Our Universe as a Black Hole in a Larger Space

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Abstract

Using present day estimates of the size and average density of the observable universe, its Schwarzschild radius is found to exceed its physical radius. This means that, if the observable universe is a representative part of a material universe included in a larger space, this material universe should be a black hole.

Approximating the material universe by a sphere filled with pressureless gas of uniform density, its geometry is described by a piece of Friedmann-Lemaître-Robertson-Walker metric embedded in a Schwarzschild metric. The matching of the two metrics at the surface of the material sphere sets the boundary conditions for the interior part.

Mathematically, the equation relating the proper time to the coordinate time in the interior of the black hole has two solutions of opposite signs. If the negative solution is adopted, matter collapsing towards the singularity is perceived as expanding by observers inside the black hole. This allows to reconcile the facts that we observe an expanding universe while we should be inside a black hole.

The proper time being the one that controls the physical processes, the physics inside the black hole universe is the same as in the usual expanding universe models. However, a fundamental change concerns the boundary conditions: the latter are not set at the singularity but at the start of the collapse, in our far future. Such a change of boundary conditions allows to propose a solution to three fundamental problems of the Big Bang cosmology: the horizon problem, the flatness problem and the particle–antiparticle asymmetry.

1 The Λ-CDM Model

Philosophers typically define the universe as the most complete physical whole [1]. To be studied without misconceptions, this definition is refined in physics. In this paper, we make a distinction between three "levels" of universes: (a) The observable universe (level 1), or the largest system possibly interacting with an observer, i.e. to which it is causally connected. That region is bounded by the particle horizon, which today is of the order of $r_{ph} \simeq 4.6 \times 10^{10}$ l.yr [2]. (b) The material universe (level 2), which contains our observable universe and those of all the observers that could possibly interact two by two. (c) The whole of spacetime (level 3), including everything that exists, thus all forms of matter and energy, and of course the two previous levels of universes. Level 3 can however coincide with level 2, but not necessarily.

The standard model of cosmology, the Λ-CDM model, is considered by most cosmologists as the current best description of our universe (level 2). This model is characterized by the emergence of a dynamical spacetime from an extremely hot and dense state 13.8 billion years ago, the Big-Bang.

In the framework of general relativity, the Λ-CDM model relies on the *cosmological principle*. It assumes that an observer's view of the universe does not depend on the direction in which he or she is looking, nor on his or her position. A good evidence for the large-scale homogeneity and isotropy of the universe comes from the remarkable isotropy of the cosmic microwave background (CMB). After correction of the dipole anisotropy, interpreted as a Doppler shift induced by a motion of the Solar System at a velocity of 368 km/s with respect to the CMB rest frame [3], the temperature fluctuations in the CMB are small $\Delta T/T \sim 10^{-5}$ [4]. Assuming that we do not occupy a particular place in space, the observed isotropy implies that the universe is also homogeneous.

The cosmological principle thereby motivates the study of simple cosmological models in which the universe is supposed to be homogeneous and isotropic. The metric of these models is called the Friedmann–Lemaître–Robertson–Walker (FLRW) metric. It takes the following form in spherical coordinates:

$$ds^2 = c^2dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right], \tag{1}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$, k takes the values $-1, 0$, or 1, depending on whether the spatial section has negative, zero or positive curvature, respectively, and $a(t)$ is the scale factor, which describes the expansion of the universe.

The geometric properties of the homogeneous and isotropic 3-space corresponding to the hypersurface $t = cst$ depend on whether $k = -1, 0$ or 1. To reveal that it is respectively the sphere, the plane and the hyperboloid, it is common to introduce a new radial coordinate, χ, defined by

$$d\chi \equiv \frac{dr}{\sqrt{1-kr^2}}. \tag{2}$$

An integration yields to

$$r = \begin{cases} \sin\chi & k = +1, \\ \chi & k = 0, \\ \sinh\chi & k = -1. \end{cases} \tag{3}$$

Using this metric in the Einstein field equations, the Friedmann equations can be derived. They can be written as

$$
\begin{aligned}
H^2 &\equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\rho_m + \rho_r + \rho_\Lambda\right) - \frac{c^2 k}{a^2}, \\
\frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}\left(\rho_m + \rho_r + \frac{3p}{c^2} - 2\rho_\Lambda\right),
\end{aligned}
\tag{4}
$$

where H is the Hubble parameter, ρ_m the matter density, ρ_r the radiation density, p the pressure and $\rho_\Lambda \equiv \Lambda c^2/8\pi G$ with Λ the cosmological constant, commonly associated to dark energy.

It is common to work with the density parameters defined as $\Omega \equiv \rho/\rho_c$, where ρ_c is the critical density: $\rho_c(t) \equiv 3H^2(t)/8\pi G \simeq 9.2 \times 10^{-27}$ kg/m^3 [4]. Their present values are measured [4] as

$$
\Omega_\Lambda^0 \simeq 0.7, \quad \Omega_r^0 \simeq 5 \times 10^{-5}, \quad \Omega_m^0 \simeq 0.3. \tag{5}
$$

The sum of the density parameters of dark energy, matter and radiation is almost equal to one, which means that our universe is very close to having a flat spatial geometry.

Despite its successes, the standard Λ-CDM model suffers from some fundamental problems. In particular, it requires an extremely accurate fine tuning of the initial conditions.

(1) To explain the high isotropy of the CMB after correction of the dipole anisotropy, while regions separated by more than 2° on the sky had no time to interact, the Big-Bang must happen with the same temperature everywhere: this is the *horizon problem.*

(2) To explain the observed flat curvature, while any deviation from the flat geometry would increase with time, the universe had to start with a very precise zero curvature: this is called the *flatness problem.* In 1981, Alan Guth proposed the primordial inflation theory to solve the horizon and flatness problems [5]. But this theory remains highly speculative because the existence of the inflaton remains hypothetical as of today.

(3) To explain that the universe contains almost exclusively matter and very little antimatter, a slight asymmetry between the production of matter and antimatter in the Big-Bang is necessary, although not predicted by the Standard Model of particle physics: this is the *asymmetry problem.*

2 Black Hole Universe

2.1 The Schwarzschild Geometry

The Schwarzschild metric specifies the geometry generated by a spherical mass distribution. In the Schwarzschild's coordinate system (t, r, θ, ϕ), the geometry takes the following form

$$
ds^2 = \left(1 - \frac{2\mu}{r}\right) c^2 dt^2 - \left(1 - \frac{2\mu}{r}\right)^{-1} dr^2 - r^2 d\Omega^2, \tag{6}
$$

where we introduced the parameter $\mu = GM/c^2$.

The Schwarzschild metric is characterized by two singularities. A genuine singularity lies at $r = 0$, and indicates a break in the model used to describe the geometry of spacetime. A coordinate singularity happens at the Schwarzschild radius $r = 2\mu \equiv r_s$, and is due to a pathology of the Schwarzschild's coordinate system near r_s.

The Schwarzschild solution is obtained considering a zero energy-momentum tensor. It therefore corresponds to a vacuum solution, exterior to any spherical mass distribution of radius R. Its validity stops at the surface of the object, and a different metric must be used to describe the body itself, i.e. for $r < R$ [6]. Yet we can consider the Schwarzschild vacuum solution for all values of r, considering it is generated by a point mass, located at the origin of the coordinate frame. Since the source is a point, r_s does not describe its radius but a critical length, as measured in the Schwarzschild's coordinate system, that divides the universe into two distinct parts: an outside region I ($r > r_s$) and an inside region II ($r < r_s$).

Another interesting feature of the Schwarzschild metric (6) is the following. In this metric, the nature of space is differentiated from that of time by the presence of the positive sign in front of the squared temporal interval. From region I to region II, however, the spatial and temporal metric components, g_{tt} and g_{rr}, change sign. The t-direction becomes spacelike whereas the r-direction becomes timelike. Inside region II, the coordinates t and r therefore reverse their character with respect to region I [7]. This feature is further discussed in the next section.

The structure of the light cones in a spacetime diagram is determined by considering the radial trajectories of incoming and outgoing photons in the Schwarzschild geometry (6). These trajectories are given by

$$ct = r + 2\mu \ln\left|\frac{r}{2\mu} - 1\right| + \text{cst} \qquad \begin{array}{l}\text{reg. I: outgoing photons}\\ \text{reg. II: incoming photons}\end{array} \tag{7}$$

and

$$ct = -r - 2\mu \ln\left|\frac{r}{2\mu} - 1\right| + \text{cst.} \qquad \begin{array}{l}\text{reg. I: incoming photons}\\ \text{reg. II: outgoing photons}\end{array} \tag{8}$$

When r tends to infinity in region I, light cones are delimited by lines of slope ± 1. The causal structure of the events of Minkowski's geometry at very large distance from the mass source is recovered because the gravitational field becomes negligible. When r tends to 2μ, the slopes of the lines tend to $\pm\infty$, and the light cones completely close at $r = 2\mu$. This suggests that incoming photons need an infinite (coordinate) time to reach the Schwarzschild radius. In region II, the light cones tilt by 90°, and are now oriented towards the singularity at $r = 0$. As illustrated in Fig. 1, all photons, incoming and outgoing, therefore inevitably end up in the center of the mass source.

The singularity at $r = 2\mu$ not being physical, the singular behaviour of the photon world lines at $r = 2\mu$ is indicative of an inappropriate choice of coordinate system. The problem can be avoided by introducing new coordinates

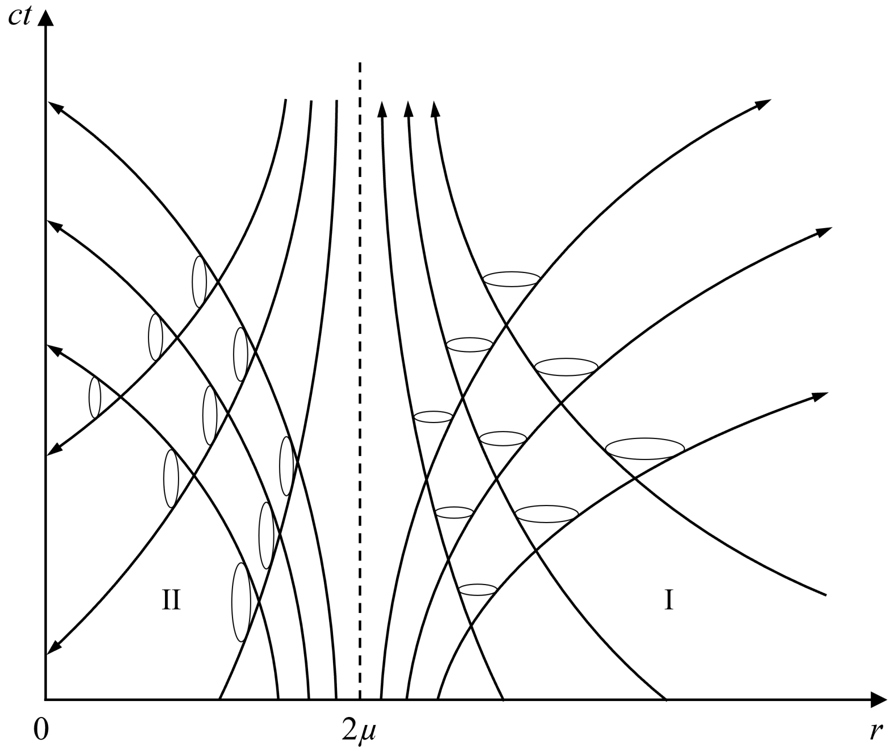

Figure 1: Spacetime diagram of the Schwarzschild solution in Schwarzschild's coordinates. Radial ingoing and outgoing null geodesics are represented to illustrate the light cone structure of the Schwarzschild solution. Adapted rom Fig. 11.1 in [15].

systems. The Kruskal-Szekeres coordinates, introduced in 1960 by both Martin Kruskal [8] and György Szekeres [9], turn out to be appropriate and cover the Schwarzschild geometry entirely. This means that all geodesics either extend to infinite values of their affine parameter or end at a past or future singularity.

To conclude this section, let us note that Birkhoff's theorem states the uniqueness of the Schwarzschild geometry as the spacetime outside a spherically symmetric body of mass M. The external spacetime is therefore necessarily static.

2.2 Motivations for a Black Hole universe

The Schwarzschild radius of a black hole is given by

$$r_s \equiv \frac{2GM}{c^2} = 2.95 \left[\frac{M}{M_\odot}\right] \text{km}, \tag{9}$$

where M is the mass of the black hole and $M_\odot$ is the mass of the Sun. Its average density is

$$\langle \rho_{BH} \rangle \equiv \frac{3M}{4\pi r_s^3} = \frac{3c^6}{32\pi G^3 M^2} \simeq 1.7 \left[\frac{M}{10^8 M_\odot}\right]^{-2} \text{g cm}^{-3}. \tag{10}$$

Thus, the larger the mass of the black hole, the smaller its average density. Small black holes have a huge density, while for large black holes, the density becomes very low. For such low densities, elementary particle interactions are irrelevant, and we do not have to worry about quantum effects [10]. As an example, the mean density of a supermassive black hole with $M \sim 10^8 M_\odot$ is only that of water.

The mass of the observable universe, whose radius is $r_{ph} \simeq 4.6 \times 10^{10}$ l.yr $\simeq 4.1 \times 10^{26}$ m [2], is of the order of $1.4 \times 10^{23} M_\odot$. The minimal mean density for that region to be a black hole is about 0.9×10^{-30} g/cm^3. The present day matter density of the universe is $\rho_m^0 \simeq 0.3\rho_c^0 \simeq 10^{-30}$ g/cm^3 [4]. So, even if the universe is not much larger than the part we can observe, it meets the criterion for being a black hole. Of course, the larger it is, the lower the critical density and the more likely it is to be a black hole.

2.3 Friedmann Universe Inside a Black Hole

The cosmic gas filling our universe is similar to a pressureless fluid of uniform density [11]. Galaxies can be seen as dust particles that interact with each other only gravitationally. For this reason and for the sake of simplicity, the universe is idealized by a Friedmann dust-filled model to investigate the black hole universe idea. These are models without cosmological constant and radiation, but with a non-zero matter content, i.e. $\Omega_\Lambda^0 = 0$, $\Omega_r^0 = 0$, and $\Omega_m^0 \neq 0$.

The dynamic of a Friedmann universe depends on its geometry. There are two possible fates for the universe depending on the value of the average density: either the universe is expanding-contracting, finite in time and space, or it is expanding indefinitely, infinite in time and space. The flat and open Friedmann universes are spatially infinite. However, if these geometries are confined in a region limited by a given radius $R = R_m$, making them spatially finite, they can be included in a portion of space that is inside a larger Schwarzschild space, as we discuss next.

To build a Friedmann black hole universe, we need to specify the world lines that the surface of a collapsing object follow in Schwarzschild geometry and also to describe its interior. To do so, rather complicated mathematics are needed [7]. However, it is simpler in the case of an object with uniform density and zero pressure. The collapse of a uniform ball of dust was first treated in details by Oppenheimer and Snyder [12] in 1939, before a clearer approach was presented by Beckedorff and Misner [13] in 1962.

It should be noted that, although the collapse of a ball of dust can hardly represent a realistic stellar collapse, it is much more appropriate to describe the collapse of a black hole universe, as the galactic gas that fills the universe is, to a good approximation, of zero-pressure and uniform density.

Birkhoff's theorem tells us that the Schwarzschild solution can be adjoined to a dynamical mass distribution, as long as the spherical symmetry is maintained. There are some questions worth asking. Could the spherical collapse of a ball of dust be modelled as the union between an inner Friedmann universe, whether open, flat or closed, and an outer Schwarzschild space? Given two universes with certain boundary manifolds, what conditions must be satisfied in order that the universes can be fitted together to form a single one by matching their boundaries? Or, can a Friedmann dust-filled universe be regarded, at any given moment, as embedded in a Schwarzschild geometry?

The union of the universes is a solution of Einstein field equations if and only if two matching conditions are satisfied [7]: the intrinsic and extrinsic geometries of the three-dimensional boundary manifold must be the same, when calculated with either metric. The intrinsic curvature of a manifold is characterized by the Riemann curvature tensor, while the extrinsic curvature depends on how a surface is embedded in a larger space. These conditions were proven in [13, 14], and their results are used. The first condition, namely the equality between the intrinsic curvatures, is met if the metrics agree on the surface. We demonstrate in the following that this is indeed the case. The second condition is not demonstrated in this work.

Because the inside part is modelled by a ball of dust, without pressure gradients, the motions of the particles that belong to its surface are not deflected. They must therefore move along radial geodesics in the exterior Schwarzschild geometry [7]. To model the interior, we consider that it is homogeneous and isotropic everywhere, except at the surface. Then, it is identical to a dust-filled Friedmann cosmological model, which can be open ($k = -1$), flat ($k = 0$), or closed ($k = 1$). The type of interior Friedmann model depends on the initial conditions of the collapse. We consider that the radius $R = R_m$ describes the boundary manifold in the Friedmann universe, and that M is the BH mass generating the Schwarzschild vacuum. Three hypothetical cases of spherical collapse are discussed:

(1) *Closed case*: If the collapse begins at rest with finite radius $r = R_m$ at $t = 0$, it is the closed case ($k = 1$) which is appropriate because an initial zero-rate of change of density at finite radius is equivalent to a moment of maximum expansion. The interior of the ball of dust is therefore identical to a portion of a closed Friedmann universe. The evolution of the scale factor is given by

$$\begin{aligned} a &= \frac{a_m}{2}(1+\cos\eta), \\ ct &= \frac{a_m}{2}(\eta+\sin\eta) \end{aligned} \tag{11}$$

and the world lines of the material universe surface in the Schwarzschild geometry are given by the parametric equations of the cycloid [15] and by the Khuri formulae [16, 7]:

$$r(\eta) = \frac{R_m}{2}(1+\cos\eta), \tag{12}$$

$$c\tau(\eta) = \left(\frac{R_m^3}{8\mu}\right)^{1/2}(\eta+\sin\eta)\,, \tag{13}$$

$$\begin{aligned} ct = & \left[\left(\frac{R_m}{2}+2\mu\right)\left(\frac{R_m}{2\mu}-1\right)^{1/2}\right]\eta \\ & +\frac{R_m}{2}\left(\frac{R_m}{2\mu}-1\right)^{1/2}\sin\eta \\ & +2\mu\ln\left|\frac{(R_m/2\mu-1)^{1/2}+\tan(\eta/2)}{(R_m/2\mu-1)^{1/2}-\tan(\eta/2)}\right| + cst, \end{aligned} \tag{14}$$

where $R_m = a\sin\chi_m$ and $\mu = GM/c^2$. Following [14], the last equation can be rewritten as

$$\begin{aligned} \frac{ct}{2\mu} &= \ln\left|\frac{\beta-\cot\eta/2}{\beta+\cot\eta/2}\right| + \beta[\eta+\alpha(\eta+\sin\eta)], \\ \alpha &\equiv \frac{R_m}{4\mu}, \beta \equiv \sqrt{2\alpha-1}. \end{aligned} \tag{15}$$

Then, we have

$$\begin{aligned} dt &= \frac{2\mu\beta}{\beta^2\sin^2\eta/2-\cos^2\eta/2} + 2\mu\beta\left[1+\alpha(1+\cos\eta)\right] \\ &= \frac{2\mu\alpha^2(1+\cos\eta)^2\beta d\eta}{\alpha(1+\cos\eta)-1}, \\ dr &= \frac{a_m\sin\chi_0\sin\eta d\eta}{2} = 2\mu\alpha\sin\eta d\eta, \\ 1-\frac{2M}{r} &= \frac{a\sin\chi_m-2\mu}{a\sin\chi_m} = \frac{\alpha(1+\cos\eta)-1}{\alpha(1+\cos\eta)}. \end{aligned} \tag{16}$$

Using the previous equations, the Schwarzschild metric (6) at the boundary manifold becomes

$$ds^2 = \left(1 - \frac{2\mu}{r}\right) c^2 dt^2 - \left(1 - \frac{2\mu}{r}\right)^{-1} dr^2 - r^2 d\Omega^2 =$$

$$\frac{4\mu^2(2\alpha-1)\alpha^3(1+\cos\eta)^3 + (1+\cos\eta)4\mu^2\alpha^3\sin^2\eta}{\alpha(1+\cos\eta)-1}$$

$$= a^2 d\eta^2 - a^2 \sin^2\chi_m d\Omega^2. \tag{17}$$

The Schwarzschild metric reduces to the closed FLRW metric at $\chi = \chi_m$. To get this result, however, the BH mass must be given by

$$M = \frac{c^2}{2G} a_m \sin^3\chi_m. \tag{18}$$

(2) *Flat case*: If the collapse begins at rest at an infinite radius, the flat case ($k = 0$) is appropriate because an initial zero-rate of change of density at infinite radius only exists in the final state of a flat universe. The interior of the ball of dust is then identical to a portion of a flat running backwards Friedmann universe. The evolution of the scale factor is given by

$$a(t) = \left(\frac{3}{2}H_0 t\right)^{2/3} = \left(\frac{8\pi G\rho_0}{3}\right)^{1/3} \left(\frac{3}{2}t\right)^{2/3}, \tag{19}$$

where we introduced the density of the flat universe as $\rho_0 = 3H_0^2/8\pi G$.

The world lines of the material universe surface in the Schwarzschild geometry are given by [15]:

$$\frac{c\tau}{2\mu} = -\frac{2}{3}\left(\frac{r}{2\mu}\right)^{3/2} + cst, \tag{20}$$

$$\frac{ct}{2\mu} = -\frac{2}{3}\left(\frac{r}{2\mu}\right)^{3/2} - 2\left(\frac{r}{2\mu}\right)^{1/2} + \ln\left|\frac{\sqrt{r/2\mu}+1}{\sqrt{r/2\mu}-1}\right| + cst, \tag{21}$$

where the radial coordinate in a flat Friedmann universe is defined by $r = a\chi$. Therefore, the surface lies at a position $r = R_m = a\chi_m$.

Then, we have

$$\frac{dr}{cd\tau} = -\left(\frac{2\mu}{r}\right)^{\frac{1}{2}}$$

$$\text{and} \quad \frac{cdt}{dr} = -\left(\frac{r}{2\mu}\right)^{\frac{1}{2}} \left(1 - \frac{2\mu}{r}\right)^{-1}. \tag{22}$$

Using the previous equations, the Schwarzschild metric (6) at the boundary manifold becomes

$$ds^2 = \left(1 - \frac{2\mu}{r}\right) c^2 dt^2 - \left(1 - \frac{2\mu}{r}\right)^{-1} dr^2 - r^2 d\Omega^2$$

$$= c^2 d\tau^2 - a^2\chi_m^2 d\Omega^2, \tag{23}$$

which corresponds to the flat FLRW metric at $\chi = \chi_m$.
(3) *Open case*: If the ball of dust collapses from infinity with a finite initial inward velocity, it corresponds to the final state of an open Friedmann universe. The open case ($k = -1$) is thus appropriate to describe the collapse. Similarly to the closed case, one can show that the Schwarzschild metric (6) at the boundary manifold becomes

$$\begin{aligned} ds^2 = \left(1 - \frac{2\mu}{r}\right) c^2 dt^2 - \left(1 - \frac{2\mu}{r}\right)^{-1} dr^2 - r^2 d\Omega^2 \\ = \frac{4\mu^2(2\alpha - 1)\alpha^3(\cosh\eta - 1)^3 + (\cosh\eta - 1)4\mu^2\alpha^3 \sinh^2\eta}{\alpha(\cosh\eta - 1) - 1} \\ = a^2 d\eta^2 - a^2 \sinh^2 \chi_m d\Omega^2. \end{aligned} \tag{24}$$

We recognize the open FLRW metric at $\chi = \chi_m$. To get this result, however, the BH mass must be given by

$$M = \frac{c^2}{2G} a_m \sinh^3 \chi_m. \tag{25}$$

Note that, in the closed case, an additional constraint comes from the second condition of junction. When demanding the extrinsic curvatures to be the same, the connection has to be made at $\chi_m < \pi/2$ [13, 14]. There is no such restriction on the flat and open models, so they may replace the interior at any χ_m.

Despite the mathematical beauty residing in the union of different geometries in hypothetical collapses, their importance lies in their physical utility. The three geometries can be truncated and included in a finite portion of space bounded by their Schwarzschild radius. In the flat and open Friedmann model cases, however, the initial conditions considered are that of an object which collapses from an infinite radius. We shall only consider the more physically relevant case of an object that starts collapsing from a finite radius.

3 Reversed Time

3.1 Internal Schwarzschild Geometry

When $r < r_s$, we have seen in Sect. 2.1 that a remarkable change occurs in the nature of the Schwarzschild coordinates. The signs before the squared temporal interval dt^2 and the radial interval dr^2 are reversed in the Schwarzschild metric (6), illustrating that coordinates t and r reverse their character when the gravitational radius is crossed. What to the outside observer is a radial coordinate becomes to the inside observer a temporal coordinate and vice versa [17].

What does it mean for r to change character from a spacelike coordinate to a timelike one? To answer this question, let us consider an explorer in a jet-powered spaceship [7]. Before arriving at r_s, the explorer always has the option to turn his jets on and change his motion from a decreasing r (infall) to an increasing r (escape). But once he crosses the Schwarzschild radius, the situation changes: the further decrease of r represents the flow of time. No command that the explorer can give to his rocket will turn time back. The invisible power that makes humans age drags the rocket in from time coordinate $r = r_s$ to the later value of $r = 0$. For the same reason that time always passes, an observer in region II cannot stay at rest, i.e. at a constant value of r, but is forced to move in towards the intrinsic singularity at $r = 0$.

Let us approach the change in the nature of the coordinates in a more quantitative way. The character of the universe within the Schwarzschild radius is better understood if we adopt a nomenclature that reflects the physical properties of the coordinates [17]. We therefore consider a transformation that replaces r by ct and ct by z in the Schwarzschild metric (6):

$$\begin{cases} r \to ct, \\ ct \to z. \end{cases} \tag{26}$$

Applying this transformation to Eq. (6) yields

$$ds^2 = \left(\frac{t_s}{t} - 1\right)^{-1} c^2dt^2 - \left(\frac{t_s}{t} - 1\right) dz^2 - c^2t^2d\Omega^2, \tag{27}$$

where we introduced $t_s \equiv r_s/c$. This equation corresponds to the Schwarzschild geometry written in the coordinate system (z, ct, θ, ϕ), which will be called the reversed coordinate system in the following. The Schwarzschild metric, static outside the gravitational radius, is time-dependent inside. It therefore describes a dynamic spacetime, which can be called a universe. The "point" source of the Schwarzschild metric is henceforth an "instant" source.

We now investigate the behaviour of clocks that remain at rest in the reversed coordinate system, i.e. $dz = d\Omega = 0$. Using the metric (27), we have

$$c^2d\tau^2 = \frac{c^2dt^2}{t_s/t - 1}. \tag{28}$$

This equation relates the proper time kept by the clocks at rest to the coordinate time. Integrating the positive root of this expression from 0 to t gives

$$\tau = t_s \left\{\pi/2 - \arccos\sqrt{t/t_s} - \sqrt{(t/t_s)(1 - t/t_s)}\right\}, \tag{29}$$

where the constant of integration has been chosen so that $\tau = 0$ when $t = 0$, and $\tau = \pi t_s/2$ when $t = t_s$. The relation between coordinate and proper times is not simple. The coordinate time, however, advances in one-to-one correspondence with the proper time [17].

The negative root can also be integrated. The result is

$$\tau = t_s \left\{\pi/2 + \arccos\sqrt{t/t_s} + \sqrt{(t/t_s)(1 - t/t_s)}\right\}. \tag{30}$$

When $t = t_s$, then $\tau = \pi t_s/2$, whereas when $t = 0$, $\tau = \pi t_s$, meaning that as the coordinate time decreases, the rest clocks proper time increases.

The exterior part of a collapsing spherical object is a piece of Schwarzschild geometry, as required by the Birkhoff's theorem. If the object is filled with a zero-pressure gas of uniform density, its interior geometry is strictly identical to that of a Friedmann universe. When the object collapses through its Schwarzschild radius, it becomes a black hole in the exterior space. The collapse ends at the central singularity, which, seen from the inside, turns out to be an instant in time rather than a place in space.

3.2 The Reversed Black Hole Universe Model

For the interior Friedmann universe to be an idealization of our observable universe, we face an apparent contradiction here: the universe is observed to expand, not to collapse. Thus, if we accept the conclusion of Sect. 2.2, that our universe lies inside a black hole, we *have* to assume that the time for an observer inside a black hole runs in the opposite direction to that of an observer outside[1]. This time, which would correspond to (30), is referred to as the reversed time in the sequel.

We thus model our material universe as a dust-filled closed Friedmann model, whose constituents are galaxies that interact only through gravity. It is collapsing inside a piece of Schwarzschild geometry, and appears as a black hole in the exterior space. The observers inside perceive an expansion, as their proper time flows in the opposite direction. The genuine singularity, considered as a central position from the outside, is seen from the inside as the beginning of time, common to all inside observers. The Schwarzschild radius is the locus of an event horizon from the outside, preventing any inside signals from reaching the outside world. From the inside, it is an event in the future, corresponding to the maximal expansion of the interior space in the exterior geometry, that it makes no sense to try to find with a telescope.

The reversed black hole universe model thus consists of three parts. Going from the outside to the inside, they are as follows. The first one is a piece of Schwarzschild space that surrounds a black hole and in which the space and time coordinates have their usual character. The second part starts at the Schwarzschild radius, where the nature of the coordinates changes. The one that described space for an outer observer describes time for an inner observer. The interior geometry is dynamical. Instead of collapsing, however, the inner universe appears to expand to observers inside it. The third part is the dust-filled closed Friedmann model. An empty dynamical space (second part) therefore separates the vacuum outside the Schwarzschild radius (first part) from the inner universe (third part). The Friedmann universe, which appears to be expanding from the inside but contracting from the outside, corresponds to level 2 (material universe) as defined in the first section. It contains our observable universe, that we see as expanding.

[1] This may not be as awkward as it appears at first sight, as the outside coordinate corresponding to the inside time is r, a spacelike coordinate. Moreover, it has no impact on the exterior space: a black hole seen by an outside observer will remain so.

4 Boundary Conditions

We have seen that a number of problems related to the initial conditions plague the standard cosmological model. But when we consider that the universe belongs to a black hole in a larger space, and that time flows in the opposite direction inside, the situation changes dramatically. Indeed, the initial conditions are not set at the singularity, but rather at the boundary between the inside and the outside of the black hole, thus at the Schwarzschild radius.

Consider first the horizon problem, i.e. the high degree of isotropy in the CMB while different regions did not have enough time to interact and reach the same temperature. In our model, there is no time limit on the existence of the material universe before it collapsed. It could have taken a very long time for the material universe to become a black hole and collapse, allowing its various parts to interact.

Let us also stress that there is no cosmological principle in our model: we are located in a random place of the material universe, whose assumed homogeneity is a simplifying assumption and not a consequence of any prior hypothesis. The observed homogeneity might just be the consequence of the fact that we can only observe a small part of a much larger material universe[2].

Our solution to the flatness problem is the following. In an expanding Friedmann model, the difference between the density parameter of matter and one (corresponding to a flat spatial geometry) is a monotonously increasing function of time. Any deviation from flatness increases with time, which sets drastic constraints on the initial spatial curvature. In our model, the boundary conditions are set at the Schwarzschild radius, thus in our far future. Any non-zero curvature at the boundary will decrease as one approaches the singularity.

Moreover, the larger the material universe, the larger its Schwarzschild radius and the further into the future the event horizon is for the inner observer. The interval over which the time coordinate decreases is longer, and the universe has more time to tend towards a flat geometry. Thus, both the homogeneity and the flatness of the observable universe suggest that it is only a small part of the material universe.

The asymmetry problem also arises from the initial conditions. In the Λ-CDM model, the universe starts from a state of extremely high temperature and density. Such a thermal background should produce equal numbers of particles and antiparticles[3]. The symmetry is maintained as the particles and antiparticles annihilate when the temperature gets sufficiently low. The universe should therefore display a matter–antimatter symmetry. But it is not so: the antimatter is missing. One has to assume a slight excess of particles, that is not predicted by the Standard Model.

This problem vanishes in our model. In the standard (inflation) model, particle–antiparticle pairs are created from energy. In the black hole model,

[2]Moreover, the high degree of isotropy of the CMB is observed after correcting for a dipole anisotropy that is interpreted as arising because of our motion with respect to the CMB rest frame, but this is also an assumption.

[3]In the inflationary model, this happens at the end of the inflation phase.

we start from matter. The question of where this matter comes from does not make more sense that the question of where the Big-Bang (pre–inflation) energy comes from in the Λ-CDM model. One has to start with something and, after all, it is well known since Einstein that matter and energy are equivalent.

5 Conclusion

The purpose of this work was twofold. First, we decided to consider the consequences of the conclusion that the density and radius of the observable universe satisfy the Schwarzschild criterion, indicating that it should be inside a black hole, if included in a larger space.

Second, we addressed the apparent contradiction of an expanding universe inside a black hole, while it would be expected to collapse. The change of the nature of the radius and time coordinates at the surface of the black hole suggested that the negative root of (30) might be the appropriate solution in this case.

The consequence of these two hypotheses (i.e. that the material universe is a black hole and that the flow of time is reversed inside it) is that the boundary conditions are to be set at the surface of the black hole rather than at the singularity, which allows to propose solutions to some fundamental problems without having recourse to inflation theory.

In this paper, we used the simplest approximation for the material universe, i.e. a uniform ball of dust. While the uniformity derives from the cosmological principle in the usual model, such a principle does not apply to our black hole universe model and should thus be considered only as a first approximation. In particular, it would be interesting to examine the consequences of a more realistic hypothesis on the evolution of the scale factor, in relation to the apparent acceleration of the expansion.

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