Interior Penalty Method for a Cartesian Discontinuous Galerkin Solver with Immersed Boundaries

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ABSTRACT

The discontinuous Galerkin method has received considerable attention in the research community because of its high accuracy combined with computational efficiency and conservativity. While the discontinuous Galerkin method is traditionally used on body-fitted meshes, its combination with immersed boundaries on octree Cartesian meshes is appealing for simulating fluid-related problems with a priori unknown, complex or moving boundaries. On the one hand, the method allows to achieve the high accuracy required to properly resolve solution features around complex geometries. On the other hand, the use of Cartesian meshes allows for significant computational efficiency gains at the level of the variational form evaluation on both entire but also cut elements. Among the many methods developed since the seminal work of Peskin, the cut-cell approach modifies locally the discretisation scheme to account for immersed boundaries described using a level-set function.

The elliptic terms are typically discretised by the interior penalty method, which enforces inter-element solution continuity weakly through "internal" Dirichlet conditions. The choice of the penalty parameter is critical, as coercivity of the bilinear form requires to surpass a certain minimal value, whereas a too high value significantly deteriorates the conditioning of the discretised system of equations. A key element of the coercivity analysis is a trace inverse inequality, which bounds the energy of the functions in the test space integrated on the element from below by that on its boundaries. While sharp estimates for the proportionality coefficient in this inequality have been found for traditional element/function space combinations [1, 2], no values have been reported for the combination of cut-cells of general shape and the tensor function space. A work around has been proposed by Kummer, who estimates the penalty parameter with proportionality coefficient for the trace inverse inequality defined on uncut element and applies a safety factor [3], at the expense of a probably too large penalisation.

This work will present a rigorous framework to study the trace inverse inequality for cut-cells and the tensor function space, and then apply it to a discontinous Galerkin/immersed boundary approach on Cartesian meshes. The immersed boundary approach will be compared to body-fitted computations on selected fluid dynamics problems.

REFERENCES

[1] Warburton T, Hesthaven JS, *On the constants in hp-finite element trace inverse inequalities*, Comput. Methods Appl. Mech. Eng., 192(25):2765–2773 (2003)

[2] Hillewaert K., *Development of the discontinuous Galerkin method for high-resolution, large scale CFD and acoustics in industrial geometries*, PhD thesis, Université catholique de Louvain, Louvain-la-Neuve, Belgium.

[3] F. Kummer, *Extended discontinuous Galerkin methods for two-phase flows: the spatial discretization*, Int. J. Numer. Meth. Engng, 109.2:259-289 (2017)