

Gravitational Lensing as a Tool: Future Observational Prospects

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Abstract

Since the discovery in 1979 of the first gravitational lens system by Walsh, Carswell and Weymann, i.e. just a little more than one decade ago, more than 1000 scientific papers have been printed on this subject. In this review, we first recall the basic principles underlying the physics of gravitational lensing and provide the reader with a concise survey of the most important astrophysical and cosmological problems associated with gravitational lensing, particularly with a view to the observational capabilities of the present and next generations of space and ground based instrumentation. We refer the reader to the book of Schneider, Ehlers and Falco (1992) and to general reviews by Blandford and Narayan (1992), Tyson (1992), Refsdal and Surdej (1993) and Soucail *et al.* (1993) for a more complete and detailed presentation of various theoretical and observational aspects of gravitational lensing.

Keywords : Gravitational lensing ; cosmology ; ground-based observations ; space observations.

1. Introduction

The general layout of this paper is organized as follows. We describe in section 2 the first multi-waveband observations of gravitational lensing. We then discuss the form of the lens equation in order to understand correctly the image properties of a distant object due to gravitational lensing (section 3). In section 4, we quote a sufficient condition for a potential deflector to produce multiple images of a distant source and we recall the important relation existing between the angular diameter of the Einstein ring and the mass of the lens. Some of the most promising applications of gravitational lensing are reviewed in section 5. We also discuss there selected observations of gravitational lens systems that have already led (or are expected to lead) to interesting astrophysical and/or cosmological applications.

2. First Multi-Waveband Observations of Gravitational Lensing

During the elaboration of his theory of General Relativity, Einstein predicted that a massive object curves the space-time in its vicinity and that any particle, massive or not (cf. the photons), move along the geodesics of this curved space. He showed in 1916 that a light ray passing near the solar limb should be deflected by an angle equal to

$$\hat{\alpha} = \frac{4GM_{\odot}}{c^2 R_{\odot}} = 1.75'',$$

where G stands for the gravitational constant, c for the velocity of light and M_{\odot} , R_{\odot} for the mass and the radius of the Sun, respectively. It is important to note here that this deflection angle turns out to be exactly twice the value derived by Soldner in 1804, in the framework of the Newtonian mechanics.

Using photographs of a stellar field taken during the solar eclipse in May 1919, and six months apart, Eddington and his collaborators (see Dyson *et al.* 1920) were able to confirm, within a 20-30% uncertainty, the deflection angle predicted by Einstein. This was not only a triumph for General Relativity but also a marvelous confirmation of the concept that light rays may undergo deflections in gravitational fields. Let us note that this

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uncertainty has been presently reduced to less than 1% thanks to radio interferometric observations of quasi-stellar sources (Fomalont and Sramek 1975a, b; Robertson *et al.* 1991).

These measurements of the bending of light rays near the Sun at both optical and radio wavelengths do actually constitute the first multi-waveband observations of 'gravitational lensing', verifying at the same time the expected 'achromaticity' of this very interesting phenomenon. It may seem paradoxical that multiply imaged distant sources often reveal chromatic effects. As we shall see in the next section, these are caused by differential magnification of a source whose size varies with wavelength and do not, therefore, violate what has just been said before.

3. The Lens Equation and Multiple Imaging

It seems that Eddington (1920) was the first to propose the possible formation of multiple images of a background star by the gravitational lensing effect of a foreground object and that Chwolson (1924) was the first to suggest that, in case of a perfect alignment between an observer and two stars located at different distances, the former should see a ring shaped image of the background star around the foreground star.

Einstein (1936) rediscovered these major characteristics of a star lensed by another star and the 'Chwolson' ring is now usually named the 'Einstein' ring.

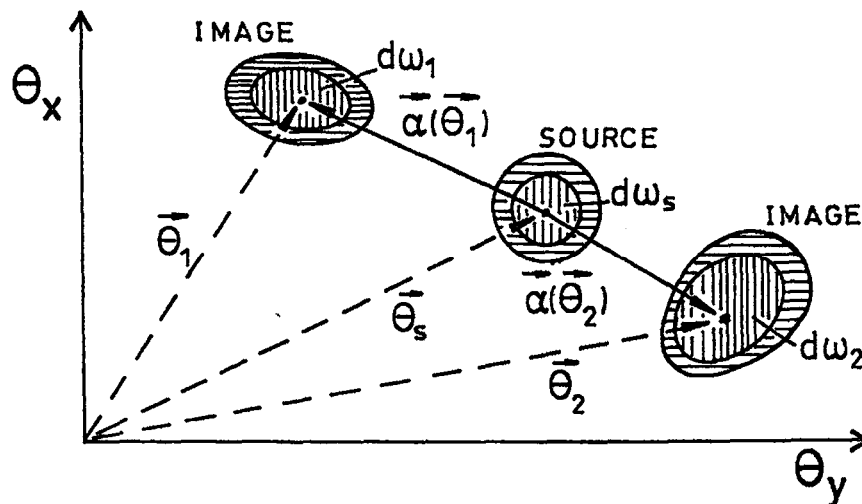
All these interesting effects may directly be understood from the lens equation (see Fig. 1). Indeed, let us define the true position of the source S on the sky by the angle $\vec{\theta}_s$ and the image position(s) by $\vec{\theta}_i$ ($i = 1, 2, \dots$). These correspond, of course, to the solutions of the *lens equation*

$$\vec{\theta} - \vec{\theta}_s = \vec{\alpha}(\vec{\theta}) = -(D_{ds}/D_{os})\vec{\alpha}(\vec{\theta}),$$

where $\vec{\alpha}$ is the displacement angle and where D_{ds} and D_{os} represent respectively the "deflector-source" and "observer-source" angular size distances.

We note that a given image position always corresponds to a specific source position whereas a given source position may sometimes correspond to several distinct image positions. Such cases of multiply imaged sources constitute of course the most spectacular and interesting aspects of gravitational lensing.

Fig. 1. Deflection of a light ray by a point mass lens (M).

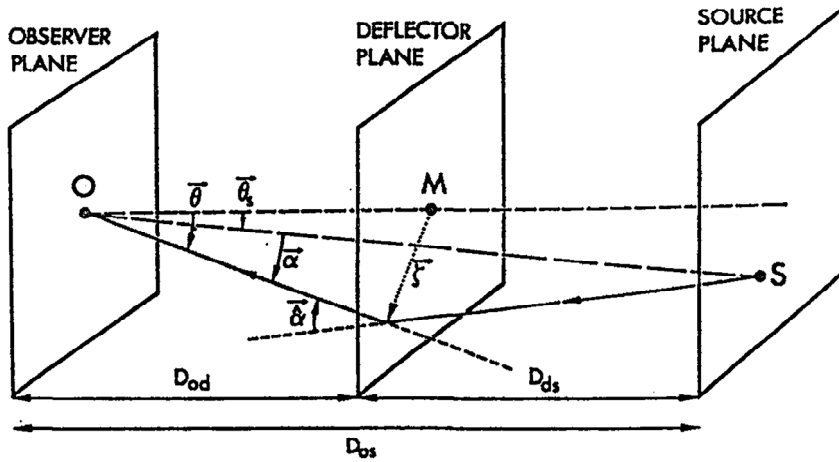


A typical lens situation is shown in Fig. 2, where source and image positions (two images in this case) are seen projected on the sky. We see again that the image position $\vec{\theta}_i$ is shifted by $\vec{\alpha}(\vec{\theta}_i)$ relative to the source position

$\vec{\theta}_s$; note, however, that $\vec{\alpha}(\vec{\theta}_i)$ is usually not constant over the source and this results in possible (de-) magnification and deformation of extended sources. It is also straightforward to understand that, since gravitational lensing preserves the surface brightness of a source, the ratio between the solid angle $d\omega_i$ covered by the lensed image and that of the unlensed source $d\omega_s$ (i.e. the magnification) immediately gives the flux amplification μ_i due to lensing.

If the source is multiply imaged and has a size which varies with wavelength, chromatic effects between the different images will result (cf. the differently magnified source regions in Fig. 2) and we see here why multi-wavelength observations of multiply imaged sources are of the greatest interest. They not only provide interesting constraints on the physical modelling of the source itself but also on the mass distribution of the distant deflector that induces the chromatic lensing effects.

Fig. 2. Images of a lensed source as seen projected on the sky.



Existing multi-wavelength observations of multiply imaged sources, resolved at arcsec and sub-arcsec angular scales, have already been successfully inverted. In several cases it has been possible from such analyses to retrieve the mass distribution of the lensing galaxy as well as the multi-wavelength shape of the source (see Wallington *et al.* 1993 and Kochanek *et al.* 1993 for a recent discussion of the numerical methods and Refsdal and Surdej 1993 for a general review of the observations).

In order to refine such astrophysical applications, it is very important in the future that not only direct imaging but also two-dimensional spectroscopy, at high angular resolution and with good dynamical range, become feasible at many different wavelengths, both from space (far UV, X-ray, γ -ray, etc.) and from the ground (visible, infra-red, radio, etc.).

4. Condition for Multiple Imaging and Angular Size of the Einstein Ring

When deriving the form of the lens equation in section 3, we have made the implicit assumption that the mass concentration which acts as a lens has a small size relative to the distances involved between the observer, the lens and the source (see Fig. 1). We have thus made use of the thin lens approximation. It is then very straightforward to show that a sufficient condition for a distant observer to see multiple images from a distant source is just that, somewhere in the lens plane, the surface mass density Σ exceeds a certain critical value Σ_c , which only depends on the relative positions of the observer, the lens and the source (Subramanian and Cowling 1986). Adopting typical cosmological distances for the deflector (redshift $z_d \approx 0.5$) and the source ($z_s \approx 2$), it is easy to show that $\Sigma_c \approx 1 \text{ g/cm}^2$.

Defining the average surface mass density $\Sigma(<R) = M/\pi R^2$ of a deflector having a typical mass M and radius R , we have listed in Table I values for the ratio $\Sigma(<R)/\Sigma_c$ considering a star, a galaxy and a cluster of galaxies as possible deflectors. We see that only stars and very compact, massive galaxies and galaxy clusters, for which $\Sigma(<R)/\Sigma_c \geq 1$, constitute promising 'multiple imaging' deflectors.

TABLE I Ratio of the average $\Sigma(< R)$ and critical Σ_c surface mass densities, angular (θ_E) and linear (ξ_E) radii of the Einstein ring for different values of the mass M , distance D_{od} and radius R of the deflector, assuming that $D_{os} = 2 \times D_{od}$ (1 parsec = 1 pc = 3.262 light years = 3.086 10^{18} cm)

Deflector	M	D_{od}	R	$\Sigma(< R)/\Sigma_c$	θ_E	$\xi_E = \theta_E D_{od}$
Star	$1 M_{\odot}$	10^4 pc	2×10^{-8} pc	2×10^6	6×10^{-4} arcsec	3×10^{-5} pc
Star	$1 M_{\odot}$	10^9 pc	2×10^{-8} pc	2×10^{11}	2×10^{-6} arcsec	10^{-2} pc
Galaxy core	$10^{12} M_{\odot}$	10^9 pc	5×10^3 pc	4	2 arcsec	10^4 pc
Cluster core	$10^{14} M_{\odot}$	10^9 pc	10^5 pc	1	20 arcsec	10^5 pc

In case of perfect alignment between the observer, a compact lens characterized by a symmetric mass distribution $M(< \xi)$ - where ξ represents the impact parameter - and the source (i.e. $\theta_s = 0$), it is clear that because of the axial symmetry, the observer will actually see a ring (the so-called 'Einstein ring') of light from a distant source. Combining the deflection (section 2) and the lens equation (section 3), the angular radius of this ring can be conveniently expressed as

$$\theta_E = \sqrt{\frac{4 G M(< \xi_E) D_{ds}}{c^2 D_{od} D_{os}}}$$

We have also listed in Table I typical values of θ_E for different types of deflectors located at various distances. We see from Table I that for a source and a lens located at cosmological distances (i.e. $z_d \simeq 0.5$ and $z_s \simeq 2$), the angle θ_E can vary from micro-arcsec (stellar deflection) to arcsec (galaxy lensing), and up to some tens of arcsec in the case of cluster lenses.

The value of θ_E derived above is very important because it can usually be used to estimate the angular separation between multiply lensed images in more general cases where the condition of a perfect alignment between the source, deflector and observer is not fulfilled or even for lens mass distributions which significantly depart from axial symmetry. Observed image separations ($\simeq 2 \theta_E$) can therefore lead to the value of M/D_{od} , or to the value of M times the Hubble parameter H_0 , if the redshifts z_d and z_s are known.

This is the simplest and most direct astrophysical application of gravitational lensing. A summary of mass estimates derived in this way for several lensing galaxies is given in Refsdal and Surdej (1993). These authors also provide a detailed list of approximately 20 proposed cases of multiply imaged distant sources (quasar-galaxy and galaxy-galaxy lensing), 6 radio rings (radio lobe - galaxy lensing) and several tens of giant luminous arcs and arclets (galaxy - cluster lensing).

5. Some Additional Promising Applications of Gravitational Lensing

In this section, we briefly review some of the other most promising astrophysical and cosmological applications of gravitational lensing.

5.1. Cosmological density of compact objects in the universe

For the case of a point mass lens, it can be shown that as the observer departs from the line of perfect alignment, the Einstein ring described in section 4 breaks up in two images and provided that the true position of the source lies inside the imaginary Einstein ring (i.e. $\theta_s \leq \theta_E$), the net magnification μ_T of the two images is larger than 1.34. This means that the cross section for significant lensing (by convention $\mu_T \geq 1.34$) is equal to $\pi \theta_E^2$, which is proportional to M (see the expression of θ_E in section 4). We can directly make use of this result when considering the maximum observed frequency of multiply imaged sources within a given sample of reference objects (e.g. quasars) to set an upper limit on the cosmological density Ω_L of compact objects in the Universe. Indeed, we just expect this frequency to be proportional to the fraction of the sky covered by the Einstein rings associated with all potential compact deflectors. Searching for multiply imaged objects within a sample of bright quasars and taking into account the magnification bias effect (Turner *et al.* 1984), Surdej *et al.* (1993a) have been able to set an upper limit of $\simeq 2\%$ on Ω_L for compact objects in the mass range $5 \times 10^{10} - 7 \times 10^{11} M_{\odot}$. Such an upper limit does already appear to be lower than that of known galaxies. Nemiroff (1993) further discusses how to make use of various types of observations of distant sources, at many different wavelengths, in order to set interesting constraints on Ω_L in the mass range $10^9 - 10^{14} M_{\odot}$.

5.2. Determination of the hubble parameter H_0

For the special case of multiply imaged quasars, one has the hope to be able to measure very accurately the time delay Δt between the arrival times of a same signal propagating along two distinct geodesics. Such a delay arises not only because the light rays coming from the source have travelled different geometrical paths, but also because the photons passing closer to the lens encounter stronger gravitational potential effects. These photons are therefore more slowed down and this effect does therefore also contribute to the time delay Δt . As early as 1964, Refsdal has shown that the time delay Δt is inversely proportional to the Hubble parameter H_0 and that if the mass distribution in the deflector plane is sufficiently well determined, this method provides astrophysicists with a unique and independent way of deriving accurately the value of the Hubble parameter H_0 . Therefore, we stress again here (see also Refsdal and Surdej 1992), the real need of dedicating in the near future, on a site with good atmospheric conditions, a medium size (2-3 m) telescope to the photometric monitoring of the multiple images of known gravitational lens systems (cf. 0957+561, PG1115+080, UM673, 1422+231, etc.).

5.3. The size of narrow absorption line clouds

Making use of the number of coincidences and anti-coincidences for the Ly α - or narrow metallic - absorption lines detected in the spectra of (at least two) distinct images of a lensed quasar, it is possible to set interesting constraints on the geometrical size of the relevant absorbing clouds. Smette *et al.* (1992) have reported a study of the Ly α forest on the basis of high resolution spectra obtained for the doubly imaged quasar UM673. By means of Monte-Carlo simulations, these authors have derived a value of $12 h_{50}^{-1}$ kpc ($q_0 = 0$) for the 2σ lower limit of the diameter of spherical Ly α clouds. Smette *et al.* have also suggested how to make use of their present results and those reported for the number of Ly α anti-coincidences in the spectra of 2345+007 A and B (Foltz *et al.* 1984) in order to assess the physical nature (binary quasar or dark gravitational lens) of the latter system. The recent discoveries of the doubly imaged quasar candidates HE1104-1805 (Wisotzki *et al.* 1993) and 1009-025 (Surdej *et al.* 1993b) offer new prospects to set more accurate sizes on (Ly α and metallic) narrow absorption line clouds. There is also no doubt that the availability of larger light collectors (cf. Keck telescopes, VLT) combined with very efficient high dispersion spectrographs will enable one to use fainter -and much more numerous - gravitational lens systems and this will of course result in improving very much our size estimates of (Ly α and metallic) narrow absorption line clouds.

5.4. Determination of the mass of a lensing cluster

The first giant luminous arcs (angular extent $\simeq 20''$, angular width $\leq 0.5''$) were discovered serendipitously in 1981 by Hoag, and quite independently in 1986 by Soucail and Fort and by Lynds and Petrosian plus their collaborators, to lie in the centres of rich clusters of galaxies (mass $\simeq 10^{14} M_{\odot}$, see Table I). As suggested by Paczyński (1987), the measurement of several arc redshifts has confirmed that they result from significant gravitational lensing (i.e. conditions of nearly perfect alignment) of distant background galaxies by rich foreground clusters. Up to now, about 20 fascinating optical giant luminous arcs, whose surface brightness is only about one tenth of the sky brightness, have been identified in rich clusters; half of them have a measured redshift which, in all cases, is larger than that of the cluster.

Much larger numbers of arclets are also found to lie around rich galaxy clusters. By definition, arclets consist of elongated images having an axis ratio larger than about 3:1, usually distributed perpendicularly to their radius vector pointing towards the center of the deflecting mass distribution. They are the result of weak gravitational lensing of very faint blue galaxies caused by an asymmetric distribution of matter around the image position, giving rise to a shear term (i.e. conditions of perfect alignment are scarcely fulfilled). By means of very deep CCD imaging, Tyson (1988) has found that there are about 300,000 of these faint blue galaxies per square degree all over the sky.

The arcs and arclets described above offer exciting possibilities for determining the mass and the mass distribution of the lensing cluster. A rough and very simple analysis can be made for systems with a single arc. Indeed, such an arc can be considered as part of an Einstein ring so that the mass inside the ring may directly be estimated from the expression of the angular Einstein radius given in section 4. Typically, the angular radius of an arc is $\simeq 20''$, corresponding to about $10^{14} M_{\odot}$. This results in the determination of a mass to luminosity ratio of about $100 M_{\odot}/L_{\odot}$, supporting the claim that dark matter is the principal constituent ($\geq 90\%$) of galaxy clusters. Additional information on the distribution of dark matter may be obtained from the faint background arclets. Because of their large number, the intrinsic distortion can be "averaged out" and a distortion pattern which traces the gravitational field of the foreground cluster up to large distances from its center can be determined. It is generally found that dark matter follows rather closely the distribution of red light of the galaxy

cluster, being also quite peaked near its center. A big challenge for the future is to obtain the redshifts of the faint blue galaxies and arclets. Detailed observations of these will only become feasible thanks to the large collecting area of future VLTs, coupled with the high efficiency of multi-object-spectrographs (MOS facilities). More accurate values for the strength of the gravitational field of selected lensing clusters will then result.

5.5. Micro-lensing, high amplification events and parallax effects

Another very interesting aspect of lensing phenomena consists of the so-called micro-lensing effects. These are induced by individual stars or compact objects having a similar or even a lower mass, usually located in a galaxy which acts as a macro-gravitational lens. This was first discussed by Chang and Refsdal (1979; see Kayser 1992 for a recent review). Since the angular sizes of quasars are smaller than, or comparable to, the Einstein ring of a star located at cosmological distances, this can lead to a splitting-up of each QSO macro-image into several micro-images, with typical angular separations of some micro-arcsec.

Of course, these micro-images are not resolvable with techniques available today; however, the integrated luminosity observed for all those micro-images will vary with time due to the relative transverse motions between the source, the star field and the observer.

Of special interest are the so-called high amplification events (HAEs) which, roughly speaking, occur when one of the micro-deflectors (star, planet, etc.) gets very near to the line-of-sight to one of the lensed macro-images (see Refsdal and Surdej 1993 for a more precise and complete description). For compact sources, one gets typical asymmetric peaks in the lightcurve of the relevant macro-image and it is easy to show that the time scale δt for the steep rise (or decline) of the lightcurve is simply given by the ratio of the source diameter to the relative transverse velocity V_r between the distant source, the micro-lens and the observer. Grieger *et al.* (1988) have shown that it is even possible to retrieve the intrinsic one-dimensional brightness profile of the source by properly analysing the lightcurve observed during an HAE.

As mentioned in section 3, gravitational lenses are basically achromatic but since the amplification factor due to micro-lensing depends on the source size, indirect chromatic effects may result if the source size depends on wavelength. In particular, the continuum source and the (much larger) broad emission-line regions of quasars may be differently amplified, causing differences between the equivalent widths of emission-lines observed in the spectra of the macro-images. It is important to note here that such micro-lens induced spectroscopic effects as well as photometric light variability have been detected for the multiply imaged quasars 2237+0305 (Irwin *et al.* 1989, Racine 1992) and H1413+117 (Kayser *et al.* 1990, Angonin *et al.* 1990). Analysis of such photometric and spectroscopic data acquired continuously during an HAE should enable one to identify the real nature (dark objects, mass in the range $10^{-5} M_{\odot}$ - $100 M_{\odot}$, etc.) of the micro-lenses as well as to set interesting constraints on the physical structure of the distant source (quasar, AGN, etc.). Because of these great challenges, photometric as well as two-dimensional spectroscopic monitoring of known multiply imaged quasars should receive more attention in the future. Access to a 2-3 m class telescope fully dedicated to the monitoring of multiply imaged quasars seems therefore well justified for the particular study of micro-lensing effects.

Another very interesting application of micro-lensing consists of the parallax effect. Indeed, because micro-lensing causes flux gradients in the observer plane, a brightness difference $\delta m = \text{grad}(m) \delta \vec{r}$ becomes measurable between two observers located at a distance $\delta \vec{r}$ from each other. This effect is often referred to as the parallax effect (Grieger *et al.* 1986). Values of $\text{grad}(m)$ are typically between 10^{-4} and 10^{-3} AU^{-1} (1 astronomical unit = $1 \text{ AU} \approx 1.5 \cdot 10^{13} \text{ cm}$), but during HAEs values up to 10^{-2} AU^{-1} may be reached (assuming that the source diameter is $\approx 10^{-3} \text{ pc}$ and that $M \approx M_{\odot}$). For two observers (1 and 2) located some AU apart, it should then be possible to determine the time lag $\delta t_{1,2}$ between the photometric lightcurves recorded during an HAE and thereby get information on the projection in the observer plane of the relative velocity V_r between the source, the deflecting stellar field and the observer. Grieger *et al.* (1988) have shown that in the presence of a third observer, a second time lag may be derived and it then becomes possible to get a precise value for V_r . Taking into account the redshifts of the source and of the lens and assuming a cosmological model, we can then determine a value for the absolute diameter of the source. Assuming V_r to be about 600 km s^{-1} , we note that this corresponds to approximately the distance to Saturn in one month; one would therefore expect a time lag of about one month between a terrestrial observer and an observer located close to Saturn.

Micro-lensing variability ($\approx 0.1 \text{ mag./month}$) has been reported for the A and B images in the Einstein Cross (Q2237+0305, see Corrigan *et al.* 1991). It is therefore clear that simultaneous observations of this system from the Earth and with an even modest space observatory passing near Saturn should allow one to measure the time lag δt , since a brightness difference of about 0.1 mag. should be expected between the two observatories.

An interesting point in connection with the measurement of a time lag δt is that it would immediately prove that we are dealing with micro-lensing variability and not with intrinsic variations since the latter ones would only produce extremely small time lags which can easily be corrected for. This is particularly important for sources with only one macro-image since then the distinction between the two types of variability is very difficult to establish by other means.

Considering baselines of the order of 100 AU or even larger, we expect that most distant quasars should show small brightness differences. Observations of a large number of quasars would then provide extremely valuable information on the mass distribution in the Universe (masses between $\simeq 10^{-5} M_{\odot}$ and $\simeq 100 M_{\odot}$). One should note that the space observatories needed to achieve these goals could be mostly dedicated to other scientific projects but that the applications suggested here would represent by-products of very great astrophysical importance.

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