Machine Learning for wall modeling in LES of separating flows

M. Boxho[*,1,4], M. Rasquin[1], T. Toulorge[1], G. Dergham[2], G. Winckelmans[3] and K. Hillewaert[1,4]

[1] Cenaero, 29, Rue des Frères Wright, 6041 Gosselies, Belgium

[2] Safran Tech, Rue des jeunes bois, Châteaufort, 78114 Magny-les-Hameaux, France

[3] Institute of Mechanics, Materials and Civil Engineering, UCLouvain, 1348 Louvain-la-Neuve, Belgium

[4] Aerospace and Mechanics dept., Université de Liège, 4000 Liège, Belgium

margaux.boxho@cenaero.be

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Large Eddy Simulations (LES) are of increasing interest for turbomachinery design since they provide a more reliable prediction of flow physics and component behavior than standard RANS simulations. However, they remain prohibitively expensive at high Reynolds numbers or realistic geometries. The cost of resolving the near-wall region has justified the development of wall-modeled LES (wmLES), which uses a wall model to account for the effect of the energetic near-wall eddies. The classical assumptions of algebraic wall models do not hold for more complex flow patterns that frequently occur in turbomachinery passages (i.e., misalignment, separation). This work focuses on the extension of wall models to the separation phenomenon. Among possibilities to solve the complex regression problem (i.e., predicting the wall-parallel components of the shear stress from instantaneous flow data and geometrical parameters), neural networks have been selected for their universal approximation capabilities. Since DNS and LES perform well on academic and several industrial configurations, they are used to produce databases to train various neural networks. In the present work, we investigate the possibility of using neural networks to improve wall-shear stress models for flows featuring severe pressure gradients and separation. The database is composed of three building-blocks flows: (1) a flow aligned turbulent boundary layer at equilibrium; (2) a turbulent boundary layer subjected to a moderate pressure gradient; and (3) a turbulent boundary layer that separates and reattaches from a curved wall. These building blocks are referred to as a channel flow at a friction Reynolds number of 950 and the two walls (i.e., the flat upper surface and the curved lower one) of the two-dimensional periodic hill at a bulk Reynolds number of 10,595, respectively.

This work is constructed around three main questions: which input points should be considered for the data-driven wall model, how should one normalize the in- and output data to obtain a unified and consistent database, and which neural networks are considered.

Input stencil The wall model input is a mix between flow field and geometry data. For the flow fields, the velocity and the pressure gradients are considered. A key piece of information for the wall model is the wall-normal distance at which the flow fields are extracted, defined as h_{wm}. The curvature K of the wall is also added as it proved to improve the prediction of the $\boldsymbol{\tau}_w$ on the curved wall. The choice of stencil is based on an analysis of space-time correlations between instantaneous flow fields and the two components of $\boldsymbol{\tau}_w$ [Boxho et *al.*, 2021]. This work has shown that the input stencil has to be enlarged to account for up and downstream data. Therefore, the relative positions have to be encoded as inputs.

Normalization The inputs described in the previous section need to be normalized to train the neural network on a unified database. The inputs and outputs of the wall model are summarized in Table 1. Recall that h_{wm} is the wall-normal distance at which the fields are measured and fed to the model. We also define $u_{\|} = \sqrt{}(u_\xi^2 + u_z^2)$, the norm of the wall-parallel velocity. The normalization of h_{wm} is based on the near wall scaling proposed by [Duprat et *al.*, 2011] combined with the work of [Zhou et *al.*, 2021]. The near-wall scaling compatible with separation uses $y_{\nu,p} = \nu/u_{\nu,p}$ with $u_{\nu,p} = \sqrt{}(u_\nu^2 + u_p^2)$ where $u_\nu = \sqrt{}(\nu\, u_{\|}/h_{wm})$ and $u_p = |({}^{\nu}\!/\!{}_{\rho})\partial_\xi p|^{1/3}$. Regarding the velocity and the pressure gradient, the former is normalized with $u_{\nu,p}$, while the latter is an analogy to the classical Clauser parameter.

Field		*Normalized*
Velocity	$\boldsymbol{u}$	$\boldsymbol{u}^\star = \boldsymbol{u}/u_{\nu,p}$
Pressure Gradients	∇p	$(\nabla p)^\star = \left(h_{wm}/\left(\rho u_{\nu,p}^2\right)\right) \nabla p$
Length scale	h_{wm}	$h_{wm}^\star = \ln\left(h_{wm}/y_{\nu,p}\right)$
Curvature	K	
Relative pos.	$\delta\xi$	$(\delta\xi)^* = \delta\xi/h$
Wall shear stress	$\boldsymbol{\tau}_w$	$\boldsymbol{\tau}_w^\star = \boldsymbol{\tau}_w/\left(\frac{1}{2}\rho\langle u_{\nu,p}^2\rangle_{\xi,z}\rangle\right)$

Table 1: Inputs and output of the data-driven wall model.

Neural Networks Since the stencil includes multiple locations, convolutional neural networks (CNNs) are preferred over multi-layer perceptrons [Indolia et *al.*, 2018]. Indeed, the translation-invariance (i.e., convolution) is encapsulated in the obtained model, which is a desirable property for convection problems. The network hyperparameters were adjusted to obtain the optimal receptive field. We call this model. The Mean Square Error (MSE) loss is selected to train the model. It has been implemented in the code Argo-DG, developed at Cenaero. The *a posteriori* validation was conducted on the channel and the two-dimensional periodic hill. The predictions on the channel are equivalent to an algebraic law of the wall (e.g., the Reichardt law of the wall). Going to the periodic hill, the model underpredicts the recirculation bubble size. Such a problem was also observed by [D. Zhou, M.P. Whitmore, K.P. Griffin and H. J. Bae, 2022]. They state that it may be explained by a directional inconsistency of the velocity at the interface and the wall shear stress. This inconsistency, in wall models, causes the velocity to go in the incorrect direction and creates a positive feedback loop, leading to the misprediction of the bubble size. The ongoing work is to correct our wall model based on this observation and to extend the *a posteriori* validation to higher Reynolds numbers.

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