SHEAR DEFORMABILITY OF COLUMN WEB PANELS IN STRONG AXIS BEAM-TO-COLUMN JOINTS AND STRUCTURAL STABILITY OF FRAMES.

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INTRODUCTION

For some years, experimental and/or theoretical research works have been devoted to the actual behaviour of beam-to-column joints in steel buildings and more especially to semi-rigid joints. This is the result of a search for simple and cheap connections with a view to a reduction of the labour cost, which grew much faster than the material cost. Thus for the sake of economy, bolted joints without any stiffener became a common practice. Unfortunately such joints have a non-linear behaviour: when subject to an applied bending moment \( M \), the axes of the connected members do not rotate at the same angle, so that there is a relative rotation \( \theta \), which is not proportional to the applied bending moment \( M \).

Both strength and stability of steel frames are affected by the semi-rigid behaviour of the joints [1]. Though several computer programmes [2], which allow for material and geometrical non-linearities - including semi-rigid connections - are available, there is an urgent need of knowledge for the \( M-\theta \) characteristics of the joints.

Present paper is aimed at presenting a mathematical approach that is likely to provide the \( M-\theta \) curve associated to the shear rotation of the unstiffened column web panels. This model is demonstrated to give results in close agreement with numerical simulations.

JOINT DEFORMABILITY COMPONENTS

The two following sources of deformability of a strong axis beam-to-column joint have to be clearly defined:

a) the deformation of the connection associated to the deformation of the connection elements (end plate, angles, bolts,...), to the slip, to the column flange deformation and to the local deformation of the column web in the tension and compression zones (respectively a lengthening and a shortening);

b) the deformation of the column web under shear associated mostly to the common presence of forces, equal and opposite, in tension and compression, carried over by the beam(s) and acting on the column web at the level of the joint.
The case of the end plate connection of figure 1 may be chosen to illustrate this. The rotation $\phi$ of the connection is mathematically defined by the difference of the two rotations $\theta_b$ and $\theta_c$ and includes the deformation of the end plate, of the bolts and of the column flange, the lengthening of the zone BC and the shortening of the zone AD of the web.

The rotation $\gamma$ of the column web under shear is defined by the difference of the rotations $\theta_c$ and $\theta_f$ where $\theta_f$ represents the flexural rotation of the column.

It is important to know that the shear in the column web is the result of the combined action of the equal but opposite forces $R_b$ in the beam flanges, which are statically equivalent to the beam moment, and of the shear forces in the column at the level of the beam flanges.

The difference between the loading of the connection and that of the column web in a same joint leads to take account separately of both deformability sources.

**NUMERICAL INVESTIGATIONS**

An important parametric study has been realized recently at the Polytechnic Federal School of Lausanne and at the University of Liège. All the results and all the conclusions of this study may be found in [3].

This study is based on numerical simulations with the non-linear FE-program FINELG [4] of the loading up to failure of welded beam-to-column joints. Material and geometrical non-linear effects are taken into account, although the latter is far less important than the former. The specimens of the chosen joints are analysed in three dimensions by using "shell" finite elements to model the webs and flanges of the profiles and "beam" finite elements to model stiffeners. The adopted finite element meshes are shown on figure 2, respectively for a "T" joint (one column, one beam) and a "cross" joint (one column, two beams).

The numerical simulations allow to study the propagation of the plasticity in the profiles and to observe the exact failure modes.

Steel is supposed to follow a piecewise linear law shown on Fig.3. The 2D elastoplastic state of stress is dealt with by using the incremental flow theory and the von MISES yield criterion. Parabolic patterns of rolling residual normal stresses in flanges and webs are taken according to the ECCS recommendations [5]. Welding imperfections are not considered. Complete data may be found in [6].

The good agreement between the numerical simulations and results of experimental tests on joints is shown in [6].
The moment-rotation curves characterizing the shear deformability and the load-introduction deformability of the column web panel have been reported for every simulation. The load-introduction deformability is the component of the connection deformability associated to the local deformation of the column web in the tension and compression zones (respectively a lengthening and a shortening).

The following parameters have been taken into account in the parametric study of the joints:

a) the type of the beam(s);
b) the type of the column;
c) the loading of the joint;
d) the transversal initial deformation of the column web;
e) the presence or not of transversal stiffeners on the column web.

Only the conclusions relative to the behaviour of the sheared column web panels are presented here.

a) The shear stresses in the column web panels may be considered as uniformly distributed; this is due to the action of the column flanges.

b) The actual value of the shear force \( V_n \) may be obtained from the equilibrium equations of the web panel [7].

It is given by the following formula (figure 4):

\[
V_n = \frac{M_{c1} + M_{c2}}{d_b} - \frac{Q_{b1} + Q_{b2}}{2} \cdot \frac{d_c}{d_b}
\]  \hspace{1cm} (1)

Some other researches refer to another formula:

\[
V_n = \frac{M_{c1} + M_{c2}}{d_b}
\]  \hspace{1cm} (2)

The validity of the proposed formula (1) has been clearly demonstrated.

c) The \( V_n - \gamma \) curve for a given joint depends on the actual loading of the joint.

Let us suppose that the two unstiffened welded nodes of figure 5 are submitted to different types of loading (figure 6) and let us report, for each node, the characteristic \( V_n - \gamma \) curve in a common diagram (figures 7 and 8).

The shear force \( V_n \) takes account of the loading of the joints; in consequence one could believe that the \( V_n - \gamma \) curves are identical for a given node. In reality a similarity exists only in the elastic range of the web panel behaviour and this demonstrates the validity of the proposed shear force definition (formula 1).
The differences between the \( V_n - \gamma \) curves in the non-elastic range of the web panel behaviour are not negligible.

The existing methods for the prediction of the shear deformability of web panels do not take the influence of the actual joint loading into account. Figures 7 and 8 show that this is questionable and has led to the elaboration of a new approach.

**THEORETICAL DEVELOPMENTS**

The theoretical developments presented in this paper are related to the study of the unstiffened column web panels. It will be referred to [3] for stiffened columns.

Let us consider a small column web element submitted to shear stresses \( \tau \) (figure 9.a) and whose material characteristic is elastic-perfectly plastic with strain-hardening (figure 10.a). The shear deformability \( \gamma \) of this element (figure 9.b) versus the shear stress \( \tau \) may be deduced [3].

The shear stresses being uniformly distributed in the web panel, a first approximation (pure shear) of the searched \( V_n - \gamma \) curve may be easily obtained by multiplying the shear stress \( \tau \) by the column web area (figure 10.b).

The theory of plasticity may be used in connection with the von Mises yield criterion [3] to modify the characteristic values \( V_{\text{ny}}(\gamma), V_{\text{nu}}, \gamma_{\text{st}} \) and \( \gamma_{\text{u}} \) (figure 10.b) with a view to account for the actual node loading.

In its most stressed zone (figure 11) an unstiffened web panel is submitted to three types of stresses:

- the shear stresses \( \tau \);
- the normal stresses \( \sigma_n \) resulting from the compression force and the bending moment in the column;
- the normal stresses \( \sigma_l \) resulting from the introduction of beam loads in the column web.

The load introduction constitutes only a local phenomena [8] which has no direct influence on the global behaviour of the web panel. The web panel deformability predicting model based on the modified value of \( V_{\text{ny}}(\gamma), V_{\text{nu}}, \gamma_{\text{st}} \) and \( \gamma_{\text{u}} \) (interaction between \( \tau \) and \( \sigma \) stresses) is applied in figures 12 and 13 to the studied nodes A and B (figures 5 and 6).

It may be seen that:

- the agreement between the values of the initial stiffness and of the strain-hardening stiffness given by the mathematical model and the numerical simulation is very good;
- the mathematically predicted plastic load is always slightly lower than the corresponding load obtained by means of the numerical simulations; the small difference which represents the bending resistance of the column flange may be neglected in the case of joints with unstiffened columns.

However the mathematical model differs from the result of the numerical simulations for what regards the length of the yield plateau and the collapse load.

It must be referred to the interaction between the web panel shear and the load-introduction effect to explain these differences:

- the strain-hardening will firstly appear in the most stressed zone of the column web (figure 11) where the three above-mentioned types of stresses interact and will depend on the importance of the $\sigma_i$ stresses relatively to the two other types of stresses;

- the collapse load of the web panel will be linked up either to the excessive shear or to the load-introduction resistance of the web (web crippling for instance); it will depend on the relative importance of $\sigma_i$ stresses too.

It is relatively easy to highlight the necessity of taking into account the interaction between shear and load-introduction by introducing the $\sigma_i$, $\sigma_n$ and $\tau$ stresses into the von Mises yield criterion to determine the shear load corresponding to the beginning of yielding in the column web.

The agreement between the calculated shear force and the results of numerical simulations is seen to be very good (figures 12 and 13).

Work is actually in progress to improve the proposed mathematical model in this way. It has already allow to enlarge the physical knowledge of the load-introduction behaviour for the column webs and to develop a mathematical approach for the prediction of this local phenomena [8].

CONCLUSIONS

Numerous numerical simulations of "T" and "cross" beam-to-column joints have been performed. The parameters investigated in this study were the type of beam(s) and of column, the loading of the joints, the transversal initial deformation of the column web and the presence or not of stiffeners on the column web.

This analysis has allowed:
1) to define accurately the shear force of the joint under any solicitation;
2) to show clearly the influence of the above mentioned parameters;
3) to enlarge the knowledge on the physical behaviour of the joint up to failure.

Theoretical developments have led to a new approach for the mathematical prediction of the non-linear shear behaviour of unstiffened column web panels.

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Figure 1 – Deformation of a strong axis joint with an end plate connection.

Figure 2 – Joint finite element meshes

Figure 3 – Stress-strain curve (mild steel)

Figure 4 – Loading of an interior joint
Figure 5 - Definition of two welded joints ("T" arrangement)

Simple bending (FS)  Pure bending (PP)  Pure bending in the beam (MF)

Figure 6 - Different types of loading

Figure 7 - Characteristic $V_n - \gamma$ curves (joint A)
Figure 8 - Characteristic $V_n - \gamma$ curves (joint B)

Figure 9 - Shear in a small web element

Figure 10 - Characteristic $\sigma - \epsilon$ and $V_n - \gamma$ curves (first approximation)

Figure 11 - Different types of stresses in a web panel
Figure 12 - Comparison between numerical simulations and model (joint A).

Figure 13 - Comparison between numerical simulations and model (joint B).