A comparison on the magnitude and complex-valued methods to detect the brain activation, application to functional MRI

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Abstract

In functional MRI studies, the significant brain activation is determined using the magnitude-only time series framework after the image reconstruction. Although this method is very popular, the information of the phase part is ignored. Most recently, a novel strategy for determining the significant brain activation is proposed which considers the magnitude and phase parts of the data simultaneously. This approach affects the quality of reconstructed image and the power of the statistical tests. In this study, the significant brain activation achieved using the complex- and magnitude-valued approaches were evaluated.

Keywords: functional MRI, Fourier Transform, K-Space, Brain Activation, Signal Processing

1. INTRODUCTION

During the last two decades, functional magnetic resonance imaging (fMRI) has become a popular tool for understanding the brain functions. fMRI experiments are used to determine precisely which part of the brain is involved in critical functions included thinking, speech, motion, sensation, and attention [5, 8]. After designing the fMRI task, the effective proton spin densities (PSD) were imaged based on the task design. Fourier transform (FT) of the PSD measures the spatial frequency spectrums, which known as k-space (raw data). After the image acquisition, the mathematical analysis were applied on the raw data to answer the questions of neuroscientists and physicians. The main goal of the computer-based analysis is to determine those parts of the brain which respond to presented stimulus. The fMRI analysis methods are composed of several basic stages: Pre-processing, signal detection, description, and extraction of the brain connectivity. In pre-processing step, different kinds of artifacts (such as systematic noise, physiological and motion artifacts, etc.) would be eliminated. Signal detection determines which voxels are activated by the stimulation. The statistical parametric mapping is commonly used to test the difference of signal levels in different stimulus. This step includes model specification, parameter estimation, hypothesis testing and the decision-making stages. The output of this step is an activation map which indicates activated parts of the brain in response to the stimulus.
The inverse Fourier transform of the MR data contains a matrix of real and imaginary components. The size (magnitude) of the complex vector is commonly considered instead of phase and Frequency. The complex -valued method was proposed to detect the brain activation [6, 7], because magnitude-only images do not consider the phase information of the raw data. Recently, a novel method introduced to detect the brain activation using the complex reconstruction method [6,7,12].
In this study, the frameworks of the MR image reconstruction using magnitude- and complex-valued approaches were evaluated and these methodologies were compared by the illustrative examples.

2. K-SPACE BACKGROUND

In this section, the K-space and mathematical framework of FT, iFT and the complex domains are described and data storage in the K-space would be explained briefly.

2.1 FOURIER TRANSFORMATIONS

Fourier transform is a famous and powerful tool in applied mathematics problems and partial differential equations. Each periodic function could be defined using the Fourier series. Assume a periodic function $y = f(x)$ defined on $x \in [-l, l]$ with the period of $p = 2l$.
The Fourier series of this function could be considered as:

$$f(x)=\frac{1}{2}a_0+\left(a_1\cos\left(\frac{\pi}{l}x\right)+b_1\sin\left(\frac{\pi}{l}x\right)\right)+\left(a_2\cos\left(\frac{2\pi}{l}x\right)+b_2\sin\left(\frac{2\pi}{l}x\right)\right)+\dots$$
$$=\frac{1}{2}a_0+\sum_{n=1}^{\infty}\left(a_n\cos\left(\frac{n\pi}{l}x\right)+b_n\sin\left(\frac{n\pi}{l}x\right)\right)$$

In other words, $a_0,(a_1,b_1),(a_2,b_2),\dots$ coefficients must be determined such that the trigonometric series converges to $f(x)$ for every x. The Fourier series is not defined for non-periodic functions. Therefore, a generalization function, called $f^*(x)$, is defined as:

$$f^*(x)=\begin{cases} f(x) & 0<x<l \\ h(x) & -l<x<0 \end{cases}$$

Where $h(x)$ is defined on its interval. Therefore, $f^*(x)$ is a $2l$ periodic function and its Fourier series coincides with Fourier series of $f(x)$ for $[0,l]$. Numerous corresponding Fourier series could be defined. But odd and even extensions of the base function $f(x)$ are commonly used. The Fourier series could also derive for any multivariate periodic or non-periodic functions in subintervals.

2.2 COMPLEX FORM OF FOURIER SERIES

If we consider the complex form of sine and cosine functions, the Fourier series of $f(x)$ with period 2π is defined by the following equation:

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left[a_n\left(\frac{1}{2}\left(e^{inx}+e^{-inx}\right)\right)+b_n\left(\frac{1}{2}\left(e^{inx}-e^{-inx}\right)\right)\right]$$
$$=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left[\left(\frac{1}{2}(a_n-ib_n)\right)e^{inx}+\left(\frac{1}{2}(a_n+ib_n)\right)e^{inx}\right]$$

Where $c_0=\frac{a_0}{2}, d_n=\frac{1}{2}(a_n+ib_n)$ and $c_n=\frac{1}{2}(a_n-ib_n)$. $f(x)$ could be defined as: $f(x)=\sum_{n=-\infty}^{\infty}c_ne^{inx}$.

Fourier transform decomposes a signal into its constructive frequencies. Although it is not restricted to the functions of time, but to have a unified function, the domain of the original function is commonly referred as time or frequency domain [3]. The inverse form of Fourier transform could be easily defined combining all of the different constructive frequencies to achieve the original function. If ω denotes the frequency, the cosine Fourier transformation of function $f(x)$ is defined as:

$$F_c(f)=\sqrt{\frac{2}{\pi}}\int_0^{\infty}f(x)\cos(\omega x)\,dx,$$

And the inverse cosine Fourier transform would be formalized as:

$$F_c^{-1}(f)=f(x)=\sqrt{\frac{2}{\pi}}\int_0^{\infty}F_c(f)\cos(\omega x)\,d\omega.$$

The Fourier transformation of f and its inverse could be defined in a similar way:

$$F_s(f)=\sqrt{\frac{2}{\pi}}\int_0^{\infty}f(x)\sin(tx)\,dx,$$

$$F_s^{-1}(f)=f(x)=\sqrt{\frac{2}{\pi}}\int_0^{\infty}F_c(f)\sin(\omega x)\,d\omega.$$

2.3 DATA ACQUISITION AND K-SPACE

Magnetic resonance imaging is an important medical diagnostic tool to observe the internal body organs and prognosis of the diseases. In addition to perform different image contrasts, MRI technique does not use ionizing radiation and has no known side effects. The structural MRI images are not suitable for understanding the complex brain behavior. In functional MR imaging (fMRI), the brain function is registered at high temporal resolutions [2]. In this section the procedure of data acquisition is briefly described.

Received signal registered as a function of frequencies. For a transverse slice, the horizontal and vertical axis are set as the frequency and phase encoding directions, respectively [2,5]. The position in K-space is related to the applied gradient across the body and it contains information of the complete scanning area. The data near to the center of K-space correspond to the contrast information of the image while edges and details of the image are related to the data in distance from the center of K-space. Figure 1 shows a schematic K-space. As illustrated, it is almost symmetric from left to right and vice versa. Therefore for some specific purposes, in order to reduce the scanning time, only slightly more than 50% of K-space could be filled and the missing lines would be filled with that of already acquired data.

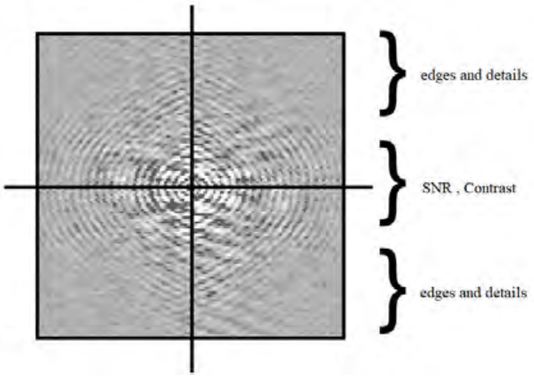

Figure 1. An illustration of K-space. Points near to the center of the K-space give information about the signal to noise ratio (SNR) and contrast of the image. In other word, points near to the center of the K-space are related to low spatial frequencies. While points in the distance from the center correspond to high spatial frequencies which contain information about the detail of the image.

The K-space is a generalization of Fourier domain. In mathematical viewpoint, it is the linear combination of the weighted sine and cosine waves in different phases, frequencies, orientations, and amplitudes (received RF signals). Inverse FT is applied to reconstruct the subject information in the spatial domain. Figure 2 demonstrates the result of the iFT for a sample MR image.

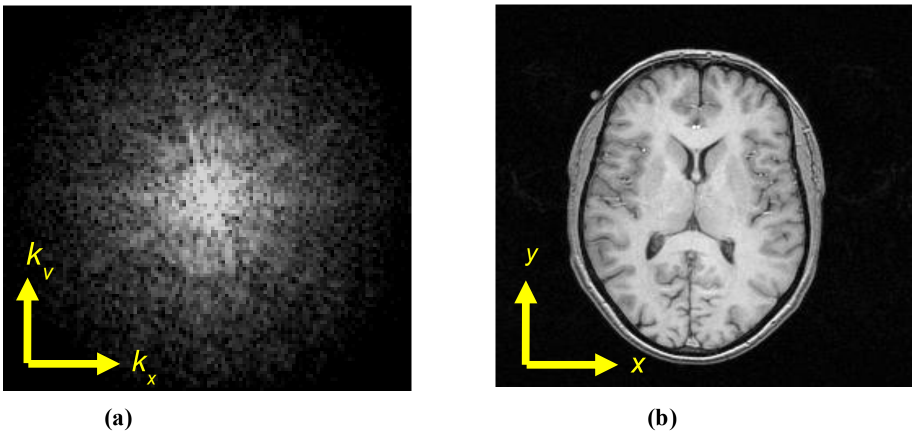

Figure2. A reconstructed MR image and its corresponding K-space. (a) An illustration of 2D K-space obtained from the scanning procedure. (b) 2D reconstructed brain image, which obtained using the inverse Fourier transform.

3. MAGNITUDE- VS. COMPLEX- VALUED APPROACHES

In this section, the magnitude- and complex- valued approaches are described. The FT and its inverse are complex-valued, so they result in complex-valued arrays. In a magnitude-valued approach, the magnitude of the measured data is considered for statistical analysis. The magnitude-valued approach is the common methodology for statistical tests. In this method, the phase information of the signal is discarded which affects the final reconstructed image [10,12]. Therefore, the complex – valued method was proposed. In this approach, the phase and magnitude of the signal are used to detect the brain activation and do the statistical analysis.
Three steps are accomplished to detect the brain activation [2,14]:

1. A measure of Association is computed based on an ideal known signal, task design and some physiological considerations.
2. An activation statistic is determined from the computed time series and a threshold would be assigned based on comparing the association measure with the measure of association resulted for the time series of the random noise.
3. Some color values are chosen for different activations and each voxel would be colored with the corresponding color.

For a one-dimensional complex-valued data with n complex-valued voxels, all n spatial frequencies corresponding to the center (base line) must be measured to achieve the whole image. $S_c = \left(s_{c1}, \dots s_{cn}\right)^T$ is considered as the transposed vector form of the measured complex-valued spatial frequencies, where the real parts of these complex values are stacked on top of their imaginary parts. It is the results of $s_c + \epsilon_c$, the noiseless complex-valued frequencies and the measured errors respectively:

$$\begin{aligned} S_c &= s_c + \epsilon_c \\ &= \left(s_R + is_I\right) + \left(\epsilon_R + i\epsilon_I\right) \\ &= \left(s_R + \epsilon_R\right) + i\left(s_I + \epsilon_I\right) \end{aligned}$$

Letting $F = F_R + iF_I$ to be a $n \times n$ complex-valued Fourier matrix. The $n \times 1$ complex-valued iFT F_c^{-1} could be written as $F_c^{-1} = F_c \times S_c$. It can also be written in matrix form as:

$$\begin{pmatrix} F_R^{-1} \\ F_I^{-1} \end{pmatrix} = \begin{pmatrix} F_R & -F_I \\ F_I & F_R \end{pmatrix} \begin{pmatrix} s_R + \epsilon_R \\ s_I + \epsilon_I \end{pmatrix}.$$

A magnitude- or complex-valued statistical distribution is needed to describe the magnitude- and complex–valued model. Here we describe the multivariate complex normal model proposed in [11].

True signal and the noise are modified using the transformed complex spatial frequencies. There is a meaningful correlation between the measurements in the frequency domain and iFT reconstructed complex-valued measurements [14,16]. For the real-valued representation of last equation, $s^* = \left(s_R^T, s_I^T\right)^T$ is considered as a multivariate normal distributed $2n$ dimensional vector, with mean and covariance matrix such as: $s = \begin{pmatrix} s_R \\ s_I \end{pmatrix}$ and

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12}^T & \Sigma_{22} \end{pmatrix}.$$

Then the distribution of F_C^{-1} is also multivariate normal with mean F_0 and covariance matrix Δ:

$$F_0 = \begin{pmatrix} F_{0R} \\ F_{0I} \end{pmatrix} = \begin{pmatrix} F_R & -F_I \\ F_I & F_R \end{pmatrix} \begin{pmatrix} s_R \\ s_I \end{pmatrix} = \begin{pmatrix} F_R s_R - F_I s_I \\ F_R s_I + F_I s_R \end{pmatrix},$$

$$\Delta = \begin{pmatrix} F_R & -F_I \\ F_I & F_R \end{pmatrix} \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12}^T & \Sigma_{22} \end{pmatrix} \begin{pmatrix} F_R^T & F_I^T \\ -F_I^T & F_R^T \end{pmatrix}.$$

Where F_c is a full rank Fourier matrix. Real-valued multivariate normal distribution is a general path for data analyzing regardless of the effect of the imaginary part, while the complex-valued multivariate normal distribution made it possible to improve the results by utilizing the complex data, true signal, and the noise. It could be performed when $\Sigma_{11} = \Sigma_{22} = \Phi$, $-\Sigma_{12} = \Theta$ and $\Sigma_{12} = \Theta$ [1,13].

A property of the complex multivariate normal distribution is that if $S \sim N\left(s, \Sigma\right)$, then $F_0 = FS$ is also normally distributed, and $F_0 \sim N\left(Fs, F\Sigma F^H\right)$. Where "H" denotes the Hermitian conjugate transpose.

It was assumed that the input is an 8×8 ideal, noiseless, gray scale image. The FT and iFT of the input image contain both real and imaginary parts. Therefore, the image could be simply represented as a complex-valued image in which, the imaginary part of it is the zero matrix. The spatial frequencies of the image could be extracted using a pre-multiplication of Fourier transform on the image arrays, and post-multiplying a transposed Fourier transform to the result. The result of this mathematical procedure is illustrated in figure 3 with 8×8 illustrative image.

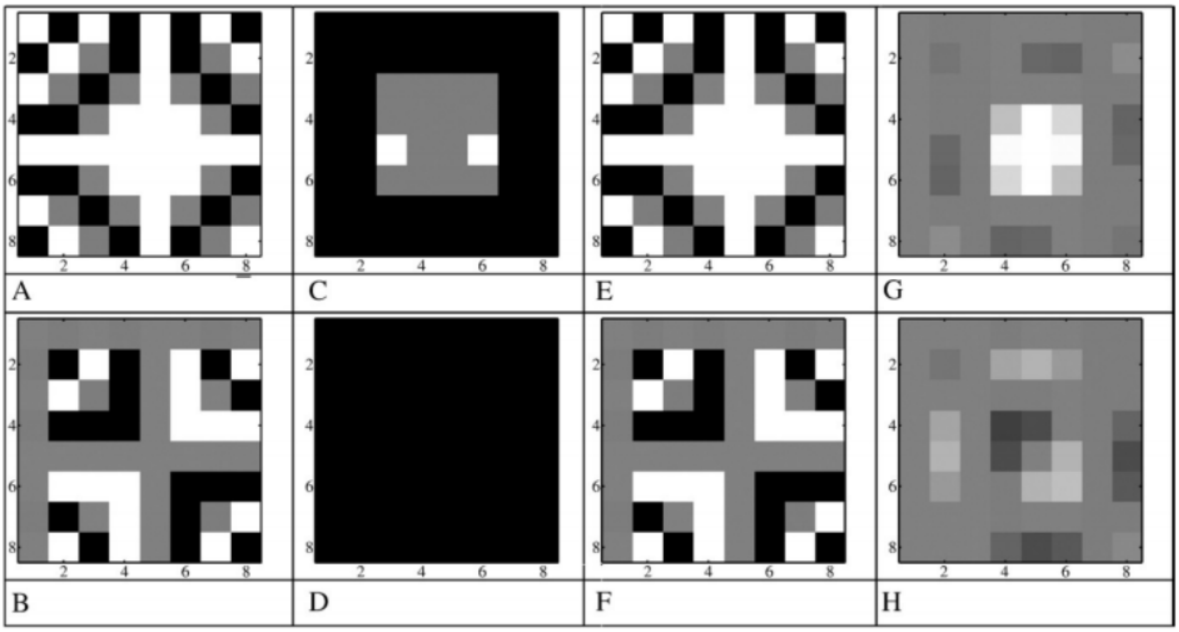

Figure 3. Complex Fourier transformation. a,b) Real and imaginary parts of the Fourier matrix c,d) Original and Zero assigned imaginary matrix e,f) Real and imaginary parts of the Fourier matrix g,h) Real and imaginary parts of the spatial frequency (k-space).

In MR imaging producer, the spatial data are frequency encoded and transformed into an image. So the inverse Fourier transformation is applied on the K-space measurements. This process is shown in figure 4 to reconstruct the same image using the iFT.

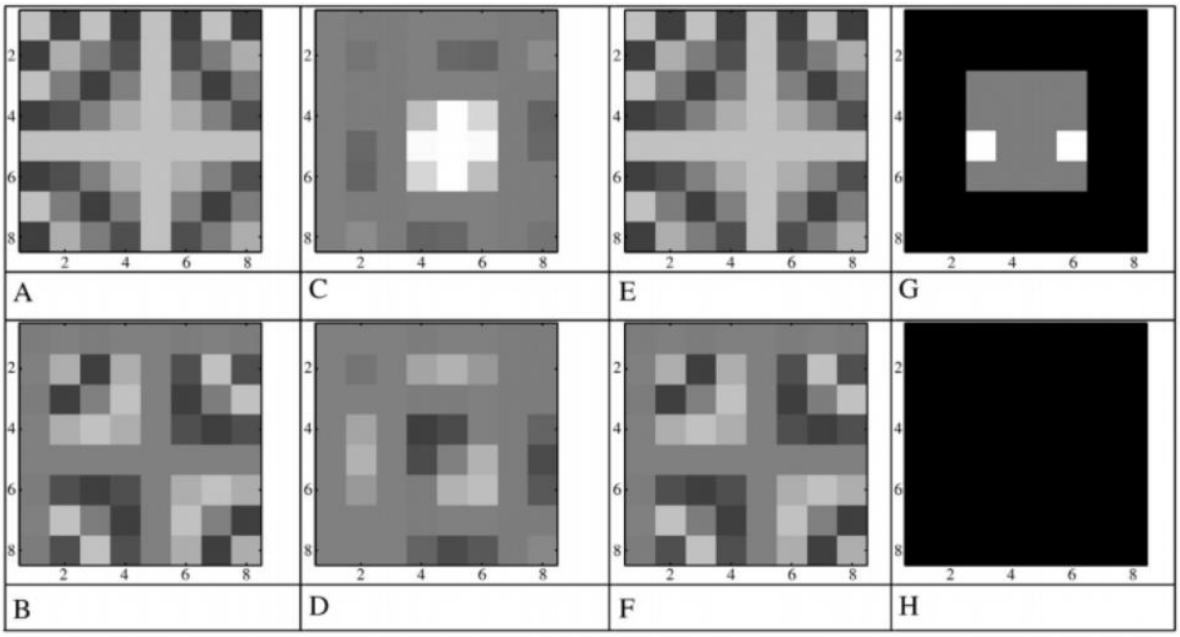

Figure 4. Complex Fourier transformation. a) Real part of the inverse Fourier matrix b) Imaginary part of the inverse Fourier matrix c) Real part of the spatial frequency d) Imaginary part of the spatial frequency e) Real part of the Fourier matrix f) Imaginary part of the Fourier matrix g) Reconstructed real part of the image h) Reconstructed imaginary part of the image.

Rowe [10] applied this strategy to demonstrate the effect of activation parameters for both methods. In Rowe's study, two methods were compared for the simulated signal with the temporal signal to noise ratio of 30 (SNR), the functional contrast ratio of 1(CNR), and the standard deviation of voxels equal to 0.05 (σ) [13]. Figure 5 shows the activation maps achieved using the magnitude- and complex-valued method with and without threshold as the illustrative examples. The results showed that the complex-valued model has better performance for estimating the true parameters. The results also indicate that complex-valued model could be a promising model to detect the brain activation, especially in low SNR.

3. DISCUSSION

In this study, a linear representation of the image has been performed and the complex reconstruction of the image was discussed mathematically. Magnitude- and complex-valued methods of the brain activation detection were evaluated for a sample data. For the magnitude-valued model, the results showed that the efficiency of the brain activation detection decreases by discarding the phase information. This degenerative effect was illustrated on an 8×8 simulated brain activation (Figure 5), in which activation maps were drawn for both magnitude- and complex-valued approaches.

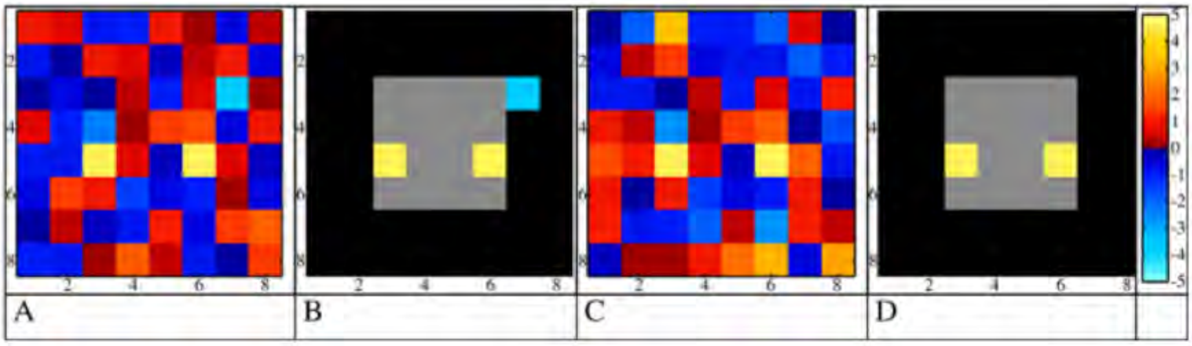

Figure 5. Activation maps. Bonferoni 5% threshold. a) Activation map achieved using the magnitude-valued method without threshold. b) Activation map achieved using the magnitude-valued method with threshold. c) Activation map achieved using the complex-valued method without threshold. d) Activation map achieved using the complex-valued method with threshold.

In conclusion, the complex-valued method is more accurate for detecting the brain activation, especially for low SNRs. In addition to the Fourier matrix and proposed linear representation, other linear reconstruction methods could be used for image reconstruction. Moreover, the effect of the different steps of the preprocessing methods could be examined and compared.

3. REFERENCES

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