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CONTRIBUTION TO THE DESIGN OF BRACED FRAMES

WITH SEMI-RIGID CONNECTIONS

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1. INTRODUCTION.

The cost of a building steel frame is considerably influenced by the nature of the choosen beam-to-column connections and particularly by their degree of stiffening. A substantial economy may be easily achieved by using bolted connections without stiffeners, the fabrication in workshop and the easy assembling on site of which ensure a minimum cost.

The use of this kind of connections for the design of steel frames compels however to account for their semi-rigid and partial strength character. The actual behaviour of bolted connections is indeed intermediate between the two idealized cases : the perfect hinge which transfers no bending moment and possesses an infinite rotation capacity, and the rigid connection which ensures full rotational continuity between the connected members at each bending moment level.
This so-called semi-rigid behaviourof the connections is governed by a non-linear relationship between the connection moment M and the associated relative rotation ϕ between the connected members (Fig. 1).

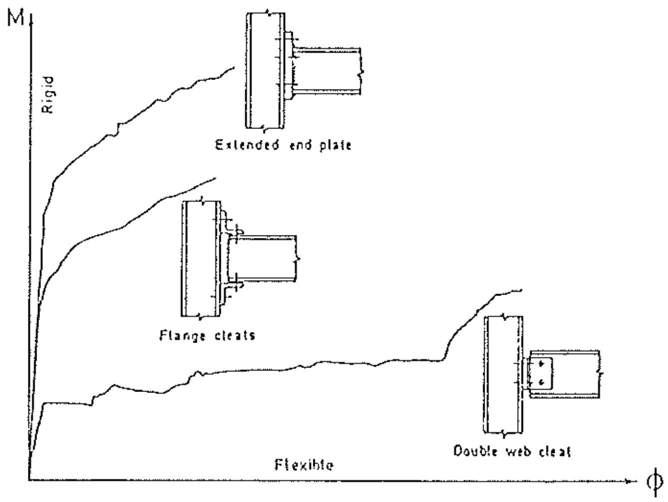

Fig. 1 - M - ϕ curves for semi-rigid joints.

The recent development of numerical programs for structural analysis [1] capable of integrating all the material and geometrical non-linearities, and particularly the actual behaviour of connections, enables presently the simulation, until collapse, of the response of the frames. However this does not prevent us from the necessity of proposing to designers simple but nevertheless accurate design methods more appropriate to daily practice.

Present paper devoted to the study of braced frames falls within this

field. It shows through a brief description of the usual design philosophies - elastic and plastic - the need of simple procedures for the evaluation of the carrying capacity of end-restrained columns under compression and particularly of their buckling length.
A survey of the main approaches in this field is then presented.
A buckling length evaluation method based on the use of buckling curves for columns differently restrained at their ends is finally proposed.

Non-linear behaviour of connections can obviously not be taken into consideration for practical design and the associated M-ϕ curves must be schematized.
The maximum bending moment M_V assumed to be carried over by the connections is represented on figure 2. This pseudo-plastic moment is physically linked up to an ultimate limit state that generally corresponds to the yielding of a connection part.
Hand calculation designmethods for the assessment of the pseudo-plastic moment are available for usually used connections [2, 3].
The connection constant stiffness considered in the stability calculation of frames is the secant stiffness (Fig. 2).
BIJLAARD and ZOETEMEIJER assert in [4] that this bi-linear representation of the semi-rigid and partial strength connection behaviour constitutes a safe approximation for the stability calculation of steel frames.

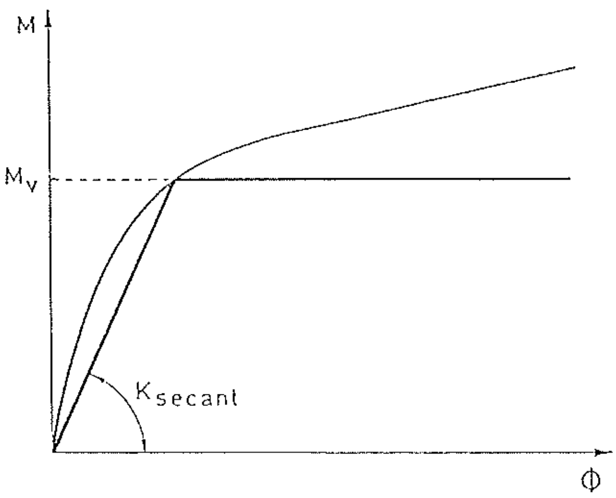

Fig. 2 - Modelisation of the connection behaviour.

2. ELASTIC AND PLASTIC DESIGN OF BRACED FRAMES.

The elastic design of a braced frame requires a first order elastic linear analysis in order to determine the internal forces.
The design in itself is achieved according to a "weak column-strong beam" criterion [5, 6] which consists to design beams and connections in such a way that their collapse never precedes that of the columns. The stability check of the whole frame is then reduced to the individual check of columns by means of usually used interaction formulae for in plane or space loaded columns [7].
The buckling length of an isolated column, useful to its stability check, may safely be choosen equal to the column height, commonly called system length [8].

As columns form however part of the frame, a more accurate estimation of their carrying capacity is obtained by considering a buckling length, called effective length [8], reduced by the presence of end restraints due to the rest of the structure and particularly to the surrounding beams and connections, whose elastic behaviour until frame collapse provides restraints with a constant character.
SNIJDER, BIJLAARD and STARK [8] however highlight the possible importance of second-order effects, whose influence on the bending moment diagram of frames may cause the premature collapse of beams and connections and consequently reduce the impact of end restraints on column buckling.
According to BIJLAARD and SNIJDER it seems that this influence of second-order effects may however be neglected when :
- the beam span to column height ratio is larger than 1.0 ;
- the moment capacity of the beam is larger than that of the column.
In short, it will be referred, for the check of column stability, to the effective buckling length when the second-order effects may be neglected at collapse and to the system length in all other cases.

As for the plastic design, it is achieved according to a "strong column-weak beam" criterion [5, 6], in which the frame collapse is associated to the formation of beam plastic mechanisms. The check of the column is performed, in a similar way to that described here above, in the structure submitted to collapse loads, a part of which remains elastic.
The problem of the rotation capacity and of the required minimum stiffness of connections for a plastic design is dealt with, among other things, in [9].

In conclusion, the use of interaction formulae [7] to check the stability of columns requires the thorough knowledge of their effective buckling length. The following section is devoted to this topic.
It must be noted that the carrying capacity of columns under compression may easily be obtained by the use of buckling curves such as the ECCS curves in Europe and the SSRC curves in North America.

3. BUCKLING LENGTH OF LINEARLY END - RESTRAINED COLUMNS.

3.1. Isolated column and actual column in a frame.

The formulae for the stability check of bent and compressed columns apply to assumed isolated columns. Their application to actual columns in braced frames needs the definition of an equivalent isolated and restrained column (Fig. 3). The effect of restraints is revealed by the presence, at the column ends, of flexural springs, the rigidity of which is defined in such a way that it equals that of the rest of the structure.
The determination of the effective buckling length of actual columns will result from the study of corresponding isolated and restrained columns.
The main problem lies obviously in the evaluation of the flexural characteristic of springs.

BJORHOVDE [10] limits the influence of the structure on the studied column to the beams (and the corresponding connections) ending at the considered extremity (Fig. 4). He proposes the following expression for the stiffness of the equivalent flexural spring at each column extremity :

$$R = \frac{2EI_g}{L_g}\left[\frac{1}{1 + \frac{2EI_g}{CL_g}}\right] \qquad (1)$$

where : E = YOUNG modulus ;
I_g = stiffness of the beam(s) ending at the considered extremity ;
L_g = length of the beam(s) ending at the considered extremity ;
C = secant stiffness of connection(s) between beam(s) and column.

This equation assumes that the beams of the substructure are bent in single curvature with equal and opposite end rotations. It may be easily modified according to the actual beam end conditions.

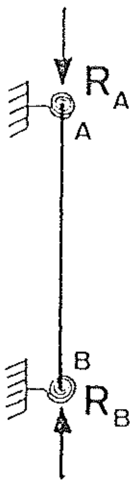

Figure 3 - Isolated column

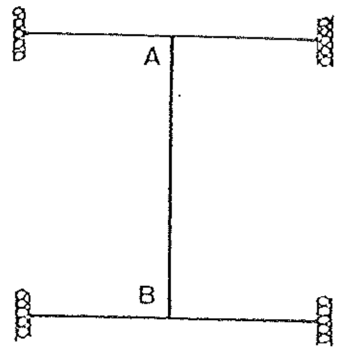

Figure 4 - Substructure

3.2. <u>Evaluation of the effective buckling length based on a study of the elastic linear stability of the column.</u>

The study of the elastic linear stability of an isolated column submitted to compression and linearly end restrained leads to an equation, the iterative solution of which gives the value of the desired effective buckling length.
Different approximated solutions of this equation are to be found in the bibliography :

- DONELL [11] defines the value of the restrained coefficient f at an extremity of the studied column as :

$$f = \frac{RL}{EI} \qquad (2)$$

where R represents the stiffness of the equivalent spring at this extremity (Fig. 3) and L (I) the column length (inertia).
The coefficient of elastic effective buckling length K_e is given by :

$$K_e = \frac{1}{\sqrt{m}} \qquad (3)$$

with :

$$m = \frac{1 + 0.446\ (f_A + f_B) + 0.170\ f_A\ f_B}{1 + 0.215\ (f_A + f_B) + 0.043\ f_A\ f_B} \tag{4}$$

- KAVANAGH [12] proposes the following approximation for the coefficient of elastic effective buckling length K_e :

$$\frac{1}{K_C^2} = \left(\frac{\pi^2 + 4f_A}{\pi^2 + 2f_A}\right) \left(\frac{\pi^2 + 4f_B}{\pi^2 + 2f_B}\right) \tag{5}$$

In this line of thought, the study of the elastic linear stability for the substructure of figure 4 provides also an equation enabling to evaluate the coefficient of elastic effective length of the column.
Its solution may be deduced either from the use of the monograms of JULIAN and LAWRENCE [13] or from that of WOOD's charts [14].
This approach requires the definition of end restraint coefficients, respectively called G_A, G_B and k_A, k_B.
The substitution in these coefficients of the stiffness of the substructure beams at each column extremity by the stiffness R of the equivalent spring (Fig. 3) enables to have two supplementary approaches at one's disposal for the evaluation of the elastic effective buckling length of an isolated column.

The four evaluation methods of K_e described here above are all based on the same study of the elastic linear stability of an isolated and end-restrained column. The few percents of difference between the different methods are due to the approximations made in order to simplify the proposed formulae.

3.3. Evaluation of the effective buckling length based on the use of buckling curves for end-restrained columns.

Buckling curves for hinged columns are well known. They express, for a given column, the variation of the ratio P/P_y between the collapse and squash loads versus its reduced slenderness $\bar{\lambda}$ (Fig. 5). The use of the above mentioned numerical programs [1] or equivalent programs limited to the study of columns [15, 16] enables to obtain buckling curves for an identical end-restrained column (Fig. 5). For each value of P/P_y, it is possible to deduce the value of the corresponding effective buckling length coefficient K_r (Fig. 5) :

$$K_r = \bar{\lambda}_{AB}/\bar{\lambda}_{AC} \tag{6}$$

Different studies [15, 16, 17] show that the K_r variation versus the reduced slenderness of the corresponding hinged column is unimportant and may be neglected in the frame of the development of simple design methods.

LUI and CHEN [15] have proposed, on the base of a parametric study of 83 columns restrained identically at both ends, the following expression for the effective buckling length coefficient :

$$K_r = 1.0 - 0.017\ \alpha \qquad \text{for } 0 \leq \alpha \leq 23 \tag{7.a}$$

$$K_r = 0.600 \qquad \text{for } \alpha > 23 \tag{7.b}$$

α represents the ratio between the rigidity of the equivalent spring (Fig.3) and the column plastic moment M_{pc} for the studied bending axis :

$$\alpha = R/M_{pc} \tag{8}$$

The envisaged parameters in this study are the column bending axis, the type of residual stresses diagram, the nature of the profiles (rolled or welded); the steel grade and, of course, the degree of end restraints.

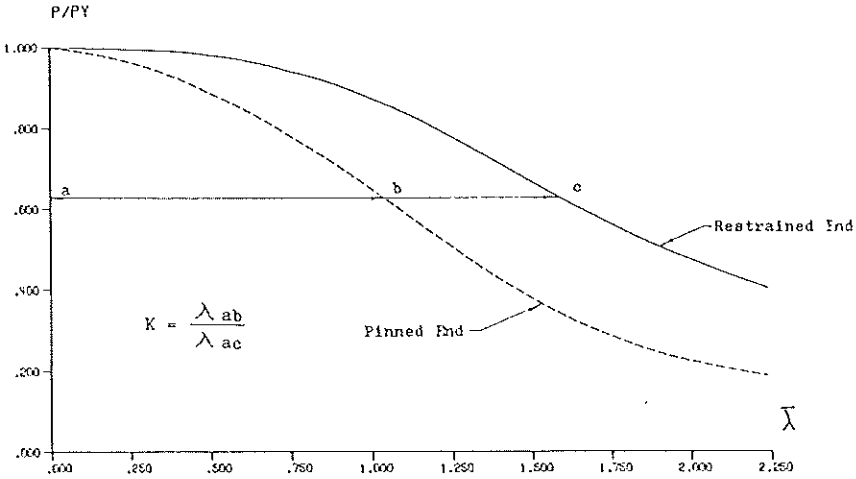

Figure 5 - Buckling curves and determination of the effective buckling length coefficient.

It must be noted that this parametric study concentrates on wide-flange columns as well as on a limited range of end restraints ($0 \leqslant \alpha \leqslant 40$), what seems to correspond to cases generally encountered in practice.
The necessity of considering the same restraints at both column ends constitutes an important limitation of the method.

The methods for the evaluation of the effective buckling length based respectively on a study of the elastic linear stability of the column and on the use of buckling curves for end-restrained columns have been compared in [19] with the results of numerous numerical simulations.

3.4. Proposition of an evaluation method for the effective buckling length of columns differently restrained at their ends.

Research works performed at the University of Liège have led to the generalization of the approach described here above to the case of columns, the different end restraints of which are characterized by α_A and α_B values (formula 8).

It may be noted by referring to the trihedral of figure 6 that the plane defined by K and α_A axes ($\psi = 0$) characterizes columns hinged at B extremity and that the median plan ($\psi = 45°$) corresponds to identically end-restrained columns.
For evident reasons of symmetry, the study of the K variation is limited to the region between these two planes ($0 \leqslant \psi \leqslant 45°$).
The adopted approach consists in studying the evaluation of the effective buckling length in a certain number of planes (K, α_c) characterized by different ratios of end restraints :

$$r = tg\ \psi = \alpha_B/\alpha_A \leq 1 \qquad (9)$$

and in which the generalized restraint α_c is defined in the following manner:

$$\alpha_c = \sqrt{\alpha_A^2 + \alpha_B^2} \qquad (10)$$

Six such planes are considered ; they correspond to ψ values respectively equal to 0°, 5°, 10°, 15°, 30° and 45°.

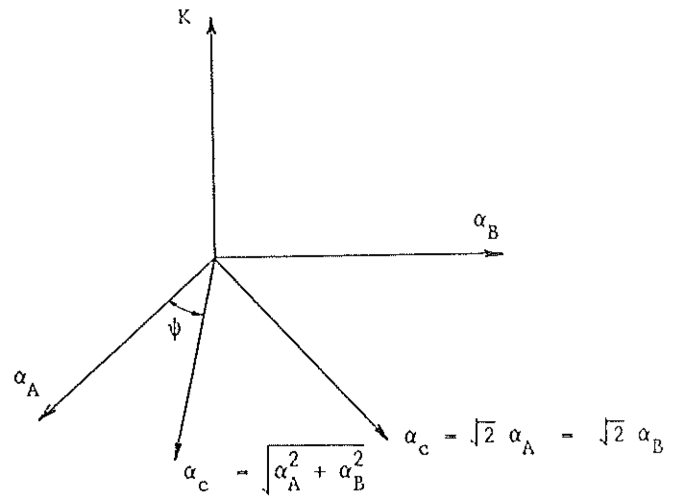

Figure 6 - (K - α_1 - α_2) trihedral.

Numerous numerical simulations of the buckling of end-restrained columns have been performed for each of these planes by means of the non-linear calculation program FLAMB 18 [16] capable to account for an initial sinusoidal geometrical deformation, a diagram of residual stresses, the actual σ-ε characteristic of steel and the linear or non-linear behaviour of end springs.

The parameters considered in this numerical study are the following :

a) the type of sections : IPE and HE rolled sections ;
b) the axis of bending : weak and strong axes ;
c) the steel grade : three values of the yield stress are considered 235 MPa (34 ksi), 275 MPa (40 ksi) and 355 MPa (51 ksi) ;
d) the generalized restraint α_c : 16 values between 5 and 800.

The σ - ε characteristic of steel is assumed elastic - perfectly plastic (E = 210000 MPa = 30480 ksi), the diagrams of residual stresses are in accordance with the ECCS recommendations [18] and the initial geometrical imperfection is equivalent to 1/1000 of the column height.

For each of the six planes (K, α_c) considered, 192 buckling curves have then been reported for a range of reduced slenderness between 0 and 2.0.
The study, for each of them, of the evaluation of the effective buckling length (Fig. 5 and formula 6) versus the reduced slenderness of the corresponding hinged column shows that this evolution is unimportant and may consequently be neglected in the frame of the development of simple design methods. This conclusion is identical to that formulated for the identically end-restrained columns.

Figures 7 and 8 present, as an example, the result, in the range of weak restraints, of the parametric study relative to the planes (K, α_c) characterized respectively by ϕ = 0° and ϕ = 30°.

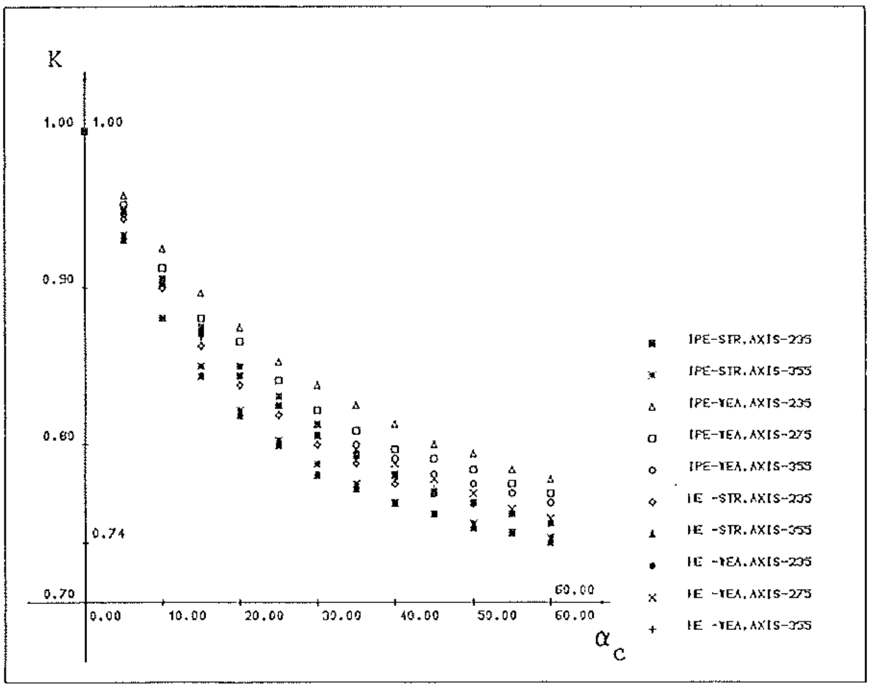

Figure 7 - Results of the parametric study plane ϕ = 0°

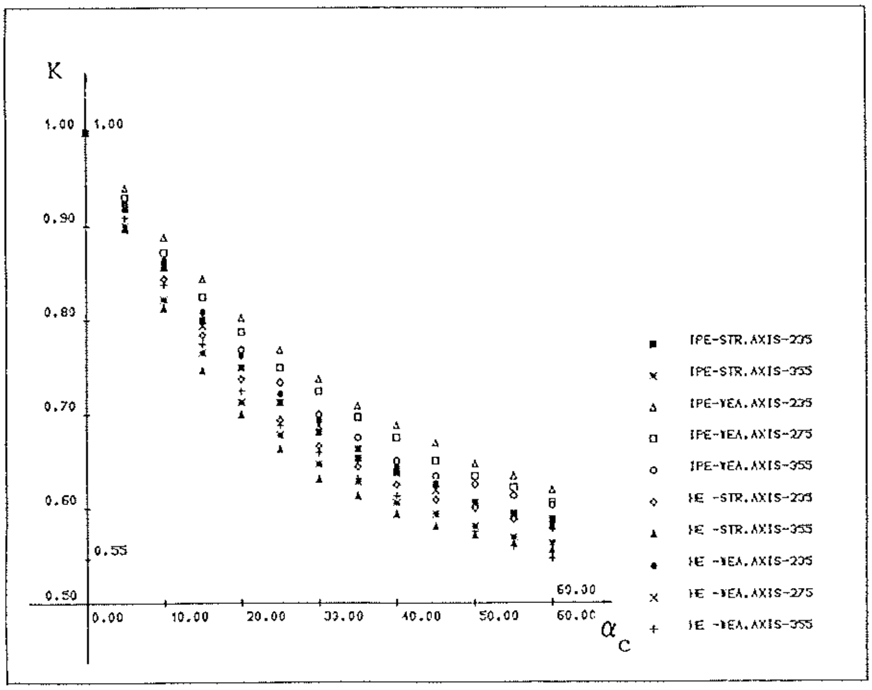

Figure 8 - Results of the parametric study plane ϕ = 30°

It may be seen that the effective buckling length is less dependent on the steel grade than on the type of section or on the bending axis. However the discrepancy between the computed K coefficients is relatively small and the evolution of the effective buckling length coefficient will be consequently modelized by means of a single mathematical expression capable of accurately predicting, for each restraint level α_c and for each plane (K, α_c), the arithmetical mean value of the actual ^cK coefficients.

These mean curves are represented on figure 9 for the six planes considered in the study.

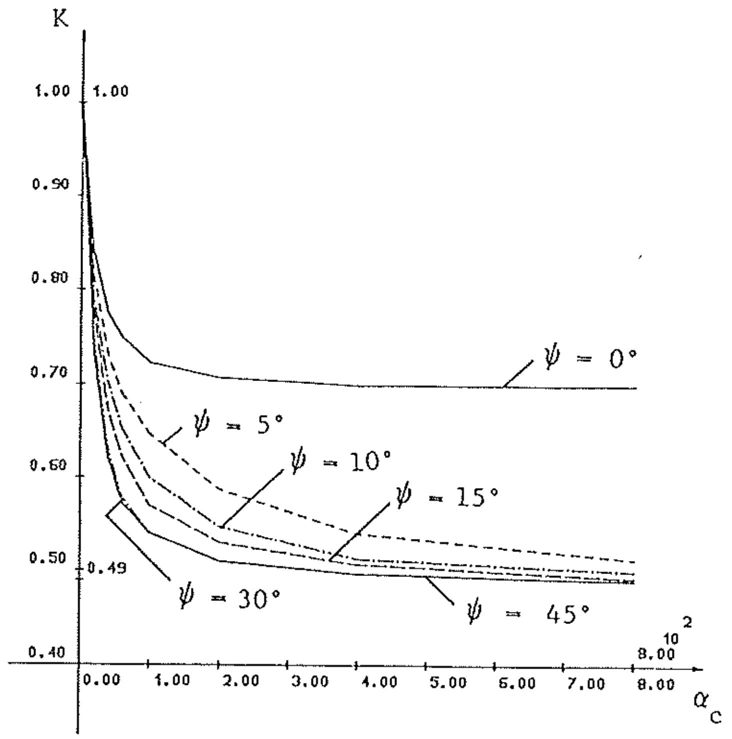

Figure 9 - Evolution of K coefficient
(mean curves)

They reveal the existence of two separate lower bounds for high values of α_c. The first of them, $K \approx 0.7$, is related to the buckling of columns hinged at one end ($\psi = 0°$) and strongly restrained at the other, and the second, $K \approx 0.5$, to that of columns restrained at both ends ($0 < \psi \leqslant 45°$), the ends of which may be considered as fixed for high values of α_c.

The formula proposed for the evaluation of the effective buckling length coefficient of columns restrained differently at their ends is as follows :

$$K = \frac{1}{\sqrt{n}} \tag{11}$$

with :

$$n = \frac{1 + 0.07\ \alpha_c f_1(r) + 0.009\ \alpha_c^2 f_2(r)}{1 + 0.034\ \alpha_c f_1(r) + 0.00225\ \alpha_c^2 f_2(r)} \tag{12}$$

In this expression :

$$f_1(r) = \frac{1 + r}{\sqrt{1 + r^2}} \qquad (13.a)$$

$$f_2(r) = \frac{r}{1 + r^2} \qquad (13.b)$$

The comparison (figure 10) of this formula with the mean curves resulting from the parametric study is seen to be quite satisfactory (maximum difference of 3 %). The use of a little more complicated formulation than that adopted by LUI and CHEN [15] for the identically end-restrained columns (formula 7) enables to cover accurately the whole possible range of restraints ($0 \leqslant \alpha_c \leqslant \infty$).

The good agreement between the K coefficients resulting from the application of formula (11) and from the results of the numerical simulation performed by LUI and CHEN on columns with rolled and welded wide-flange sections (in the plane $\psi = 45°$) leads to the conclusion that the proposed formula is applicable to all the types of sections.

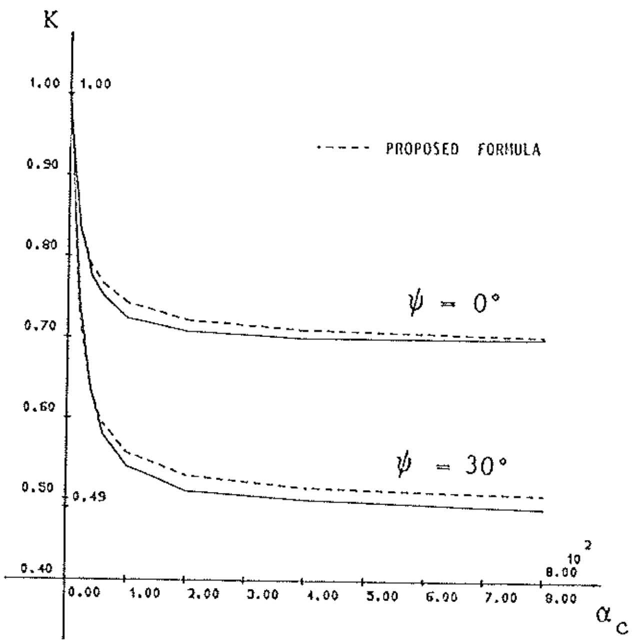

Figure 10 - Comparison of proposed the formula with numerical results (mean curves).

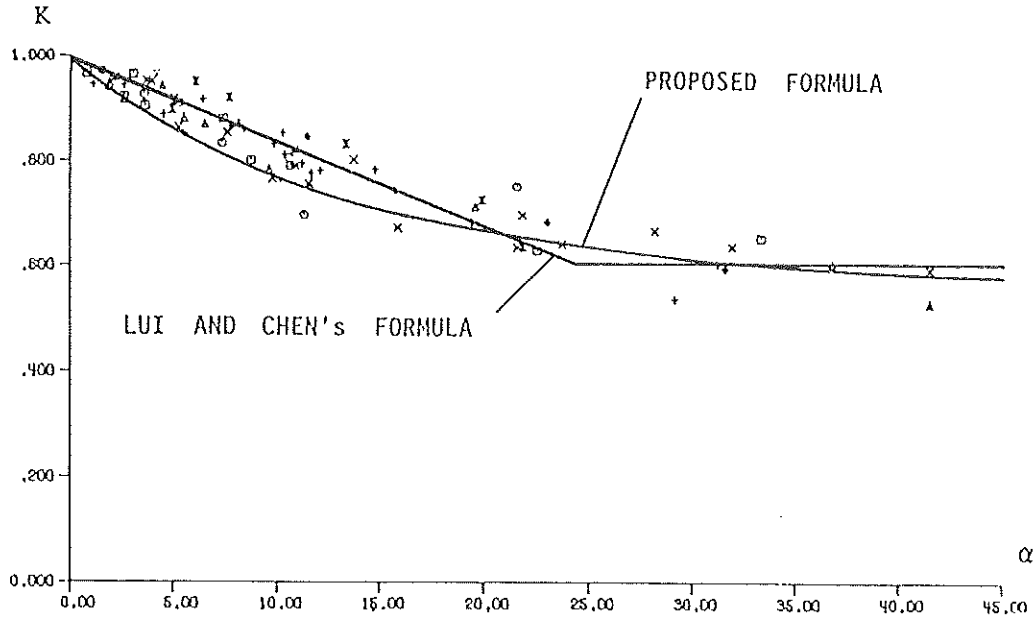

Figure 11 - Comparison of the proposed formula with LUI and CHEN's results

4. CONCLUSIONS

Recent developments presented in this paper have led to the proposition of a formula for the evaluation of the effective buckling length of columns restrained differently at their ends. The comparison of this formula with the results of numerous numerical simulationshas led to the conclusion that it may be accurately used for different types of sections, both bending axes, all steel grades, all restraint levels and all columns slendernesses.

5. REFERENCES

1. JASPART, J.P. and de VILLE de GOYET, V., "Etude expérimentale et numérique du comportement des structures composées de poutres à assemblages semi-rigides", Construction Métallique, n° 2, 1988.

2. "Steinfenlose Stahskeletttragwerke und dünnwandige Vollwandträger, Berechnung und Konstruktion", Europaïshe Konvention für Stahlbau, 1977.

3. ZOETEMEIJER, P., "Samenvatting van het onderzoek op geboute balkkolom verbindingen", Report 6-85-7, Delft University of Technology, Stevin Laboratory, 1983.

4. BIJLAARD, F.S.K. and ZOETEMEIJER, P., "Influence of joint characteristics on the structural response of frames", presented at the International Conference "Steel Structures, Recent Advances and their Application to Design", Budva, Yugoslavia, 29 Sept. - 1 Oct. 1986.

5. VINNAKOTA, S., "Design of columns in planar frames. A few comments", presented at the National Conference on Tall Buildings, held at New-Delhi, 1973.

6. VINNAKOTA, S., "Design of columns as part of frames - Some remarks", Paper submitted to TG 3 : SSRC, 1983.

7. EUROCODE 3, "Common unified rules for steel structures". Directorate General for Internal Market and Industrial Affairs. Commission of the European Communities, 1984.

8. SNIJDER, H.H., BIJLAARD, F.S.K. and STARK, J.W.B., "Use of the elastic effective length for stability checks of columns and consequences for checks on beams in braced frames", Proceedings of the Michael R. HORNE Conference, edited by L.J. MORRIS, London, Granada, 1983.

9. STARK, J.W.B. and BIJLAARD, F.S.K.,"Monograph on beam-to-column connections . Chapter 17 : Design rules for beam-to-column connections in Europe", Institute TNO for Building Structures and Building Materials, Delft, The Netherlands, 1984.

10. BJORHOVDE, R., "Effect of end restraint on column strength - Practical applications", Engineering Jl. A.I.S.C., to be published.

11. DONELL, L.H., Reissner Anniversary Volume.

12. KAVANAGH, T.C. "Effective length of framed columns", Transactions of the A.S.C.E., Vol. 127, part II, 1962.

13. Guide to Design Criteria for Metal Compression Members, Column Research Council, 2nd edition by B.G. JOHNSTON, J. WILEY & Sons, Inc. New-York, London, Sydney, 1966.

14. "Longueurs de flambement des éléments de structures". Notes du Centre de Recherches Scientifiques et Techniques de l'Industrie des Fabrications Métalliques, to be published.

15. LUI, E.M. and CHEN, W.F., "Strength of H-columns with small end restraints", The Structural Engineer, Vol. 61B, N° 1, Part B, March 1983.

16. JASPART, J.P., "Sur le comportement des assemblages semi-rigides et leur influence dans l'étude des structures en acier", Annales des Travaux Publics de Belgique, N° 2, 1986.

17. JONES, S.W., KIRBY, P.A. and NETHERCOT, D.A., "Modelling of semi-rigid connection behaviour and its influence on steel column behaviour". Joints in Structural Steelwork, ed.HOWLETT, JENKINS & SAINSBY, Pentech Press, London 1981.

18. ECCS, "Ultimate limit state calculations of sway frames with rigid joints", Technical Committee 8 - Structural Stability Working Group 8.2. System, Publication n° 33, 1st ed., 1984.

19. AMOUR, A., "Contribution à l'étude de la capacité portante des colonnes de structures contreventées à noeuds semi-rigides", Maîtrise en Sciences Appliquées, Service M.S.M., Université de Liège, 1988.