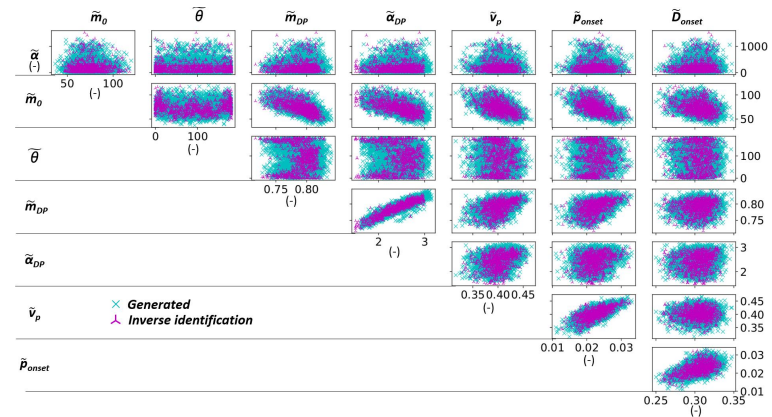
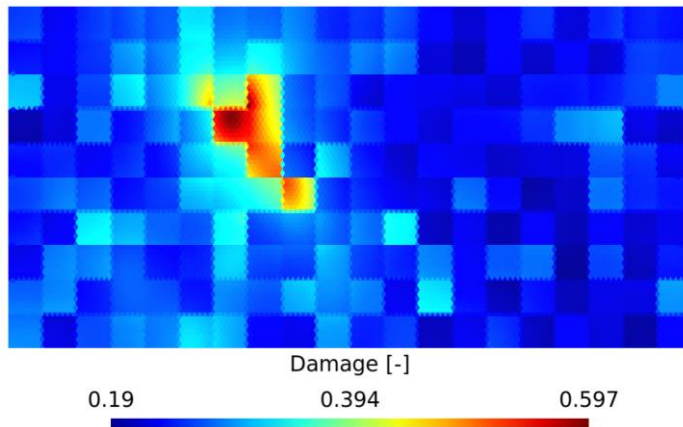


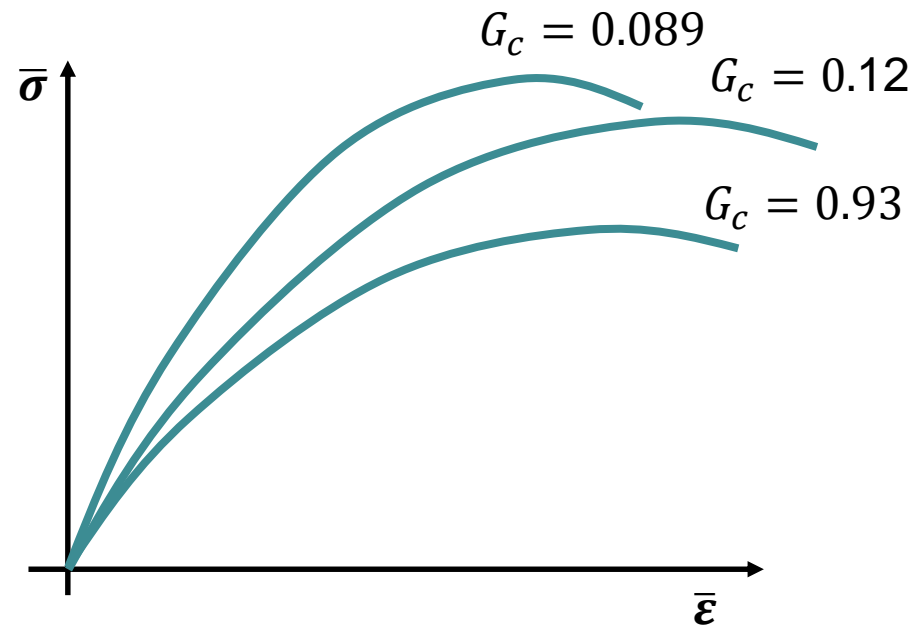
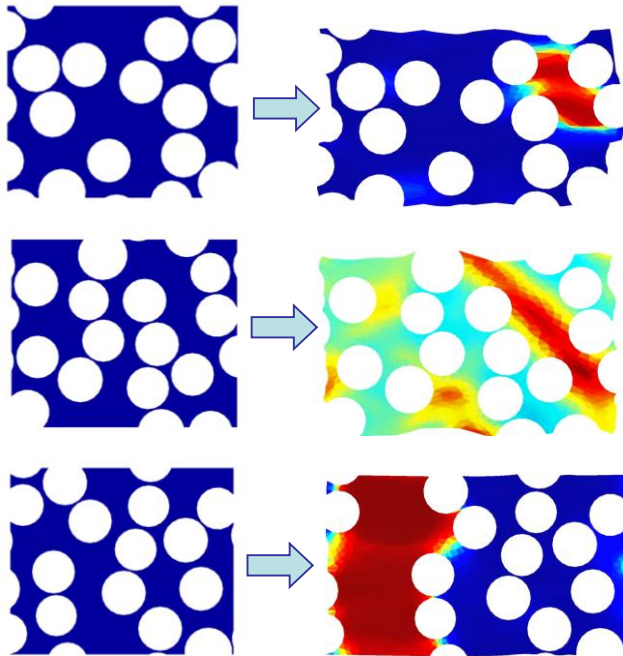
Juan Manuel Calleja Vázquez

ACOMEN - Liège 2022:

Pressure-dependent multiscale stochastic simulations using a
MFH model constructed from full-field SVE realizations

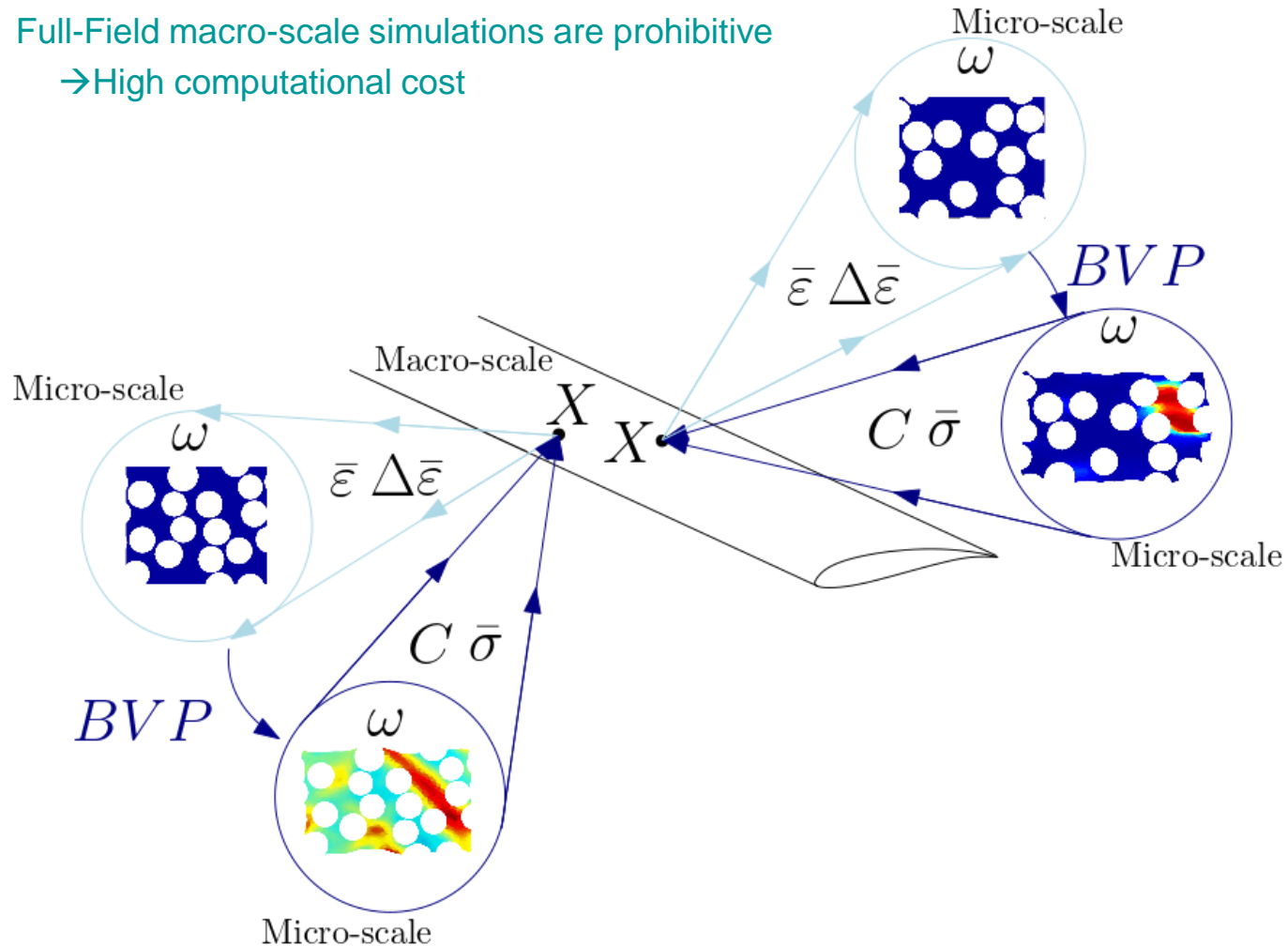


- A look into the microstructure:
 - Non determinisms affect the SVE response → Each SVE has a different behavior
 - Geometrical
 - Fibre-matrix bonding
 - ...



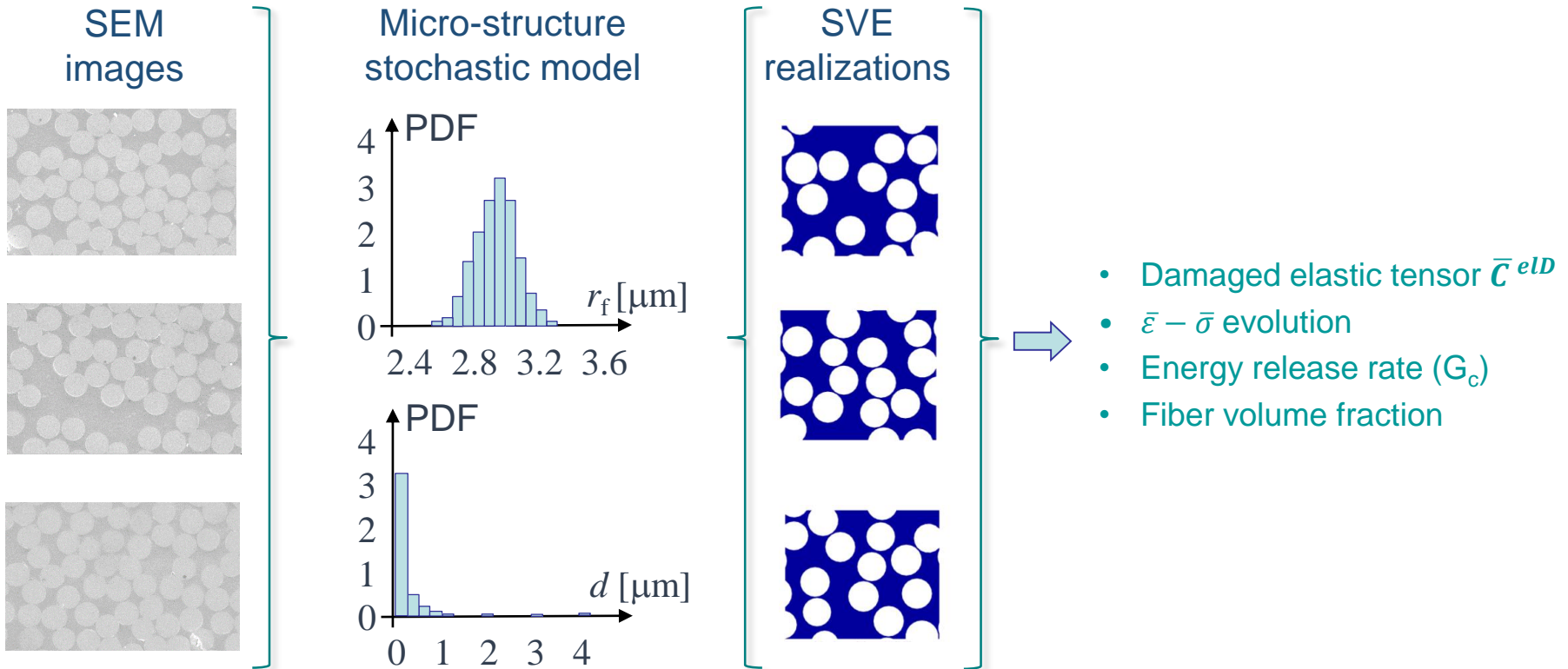
Motivation

- A look into the microstructure:
 - Need of a stochastic model in order to scale these uncertainties to the macroscale
 - Efficient model for the simulation is needed
 - Full-Field macro-scale simulations are prohibitive
→ High computational cost



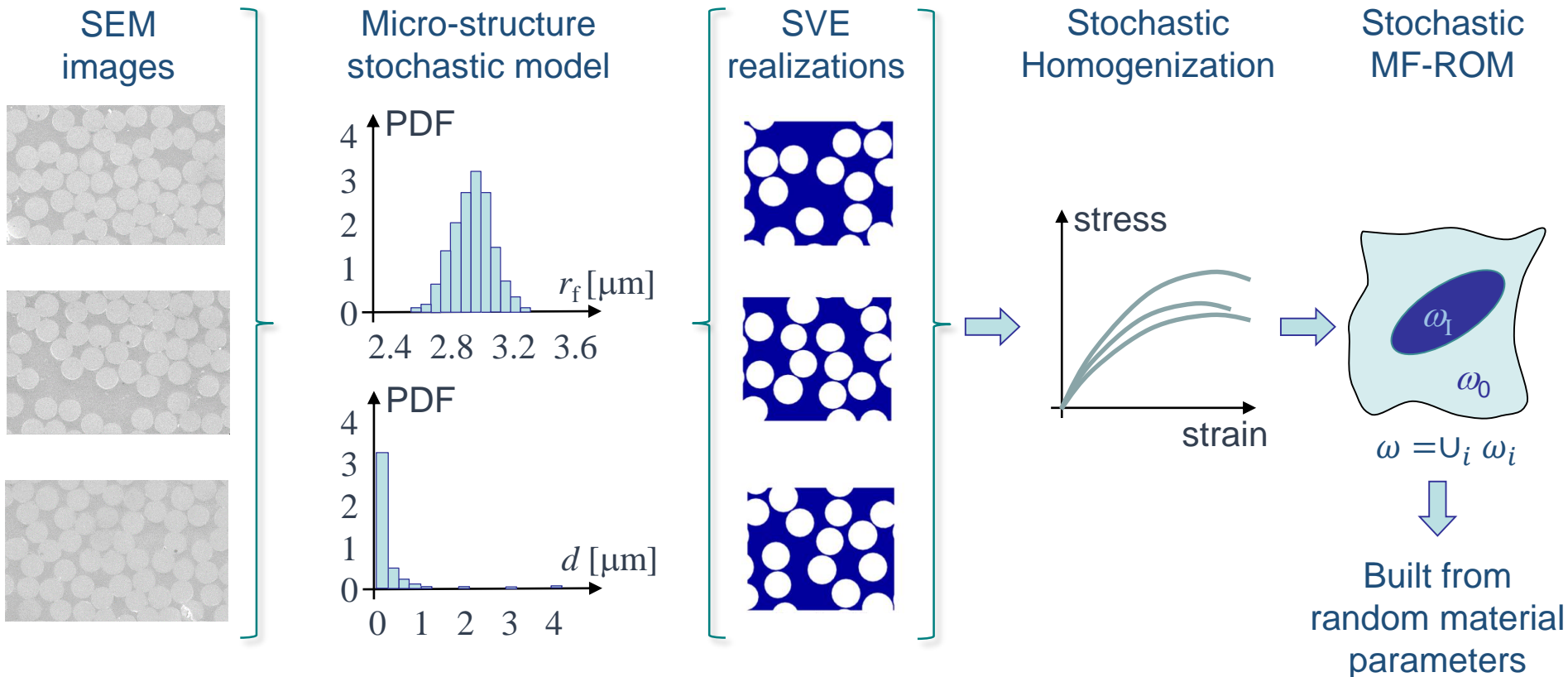
Methodology Overview: Stochastic MF-ROM

- Methodology scheme: From SEM images to efficient stochastic simulations



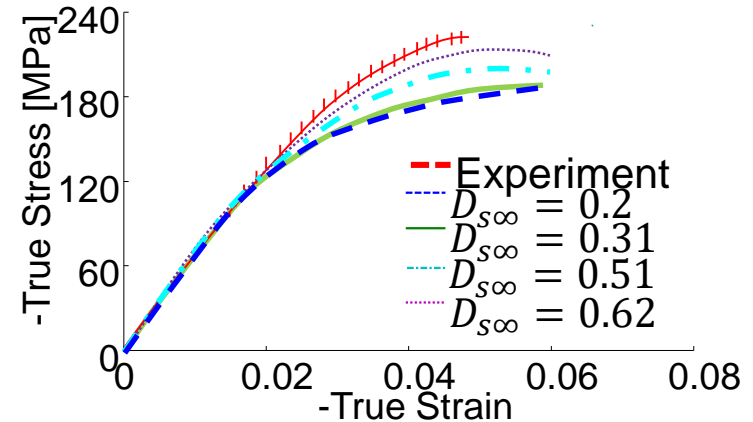
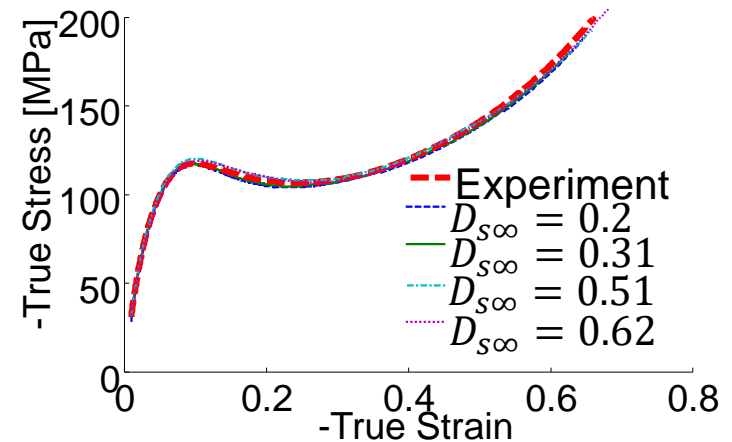
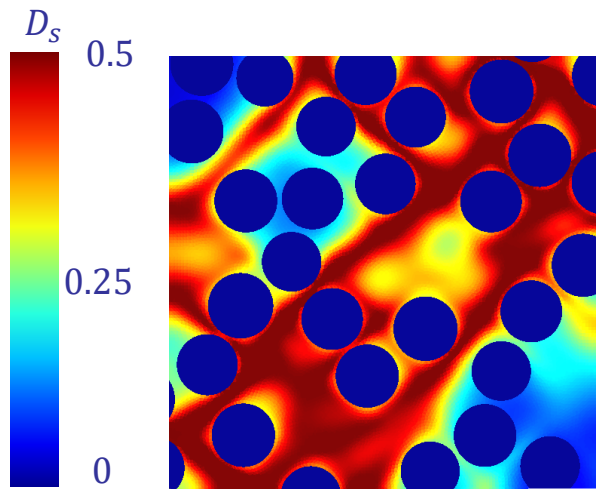
Methodology Overview: Stochastic MF-ROM

- Methodology scheme: From SEM images to efficient stochastic simulations



Full-Field Realizations: Large Strain Matrix Model

- UD Composites with RTM6 epoxy matrix
 - Full-field simulations performed with identified matrix material behaviors
 - Hyperelastic viscoelastic-viscoplastic constitutive model enhanced by a multi-mechanism non-local damage model
- Capable of accurately represent the behavior of the high crosslinked epoxy
- Represents the epoxy behavior up to its complete failure thanks to G_c calibration after the localization onset



V.-D. Nguyen, L. Wu, L. Noels, A micro-mechanical model of reinforced polymer failure with length scale effects and predictive capabilities. Validation on carbon fiber reinforced high-crosslinked RTM6 epoxy resin

Incremental Secant MFH Scheme: Elasticity

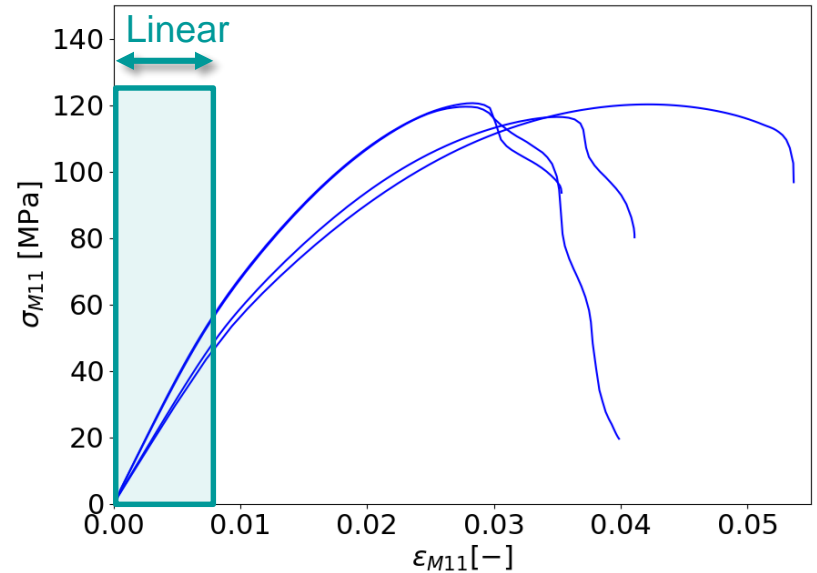
- Elastic Stage

$$\bar{\boldsymbol{\varepsilon}} = \nu_0 \boldsymbol{\varepsilon}_0 + \nu_I \boldsymbol{\varepsilon}_I$$

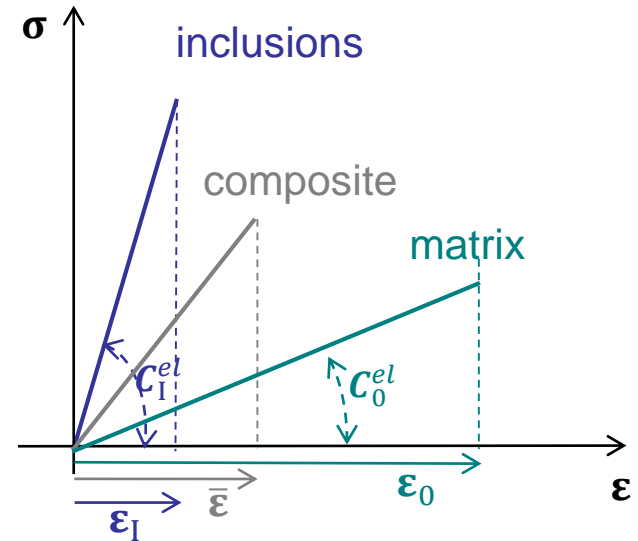
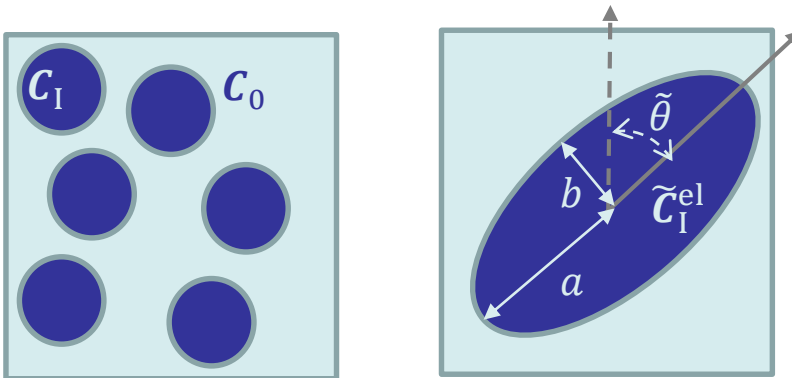
$$\bar{\boldsymbol{\sigma}} = \nu_0 \boldsymbol{\sigma}_0 + \nu_I \boldsymbol{\sigma}_I$$

$$\boldsymbol{\varepsilon}_I = \mathbf{B}^\varepsilon(I, \mathbf{C}_0, \mathbf{C}_I, \nu_I) : \boldsymbol{\varepsilon}_0$$

- Where the Mori-Tanaka assumption is used for \mathbf{B}^ε
- How do we identify the elastic random vectors?

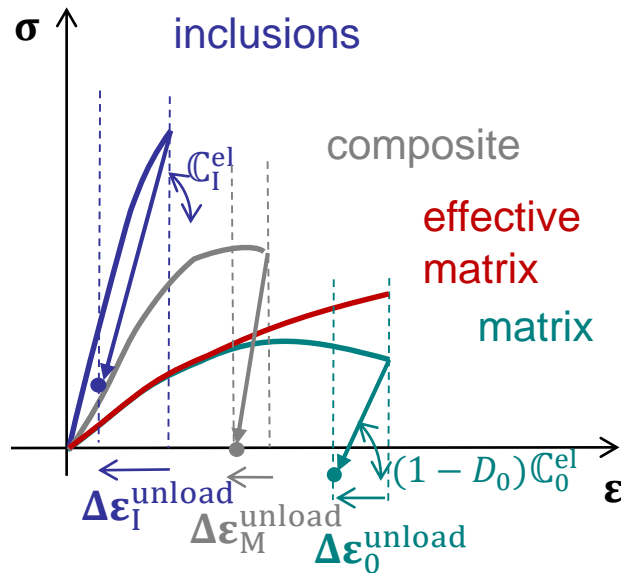
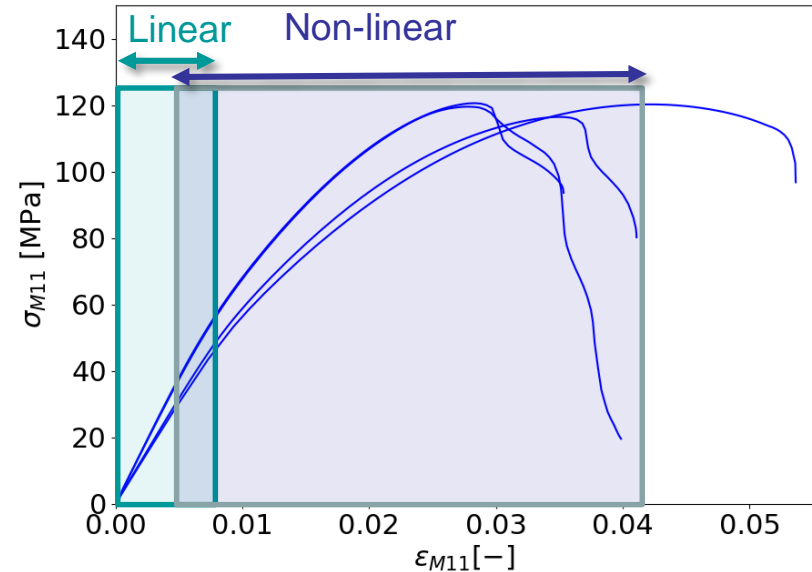


$$\min_{\tilde{I}, \tilde{\theta}, \tilde{E}_0, \tilde{\nu}_0} \left\| \left(\tilde{\mathbf{C}}^{el}(\tilde{I}, \tilde{\theta}, \tilde{\mathbf{C}}_0^{el}(\tilde{E}_0, \tilde{\nu}_0)) \right) - \bar{\mathbf{C}}^{el} \right\|$$



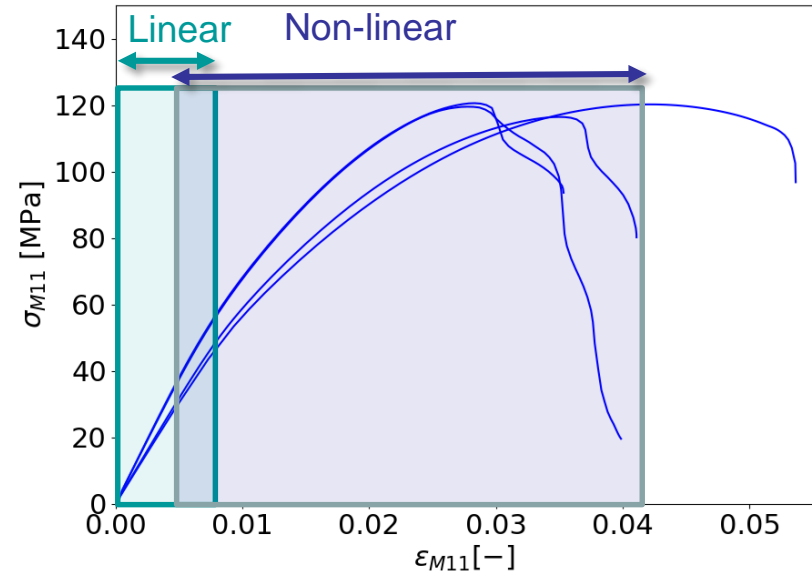
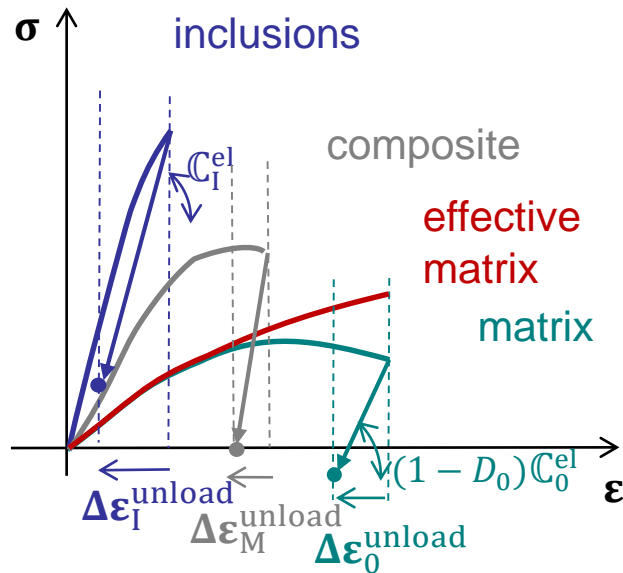
Incremental Secant MFH Scheme: Plasticity

- Damage-enhanced plasticity
 - Use of the incremental-secant MFH
 - Virtual elastic unloading from the previous state
 - Composite material unloaded to reach the stress-free state
 - Residual stress in components
 - Damage is taken into account in the matrix phase



Incremental Secant MFH Scheme: Plasticity

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- $\tilde{\mathcal{C}}_n^{elD}$ is extracted from full-field SVE realizations

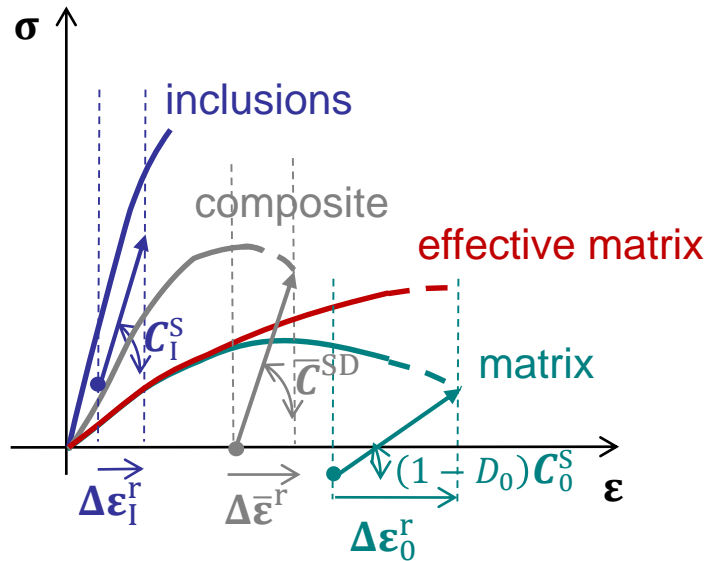
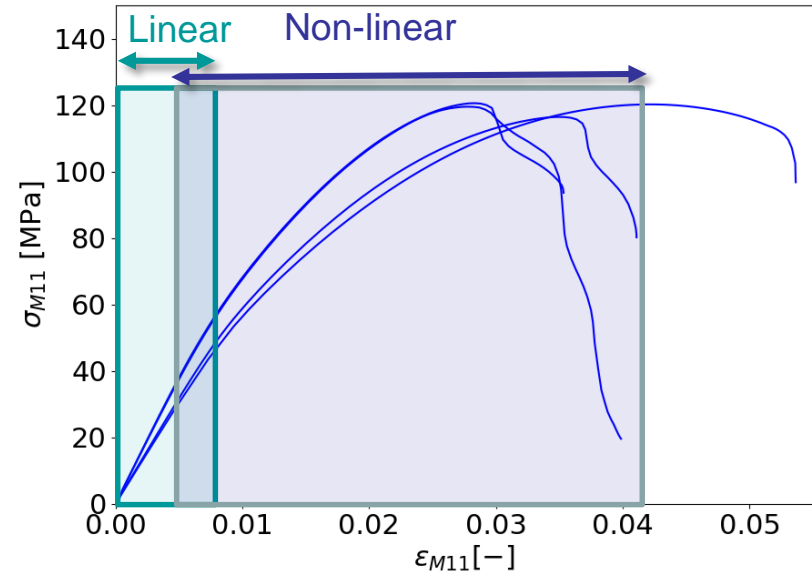
- \tilde{D}_{0n} can be identified through a minimization process:

$$\min_{\tilde{D}_{0n}} \left\| \tilde{\mathcal{C}}_n^{elD} \left(\tilde{\mathcal{C}}_0^{elD}(\tilde{D}_{0n}) \right) - \bar{\mathcal{C}}_n^{elD} \right\|$$

- \tilde{D}_0 is then fitted through a dedicated damage evolution law

Incremental Secant MFH Scheme: Plasticity

- Damage-enhanced plasticity
 - Use of the incremental-secant MFH
 - Reload of the composite:
 - The composite material is reloaded to reach the next stress-strain state
 - Definition of the **LCC**
 - Damage is taken into account in the matrix phase



$$\Delta \bar{\boldsymbol{\epsilon}}_{n+1}^r = v_0 \Delta \boldsymbol{\epsilon}_{0n+1}^r + v_I \Delta \boldsymbol{\epsilon}_{In+1}^r$$

$$\bar{\boldsymbol{\sigma}}_{n+1} = v_0 \boldsymbol{\sigma}_{0n+1} + v_I \boldsymbol{\sigma}_{In+1}$$

$$\Delta \boldsymbol{\epsilon}_{In+1}^r = \mathbf{B}^\epsilon : \Delta \boldsymbol{\epsilon}_{0n+1}^r$$

- Incremental secant operator

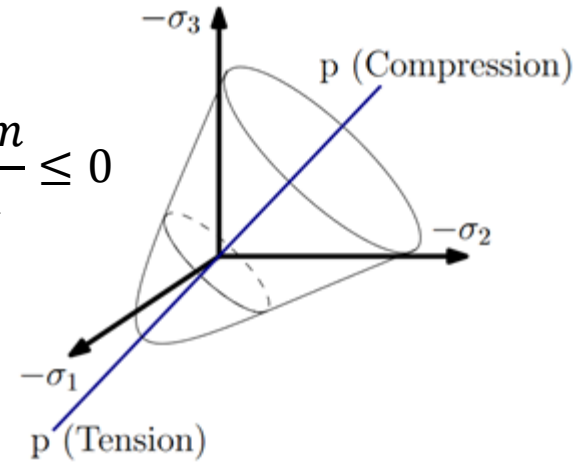
$$\bar{\boldsymbol{\sigma}}_{n+1} = \bar{\boldsymbol{\sigma}}_n^{res} + \bar{\mathbf{C}}^{SD} (I, (1 - D_0) \mathbf{C}_0^S, \mathbf{C}_I^S, v_I) : \Delta \bar{\boldsymbol{\epsilon}}^r$$

LCC: Linear Comparison Composite

- Enhanced Drucker-Prager yield function with non-associated plastic flow:

$$f(\hat{\boldsymbol{\sigma}}_{n+1}, p_n) = \frac{((\hat{\boldsymbol{\sigma}}_{n+1})^{eq})^\alpha}{\sigma_c^\alpha} - 3 \frac{m^\alpha - 1}{(m + 1)\sigma_c} \hat{\phi}_{n+1} - \frac{m^\alpha + m}{m + 1} \leq 0$$

$$\sigma_c = \sigma_c^0 + R(p) \quad \hat{\phi} = \frac{1}{3 \text{tr}(\hat{\boldsymbol{\sigma}})}$$



- \mathcal{C}^{Sr} is found to be isotropic, being possible to write it in terms of μ_s^r and κ_s^r as:

$$\mathcal{C}^{Sr} = 3\kappa_s^r I^{vol} + 2\mu_s^r I^{dev}$$

– Where

$$\kappa_s^r = \kappa_s^r(\kappa^{el}, \beta, \Gamma) \quad \mu_s^r = \mu_s^r(\mu^{el}, \Gamma)$$

Stochastic MF-ROM: Parameter identification

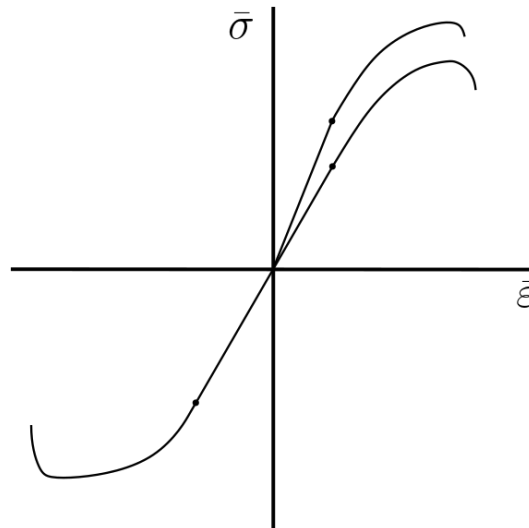
- 16 random material parameters needed to describe the composite:

Already identified!

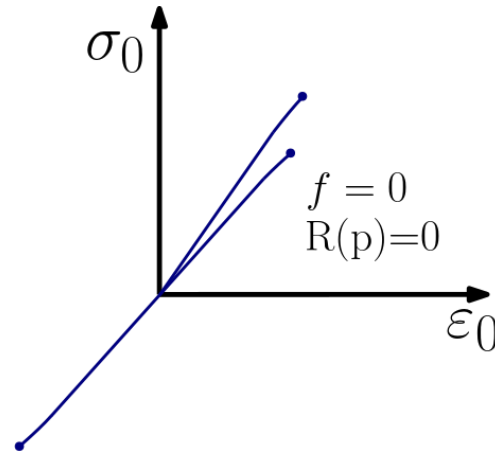
$$[\tilde{\nu}_I, \tilde{I}, \tilde{\theta}, \tilde{E}_0, \tilde{\nu}, \tilde{\sigma}_y, \tilde{h}_0, \tilde{h}_1, \tilde{m}_0, \tilde{m}, \tilde{\nu}_p, \tilde{\alpha}, \tilde{p}_{onset}, \tilde{D}_{onset}, \tilde{\alpha}_{dam}, \tilde{\beta}]$$

Elastic Hardening Pressure Damage

- In order to fully describe the pressure-dependent model:
 - 3 full-field tests are needed for each SVE
 - Uniaxial tension, uniaxial compression and biaxial tests are used



- Once the matrix stress-state is computed at the plasticity onset:
 - The accumulated plastic strain is supposed to be zero at this state
 - Solving the yield surface for the 3 tests allows to obtain the 3 parameters involved in its definition
 - $\tilde{\sigma}_y, \tilde{m}, \tilde{\alpha}$



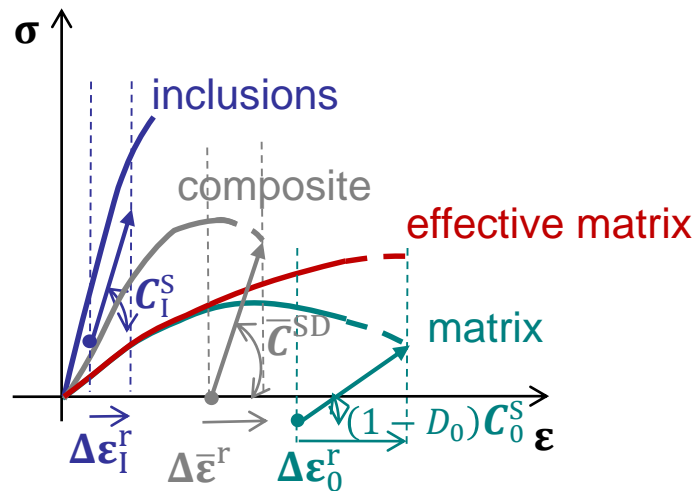
$$\min_{\alpha, m, \sigma_y} \left\{ \left\| \sum f(\alpha, m, \sigma_y) \right\| \right\}$$

- Only one of the tests is needed for the remaining of the identification process
 - Uniaxial tension is used

- The matrix secant tensor is optimized at each step
 - The bulk and shear moduli remain the only unknown in its definition

$$\min_{\tilde{\mu}_s^{Dr}; \tilde{\kappa}_s^{Dr}} \left\{ \left\| \tilde{\mathbf{C}}^{SD}(\tilde{\mathbf{C}}_0^{SD}(\tilde{\mu}_s^{Dr}; \tilde{\kappa}_s^{Dr})) : \Delta \bar{\boldsymbol{\varepsilon}}_n^r - \bar{\boldsymbol{\sigma}}_{n+1} \right\| \right\}$$

$$\mu_s^r = \mu_s^r(\mu^{el}, \Gamma) \quad \kappa_s^r = \kappa_s^r(\kappa^{el}, \beta, \Gamma)$$

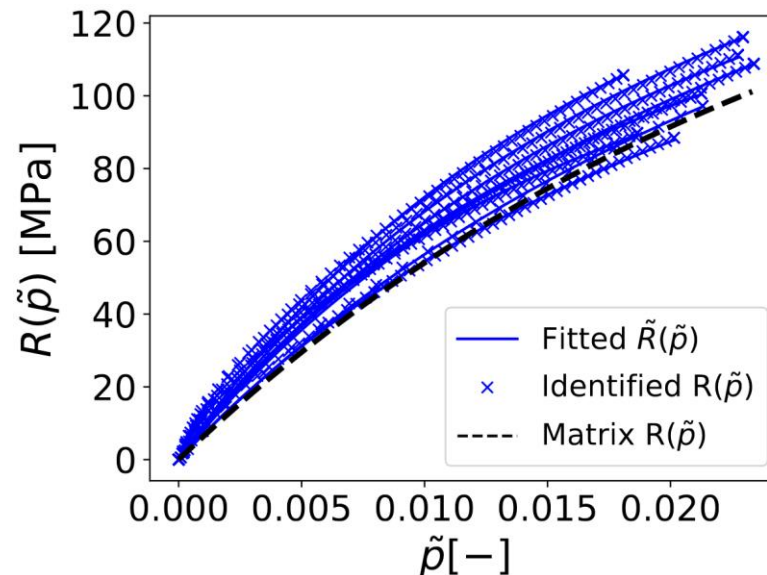


- The plastic multiplier allows then to compute the plastic strain evolution at each iteration

- With $\tilde{\sigma}_y, \tilde{m}$ and $\tilde{\alpha}$ identified, $\tilde{\rho}_{0n}$ and σ_{0n} computed:
 - R_n remains the only unknown in the yield surface definition
 - During plasticity $f = 0$. R_n can be identified by the minimization problem:

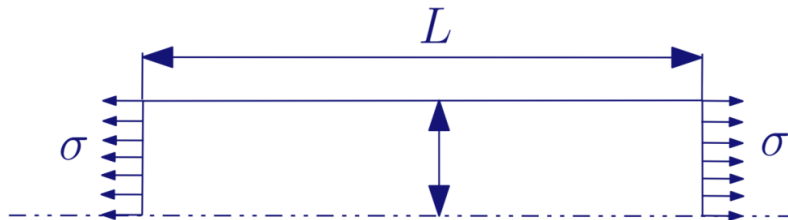
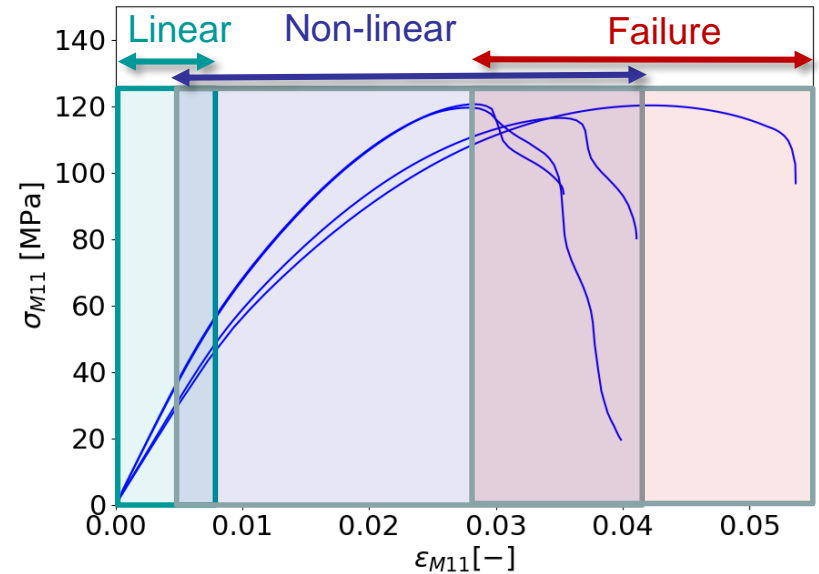
$$\min_{R_n} \{ \|f_n(R_n)\| \}$$

- A curve fitting process is used to identify the dedicated linear exponential hardening law parameters: $\tilde{h}_0, \tilde{h}_1, \tilde{m}_0$



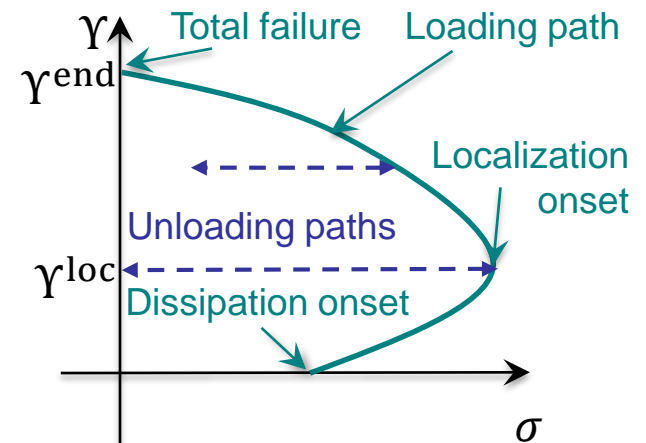
- Failure stage

- Last stage in the MFH inverse analysis
 - Failure → Loss of size objectivity
 - Use of the energy release rate G_c as quantity that allows to recover the size objectivity
- $L \gg l$; $W \ll l$ cylinder to obtain MFH non-dimensional G_c
- Optimization problem to recover SVE G_c value



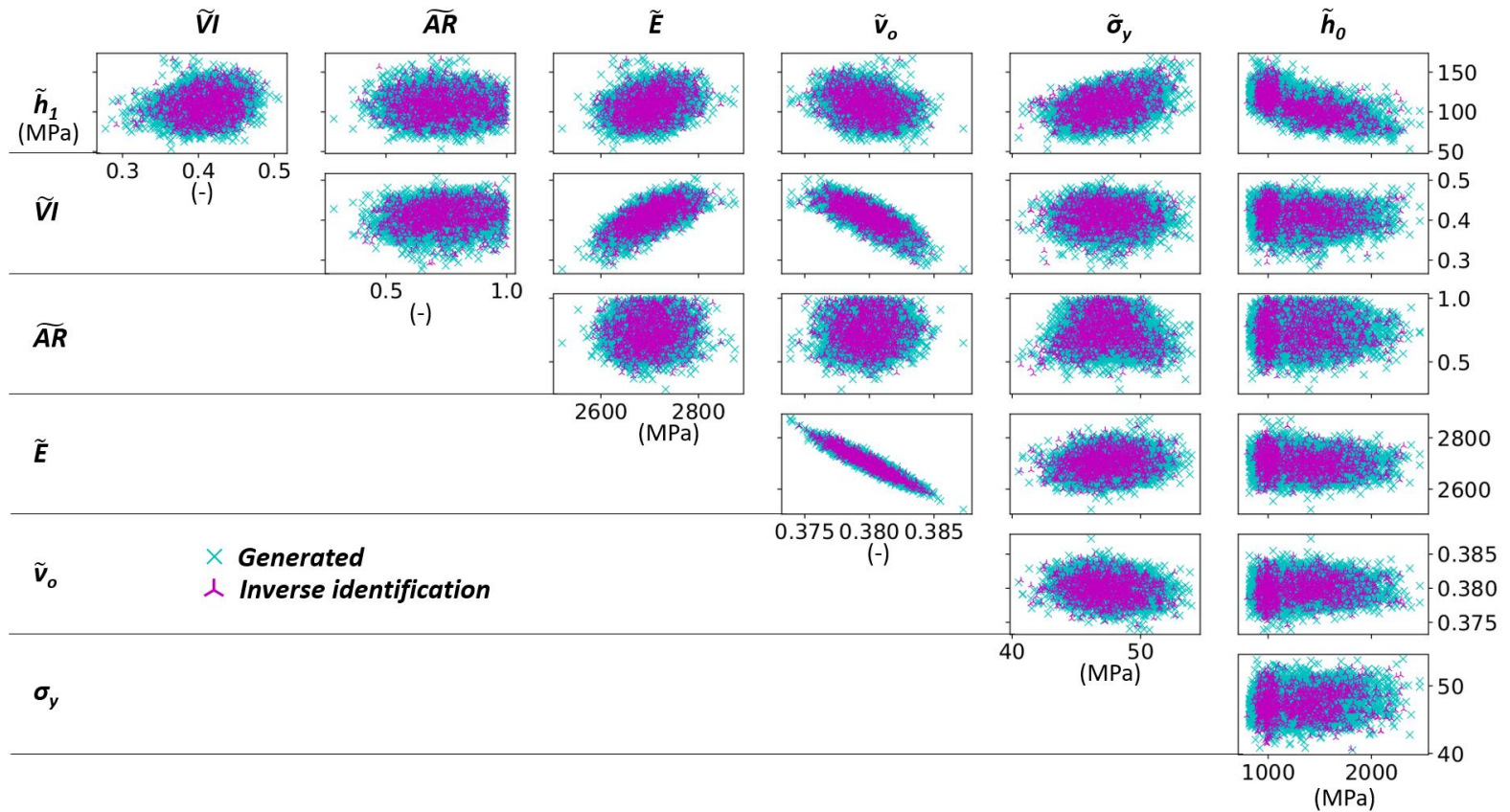
W

$$G_c = \frac{\mathcal{D}_{V_0}^{end} - \mathcal{D}_{V_0}^{loc}}{A_0}$$



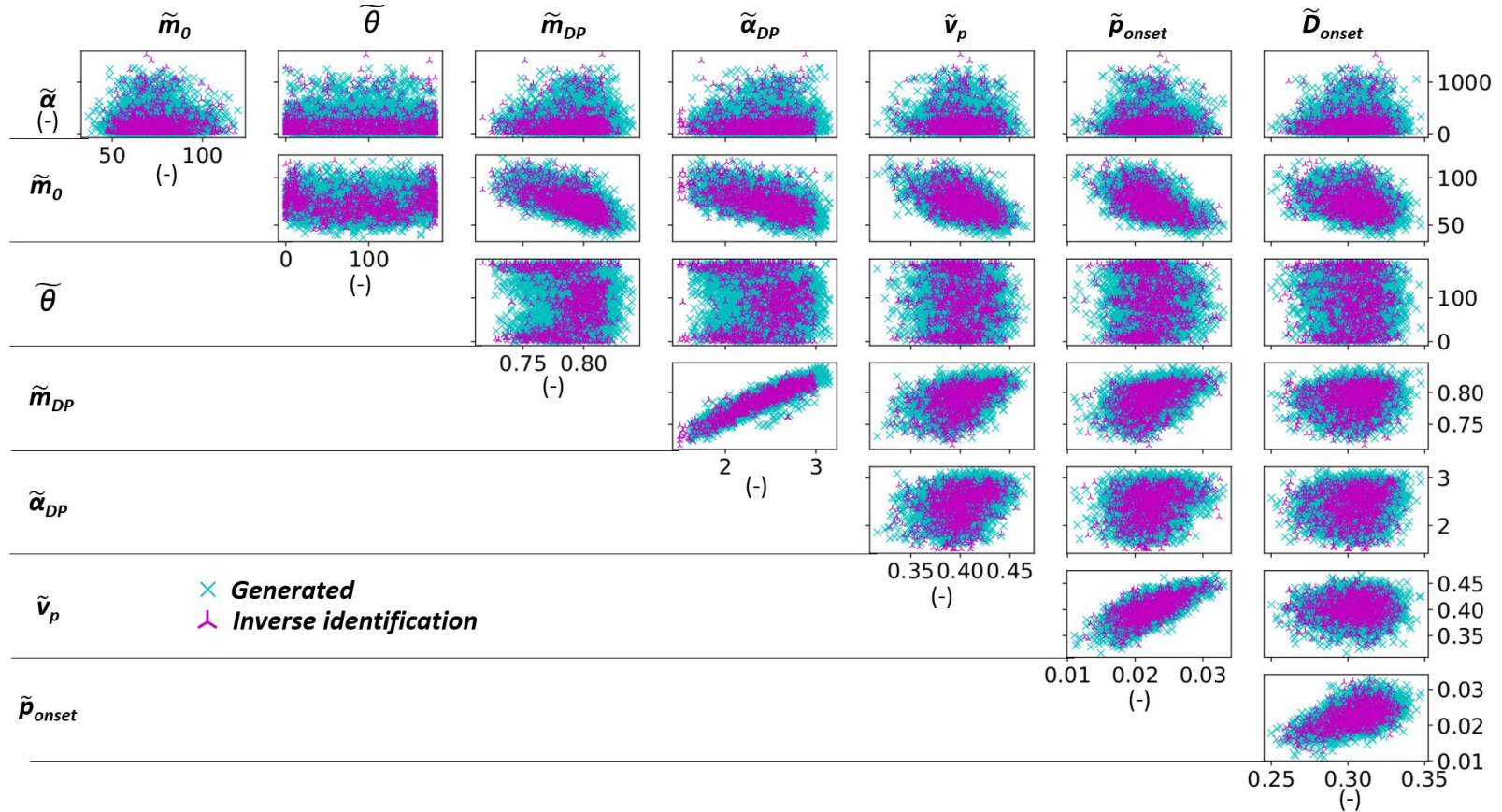
Stochastic MF-ROM

- Parameters are identified for 1050 SVE
- New data is generated using a Markov chain Monte Carlo (MCMC) process in order to obtain proper random fields



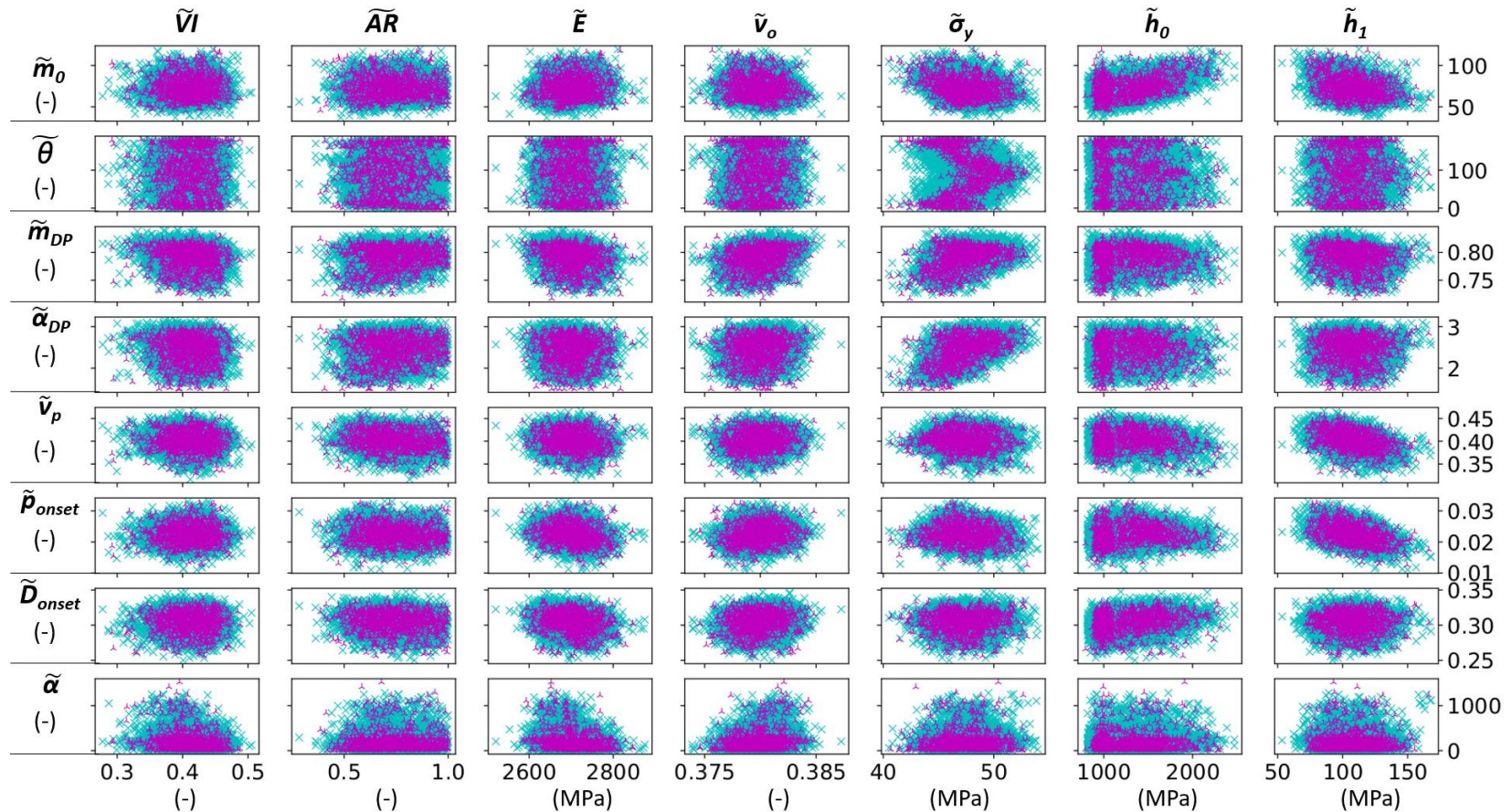
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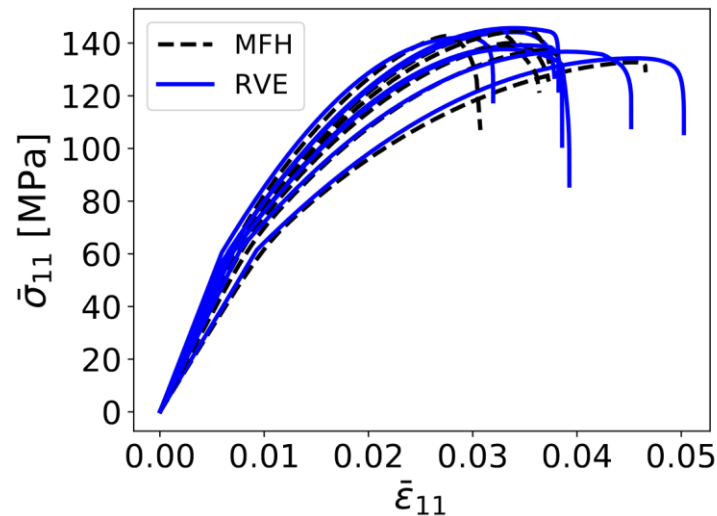
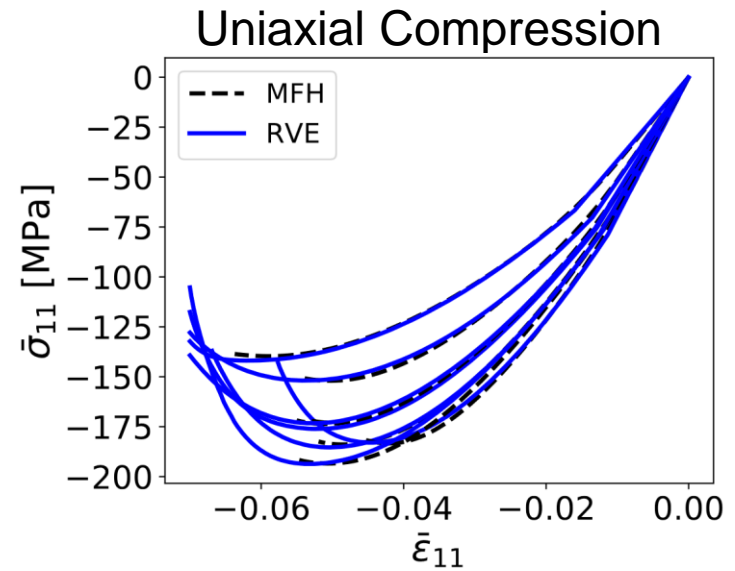
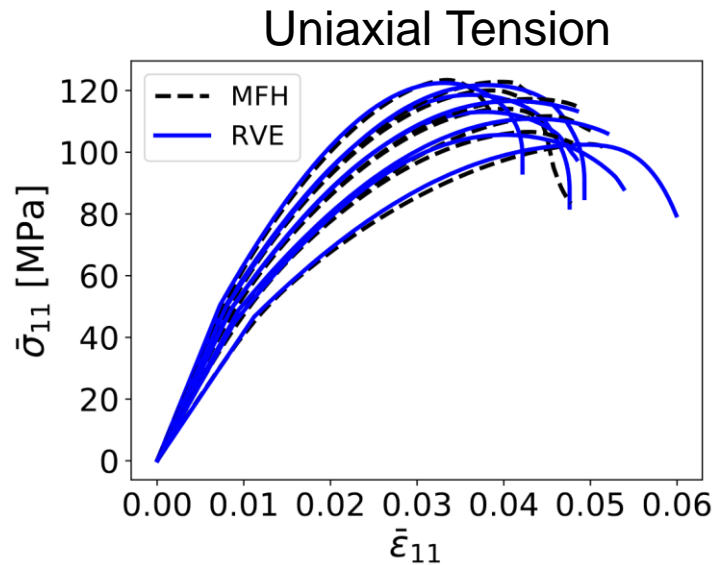


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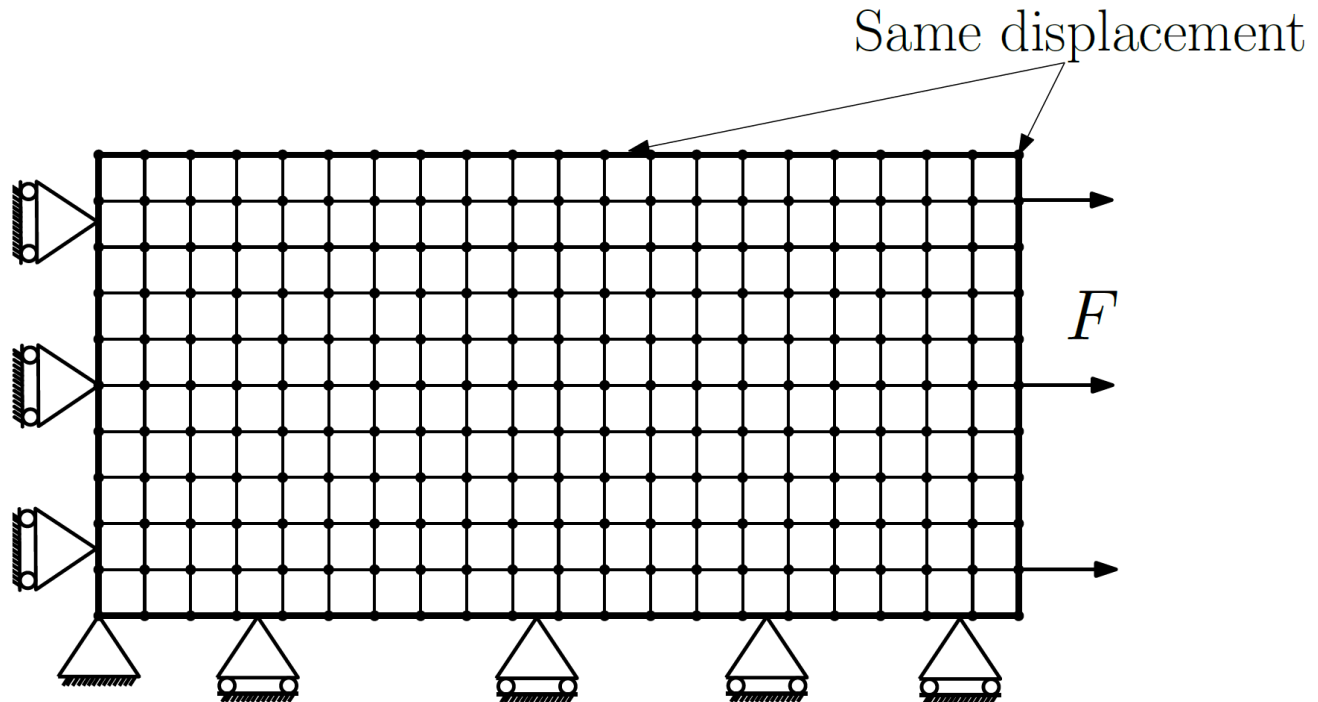
- The identified parameters yield good results for the wide range of SVE used



Biaxial

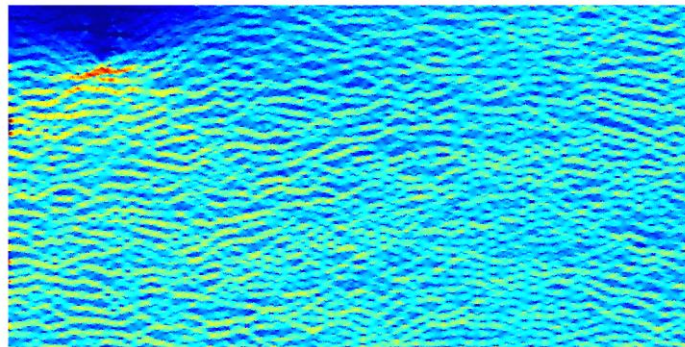
Stochastic MF-ROM Verification

- Tensile tests on $500 \times 250 \mu\text{m}$ composite ply are performed using full-field and stochastic MFH simulations

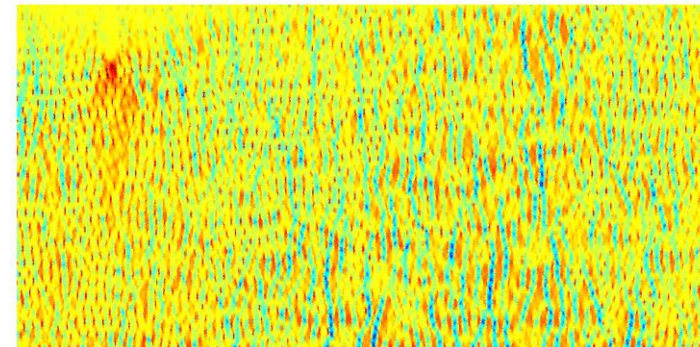


Stochastic MF-ROM Verification

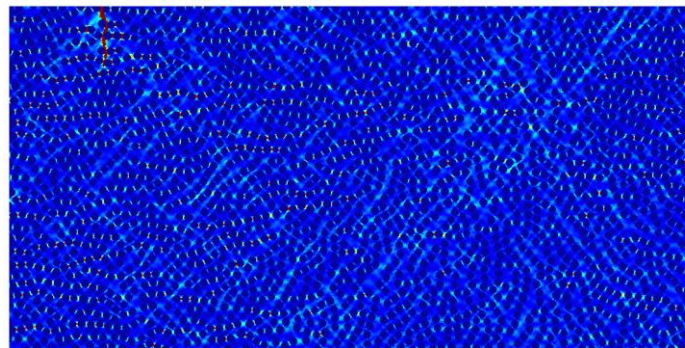
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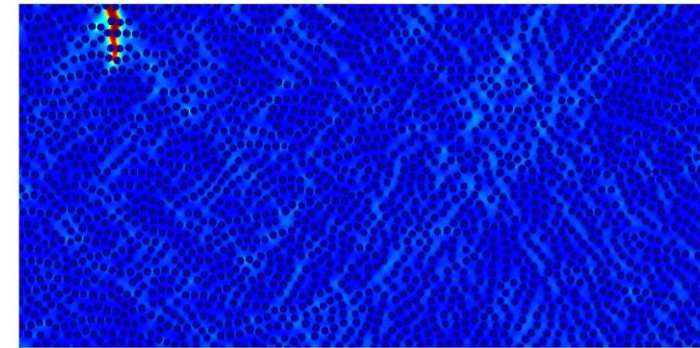
Stress 11 [MPa]
0 100 200



Stress 22 [MPa]
-150 -25 100



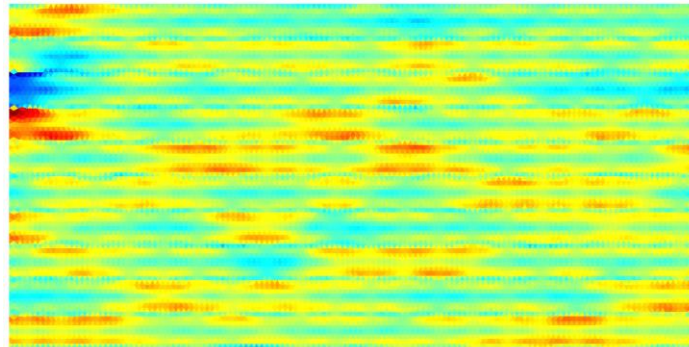
Accumulated plastic strain [-]
0 0.05 0.1



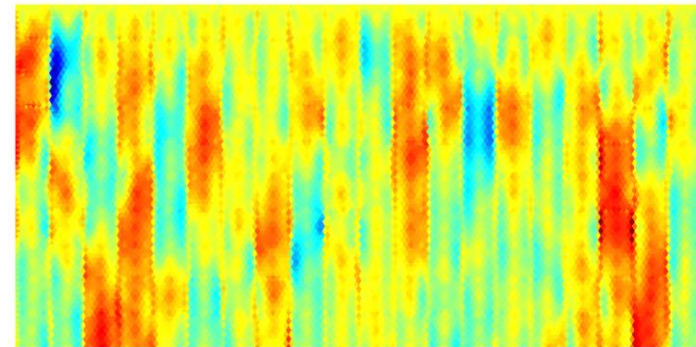
Damage [-]
0 0.5 1

Stochastic MF-ROM Verification

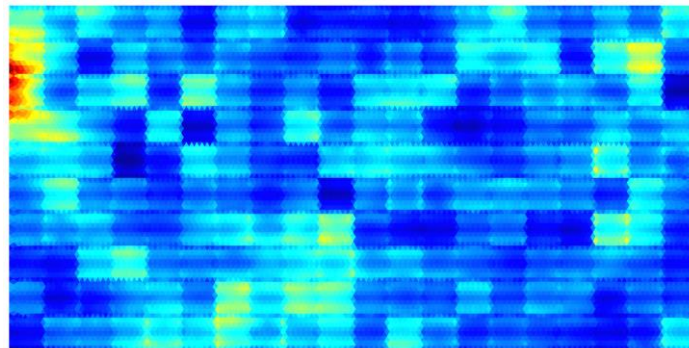
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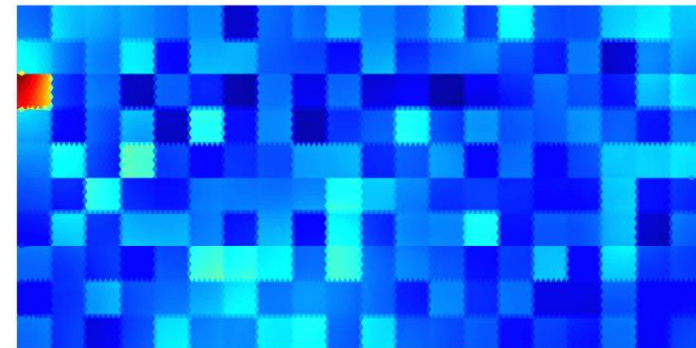
Stress 11 [MPa]
90 113 135



Stress 22 [MPa]
-15.3 -2.32 10.6



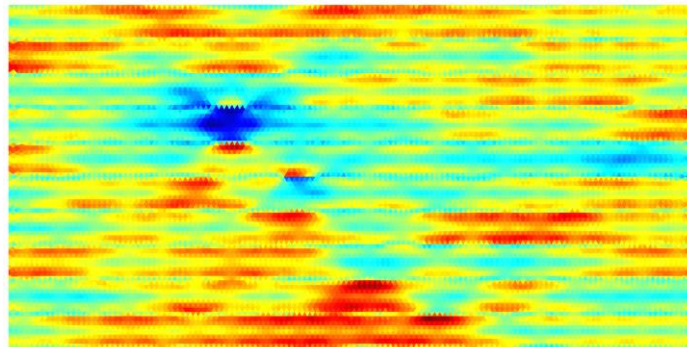
Accumulated plastic strain [-]
0.0122 0.024 0.0357



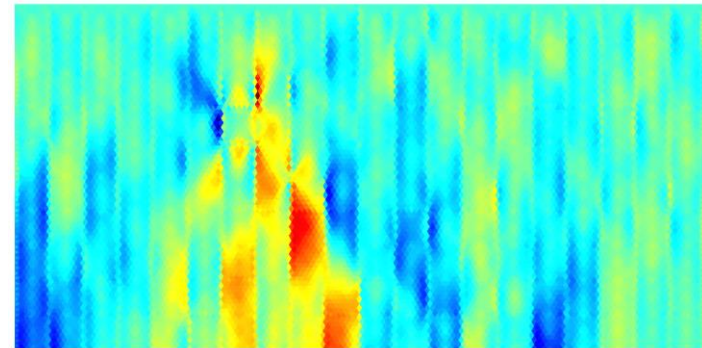
Damage [-]
0.175 0.35 0.525

Stochastic MF-ROM Verification

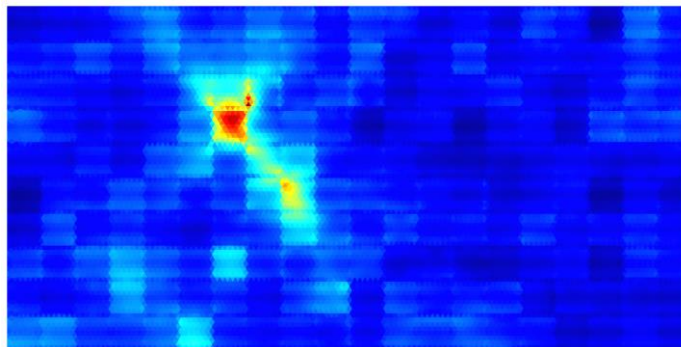
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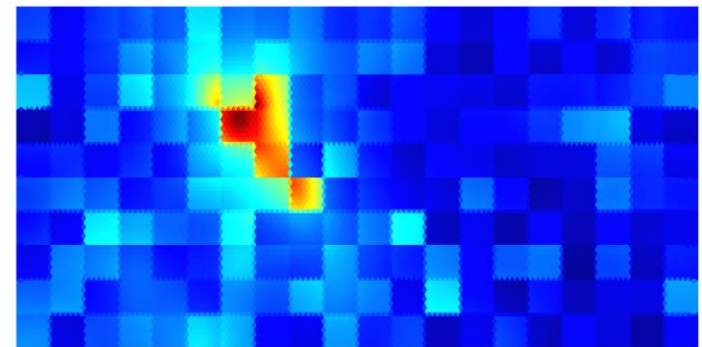
Stress 11 [MPa]
89.3 109 129



Stress 2 [MPa]
-17.7 3.07 23.8



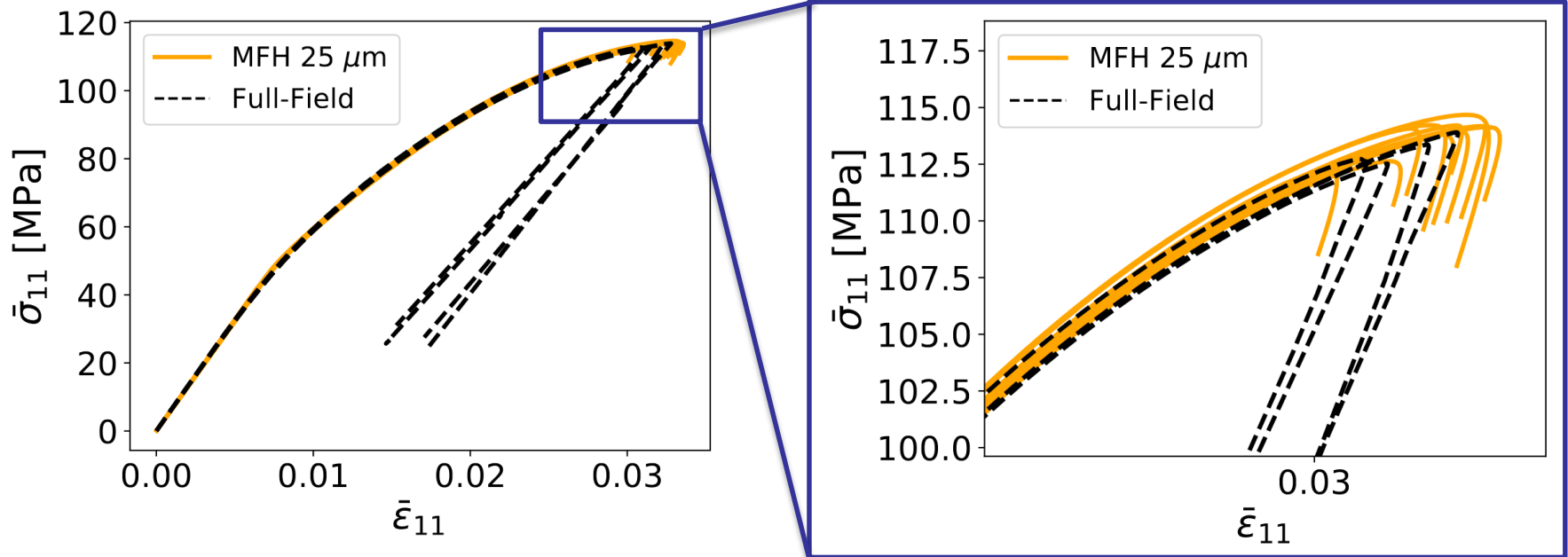
Accumulated plastic strain [-]
0.012 0.037 0.0619



Damage [-]
0.19 0.394 0.597

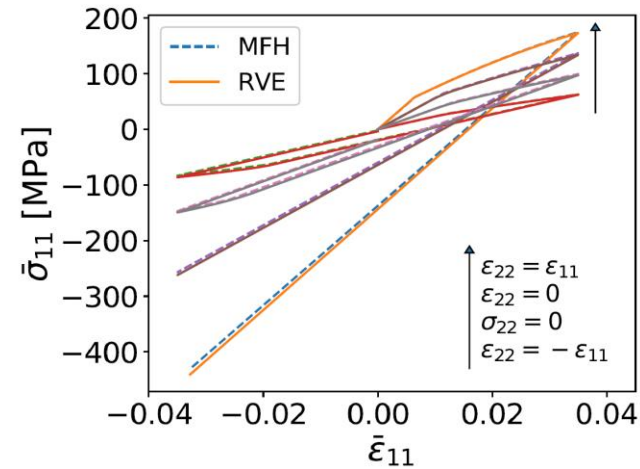
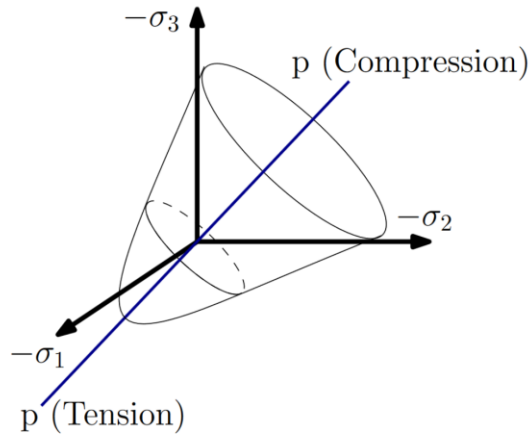
Stochastic MF-ROM Verification

- Tensile tests on 500 x 250 μm composite ply are performed using full-field and stochastic MFH simulations



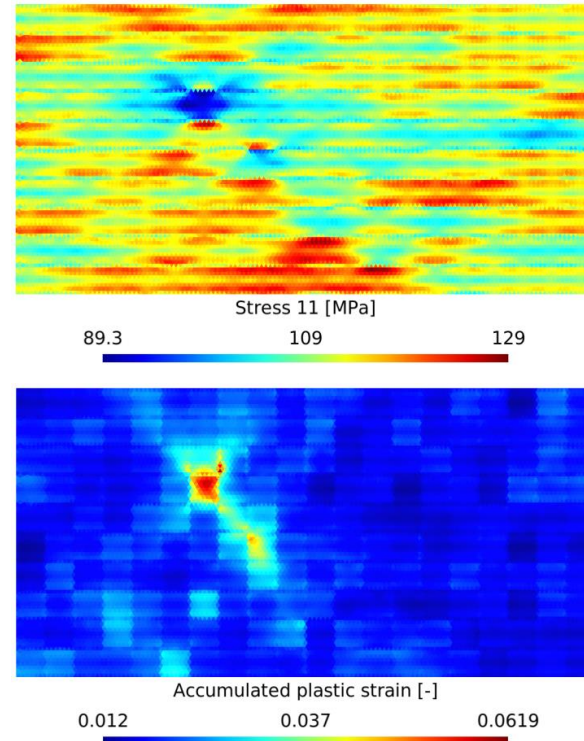
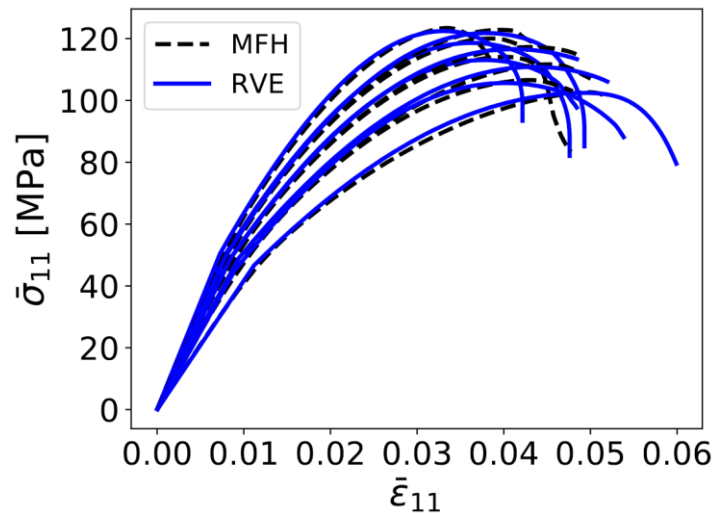
Conclusion and Future Work

- The MFH implementation of the pressure dependent plasticity model shows good agreement between the full-field simulations and the MFH simulations for all test cases



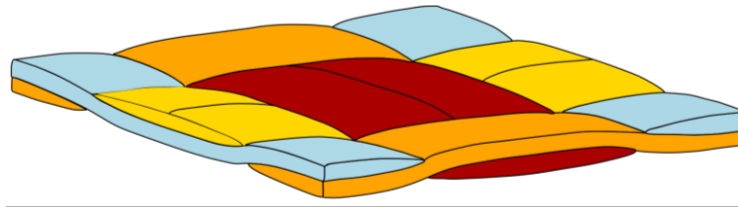
Conclusion and Future Work

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- The parameter identification process is robust and is capable of successfully represent a wide variety of SVE up to their total failure



Conclusion and Future Work

- The MFH implementation of the pressure dependent plasticity model shows good agreement between the full-field simulations and the MFH simulations for all test cases
- The parameter identification process is robust and is capable of successfully represent a wide variety of SVE up to their total failure
- The build process of the stochastic MF-ROM offers high flexibility, allowing to further enrich it in future works with more complex hardening and damage evolution laws
- This MF-ROM could be used in the modelling of multiscale woven composites



Special thanks to:



Wallonie
STOMMMAC project



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- V.-D. Nguyen, L. Wu, L. Noels, A micro-mechanical model of reinforced polymer failure with length scale effects and predictive capabilities. Validation on carbon fiber reinforced high-crosslinked RTM6 epoxy resin , *Mech. Mater.* (ISSN: 0167-6636) 133 (2019) 193-213, <https://dx.doi.org/10.1016/j.mechmat.2019.02.017>

Q & A

The image features the text "Q & A" rendered in a bold, three-dimensional, red font. The characters are thick and blocky, with a slight shadow cast beneath them on the light gray surface they rest on. The background is a plain, light gray wall that transitions to a white surface at the bottom, creating a clean, minimalist aesthetic.

- The plastic correction reads:

$$\Delta p = k\Gamma\sqrt{\mathbf{N}:\mathbf{N}} = k\Gamma\sqrt{6((\hat{\boldsymbol{\sigma}}_{n+1} - \hat{\boldsymbol{\sigma}}_n^{res})^{eq})^2 + \frac{4}{3}\beta^2(\hat{\phi}_{n+1} - \hat{\phi}_n^{res})^2}$$

$$(\hat{\boldsymbol{\sigma}}_{n+1} - \hat{\boldsymbol{\sigma}}_n^{res})^{dev} = (\hat{\boldsymbol{\sigma}}_{n+1}^{tr} - \hat{\boldsymbol{\sigma}}_n^{res})^{dev} - 2\mu^{el}\Gamma(3(\hat{\boldsymbol{\sigma}}_{n+1} - \hat{\boldsymbol{\sigma}}_n^{res})^{dev}) = \frac{(\hat{\boldsymbol{\sigma}}_{n+1}^{tr} - \hat{\boldsymbol{\sigma}}_n^{res})^{dev}}{1 + 6\mu^{el}\Gamma}$$

$$(\hat{\boldsymbol{\sigma}}_{n+1} - \hat{\boldsymbol{\sigma}}_n^{res})^{vol} = (\hat{\boldsymbol{\sigma}}_{n+1}^{tr} - \hat{\boldsymbol{\sigma}}_n^{res})^{vol} - 3\kappa^{el}\Gamma\left(\frac{2\beta}{3}(\hat{\boldsymbol{\sigma}}_{n+1} - \hat{\boldsymbol{\sigma}}_n^{res})^{vol}\right) = \frac{(\hat{\boldsymbol{\sigma}}_{n+1}^{tr} - \hat{\boldsymbol{\sigma}}_n^{res})^{vol}}{(1 + 2\kappa^{el}\Gamma\beta)}$$

$$\hat{\boldsymbol{\sigma}}^{vol} = \hat{\phi}\mathbf{I} \quad \nu_p = \frac{9 - 2\beta}{18 + 2\beta}$$

- It is then possible to define \mathbf{C}^{sr} , which writes:

$$\mathbf{C}^{sr} = \mathbf{C}^{el} - \frac{6\mu^{el}\Gamma}{1 + 6\mu^{el}\Gamma} (\mathbf{I}^{dev} : \mathbf{C}^{el}) - \frac{2\beta\kappa^{el}\Gamma}{1 + 2\kappa^{el}\Gamma\beta} (\mathbf{I}^{vol} : \mathbf{C}^{el})$$

- \mathbf{C}^{sr} is found to be isotropic, being possible to identify μ_s^r and κ_s^r as:

$$\kappa_s^r = \kappa^{el} - \frac{2\beta\kappa^{el^2}\Gamma}{1 + 2\kappa^{el}\Gamma\beta} \quad \mu_s^r = \mu^{el} - \frac{6\mu^{el^2}\Gamma}{1 + 6\mu^{el}\Gamma}$$

- It is then possible to compute the damage-enhanced residual-incremental secant operator and the final stress as:

$$\mathbf{C}^{SDr} = (1 - D_{n+1})\mathbf{C}^{Sr}$$

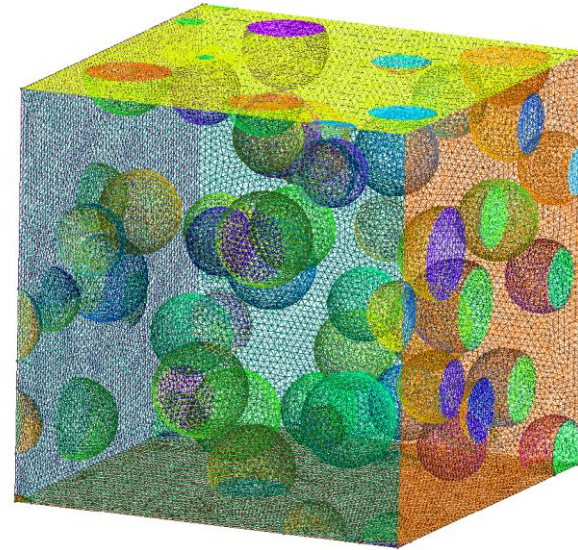
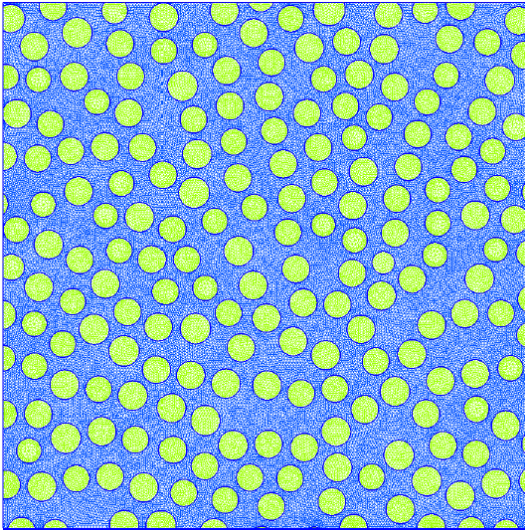
$$\boldsymbol{\sigma}_{n+1} = (1 - D_{n+1})\widehat{\boldsymbol{\sigma}}_n^{res} + \mathbf{C}^{SDr} : \Delta\boldsymbol{\varepsilon}_{n+1}^r$$

- Consequently, the damaged bulk and shear moduli read:

$$\kappa_s^{Dr} = (1 - D_{n+1}) \left(\kappa^{el} - \frac{2\beta\kappa^{el^2}\Gamma}{1 + 2\beta\kappa^{el}\Gamma} \right)$$

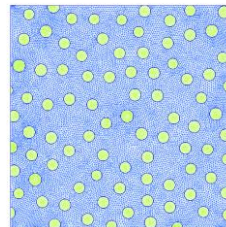
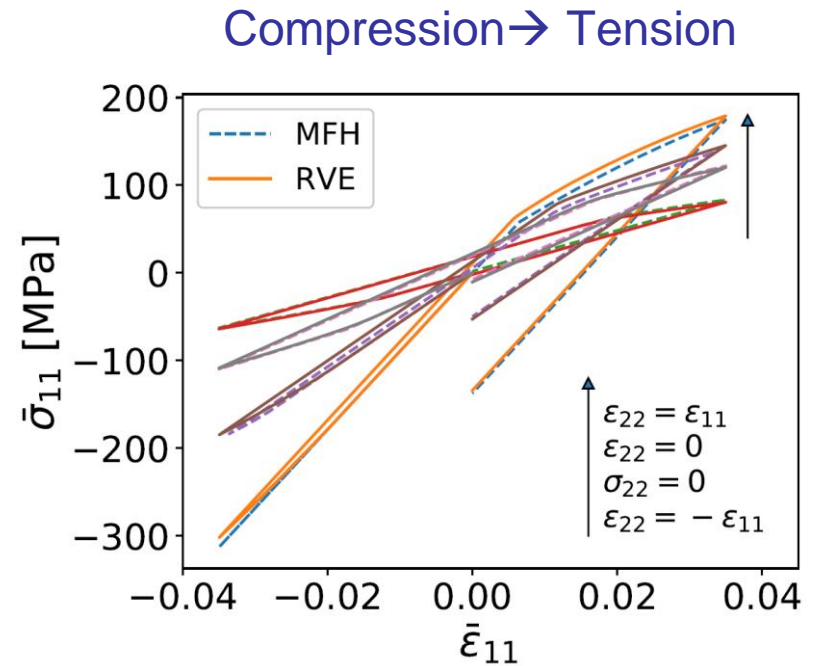
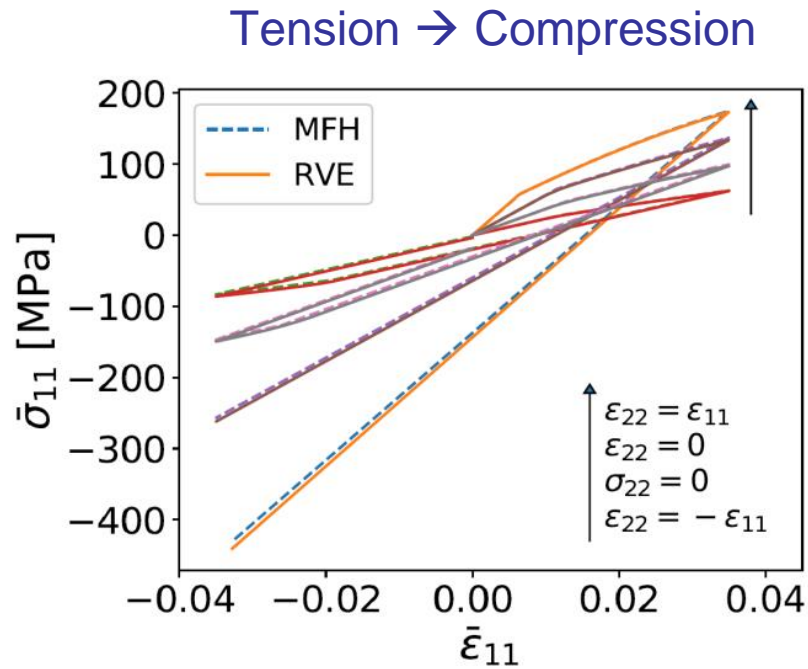
$$\mu_s^{Dr} = (1 - D_{n+1}) \left(\mu^{el} - \frac{6\mu^{el^2}\Gamma}{1 + 6\mu^{el}\Gamma} \right)$$

- The MFH scheme was tested using Uni-Directional (UD) composite and spherical inclusions-reinforced matrix using periodic BC
 - 150x150 μm UD RVE 18, 28 and 40% fiber volume fraction
 - 100x100x100 μm spherical inclusions-reinforced RVE 20% fiber volume fraction



MFH verification : No damage

- Pressure Dependency:

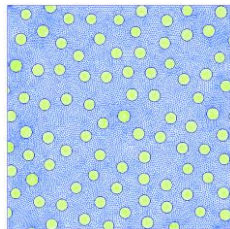
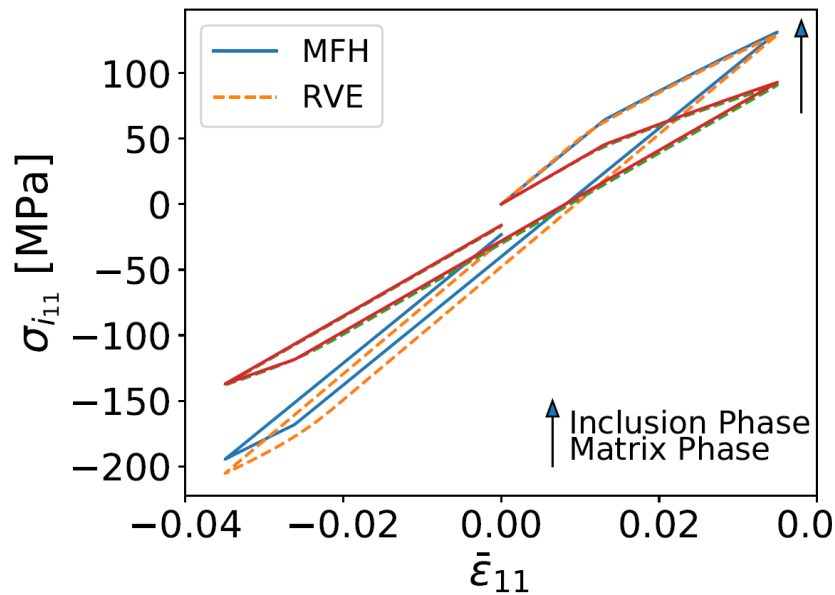


18%
Volume
Fraction

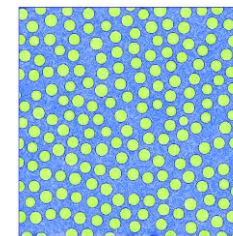
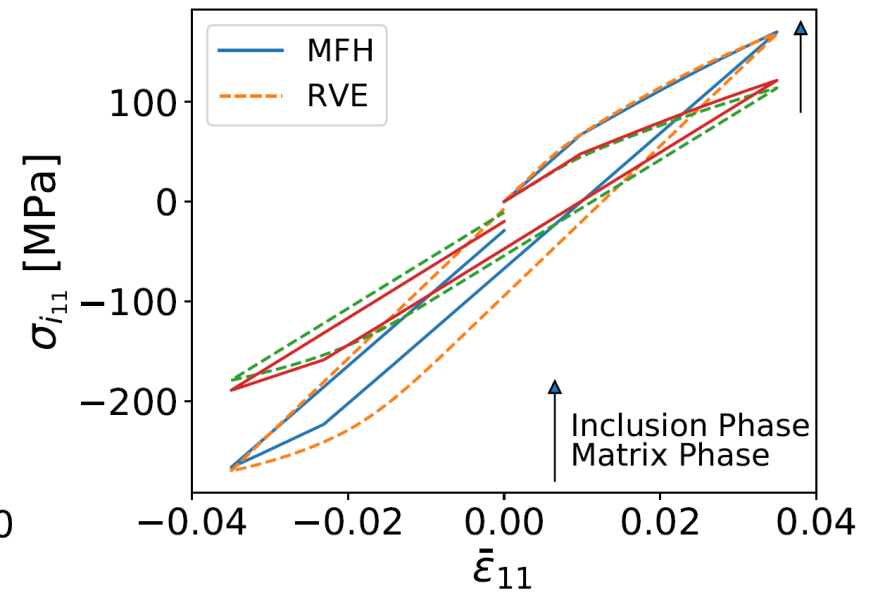
MFH verification : No damage

- Volume Fraction Dependency: Uniaxial Example

Phases

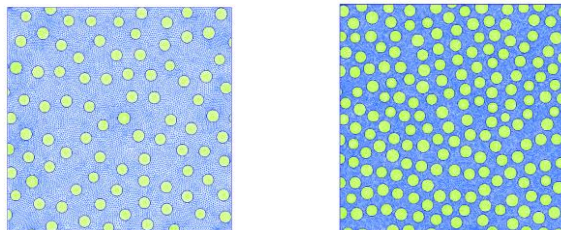
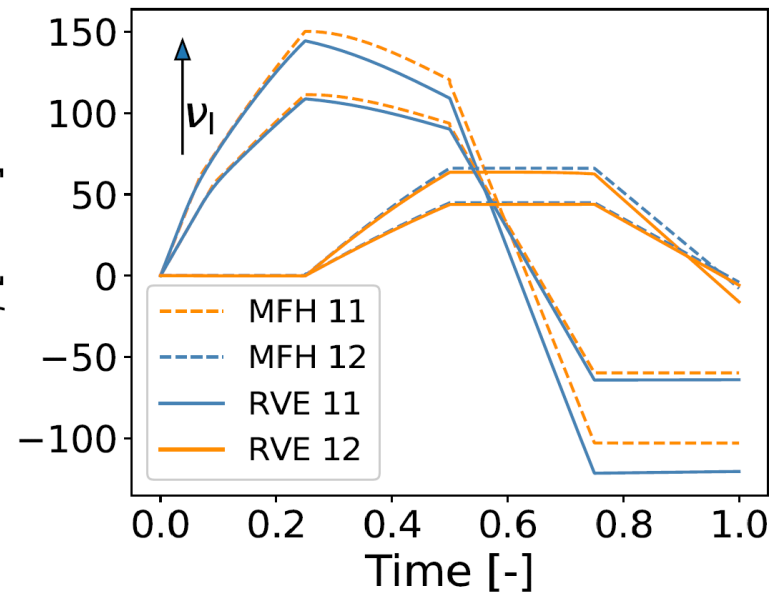
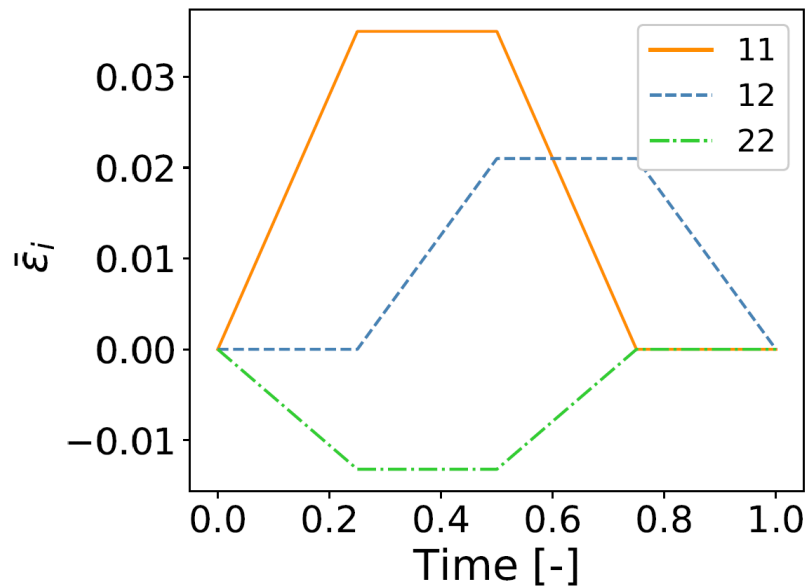


18%
Volume
Fraction



40%
Volume
Fraction

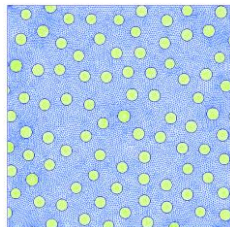
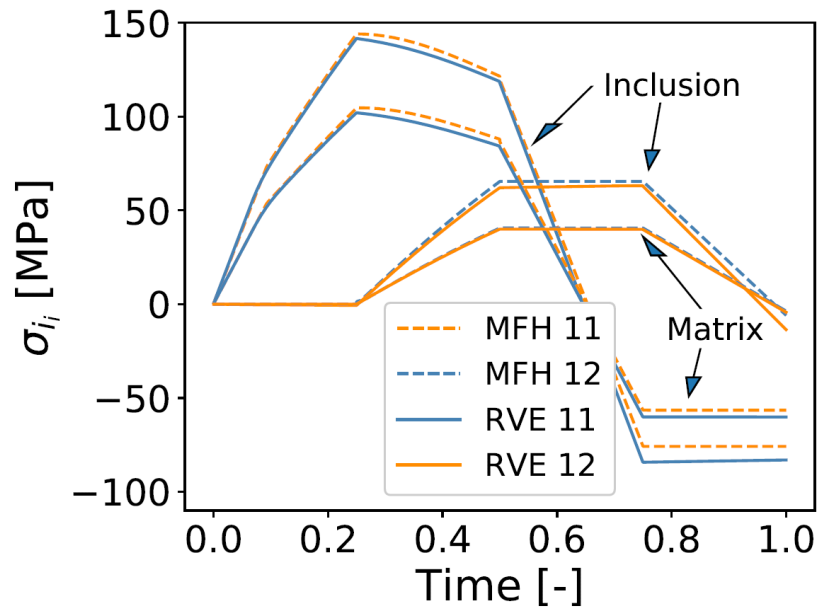
- Non-Proportional Loading



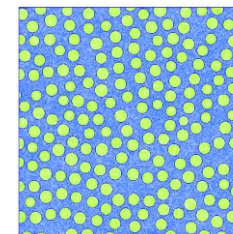
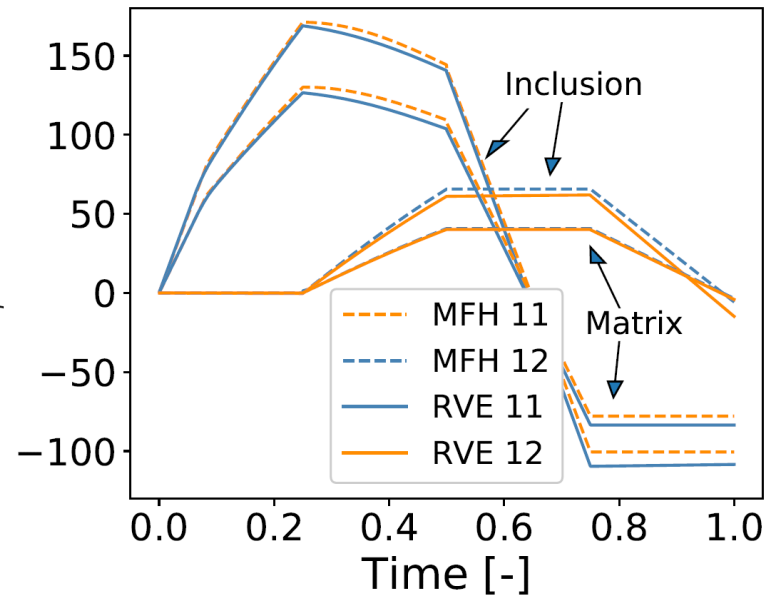
18% and 40% Volume Fractions

- Non-Proportional Loading

Phases



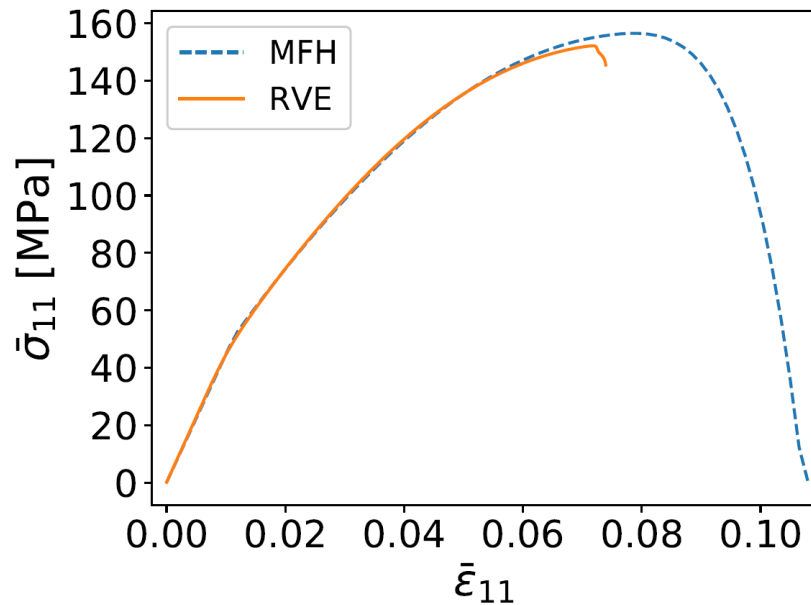
18%
Volume
Fraction



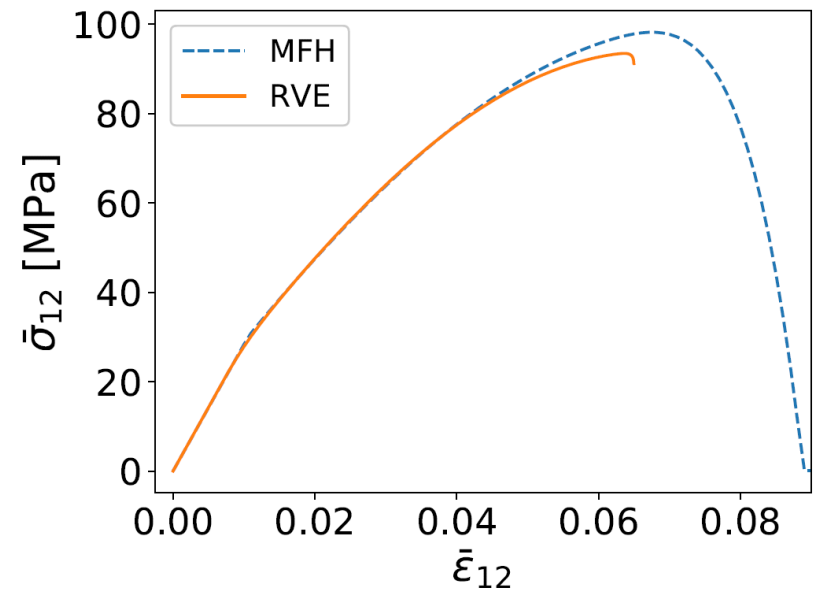
40%
Volume
Fraction

- 40% Composite

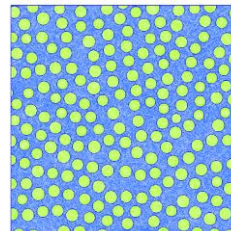
Composite



Uniaxial Tension

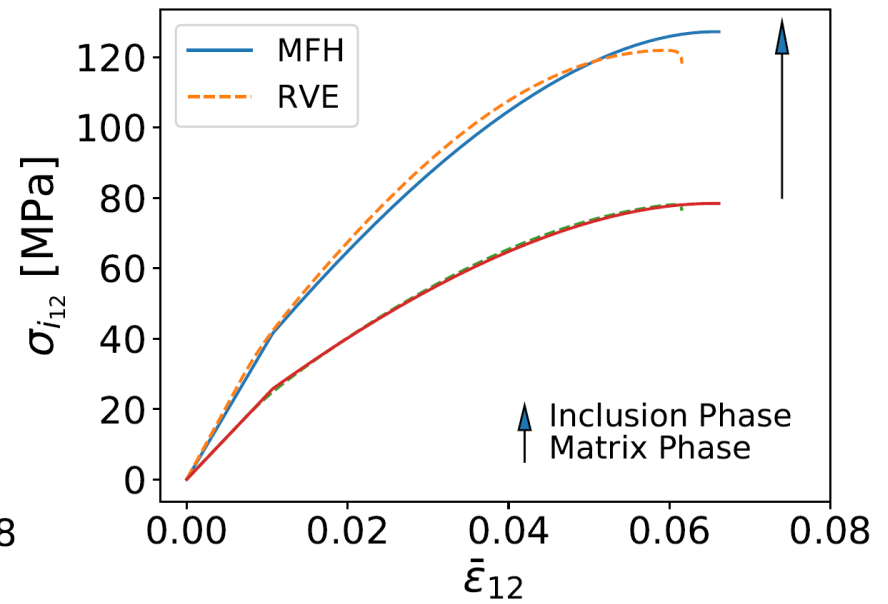
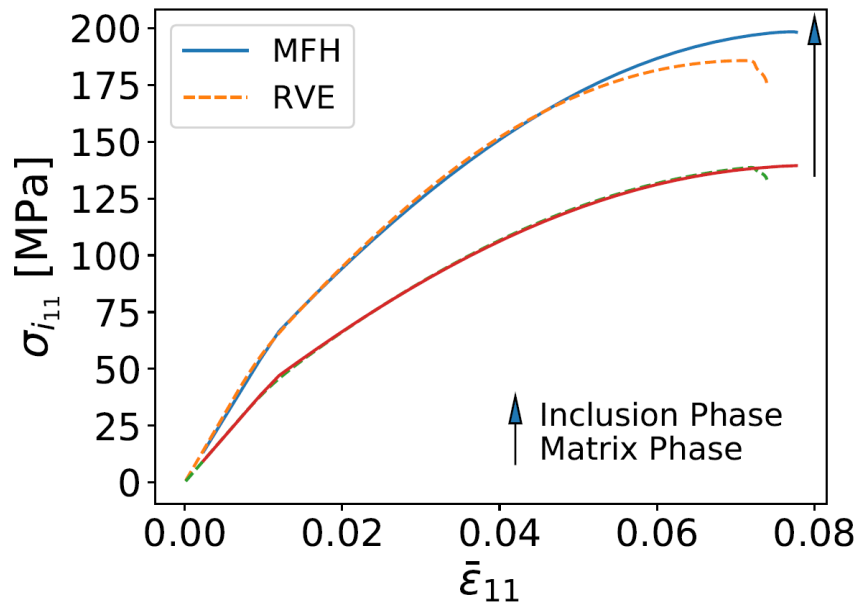


Shear Tension

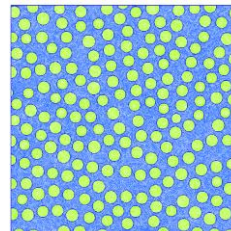


- 40% Composite

Phases



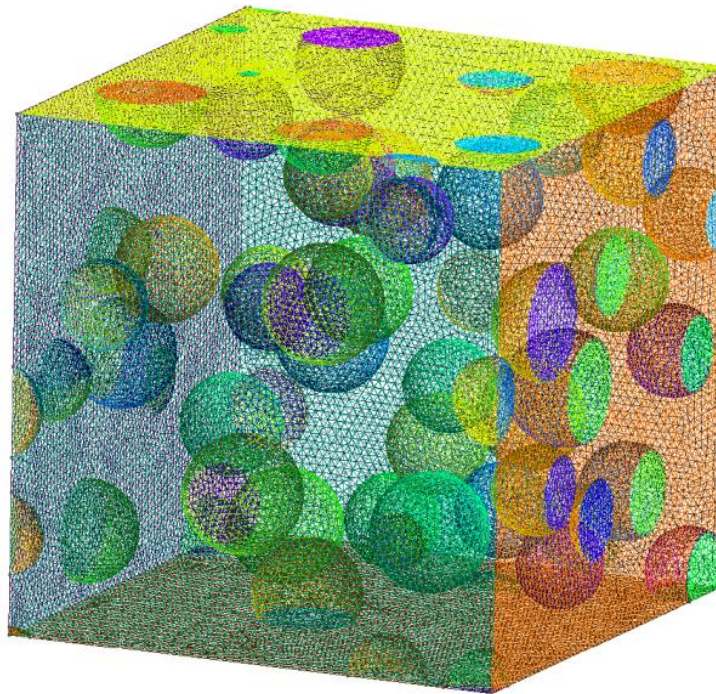
Uniaxial Tension



Shear Tension

MFH verification: 3D

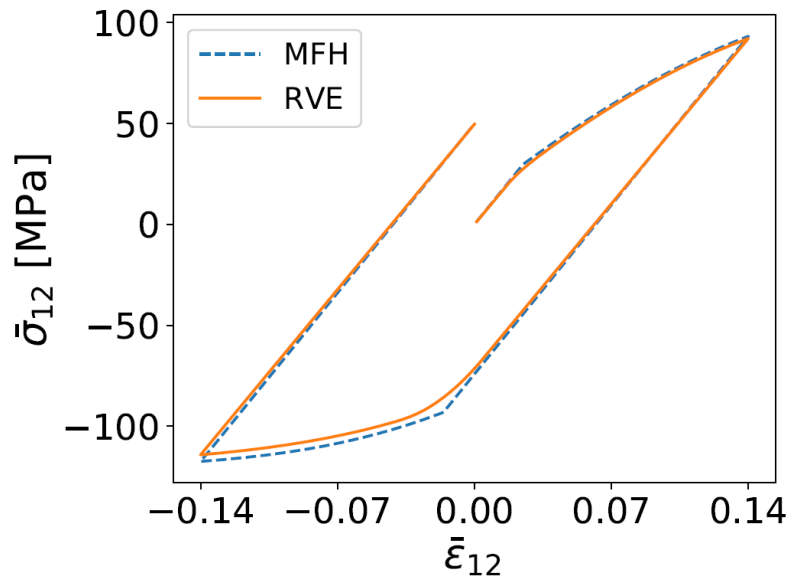
- Spherical inclusions-reinforced matrix
- 100x100x100 μm RVE 20% fiber volume fraction
- Periodic boundary conditions under different loadings including uniaxial, triaxial, non-proportional and shear



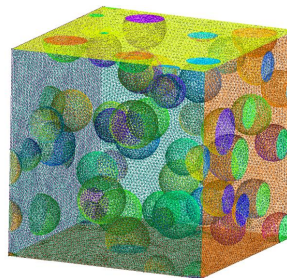
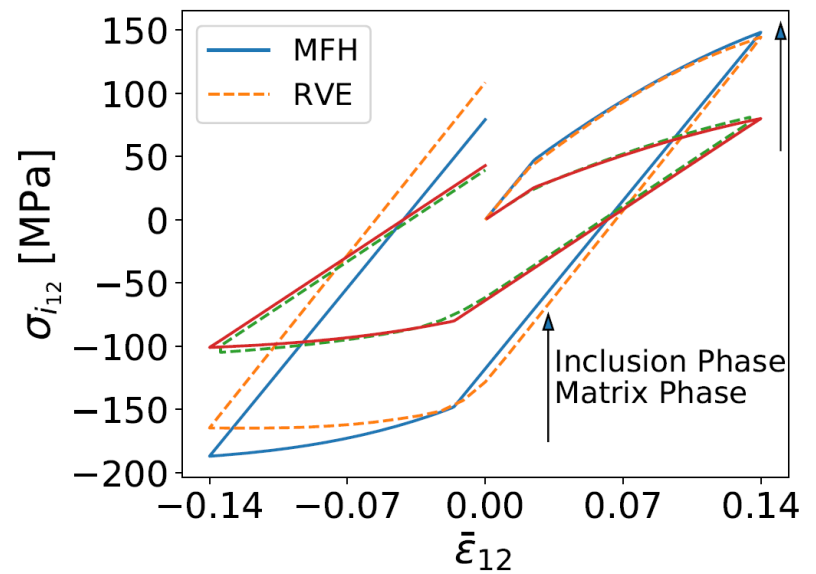
MFH verification : No damage

- Shear Loading

Composite



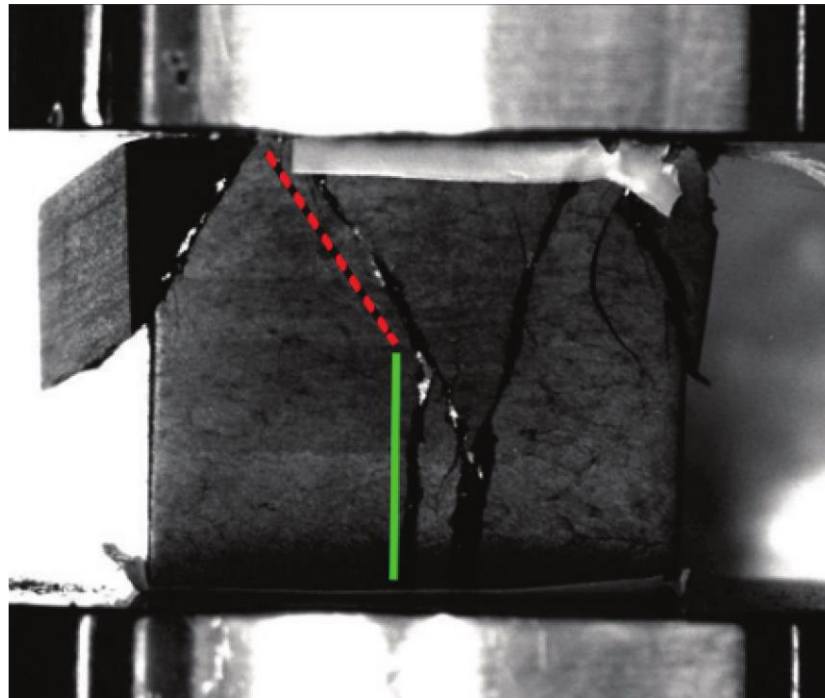
Phases



20% Volume Fraction

MFH verification: Experimental Compression Test

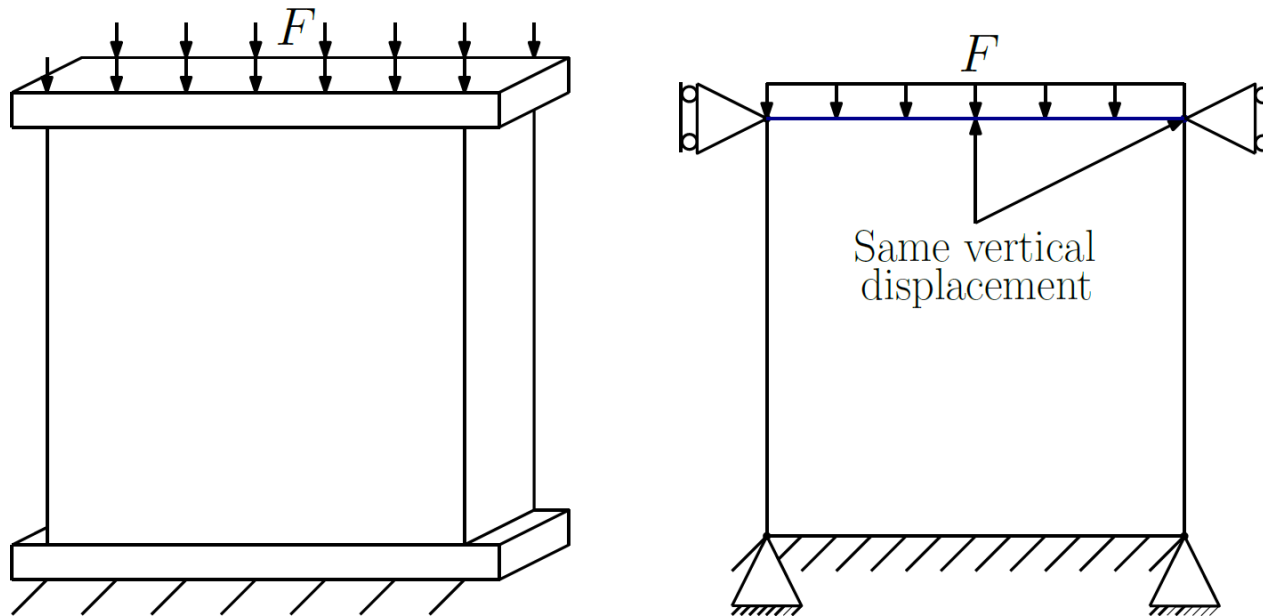
- Experimental test carried out in J. Chevalier et al. on UD composite
- 10x10x10 mm cubic sample with 40% fiber volume fraction
- Transverse compression test at a loading rate of 1^{-4}s^{-1}



Chevalier J, Camanho P, Pardoën T. Multi-scale characterization and modelling of the transverse compression response of unidirectional carbon fiber reinforced epoxy. *Comp. Struct.* 2019

MFH verification: Experimental Compression Test

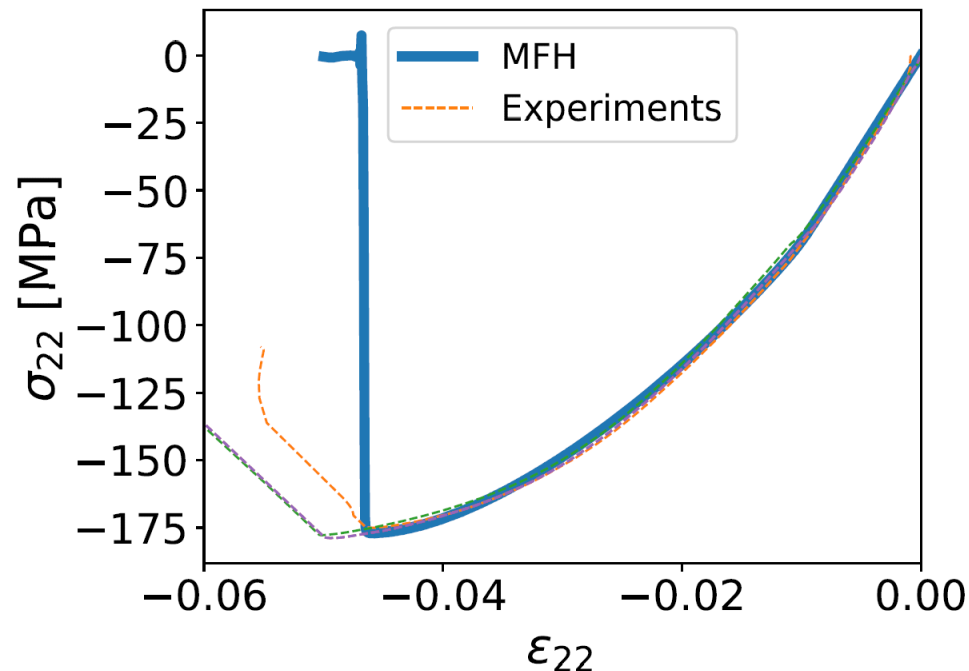
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Chevalier J, Camanho P, Pardoën T. Multi-scale characterization and modelling of the transverse compression response of unidirectional carbon fiber reinforced epoxy. *Comp. Struct.* 2019

MFH verification: Experimental Compression Test

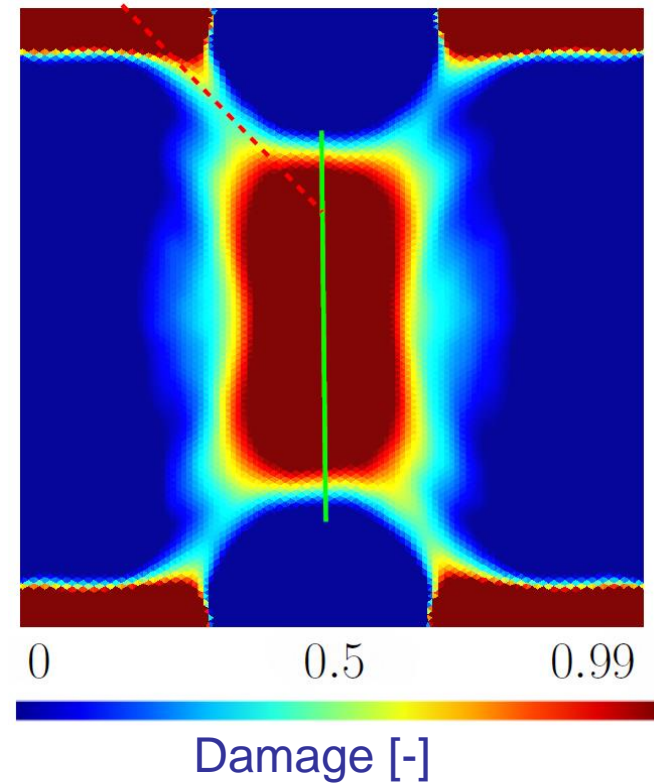
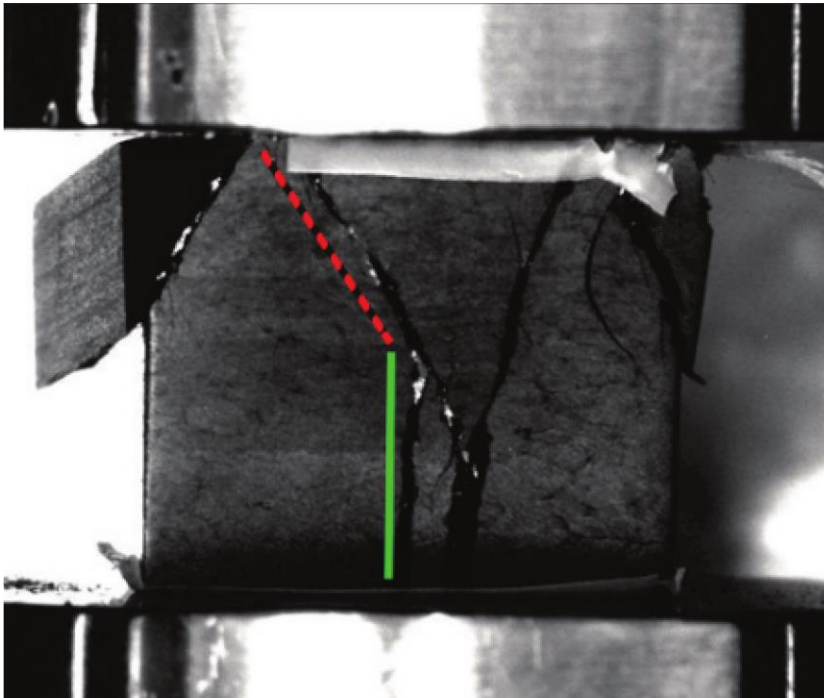
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Chevalier J, Camanho P, Pardoën T. Multi-scale characterization and modelling of the transverse compression response of unidirectional carbon fiber reinforced epoxy. *Comp. Struct.* 2019

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Chevalier J, Camanho P, Pardoën T. Multi-scale characterization and modelling of the transverse compression response of unidirectional carbon fiber reinforced epoxy. *Comp. Struct.* 2019

- Parameter identification of the two-part function damage model $(\tilde{p}_{onset}, \tilde{D}_{onset}, \tilde{\alpha}_{dam}, \tilde{\beta}_{dam})$:

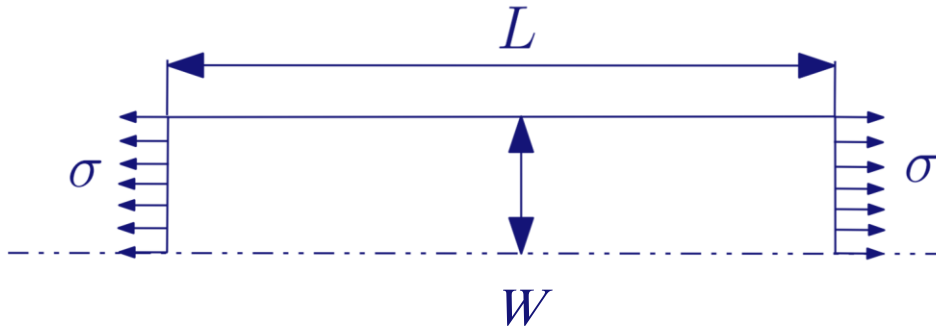
- Linear increase of damage up to the softening onset point
 - Rate of the damage evolution depends on the damage and the accumulated plastic strain at onset point $\rightarrow \tilde{p}_{onset}$, and \tilde{D}_{onset} obtained directly from the identified values at the softening onset

$$\dot{\tilde{D}}_0 = \frac{\tilde{D}_{onset}}{\tilde{p}_{onset}} \Delta\tilde{p}_0$$

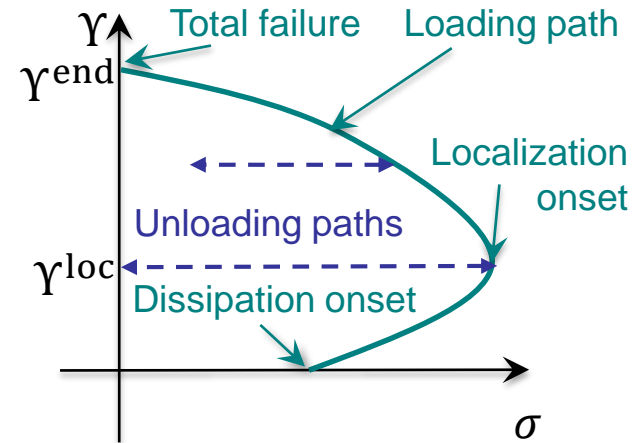
- After the softening onset, the parameters are identified to recover the SVE energy release (G_c), as it becomes the only objective value
- $\min_{\tilde{f}_{Dam}} \{ |\Delta G_{CMFH}(\tilde{f}_{Dam})| \}$ where $\Delta G_{CMFH}(\tilde{f}_{Dam}) = G_{CMFH}(\tilde{f}_{Dam}) - G_{CSVE}$
- At each minimization step, an MFH simulation is performed in order to obtain this value

Damage-Enhanced Elasto-Plastic Identification

- In order to recover the equivalent MFH SVE G_c
 - $L \gg l$; $W \ll l$ to obtain non-dimensional G_c



$$G_c = \frac{D_{V_0}^{end} - D_{V_0}^{loc}}{A_0}$$



- This optimization problem allows to identify the damage model parameters $(\tilde{\alpha}_{dam}, \tilde{\beta})$ of the new definition of the two-part function:

$$\dot{\tilde{D}}_0 = \tilde{\alpha}_{dam} (\tilde{p}_0 + \Delta\tilde{p}_0 - \tilde{p}_{onset})^{\tilde{\beta}} \Delta\tilde{p}_0$$

- Less than 1% difference between full field and homogenized $G_c \rightarrow$ Good representation of SVE behavior up to total failure