SIMPLE DESIGN METHOD FOR SWAY FRAMES WITH SEMI-RIGID CONNECTIONS

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SUMMARY

Present paper is concerned with serviceability and ultimate limit states of sway frames with semi-rigid connections. More especially for what regards the ultimate carrying capacity, it is referred to a generalization of the Merchant-Rankine formula to the case of frames with semi-rigid connections. Simple procedures are suggested for the computation of the critical and first order plastic multipliers. The validity of this approach is supported by the comparison with a wealth of results get from numerical simulations and the range of validity is peculiarly stressed.

1. INTRODUCTION

In accordance with the philosophy of limite states [1], any structure must be designed so that it not only offers a specified safety against collapse under factored loads but also complies with durability conditions under service loads over its whole presumed life duration.

It is while stressing that no restriction regarding the degree of plastification of structures under service conditions is mentioned explicitely amongst the principles governing the design and the check of civil engineering structures. That seems to be somewhat contradictory with the daily practice. In this respect, it is advisable to distinguish between braced and unbraced frames :

a) when loaded, an unbraced frame presents a progressive increasing horizontal deflection, usually designated by the wording "sway". It is generally acknowledged [2] that the sway check under service loads constitutes, much more often than the ultimate limite state, the commanding design citerion.
b) whereas the control of a braced frame can usually be reduced to the check of its individual components, it is generally not permitted to do so for an unbraced frame. In addition, it appears that the global structural analysis of unbraced frames ought reasonably be conducted in the frame of a linear elastic behaviour of the material only.

As a result, it could appear quite justified to conduct the design of unbraced frames under service loads by using a geometrically non linear elastic analysis, i.e. with account taken of the second order effects due to sway.

This design philosophy can however become disputable when the designer wants to take account of the actual semi-rigid behaviour of the connections. Indeed, when subject to bending mainly, such connections allow for a relative rotation between the respective axis of the connected members ; the connection behaviour is thus characterized by a M-ϕ curve where the relative rotation ϕ is plotted against the applied bending moment M (figure 1).

The connection rigidity $K = M/\phi$ is evolving during the loading : it decreases with increasing bending moment. In addition, a pseudo-plastic moment M_v of the connection can be defined, which will be used for practical purpose : for any bending moment exceeding M_v, it is indeed advisable, for sake of design, to substitute the connection with a plastic hinge having a resistant moment M_v [3] (figure 1). Both factors - K and M_v - are thus two main characteristics of any connection.

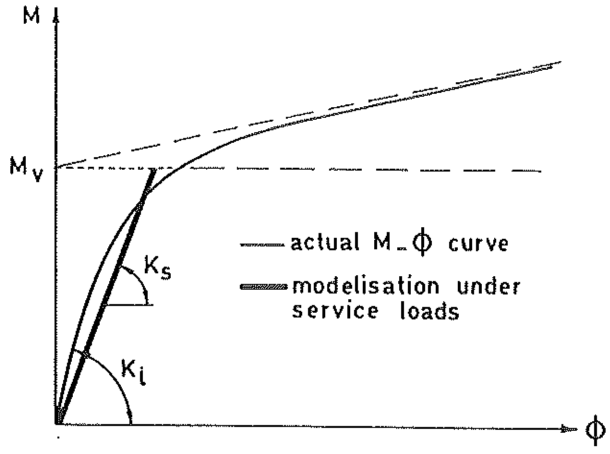

Figure 1 - Connection M-ϕ curve : definition of the initial rigidity K_i, the constant fictitious rigidity K_s and the pseudo-plastic moment M_v.

Because of the governing role of the sway check when designing unbraced frames under service loads, the bending moment in any connection is not allowed to exceed the corresponding value M_v ; with a view to limit the sway amplitude, connections with a still appreciable rigidity when the loading increases are requested.

That being, the behaviour of these connections in the practical range of use ($M < M_v$) reveals to be more or less highly non linear (figure 1) and it must be accounted for. In the following chapter, it is proposed to replace the actual M - ϕ curve by a straight line, the slope of which represents a constant fictitious rigidity K_s (figure 1). Using this substitute rigidity when carrying out a geometrically non linear elastic structural analysis provides the designer with a structural response under service loads which can be expected to be very close to the actual one.

2. STRUCTURAL ANALYSIS OF UNBRACED FRAMES UNDER SERVICE LOADS

The check of unbraced frames under service loads will be conducted by using a geometrically non linear elastic analysis taking into account the second order effects as well as the behaviour of the connections (constant fictitious rigidity K_s).

Vandepitte has shown [4] that the second order effects on a sway frame with rigid joints may be taken into account in a very simple and direct manner by means of an amendment of the slope-deflection method - thus avoiding the usual iterative analysis - and even that the decrease in stiffness due to the thrust in the columns can be incorporated into the computation. The extending of the slope-deflection method to the analysis of frames with semi-rigid connections has been introduced in 1941 by Johnston and Mount [5]. In consequence, the analysis of unbraced frames under service loads will be easily performed provided we define the value of the constant fictitious rigidity K_s.

Figure 2 shows the evolution of the second order elastic transversal deformation of an unbraced frame with semi-rigid connections of constant rigidity K versus the rigidity K. Theoretically speaking, K_s is defined as the particular value of rigidity K for which this transversal deformation of the frame is equal to the actual one, V, under service loads.

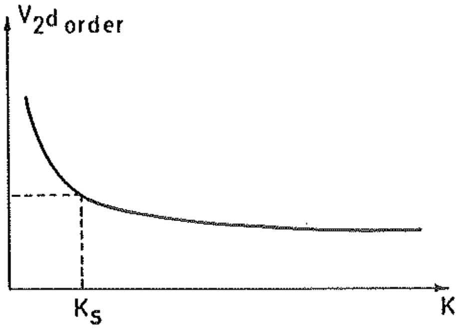

Figure 2 - Definition of the connection constant fictitious rigidity K_s

Indeed, it may be shown that the diagrams of internal forces under servive loads obtained from the geometrically non linear elastic analysis of the frame with connections of constant rigidity K_s are always quite similar to the actual ones [7]. Studies are actually in progress in Liège to predict easily and accurately the value of this constant rigidity K_s.

3. STRUCTURAL ANALYSIS OF UNBRACED FRAMES UNDER FACTORED LOADS

The check of ultimate limit states consists, in warranting that the actual ultimate carrying capacity of the structure remains at least equal to the design factored loads.

In a recent paper [3], the junior author has suggested a generalization of the well-known Merchant-Rankine formula, when assessing, in a simple way, the ultimate load of an unbraced frame fitted with semi-rigid connections. According to Wood [6] the aforementioned formula writes :

$$\frac{1}{\lambda_f} = \frac{1}{\lambda_{cr}} + \frac{0.9}{\lambda_p} \qquad (1)$$

with :
λ_f : collapse load factor (collapse multiplier) ;
λ_{cr} : linear elastic critical load factor (critical multiplier) ;
λ_p : first order plastic collapse load factor (plastic multiplier).

The critical multiplier λ_{cr} is derived from the linear elastic instability analysis conducted on the unbraced frame by assuming that each of the connections is characterized by a rigidity specified equal to the initial stiffness K_i (figure 1).

With a view to determine the plastic multiplier λ_p, a first order plastic analysis is carried out. It is assumed that at the location of any specified connection, the plastic moment is equal either to that of the connected beam or to that of the connection (M_v, see figure 1), according to which is the lesser.

The range of validity of the Merchant-Rankine formula, as slightly modified by Wood to account for the unavoidable influence of strain hardening through numerical factor 0.9, is usually specified as :

$$4 \leq \lambda_{cr}/\lambda_p \leq 10 \qquad (2)$$

Obviously the advantage to refer to the Wood-Merchant-Rankine formula is linked to the availability of simple methods allowing for the assessment of both critical and plastic multipliers. In this respect, simple, fast and accurate procedures have been recently developed at the University of Liège for what regards unbraced frames with semi-rigid connections [7]. The validity of these procedures is demonstrated on the ground of an extended comparison with the results of numerical simulations conducted on a wealth of frames (figure 3). The design tool, that has been used for this purpose, is the FINEL G programme [8], which allows for both geometrically and materially non linear computations and was recently improved in such a way that the semi-rigid behaviour of the connections can be accounted for [9]. From the aforementioned comparison, conclusions are drawn for what regards the influence of different parameters, such as the type of connections, the nature of the loading, the vertical to horizontal loads ratio,... The range limits for each parameter was choosen in accordance with conditions usually met in the daily practice.

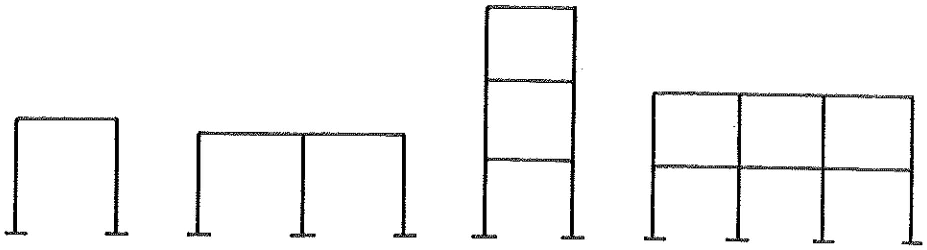

Figure 3 - Semi-rigid frames used for the numerical simulations.

3.1. Assessment of the critical multiplier λ_{cr}

Figure 4 is aimed at presenting the comparison between the results of simple design approaches for the critical multiplier λ_{cr} and those get numerically by means of the FINEL G programme. The ratios between hand and numerically computed values are plotted in these figures.

Method 1 has been largely explained in [3]. It consists in replacing the actual semi-rigid structure by a sensibly equivalent Ginter single bay frame with rigid joints, the critical multiplier of which is computed as the lowest of the critical multipliers associated respectively to each storey considered separately.

Though based on a similar background as Method 1, Method 2 differs for what regards :
- a more accurate definition of the characteristics of the equivalent Ginter frame, according to what is largely justified and explained in [7];
- the account of the very actual continuity of the storeys when computing the critical load of each of the latter.

Last, Method 3 is aimed at extending the specifications of the British Code BS 5950 [10] dealing with structures with rigid joints to structures with semi-rigid connections.

Amongst the three methods in consideration, Method 1 is far the most fast one. Indeed Methods 2 and 3 first require the determination of the linear elastic deflection of the structure.

Based on the critical appraisal of the results, some conclusions can be drawn :
a) all the three methods yield results which are either safe, or unsafe, compared to the ones get from numerical simulation ;
b) the most accurate assessment - with a discrepancy, with respect to the actual value, less than 10 % for the whole wealth of frames - is got from Method 2. Therefore the use of the latter is recommended.

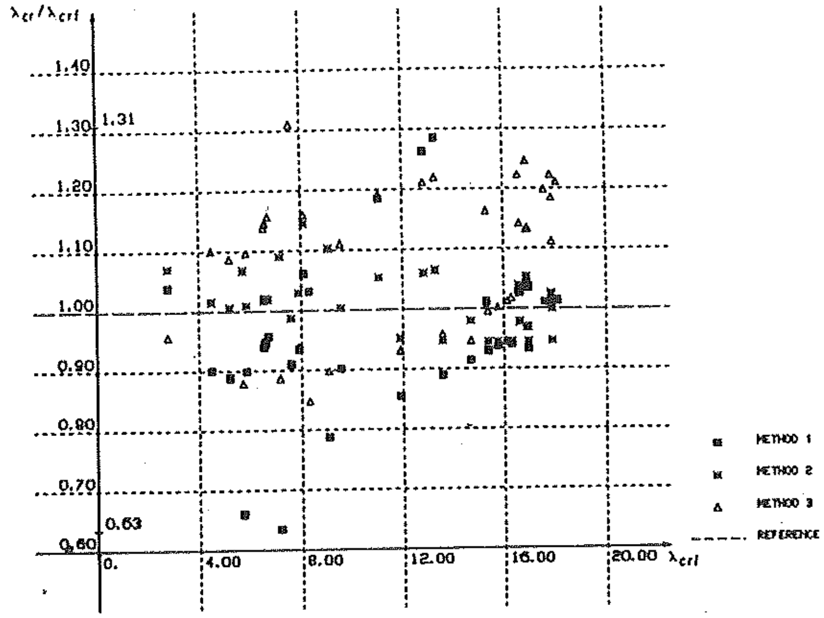

Figure 4 - Comparison between hand (λ_{cr}) and numerically (λ_{crf}) computed values of the critical multiplier.

Anyway one must stress the fact that, due to the range of validity of the Merchant-Rankine formula (specified by (2)), even an appreciable error on the value of the critical multiplier λ_{cr} affects only slightly the value of the collapse multiplier λ_f. Therefore a larger freedom is offered when choosing the method aimed at assessing λ_{cr}.

3.2. Assessment of the plastic multiplier λ_p

In contrast to what regards the critical multiplier λ_{cr}, any error made when assessing the plastic multiplier λ_p will result in a closely similar error on the collapse load, when the latter is derived from the use of the Merchant-Rankine formula. Therefore a rather accurate value of the plastic multiplier λ_p is required.

Applying the kinematic theorem in conjunction with the method of combination of plastic mechanisms [11] will obviously yield the proper exact value of λ_p. Because this procedure is rather long and fastidious, it is not at all suitable for a preliminary design, as far as the designer has no access to adequate computer programmes. The use of the static theorem is not likely to offer a better solution in this respect [11].

The assessment method of λ_p, that is suggested by the authors, is detailed in [7]. It consists in a two steps procedure. First, the structure is divided in substructures, each of which is associated to one storey (figure 5) and defined on base of the assumption that points of contraflexure are located at mid-depth of each column when unbraced frames are loaded [12]. The values of the plastic multipliers of all the substructures defined accordingly are then computed; the smallest one is aimed at assessing the plastic multiplier of the whole structure provided it remains lower than the plastic multiplier associated to the formation of any panel mechanism in the whole structure. The latter is the aim of the second step; it is obtained by combining independent panel mechanisms and node mechanisms in the whole structure till a minimum value of the plastic multiplier be reached.

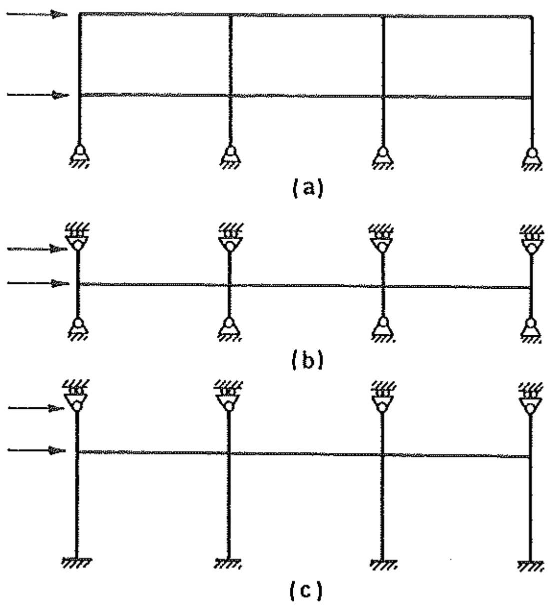

Figure 5 - Definition of substructures for the evaluation of λ_p :
a) upper storey ; b) intermediate storey ; c) lower storey.

The comparison of the results obtained in accordance with this simplified approach on the one hand and with exact computations on the other one results in the validity of the above assumptions and in the accuracy of the authors' suggestion (figure 6).

3.3. Assessment of the collapse multiplier λ_f

The generalization of the Merchant-Rankine formula to frames with semi-rigid connections is proved to be quite justified on base of the results plotted in figure 7. Each plot is representative of a numerical simulation conducted by means of the FINEL G programme with account taken of standard residual stresses, out-of-straightness and out-of-plumb, of geometrical second order effects, of actual behaviour of connections, of material yielding and of material strain hardening [8].

The influence of the type of plastic mechanism associated to λ_p on the accuracy of the generalized Merchant-Rankine formula is clearly shown on the figure. Some conclusions can be drawn for what regards the procedure :

- it is very accurate as far as the collapse mechanism is a complete one,i.e. of the combined type ;
- it is slightly conservative when a partial beam mechanism is governing ;
- it is generally largely unsafe when a panel mechanism is commanding.

Such conclusions can be physically justified by the variable influence of the second order effects, according to the type of collapse mechanism, on the value of the plastic multiplier when the structure sways progressively (angle ϕ - figure 8).

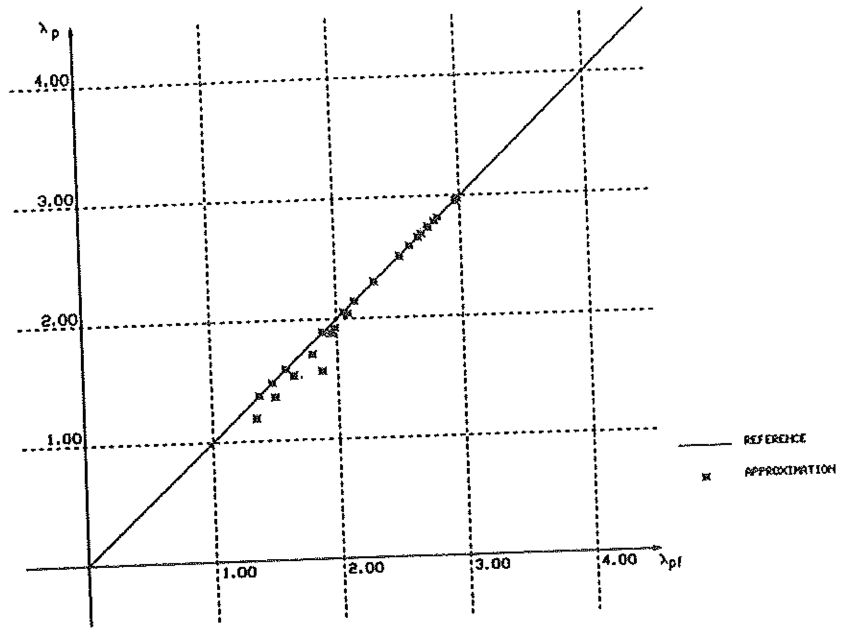

Figure 6 - Comparison between hand (λ_p) and exact (λ_{pf}) computed values of the plastic multiplier.

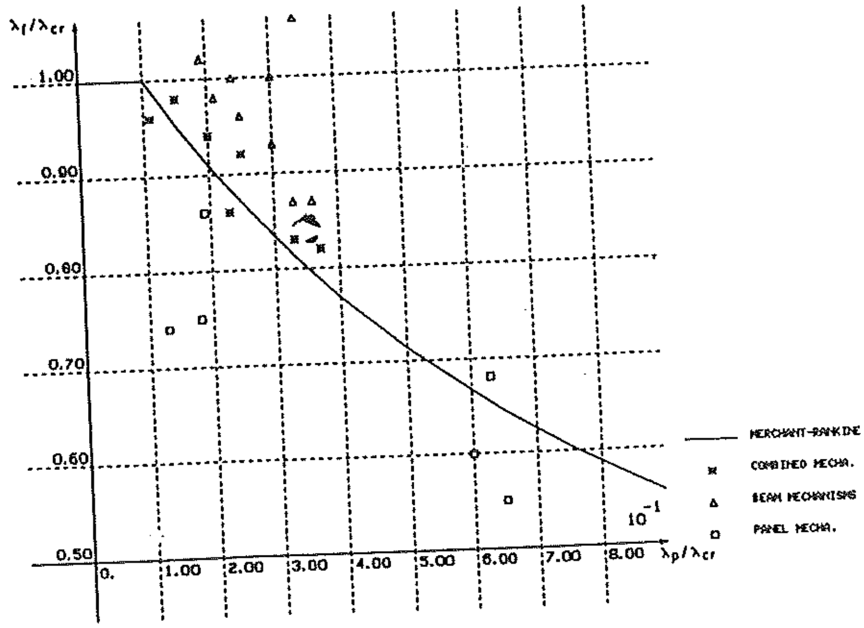

Figure 7-Comparison between hand and numerically computed values of the collapse multiplier.

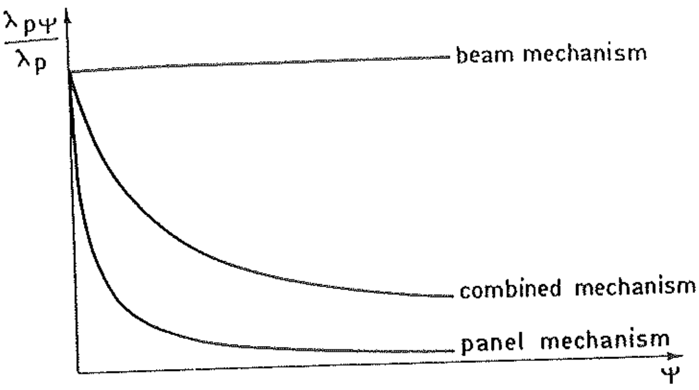

Figure 8-Influence of the second-order effects, according to the type of collapse mechanism, on the value of the plastic multiplier.

It can thus be concluded that the Merchant-Rankine formula, used with the suitable changes required by the semi-rigid character of the connections, is able to predict the collapse load of sway frames with a very good accuracy as far as the first order plastic collapse load is not associated to a panel mechanism.

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