University of Liège, Belgium.
Aerospace and Mechanical Engineering Department,
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Application of the Generalized-alpha time integration scheme in PFEM for solving the incompressible Navier-Stokes equations

ECCOMAS 2022

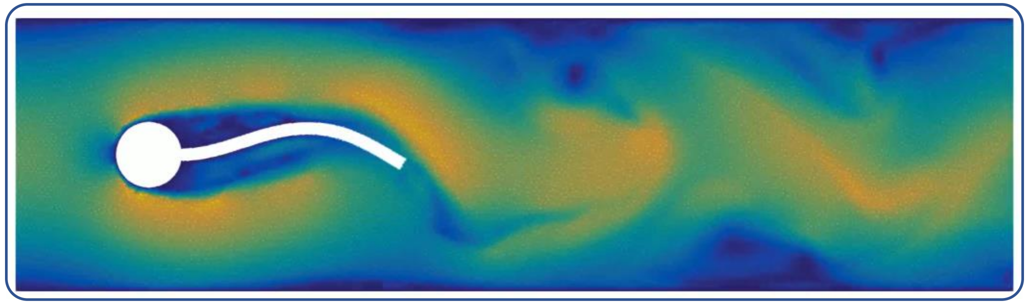
Eduardo Fernández, Simon Février, Martin Lacroix, Romain-Boman, Jean-Phillipe Ponthot

7 / 06 / 2022

This work was supported by the ALFEWELD project (convention 1710162) funded by the WALInnov program of the Walloon Region of Belgium.

INTRODUCTION : Motivation

Fluid-Structure Interaction (FSI) simulations



INTRODUCTION : Motivation

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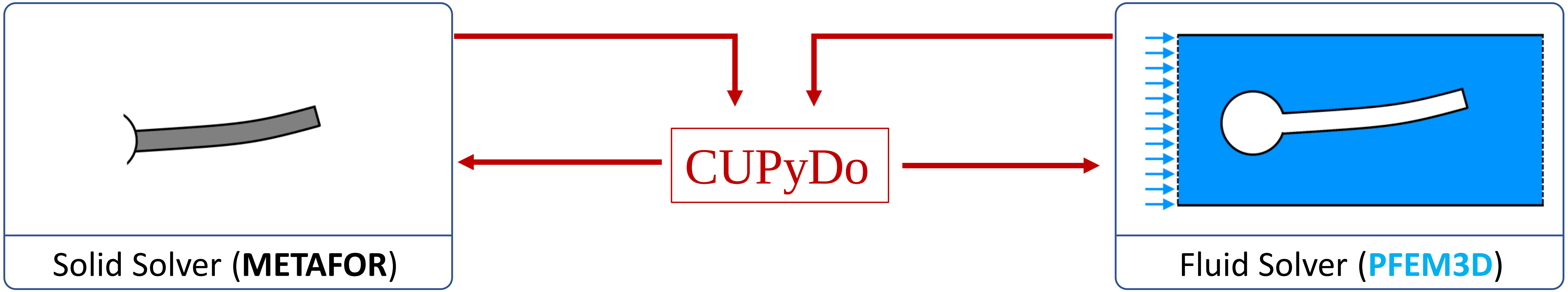
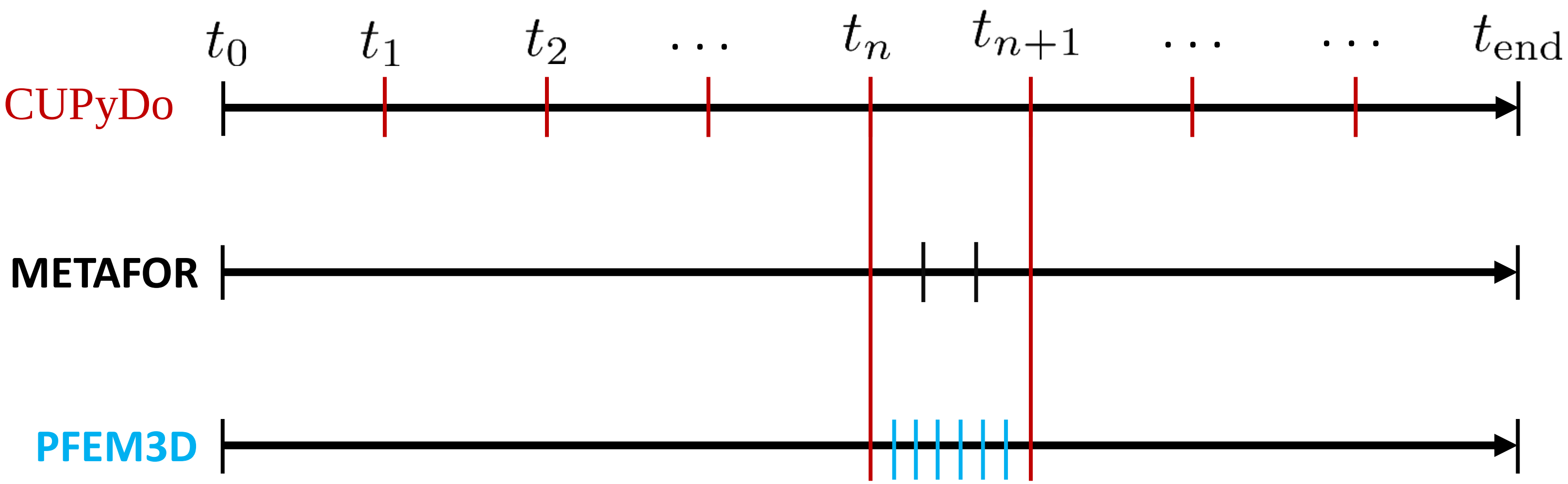




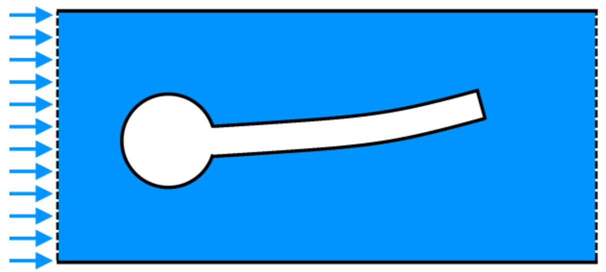
INTRODUCTION : Motivation

Fluid-Structure Interaction (FSI) simulations

Solid Solver (**METAFOR**)

CUPyDo

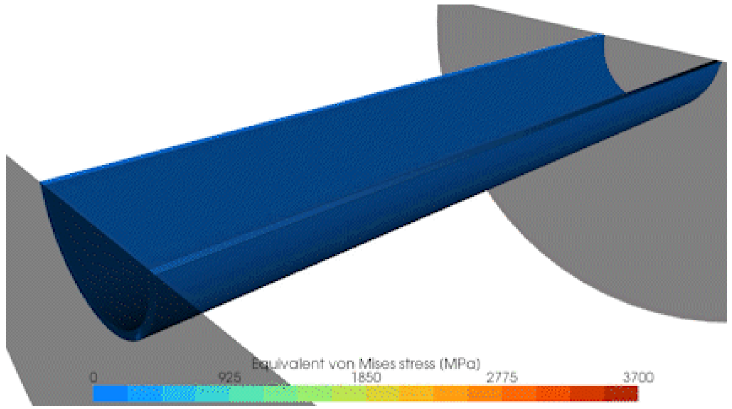
Thomas D et.al. (2019). CUPyDO-An integrated Python environment for coupled fluid-structure simulations. *Advances in Engineering Software*, *128*, 69-85.



Fluid Solver (**PFEM3D**)

More About METAFOR, visit:

http://metafor.ltas.ulg.ac.be/



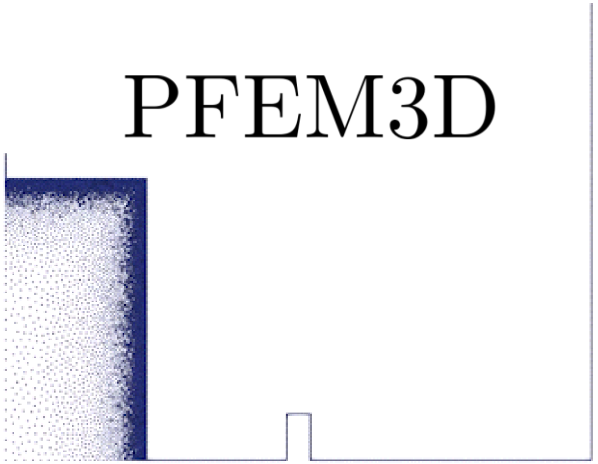
More about PFEM3D in ***ECCOMAS 2022*** *:*

Simon FÉVRIER

PFEM for simulations of Selective Laser Melting with vaporization.

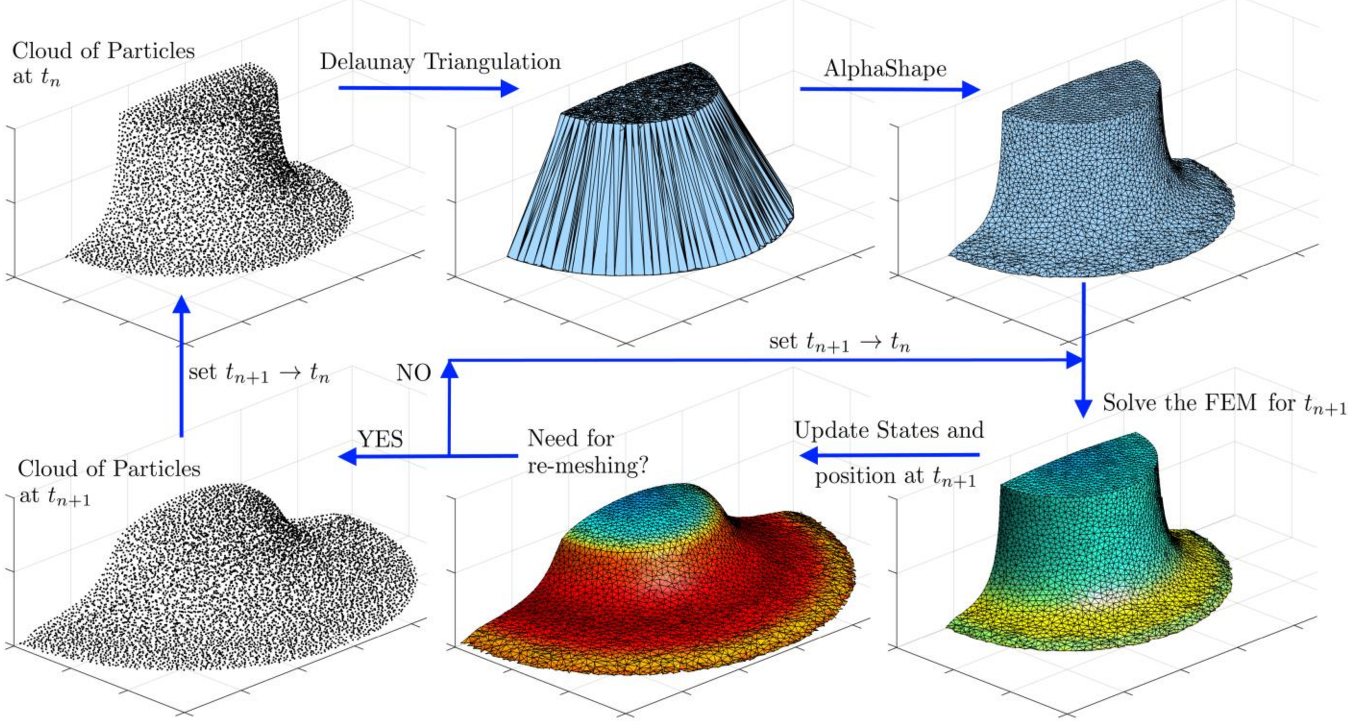
Martin LACROIX

PFEM for 2D/3D Fluid-Structure Interactions including Contact Interactions.



INTRODUCTION : Particle Finite Element Method

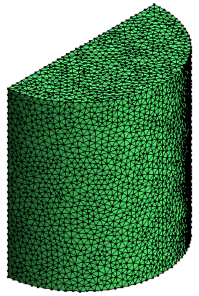
PFEM workflow

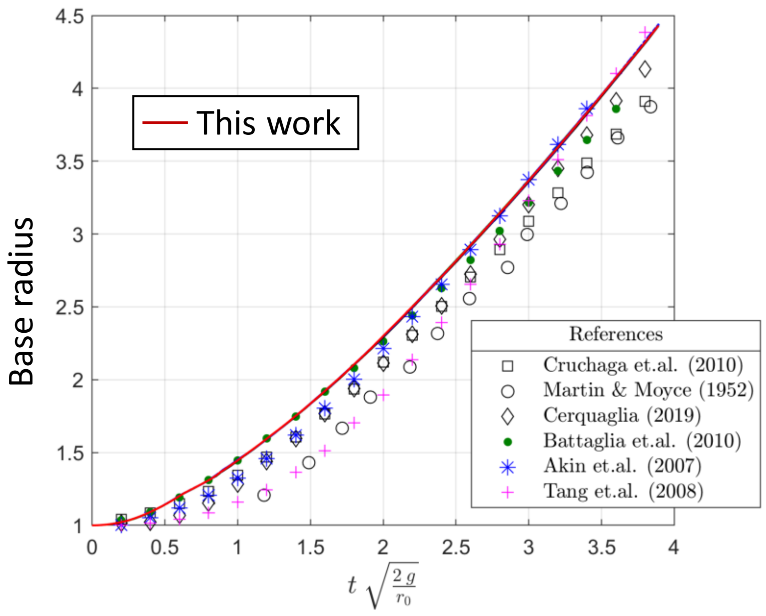


Proposed by Oñate, Idelsohn & Del Pin, 2004

Idelsohn, S. R., Oñate, E., & Pin, F. D. (2004). The particle finite element method: a powerful tool to solve incompressible flows with free-surfaces and breaking waves. *IJNME*, *61*(7), 964-989.

Collapse of a Cylindrical Water Column

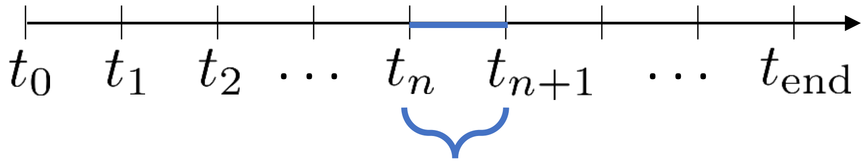




Outline

1. Introduction : Motivation / PFEM
2. **Time Integration schemes**
3. Numerical example
4. Conclusions

Time Integration Schemes : Backward Euler



$$\boldsymbol{f}^{\text{dyn}}(\dot{\mathrm{v}}_{n+1}) + \boldsymbol{f}^{\text{int}}(\mathrm{v}_{n+1}, \mathrm{p}_{n+1}) = \boldsymbol{f}^{\text{ext}}$$

+

Backward Euler

(Most common scheme in the PFEM literature)

$$\dot{\mathrm{v}}_{n+1} = \frac{\mathrm{v}_{n+1} - \mathrm{v}_n}{\Delta t}$$

$$\mathrm{v}_{n+1} = \frac{\mathrm{x}_{n+1} - \mathrm{x}_n}{\Delta t}$$

CONTEXT :

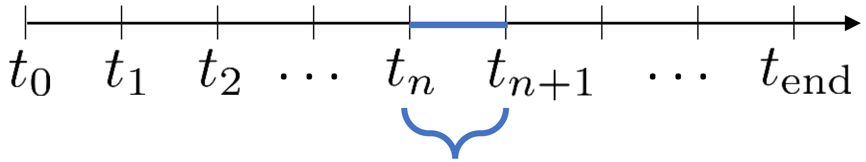
Incompressible Newtonian Fluid

Monolithic scheme

PSPG stabilization.

Our previous implementation and common scheme in the PFEM literature

Time Integration Schemes : Generalized-α



$$\boldsymbol{f}^{\text{dyn}}(\dot{\mathrm{v}}_{n+1}) + \boldsymbol{f}^{\text{int}}(\mathrm{v}_{n+1}, \mathrm{p}_{n+1}) = \boldsymbol{f}^{\text{ext}}$$

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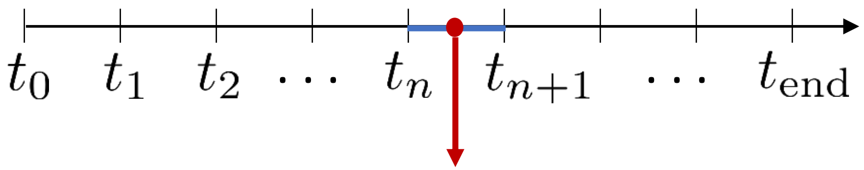
Our previous implementation and common scheme in the PFEM literature



$$\boldsymbol{f}^{\text{dyn}}(\dot{\mathrm{v}}_{n+\alpha_m}) + \boldsymbol{f}^{\text{int}}(\mathrm{v}_{n+\alpha_f}, \mathrm{p}_{n+\alpha_f}) = \boldsymbol{f}^{\text{ext}}_{n+\alpha_f}$$

where:

$$t_{n+\alpha_m} = (1-\alpha_m)\, t_n + \alpha_m\, t_{n+1}$$
$$t_{n+\alpha_f} = (1-\alpha_f)\, t_n + \alpha_f\, t_{n+1}$$

+

Newmark

$$\dot{\mathrm{v}}_{n+1} = \frac{\mathrm{v}_{n+1} - \mathrm{v}_n}{\gamma \Delta t} - \frac{1-\gamma}{\gamma}\dot{\mathrm{v}}_n$$

$$\mathrm{v}_{n+1} = \frac{\mathrm{x}_{n+1} - \mathrm{x}_n}{\beta \Delta t^2} + \frac{\mathrm{v}_n}{\beta \Delta t} - \frac{1-2\beta}{2\beta}\dot{\mathrm{v}}_n$$

+

Assumption at $t_{n+\alpha_m}$ and $t_{n+\alpha_f}$

Generalized-α : Implementation Approaches

Chung, J., & Hulbert, G. (1993). A time integration algorithm for structural dynamics with improved numerical dissipation: the generalized-α method.

Jansen, K. E., Whiting, C. H., & Hulbert, G. M. (2000). A generalized-α method for integrating the filtered Navier–Stokes equations with a stabilized finite element method. *CMAME*, *190*(3-4), 305-319.

GA-I

$$\dot{\mathrm{v}}_{n+\alpha_m} = (1-\alpha_m)\,\dot{\mathrm{v}}_n + \alpha_m\,\dot{\mathrm{v}}_{n+1}$$

$$\mathrm{v}_{n+\alpha_f} = (1-\alpha_f)\,\mathrm{v}_n + \alpha_f\,\mathrm{v}_{n+1}$$

$$\mathrm{d}_{n+\alpha_f} = (1-\alpha_f)\,\mathrm{d}_n + \alpha_f\,\mathrm{d}_{n+1}$$

$$\mathrm{p}_{n+\alpha_f} = (1-\alpha_f)\,\mathrm{p}_n + \alpha_f\,\mathrm{p}_{n+1}$$

Wood, W. L., Bossak, M., & Zienkiewicz, O. C. (1980). An alpha modification of Newmark's method. *International journal for numerical methods in engineering*, *15*(10), 1562-1566.

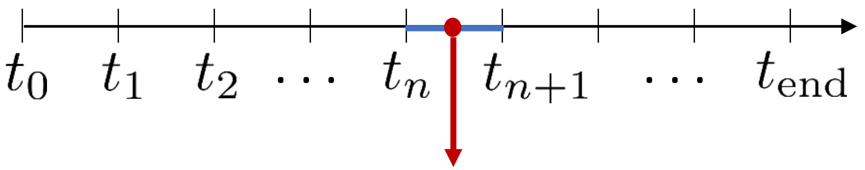
Hilber, H. M., Hughes, T. J., & Taylor, R. L. (1977). Improved numerical dissipation for time integration algorithms in structural dynamics. *Earthquake Engineering & Structural Dynamics*, *5*(3), 283-292.

GA-II

$$\boldsymbol{f}^{\mathrm{dyn}}_{n+\alpha_m} = (1-\alpha_m)\,\boldsymbol{f}^{\mathrm{dyn}}_n + \alpha_m\,\boldsymbol{f}^{\mathrm{dyn}}_{n+1}$$

$$\boldsymbol{f}^{\mathrm{int}}_{n+\alpha_f} = (1-\alpha_f)\,\boldsymbol{f}^{\mathrm{int}}_n + \alpha_f\,\boldsymbol{f}^{\mathrm{int}}_{n+1}$$

$$\boldsymbol{f}^{\mathrm{ext}}_{n+\alpha_f} = (1-\alpha_f)\,\boldsymbol{f}^{\mathrm{ext}}_n + \alpha_f\,\boldsymbol{f}^{\mathrm{ext}}_{n+1}$$



$$\boldsymbol{f}^{\mathrm{dyn}}(\dot{\mathrm{v}}_{n+\alpha_m}) + \boldsymbol{f}^{\mathrm{int}}(\mathrm{v}_{n+\alpha_f}, \mathrm{p}_{n+\alpha_f}) = \boldsymbol{f}^{\mathrm{ext}}_{n+\alpha_f}$$

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+

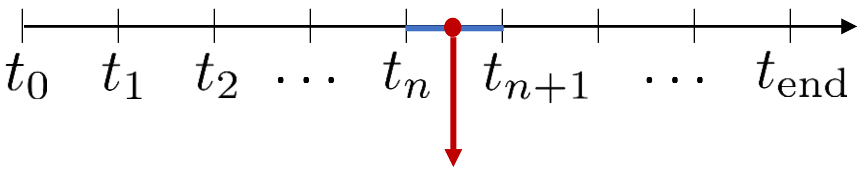
Newmark $$\dot{\mathrm{v}}_{n+1} = \frac{\mathrm{v}_{n+1} - \mathrm{v}_n}{\gamma \Delta t} - \frac{1-\gamma}{\gamma}\dot{\mathrm{v}}_n$$

$$\mathrm{v}_{n+1} = \frac{\mathrm{x}_{n+1} - \mathrm{x}_n}{\beta \Delta t^2} + \frac{\mathrm{v}_n}{\beta \Delta t} - \frac{1-2\beta}{2\beta}\dot{\mathrm{v}}_n$$

+

Assumption at $t_{n+\alpha_m}$ and $t_{n+\alpha_f}$

Generalized-α : Implementation Approaches



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$$\boldsymbol{f}^{\text{dyn}}(\dot{\mathrm{v}}_{n+\alpha_m}) + \boldsymbol{f}^{\text{int}}(\mathrm{v}_{n+\alpha_f}, \mathrm{p}_{n+1}) = \boldsymbol{f}^{\text{ext}}_{n+\alpha_f}$$

Liu, J., Lan, I. S., Tikenogullari, O. Z., & Marsden, A. L. (2021). A note on the accuracy of the generalized-α scheme for the incompressible Navier-Stokes equations. *International Journal for Numerical Methods in Engineering*, *122*(2), 638-651.

+

Newmark $\dot{\mathrm{v}}_{n+1} = \dfrac{\mathrm{v}_{n+1} - \mathrm{v}_n}{\gamma \Delta t} - \dfrac{1-\gamma}{\gamma}\dot{\mathrm{v}}_n$

$$\mathrm{v}_{n+1} = \frac{\mathrm{x}_{n+1} - \mathrm{x}_n}{\beta \Delta t^2} + \frac{\mathrm{v}_n}{\beta \Delta t} - \frac{1-2\beta}{2\beta}\dot{\mathrm{v}}_n$$

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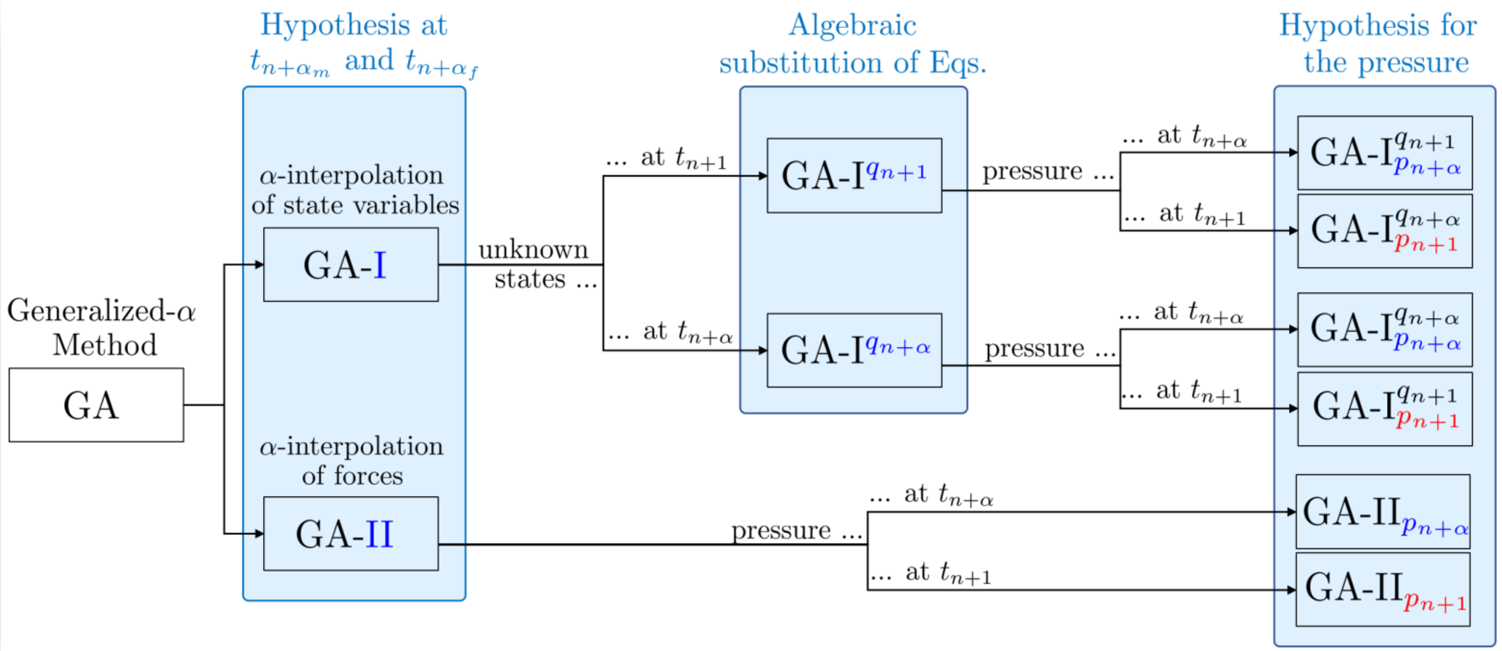
Assumption at $t_{n+\alpha_m}$ and $t_{n+\alpha_f}$

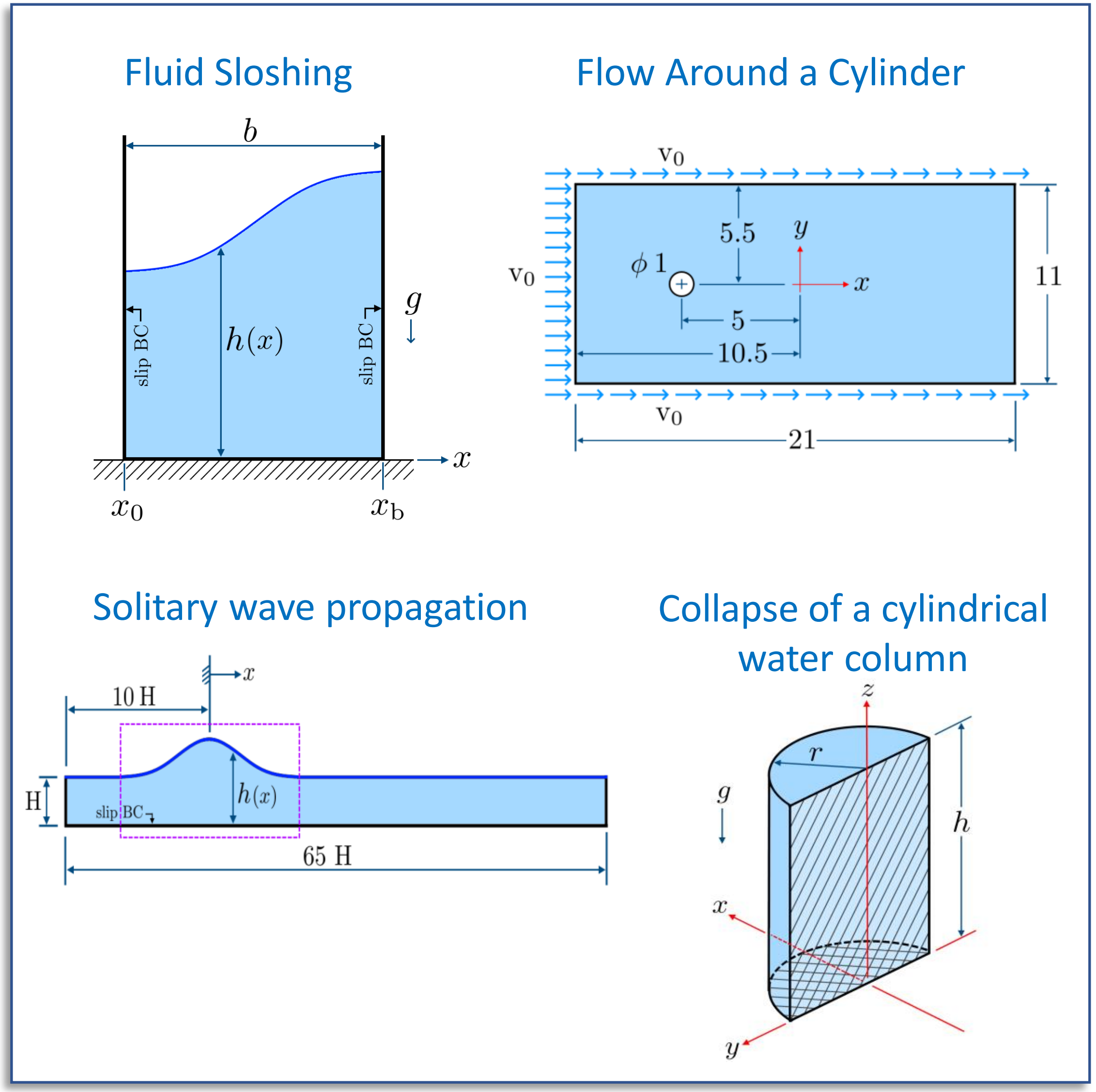


Generalized-α : Implementation Approaches

Fernandez E, Février S, Lacroix M, Boman R, Ponthot JP. Generalized-α scheme in the PFEM for velocity-pressure and displacement-pressure formulations of the incompressible Navier-Stokes equations (Under Review : IJNME)

- Detailed implementation of Generalized-α in PFEM.
- All implementation approaches are compared.
- Velocity-based and Displacement-based formulations are considered.
- Comparison of different time integration schemes (Backward Euler, Trapezoidal, Newmark, Generalized-α)

CONTEXT :
- Incompressible Newtonian Fluid
- Monolithic scheme
- PSPG stabilization.



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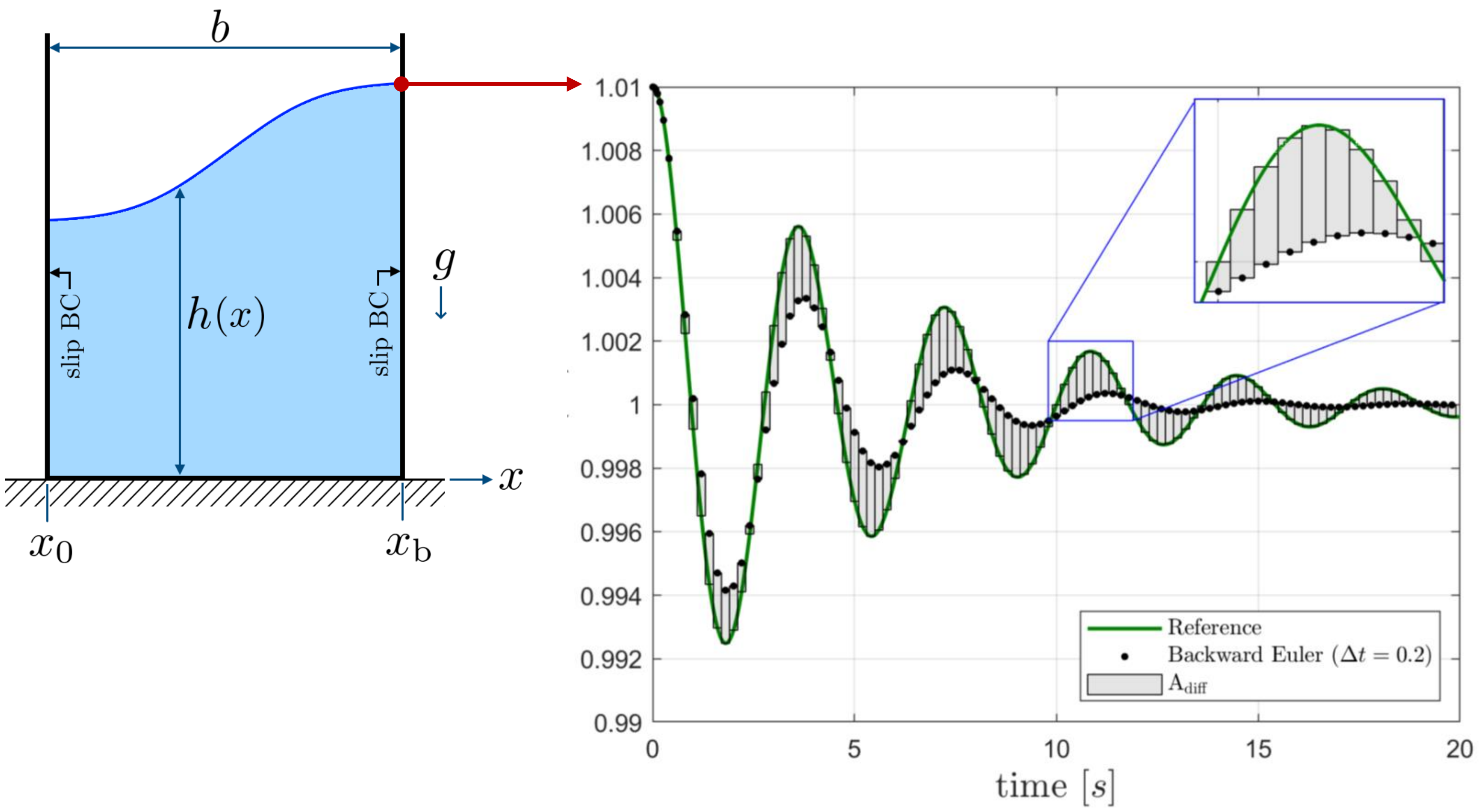
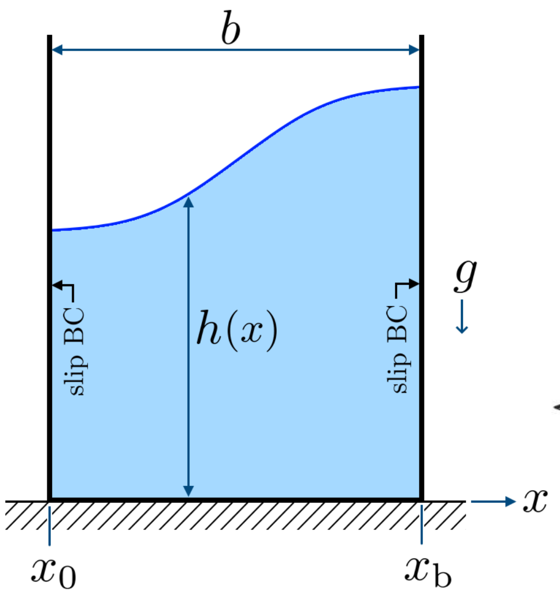
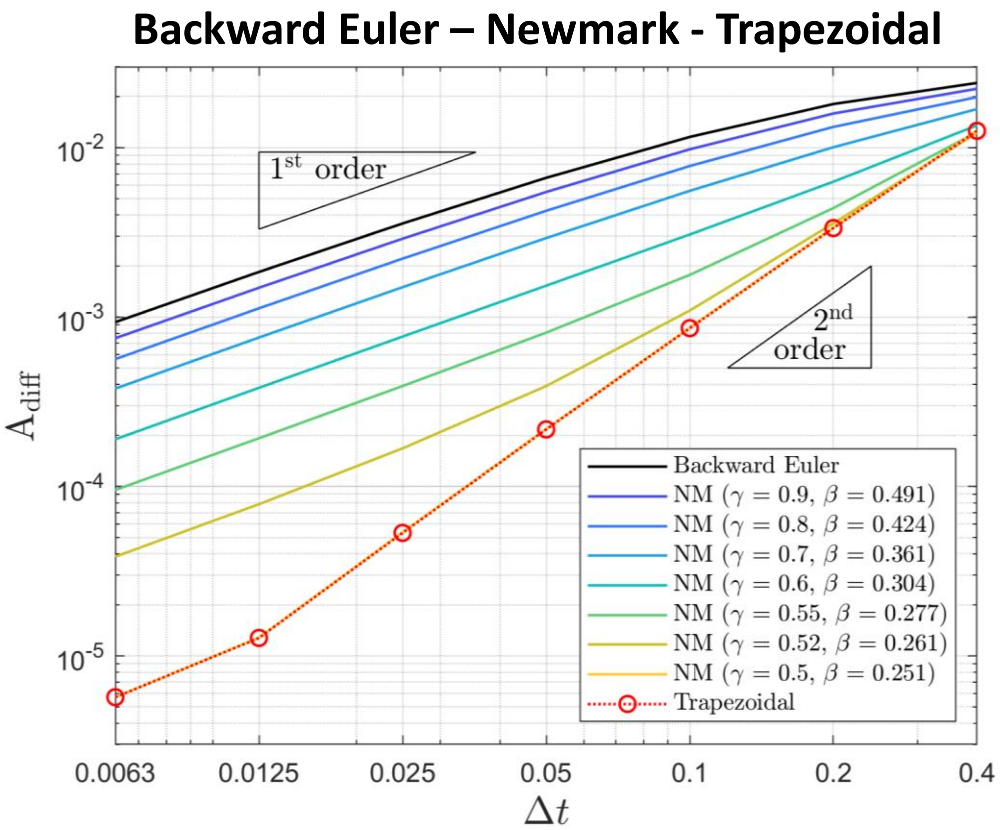
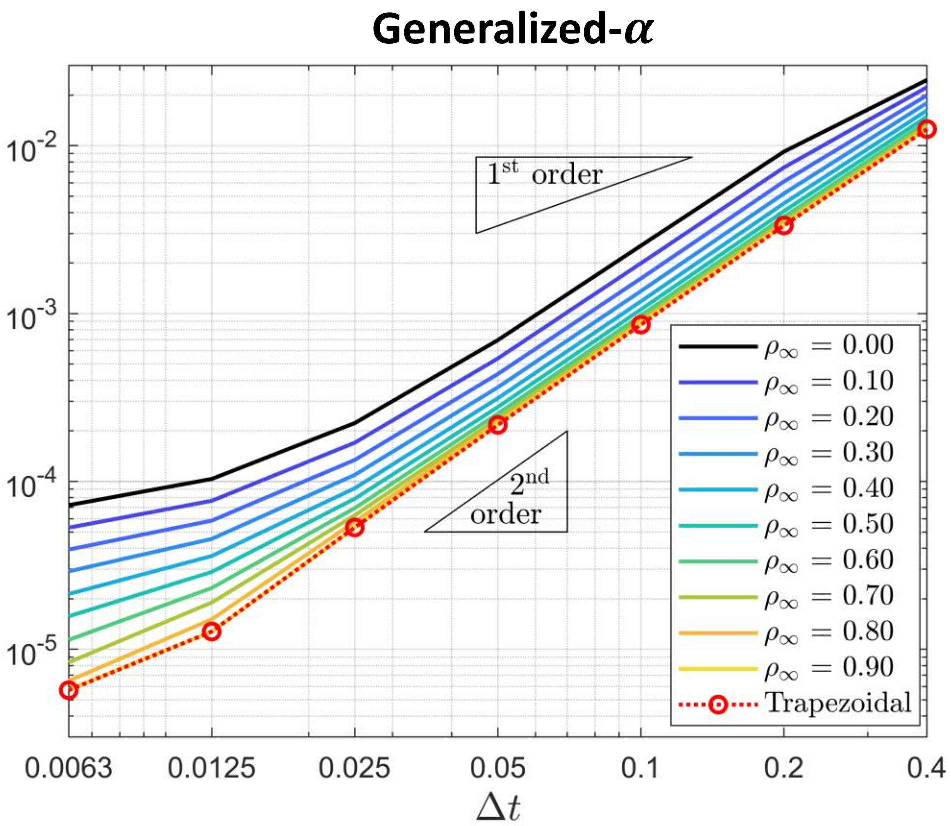
Numerical Example: Fluid sloshing



Numerical Example: Fluid sloshing







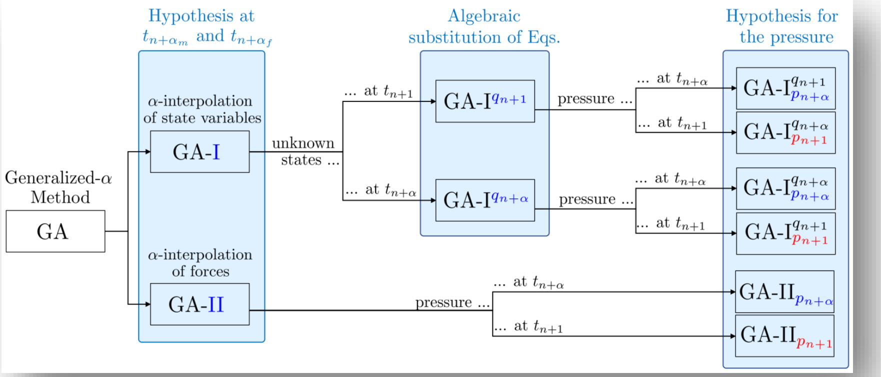
(classical parameterization for stability of linear problems)

For Newmark: $\gamma \geq 0.5\ ,\ \beta = \frac{1}{4}\left(\gamma + \frac{1}{2}\right)^2$

Generalized-α: $\alpha_m = \frac{1}{2}\left(\frac{3-\rho_\infty}{1+\rho_\infty}\right)\ ,\quad \alpha_f = \frac{1}{1+\rho_\infty} \quad \gamma = \frac{1}{2} + \alpha_m - \alpha_f\ ,\ \beta = \frac{1}{4}\left(\gamma + \frac{1}{2}\right)^2$

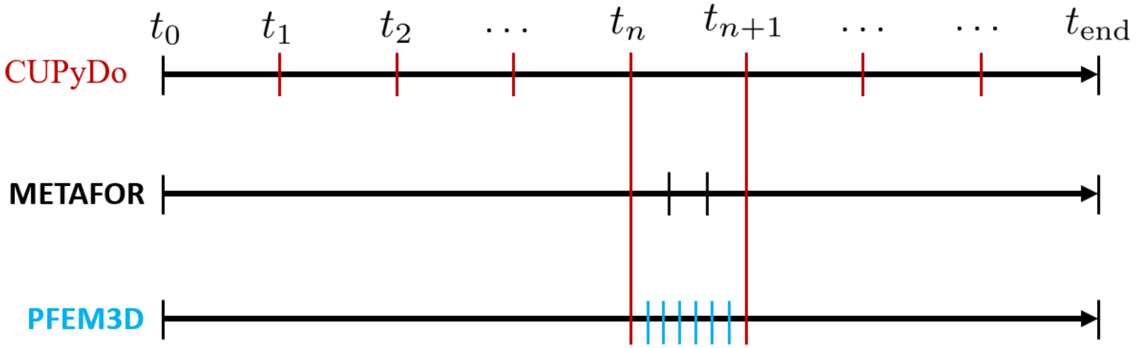
Conclusions

- Generalized-Alpha outperforms Newmark and Backward Euler in the PFEM.
- Similar results are obtained either by α-interpolating the state variables or the equilibrium forces (GA-I ≈ GA-II).



- Observations are also valid for displacement-based formulation.

Perspective : To study the performance of time integration schemes in the PFEM for Fluid-Structure Interactions.



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