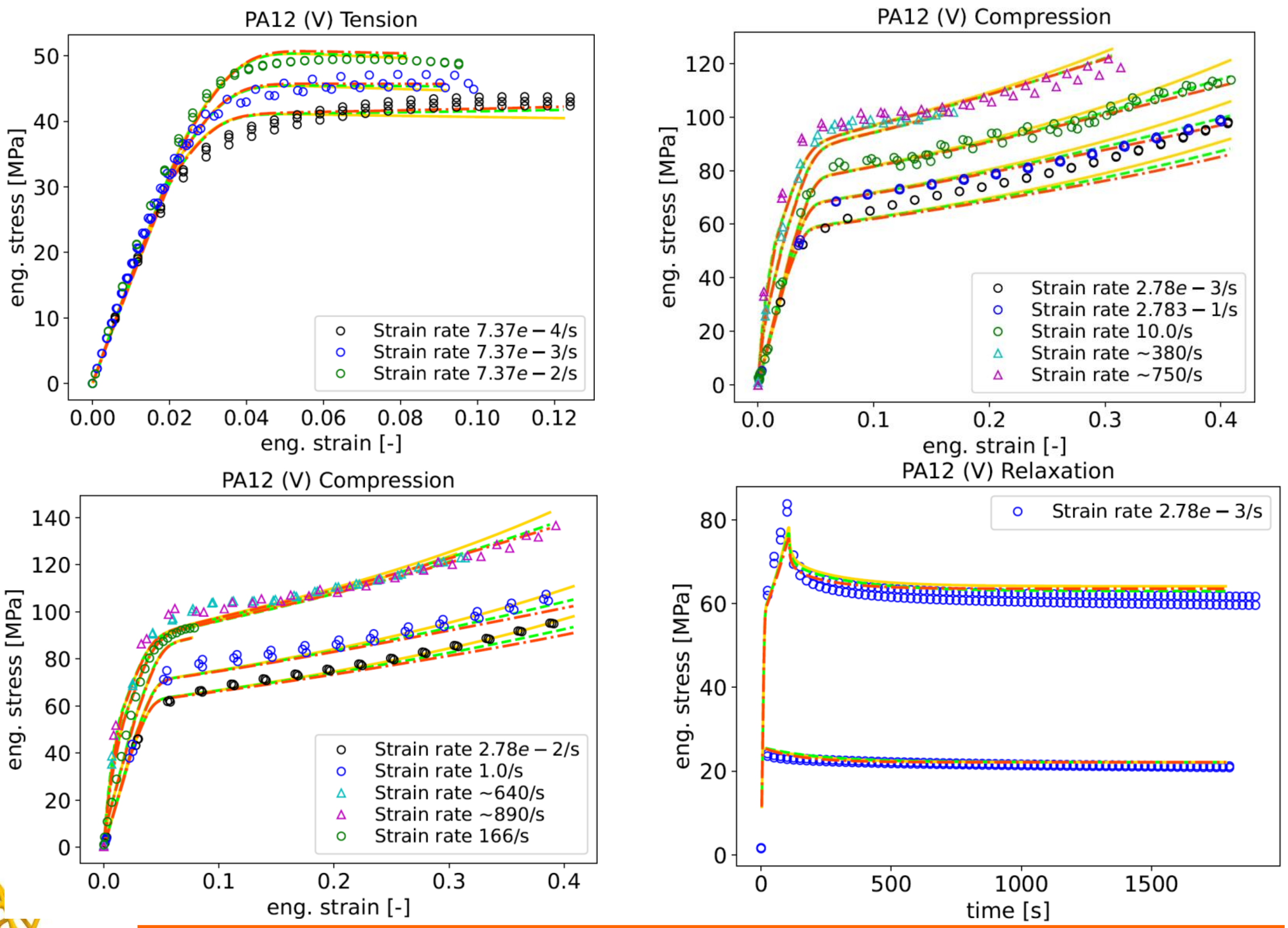


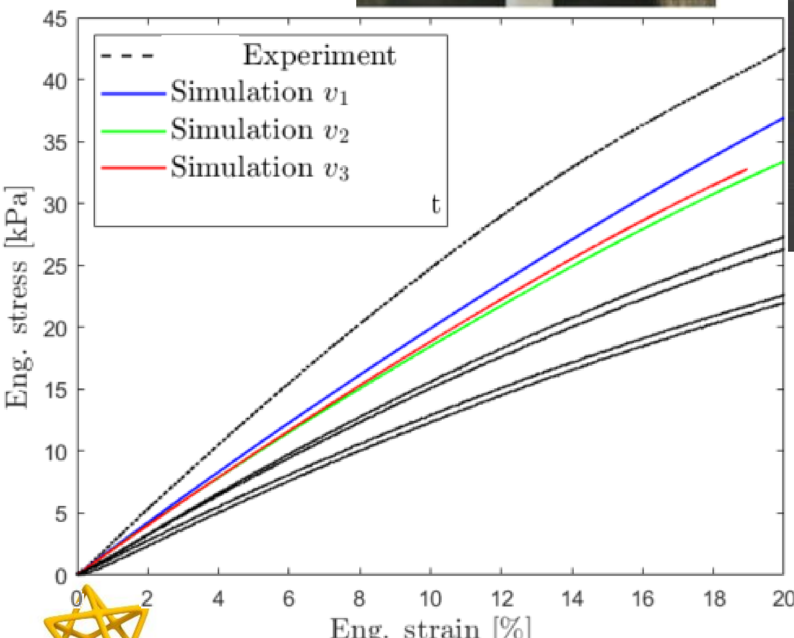
- Numerical modelling

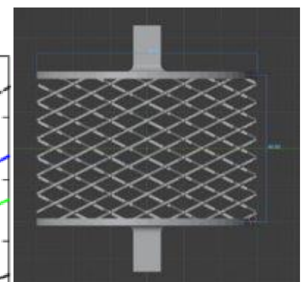


- Preliminary work: simulate a complex structure
  - Validate the infer parameters

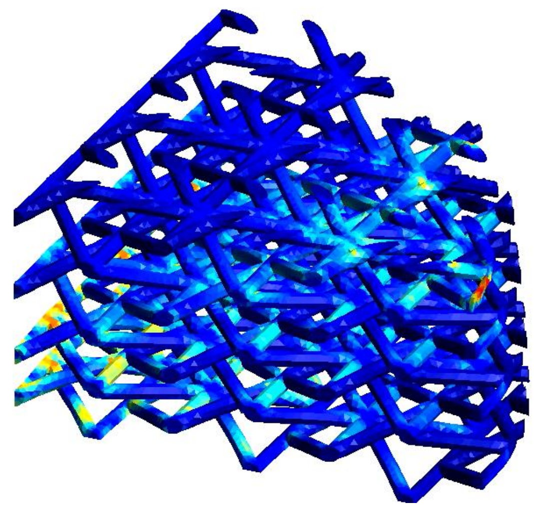
Underlying USF cell

Experimental test JKU





Numerical simulation



References

- More on
  - www.moammm.eu
  - High-Dimensional Parameter Identification with Bayesian Inference for Finite-Strain Visco-Elastic-Visco-Plastic Modeling of Selective Laser Sintering (Sls) Polyamide 12 (Pa12) and Neural Net-Work (Nnw)-Based Material Parameter Generator, http://dx.doi.org/10.2139/ssrn.4457392
  - Data on https://dx.doi.org/10.5281/zenodo.7998798