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GUIDELINES FOR THE DESIGN OF BRACED FRAMES WITH SEMI-RIGID CONNECTIONS.

Summary : This paper describes the mode of application of the two usual design philosophies - elastic and plastic - to the braced frames with semi-rigid connections.
The attention of the designer is particularly drawn on the influence of the second-order effects on the frame stability. Practical recommendations which allow to define the collapse load of the frame in the most convenient way are proposed. Examples illustrate the stated design principles.

1. INTRODUCTION

The cost of a building steel frame is considerably influenced by the nature of the choosen beam-to-column connections and particularly by their degree of stiffening. A substantial economy may be easily achieved by using bolted connections without stiffeners, the fabrication in workshop and the easy assembling on site of which ensure a minimum cost.

The use of this kind of connections for the design of steel frames compels however to account for their semi-rigid and partial strength character. The actual behaviour of bolted connections is indeed intermediate between the two idealized cases : the perfect hinge which transfers no bending moment and possesses an infinite rotation capacity, and the rigid connection which ensures full rotational continuity between the connected members at each bending moment level.
This so-called semi-rigid behaviour of the connections is governed by a non-linear relationship between the connection moment M and the associated relative rotation ϕ between the connected members (Fig. 1).

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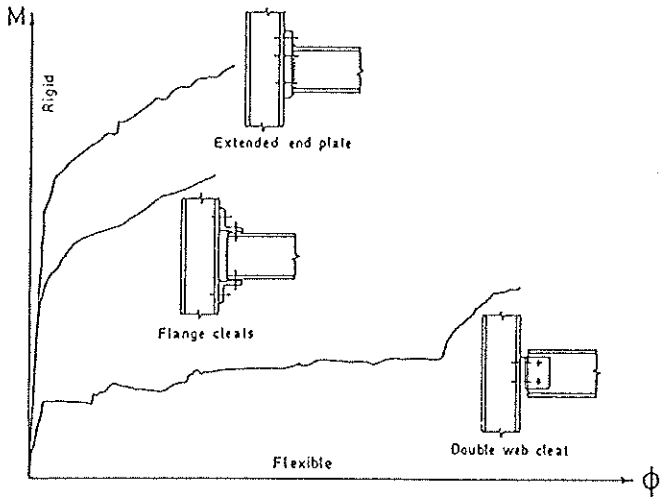


Figure 1 - M - ϕ curves for semi-rigid joints.

The recent development of numerical programs for structural analysis [1] capable of integrating all the material and geometrical non-linearities, and particularly the actual behaviour of connections, enables presently the simulation, until collapse, of the response of the frames. However this does not prevent us from the necessity of proposing to designers simple but nevertheless accurate design methods more appropriate to daily practice. Present paper devoted to the study of braced frames falls within this field.

The non-linear behaviour of connections can obviously not be taken into consideration for practical design and the associated M - ϕ curves must be schematized. The maximum bending moment M_v assumed to be carried over by the connections is represented on figure 2. This pseudo-plastic moment is physically linked up to an ultimate limit state that generally corresponds to the yielding of a connection part.
The connection constant stiffness considered in the stability calculation of frames is the secant stiffness (Fig. 2).
Hand calculation methods for the assessment of the pseudo-plastic moment and of the secant stiffness are available for usual connections [2, 3]. BIJLAARD and ZOETEMEIJER assert in [4] that this bi-linear representation of the semi-rigid and partial strength connection behaviour constitutes a safe approximation for the stability calculation of steel frames.

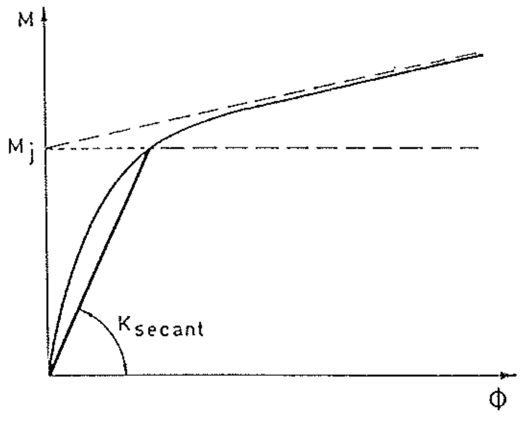


Figure 2 - Modelisation of the connection behaviour.

2. ELASTIC DESIGN OF BRACED FRAMES.

2.1. Design principles.

The elastic design of a braced frame requires a first order elastic linear analysis in order to determine the internal forces.
The extending to the analysis of frames with semi-rigid connections of classical elastic linear methods such as the slope-deflection method and the moment-distribution method has been introduced in 1942 by JOHNSTON and MOUNT [5].
The design in itself is achieved according to a "weak column-strong beam" criterion [6, 7] which consists to design beams and connections in such a way that their collapse never precedes that of the columns. The stability check of the whole frame is then reduced to the individual check of columns by means of usual interaction formulae for in plane or space loaded columns [8].

The buckling length of an isolated column, useful to its stability check, may safely be choosen equal to the column height, commonly termed "system length" [9]. As columns form however part of the frame, a more accurate estimation of their carrying capacity is obtained by considering a buckling length, termed effective length [9], smaller than the system length. This reduction results from the presence of end restraints due to the rest of the structure and particularly to the surrounding beams and connections, whose elastic behaviour until frame collapse provides restraints with a constant character.

2.2. Buckling length of linearly end-restrained columns.

The formulae for the stability check of bent and compressed columns apply to assumed isolated columns. Their application to actual columns in braced frames needs the definition of an equivalent isolated and restrained column (Fig. 3). The effect of restraints is revealed by the presence, at the column ends, of flexural springs, the rigidity of which is defined in such a way that it equals that of the rest of the structure.
The determination of the effective buckling length of actual columns will result from the study of corresponding isolated and restrained columns.
The main problem lies obviously in the evaluation of the flexural characteristic of springs.

BJORHOVDE [10] limits the influence of the structure on the studied column to the beams (and the corresponding connections) ending at the considered extremity (Fig. 4). He proposes the following expression for the stiffness of the equivalent flexural spring at each column extremity :

$$R = \frac{2EI_g}{L_g}\left[\frac{1}{1+\dfrac{2EI_g}{CL_g}}\right] \qquad (1)$$

where : E = YOUNG modulus ;
I_g = stiffness of the beam(s) ending at the considered extremity ;
L_g = length of the beam(s) ending at the considered extremity ;
C = secant stiffness of connection(s) between beam(s) and column.

This equation assumes that the beams of the substructure are bent in single curvature with equal and opposite end rotations. It may be easily modified according to the actual beam end conditions.

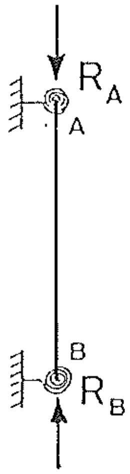


Figure 3 - Isolated column

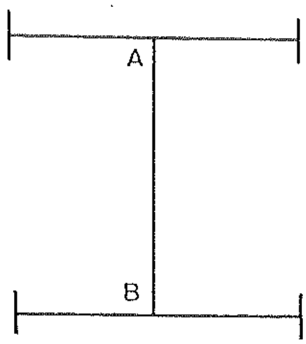


Figure 4 - Substructure

The practical assessment of the effective buckling length of isolated and linearly end-restrained columns may be achieved by means of simplified formulae resulting from a study of elastic linear stability or from the use of buckling curves for end-restrained columns.
A survey of the main existing approaches, as well as an original buckling length evaluation method for columns with different restraints at the ends are proposed in [11].

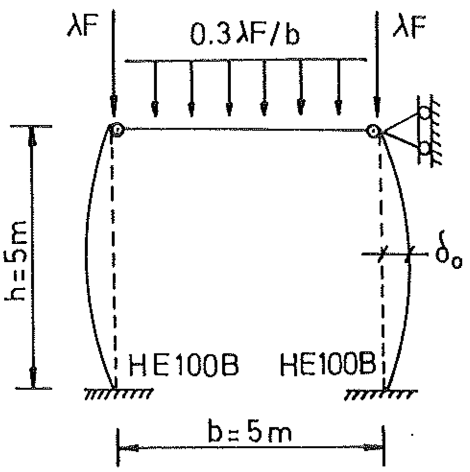
2.3. <u>Second-order effects.</u>

SNIJDER, BILAARD and STARK [9] have highlighted the possible importance of second-order effects on the behaviour of braced frames. Indeed the compression axial forces acting in the columns produce a decrease in flexural stiffness ; this so-called "ϵ effect" has an influence on the bending moment diagram and may cause the premature collapse of beams and/or connections, what results, for the columns, in a reduction of the amount of restraint at their ends and in a modification of their loading.
According to BIJLAARD and SNIJDER, the influence of these second-order effects could be neglected when :
- the beam span to column height ratio is larger than 1.0 ;
- the moment capacity of the beam is larger than that of the column.

However, studies performed in Liège [12, 13] have not allowed to confirm these conclusions.
As long as more reliable criteria are not available, it is suggested :
- to design the frame according to the principles expressed here above by referring to the first order elastic linear analysis of the whole frame ;
- then to check that the second-order moments in the frame at collapse do not exceed the plastic moment M_{pb} of the section in the beams and the pseudo plastic moment M_v in the connections. It must be noted that the second-order elastic linear analysis of a braced frame may be achieved in a simple, accurate and non iterative way by means of the modified slope-deflection method developed by VANDEPITTE [14].

2.4. <u>Examples of application.</u>

The described approach has been applied (Table 1) to the planar frame of figure 5 in two different cases (HE140B and HE100B beam) and the computed collapse load multipliers have been compared with those resulting from a numerical simulation of the frame behaviour by means of the finite element program FINELG [1] which accounts for all the material and geometrical non linearities.



. beams and columns bent about strong axis ;

. F = 100 kN ;

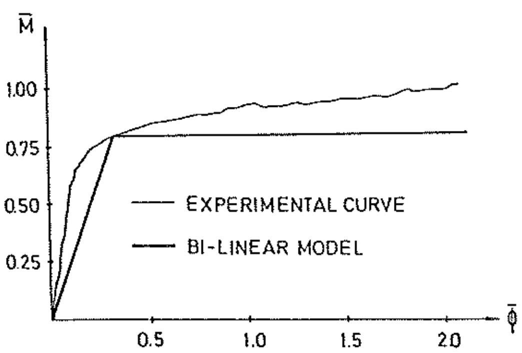
. λ = 1 corresponds to the service loads ;

. δ_o/h = 1/1000 (sinusoïdal initial déformation) ;

. elastic-perfectly plastic stress-strain diagram of steel (f_y = 235 MPa) ;

. non-dimensional characteristic curves of the beam-to-column connections (end plate): see figure 6.

Figure 5 - Frame for application



. $\bar{M} = \frac{M_c}{M_{pb}}$

with M_c = moment in the connection

M_{pb} = plastic moment of the beam

. $\bar{\phi} = \frac{\phi_c}{\phi_p}$

with ϕ_c = connection relative rotation

$\phi_p = \frac{M_{pb}}{\frac{EI_p}{5h_b}}$ where I_p = beam inertia

h_b = beam depth

Figure 6 - Connection non-dimensional M-ϕ curves

From the first order analysis of the frame,it may be stated in both cases that the buckling of the column precedes the yielding of the beams and of the connections and determines consequently the collapse of the frame.

The second-order elastic linear analysis allows however to point out the non negligible influence of the axial loads in the columns on the maximum bending moment in the beam and in the connections.
In the frame with a HE140B beam, the "ϵ effect" is not sufficient to give rise to plastic hinges in the beam at mid-span. The assessment of the collapse load multiplier based on the first order analysis may be considered as a valuable appraisal of the actual frame collapse multiplier (see table 1) despite the modification of the column loading due to the second-order effects. The situation is quite different for what regards the frame with a HE100B beam, where the premature development of a plastic

hinge in the beam during the loading sequence reduces the ultimate strength of the frame.

Type of analysis	Results	HE140B beam : M_{pb} = 57,8 kNm Connections: M_v = 46,2 kNm	HE100B beam : M_{pb} = 24,4 kNm Connections: M_v = 19.5 kNm
First order elastic linear analysis	Collapse load multiplier for beams and/or connections	4.05	2.30
	Collapse load multiplier for columns	3.02	2.29
	Frame collapse load multiplier	3.02	2.29
	Moment in the beam at mid-span(kNm)at collapse	43.1 < M_{pb}	24.3 < M_{pb}
	Moment in the connections (kNm) at collapse	13.5 < M_v	18.7 < M_v
Second-order elastic linear analysis	Moment in the beam at mid-span (kNm) at "first order" collapse	49.7 < M_{pb}	35.7 > M_{pb}
	Moment in the connections (kNm)at "first order" collapse	6.9 < M_v	7.3 < M_v
Non-linear analysis FINELG	Frame collapse load multiplier	3.1	2.06

Table 1 - Collapse load multipliers (elastic design).

The load multiplier associated to the formation of this hinge has been evaluated in [12] (λ = 1.7); it constitutes the ultimate elastic resistance of the frame and has to be normally considered as its design resistance.
A less safe estimation of the collapse load may however be calculated by dtermining the buckling resistance of the column assumed to be hinged at its upper extremity and subjected to the first order internal forces.
This approach leads to a value of the frame collapse load multiplier λ equal to 2.16; this slightly unsafe result (the actual collapse multiplier equals 2.06 - see table 1) is principally linked up to the degree of accuracy of the stability check formula. However, it must be noted that the formation of a hinge in the beam at mid-span for λ = 1.7 deletes actually the restraint at the upper column extremity in the particular frame studied (figure 7); this will be rarely encountered in practical cases so that this approach may be generally considered as a safe one.

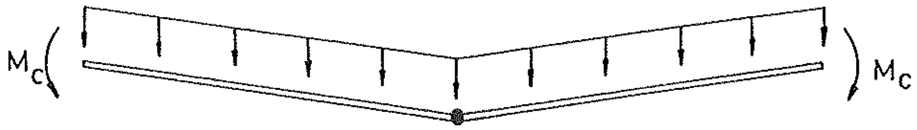


Figure 7 - Reduction to zero of the beam stiffness after the formation of a hinge at mid-span.

3. PLASTIC DESIGN OF BRACED FRAMES.

The plastic design is achieved according to a "strong column-weak beam" criterion [6, 7], in which the frame collapse is associated to the formation of beam plastic mechanisms. The check of the column is performed, in a similar way to that described here above, in the structure submitted to collapse loads, a part of which remains elastic.
The problem of the rotation capacity and of the required minimum stiffness of connections for a plastic design is dealt with, among other things, in [15].

The plastic design approach has been applied to the frame defined in figure 5 constituted in this case of HE200B columns and of a HE140B beam.
The collapse load multipliers are reported in table 2. The plastic collapse of the beam precedes that corresponding to the column instability and determines therefore the frame collapse. The column collapse multiplier has been evaluated by assuming hinge conditions at upper extremity of the columns which are each subjected there to an increasing axial load and to a concentrated constant bending moment equal to the plastic moment of the connection (M_v) or to that of the beam (M_{pb}), according to which is lesser. The agreement between the hand computed collapse multiplier and that resulting from a numerical simulation by means of the FINELG program is seen to be excellent.

Type of analysis	Results	HE140B beam: M_{pb} = 57,8 kNm Connections: M_v^p = 46,2 kNm
First order	Collapse load multiplier for the beam	5.01
plastic	Collapse load multiplier for the columns	6.18
analysis	Frame collapse load multiplier	5.01
Non-linear analysis FINELG	Frame collapse load multiplier	5.1

Table 2 - Collapse load multipliers (plastic design).

4. CONCLUDING REMARK

The supplementary calculation in Liège of a great deal of different structures through the FINELG and the described design methods will enable to define with more precision the range of validity and also the general degree of accuracy of the proposed methods.

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