## MOMENT ROTATION CURVES FOR BOLTI CONNECTIONS<sup>a</sup>

Discussion by René Maquoi3 and Jean-Pierre Jasparts

The authors have successfully presented a plain mathematical model for the nonlinear behaviour of semirigid, moment-transmitting, end-plate eave connections. Their analysis advantageously allows prediction of momentrotation curves without experimentation by using quantifying factors which have a clear physical meaning. The authors have to be congratulated for their valuable paper.

It appears, however, that this analysis is still open to improvement, especially in regards to the maximum transmittable bending moment in the connection. For instance, Eq. 10 yields a conservative value of the maximum load in the bolts around the tension flange. An investigation of several admissible yield-line mechanisms in the column flange and end-plate (22,24) would supply the designer with a more accurate estimate of this load. In the same respect, allowance could be made for the strength of the beam-to-end-plate fillet welds and for the effect of the relative axial load in the column on the carrying capacity of the column flange (25) and on the web crippling (22).

The symbol b that appears in Fig. 2 should be clearly and completely defined as the distance between the bolt center line and the outer face of the beam flange, less the fillet weld leg length l. According to this definition, Eq. 22 would take account of the stiffness provided by the fillet weld legs; unfortunately, Fig. 5 is ambiguous in this respect. The first part of Eq. 34 is quite similar to Eq. 22; however, b' is more precisely defined so that unlike b, it does not take into consideration the rigidity effects provided by the radius of fillet or, for welded columns, by the fillet weld leg. The writers believe that, for sake of consistency, both quantities b and b' should be defined similarly.

The writers would like to comment somewhat more on the section devoted to the determination of rigidities  $K_i$  and  $K_p$ , and especially on the definition of parameter C.

First of all, Eq. 24 has probably been printed incorrectly; indeed, the factor  $t_{ef}^3$  must be substituted to  $t_{ef}^3$ . More essential are the corrections to Eqs. 27, 30, 33. The formulation of bolt force B is obtained by equating the bolt elongation  $\delta_{p2}$  and the total deflection  $\delta_{p1}$  of the end-plate and of the column flange. Although both Eqs. 25 and 26 are correct for  $\delta_{p1}$  and  $\delta_{p2}$ , respectively; Eq. 27 is incorrectly derived. The latter should read as follows:

$$B = \frac{F(Z_{ep} \alpha_{ep1} + Z_{cf} \alpha_{cf1})}{4\left(Z_{ep} \alpha_{ep2} + Z_{cf} \alpha_{cf2} + \frac{k_1 + 2k_4}{2A_s}\right)}$$
 (53)

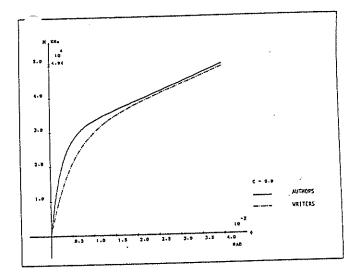


FIG. 14. Effect of Alterations on M-& Curve

The consequential effects are that Eqs. 30 and 33 become, respectively:

$$q_{s} = \frac{Z_{ep} \alpha_{ep1} + Z_{cf} \alpha_{ef1}}{\left[Z_{ep} \alpha_{ep2} + Z_{cf} \alpha_{ef2} + \frac{k_{1} + 2k_{4}}{2A_{s}}\right]}$$
(54)

$$q_{T} = \frac{Z_{ep} \alpha_{ep1} + Z_{cf} \alpha_{cf1}}{\left[ Z_{ep} \alpha_{ep2} + Z_{cf} \alpha_{cf2} + \frac{k_{2} k_{3}^{*}}{2A_{s}(k_{2} + k_{3}^{*})} \right]}$$
(55)

The repercussion of these alterations on the M- $\phi$  curve is plotted in Fig. 14 for a specified moment-transmitting, end-plate eave connection with C=0.

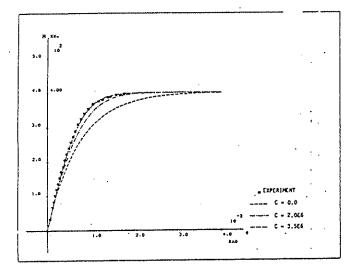
The writers' major criticism will affect the definition of the parameter C that governs the moment-relative rotation relation (7).

It stands to reason that the argument of the exponential term is dimensionless, because  $\theta$  is, itself, dimensionless;  $K_l$ ,  $K_p$ , and C have to be expressed with the same unit as the bending moment capacity  $M_p$ . Unlike  $K_l$ ,  $K_p$ , and  $M_p$  which are dimensionally correct, the values of factor C are given as dimensionless numbers in Table 3 as well as in Fig. 12. Thus, C wrongly appears to be independent of any unit system. Regarding test K7, the writers' results of mathematical and experimental modeling agree closely with the authors' results (Fig. 12), using  $C = 3,5.10^6$  kNm as in Fig. 15(a) and the ordinates of the M- $\phi$  curve quoted in kNm.

This implies that the values of C as given in Table 3 should be multiplied by a 10<sup>6</sup> correction factor and expressed in kNm. To obtain a correspond-

<sup>\*</sup>March, 1986, Vol. 112, No. 3, by Yoke Leong Yee and Robert E. Melchers (Paper 20449).

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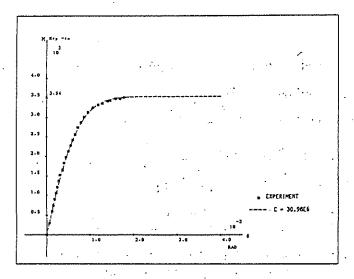


FIG. 15. Writers' Fitting of C Parameter for Specimen K7 (a) unit for M; kNm (b) unit for M: klp-in

ing agreement when the ordinates are quoted in kip-in, it would then seem necessary to convert the factor C into the imperial units, i.e.,  $C = 3.5 \cdot 10^6$  $\times$  8,85 = 30.98 106 kip-in. Calculations performed accordingly are conclusive with respect to experimental results; see Fig. 15(b).

Lastly, it must be kept in mind that the authors' model takes account of strength and shear deformability of the web of the column. Therefore, the

rotation  $\theta$  include. Ine deformation not only of the connection proper but also of the web of the column. That may be of great importance to the several possible definitions of the term "relative rotation  $\phi$ ".

In a comparison between the experimental results and the output of the authors' physically based mathematical model, one must be cautious to remain in the bounds within which the relationship must lie. For instance, the latter is invalid without alteration, when: the end-plate is flush, when more than four bolts are in tension, and when there is minor axis bending. Such extents could be the aim of future research work.

## APPENDIX. REFERENCES

25. Zoetemeijer, P. (1983). "Samenvatting van het Onderzoek op Geboute Balkkolom Verbindingen." Doc 6-85-7, Th.Delft, Stevins lab., Delft Univ. of Tech., Delft, the Netherlands, Nov. I have been a properly the section to be seen a let to be

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Superior Control of the Control of the State of the Control of the State of the Control of the State of the Control of the Con The writers wish to acknowledge Maquoi and Jaspart for the discussion and would like to comment briefly on the points raised in the discussion.

On the use of a value of 1.33 as the bolt force prying factor  $\gamma_b$ , the writers feel that while there have been several investigations into the behavior of the column flange and end-plate which have led to a better conceptual understanding of the prying force. The accurate quantification of this force still appears to be a subject that requires further study. This is evident, as mentioned by the writers in their paper, in the wide variations in value of the force reported. The value of  $\gamma_b = 1.33$  appears to be a suitable upper-bound solution.

The present study pays particular attention to bolted end-plate connections that are used for eave connections in low-rise portal frames. In such cases, the axial load effect in the column is not generally appreciable at the eave connections. However, in connections where this force is significant, notably in beam-to-column connections in the lower stories of high-rise frames, its effect should be incorporated in the determination of the parameters for Eq. 7.

The writers agree with the discussers that the presence of the fillet weld legs accounts for some proportion of the overall stiffness of the connection. The exact amount by which the value of b should be reduced because of this effect is likely to be less than the fillet leg length, because the fillet weld is unlikely to be as stiff as the flange of the beam (in the case of the end-plate region). As such, for the purpose of determining the stiffness of the connection, the writers did not take this effect into account.

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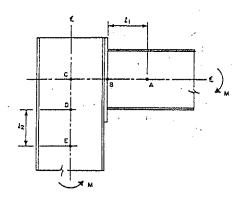


FIG. 16. Reference Points in Typical Eave Connection

The writers agree that the value of C in Eq. (7) is indeed dimension-dependent and, as correctly pointed out by the discussers, the values of C given in Table 3 are expressed in kNm  $\times$  10<sup>-6</sup>.

On the subject of the definition of connection rotation, Jones et al. (8) in a review of previous work on connection testing, pointed out the discrepancies that exist in the literature regarding the accurate representation of connection rotation. Consider the connection shown in Fig. 16. In the present study, the connection was assumed to be made up of the column web panel and the end-plate. Under the application of a moment M the elements within the connection deformed resulting in a relative rotation between point B and point D. This relative rotation was defined as the connection rotation. Because of the large local distortions, the rotations at point B and point D could not be directly measured. It was therefore necessary to measure the rotations at points sufficiently far away such that these distortions do not affect the overall readings. Point A and point E were chosen for this purpose. The distances  $l_1$  and  $l_2$  varied depending on the size of the connection. It is important to note that in the analysis of the resulting data, the flexural deformations for the length A to B and for the length D to E had to be taken into account. To minimise errors in measurement,  $l_1$  and  $l_2$  were chosen to be as short as possible without being affected by local distortion. The following relationships were adopted throughout the tests:

$$\frac{l_1 + t_{ep}}{D_b} = 0.57. ag{56}$$

$$\frac{l_2}{D_e} = 0.45$$
 (57)

The following equation relates the component rotations:

$$\theta_{\text{TOTAL}} = \theta_{\text{AE}} = \theta_{c} + \theta_{\text{AB}} + \theta_{\text{DE}}$$
 (58)

in which  $\theta_{TOTAL}$  = relative rotation between A and E;  $\theta_{AB}$  = rotation due to flexural deformation of beam stub AB;  $\theta_{DE}$  = rotation due to flexural

deformation column stub DE; and  $\theta_c$  = connection rotation. The method of taking  $\theta_{AB}$  and  $\theta_{DE}$  into account is discussed in detail in Ref. 23.

Finally the writers would also like to thank Maquoi and Jaspart for pointing out a number of corrections to the original paper which are mentioned in the accompanying Errata.

Errata. The following corrections should be made to the original paper: Page 623, Eq. 24: Should read

$$\Delta_{cf} = \frac{Pl_{cf}^{3}}{2Ew_{cf}t_{cf}^{3}} - \frac{2Bl_{cf}^{3}}{Ew_{cf}t_{cf}^{3}} \left(\frac{3\alpha_{cf}}{4} - \alpha_{cf}^{3}\right)$$

Page 623, Eq. 27: Should read

$$B = \frac{F(Z_{ep}\alpha_{ep1} + Z_{cf}\alpha_{cf1})}{4\left(Z_{ep}\alpha_{ep2} + Z_{cp}\alpha_{cf2} + \frac{k_1 + 2k_4}{2A_*}\right)}$$

Page 624, Eq. 30: Should read

$$q_{s} = \frac{Z_{ep} \alpha_{ep2} + Z_{ef} \alpha_{ef1}}{(Z_{ep} \alpha_{ep2} + Z_{ef} \alpha_{ef2} + \left(\frac{k_{1} + 2k_{4}}{2A_{s}}\right)}$$

Page 625, Eq. 33: Should read

$$q_T = \frac{Z_{ep} \alpha_{ep1} + Z_{cf} \alpha_{cf1}}{[(Z_{ep} \alpha_{ep2} + Z_{cf} \alpha_{cf2}) + \frac{k_2 k_3^*}{2A_s (k_2 + k_3^*)}]}$$

Page 629, Table 3: Values of C in column 2 should be multiplied by  $10^6$  and have units of kN-m.

## EFFECTIVENESS FACTOR FOR SHEAR IN CONCRETE BEAMS<sup>a</sup>

Discussion by Theodor Krauthammer, 4 M. ASCE

Shear strength of reinforced concrete structures has been one of the most important issues faced by structural engineers for several decades, and the research performed by the authors is another indication of this fact. The valuable information presented in this paper, obtained from an

June, 1986, Vol. 112, No. 6, by B. deV. Batchelor, H.K. George, and T.I. Campbell (Paper 20705).

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