Model Updating Based on Frequency Response Functions Using A General Finite Element Code

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Abstract

Model updating techniques using frequency response function (FRF) data are studied in this paper. The numerical techniques are discussed for implementation with a large commercial finite element (FE) code. System equivalent reduction expansion process is adopted to reduce the complete FE solutions onto the experimental degrees of freedom. The rank-deficiency difficulty with this method is overcome using either of two numerical techniques: diagonal perturbation and singular value decomposition. This second technique is also used in solving the updating equation. Experimental FRF data are compared with the FE solutions, and the updated model parameters are obtained via an iteration procedure. A simplified frequency domain assurance criterion is proposed to evaluate the correlation between the FE model and the measured structure at the chosen frequencies. After verifying the efficiency of the methods with several benchmark tests, the program is applied to an aeroplane model test. Some conclusions are given and remaining problems illustrated.

Keywords : Model updating; FRF updating; Error localisation; Structural dynamics.

1. Introduction

Precise description of the dynamic behaviour of aerospace, mechanical and civil engineering structures is very concerned. Due to the fact that experimental analysis is usually expensive and time consuming, numerical simulation by FE method is promoting. However, the existing differences in geometry and material between the FE models and real structures may make the simulation undependable. In recent years, a significant amount of work has dealt with evaluating and reducing the distance between the numerical models and the experimental structures in terms of their dynamic behaviour [1]. On the other hand, using an updated FE model, damage identification of structures may be performed.

Generally, model updating may be performed by use of global methods considering model system matrices as abstract mathematical operators in an optimisation procedure. However the engineer is not able to understand which is the related physical parameter changes corresponding to the correction. This reduces seriously the reliability of updating procedure. In this paper we use an alternative approach (local method) by estimating the uncertain physical parameters while preserving the principal FE properties of the initial model. The identified modification of model is associated clearly to physical parameter changes. With an updated model, one may predict the dynamic behaviour of structures for different loading condition and for a modified structure.

Historically, modal parameters have been preferred as input for model updating since natural frequencies constitute data obtained with great confidence, and mode shapes are a highly condensed database on deformations suffered in structures. During the last years an increasing group of researchers have used the frequency domain data such as FRF that are direct results of experimental analysis, so modal analysis (necessary for obtaining modal parameters) and the errors with this analysis are avoided. The first implementation of frequency domain least squares updating was proposed by Lin and Ewins [2]. To overcome the coordinate incompleteness in the measured responses, analytical counterparts at slave (unmeasured) degrees of freedom (DOF) were adopted as a very simple form of FRF expansion. Along a contraire direction, the reduction of the FE model to the measured DOF in different ways has been advocated, see D'Ambrogio *et al.* [3], Lammens *et al.* [4], Pascual *et al.* [5], for example. Although theoretical method of updating with frequency data has been well developed, the numerical implementation and tests were performed up to now mainly with some simple academicals examples where the FE modelling and data transformation may be easily performed by using some simple programming tools. Obviously, numerical implementation with a large FE code is necessary for a practical application to engineering problems, which remains a challenge.

In this paper, we study FE-model updating method using FRF data with a commercial finite element code SAMCEF [6]. The related numerical techniques are discussed. System equivalent reduction expansion process (SEREP) is adopted to map the complete FE eigenvectors onto the experimental slave DOF. The rank-deficiency difficulty with this method is overcome with the proposed numerical techniques. The FRF data of the FE model and experimental analysis are correlated and the model parameter updating is explicitly formulated. The updated model is obtained via an iteration procedure. A simplified frequency domain assurance criterion (SFDAC) is proposed to evaluate the correlation between the updated model and the measured structure at the chosen frequencies. The efficiency of the method is examined with several benchmark tests.

2. Theoretical formulation

2.1 Model updating using FRF data

From the point of view of FE modelling, the dynamic behaviour of structures without damping is determined by a stiffness matrix $\mathbf{K} \in \Re^{N_{fe} \times N_{fe}}$ and a mass matrix $\mathbf{M} \in \Re^{N_{fe} \times N_{fe}}$, where N_{fe} is the DOF number of the FE model. Starting from the current iteration n with the current matrices ($\mathbf{K}^n$, $\mathbf{M}^n$), we search for the updated ones ($\mathbf{K}^{n+1}$, $\mathbf{M}^{n+1}$) at iteration n+1. The dynamic equilibrium equation for the updated model is

$$\left(\mathbf{K}^{n+1} - \omega_i^2 \mathbf{M}^{n+1}\right)\bar{\mathbf{h}}_i = \mathbf{f} \quad \text{or} \quad \mathbf{Z}_i^{n+1}\bar{\mathbf{h}}_i = \mathbf{f} \tag{1}$$

and for the current FE model is:

$$\left(\mathbf{K}^{n} - \omega_i^2 \mathbf{M}^{n}\right)\mathbf{h}_i^n = \mathbf{f} \quad \text{or} \quad \mathbf{Z}_i^{n}\mathbf{h}_i^n = \mathbf{f} \tag{2}$$

where ω_i denotes the chosen i frequency; $\bar{\mathbf{h}}_i, \mathbf{h}_i^n$ are the FRF of the experimental and current models, respectively; $\mathbf{f}$ is the excitation vector ($\mathbf{f}=\mathbf{e}^k$, that means a unit force on k-th DOF of FE model). From Eq. (1-2) one has

$$(\mathbf{Z}_i^{n+1} - \mathbf{Z}_i^n)\bar{\mathbf{h}}_i = \mathbf{Z}_i^n(\mathbf{h}_i^n - \bar{\mathbf{h}}_i) \tag{3}$$

or

$$\mathbf{H}_i^n \Delta\mathbf{Z}_i \bar{\mathbf{h}}_i = \mathbf{h}_i^n - \bar{\mathbf{h}}_i \tag{4}$$

with

$$\mathbf{H}_i^n = (\mathbf{Z}_i^n)^{-1}, \quad \Delta\mathbf{Z}_i = \mathbf{Z}_i^{n+1} - \mathbf{Z}_i^n \tag{5}$$

The updated dynamic stiffness matrix, $\mathbf{Z}_i^{n+1}$, is defined as a function of the updating parameters $\mathbf{p}$, and can be expressed as a Taylor expansion of the current dynamic stiffness matrix $\mathbf{Z}_i^n$ as follows:

$$\mathbf{Z}_i^{n+1} = \mathbf{Z}_i^n + \Delta\mathbf{Z}_i = \mathbf{Z}_i^n + \frac{\partial \mathbf{Z}_i^n}{\partial \mathbf{p}}\mathbf{p} + o(\mathbf{p}^2) \tag{6}$$

Retaining only first order terms and substituting for $\Delta\mathbf{Z}_i$ in (4) lead to

$$\mathbf{A}_i \mathbf{p} = \mathbf{b}_i \tag{7}$$

with

$$\mathbf{A}_i = \mathbf{H}_i^n \frac{\partial \mathbf{Z}_i^n}{\partial \mathbf{p}}\bar{\mathbf{h}}_i = \mathbf{H}_i^n \left[\frac{\partial \mathbf{Z}_i^n}{\partial p_1}\bar{\mathbf{h}}_i,\ \frac{\partial \mathbf{Z}_i^n}{\partial p_2}\bar{\mathbf{h}}_i,\ \ldots,\ \frac{\partial \mathbf{Z}_i^n}{\partial p_{Np}}\bar{\mathbf{h}}_i\right] \tag{8}$$

$$\mathbf{b}_i = \mathbf{h}_i^n - \bar{\mathbf{h}}_i \tag{9}$$

Convergence is performed when the FRF value $\mathbf{h}_i^n$ of the updated model becomes ideally identical or close to the measured $\bar{\mathbf{h}}_i$, corresponding to a minimization of output residue at any frequencies ω_i:

$$\min_{\mathbf{p}} \left\|\bar{\mathbf{h}}_i - \mathbf{h}_i^n\right\|^2 \tag{10}$$

In order to exploit the redundancy of the experimental information, Eq. (7-9) should be repeated for a set of frequencies ω_i, i =1…N_{fr} (N_{fr} is the number of chosen frequencies) spanning extensive frequency range. This leads to the following system

$$\begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \\ \cdots \\ \mathbf{A}_{N_{fr}} \end{bmatrix} \mathbf{p} = \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \cdots \\ \mathbf{b}_{N_{fr}} \end{bmatrix} \tag{11}$$

which can be simply written as

$$\mathbf{A}_h \mathbf{p} = \mathbf{b}_h \tag{12}$$

Here, the index h means updating with FRF data $\mathbf{h}$. Each row of the sensitivity matrix $\mathbf{A}$ in (8), defines the sensitivity of responses at a particular DOF to the updating parameter $\mathbf{p}$. More description and discussions are referred to the literature (e.g. [7]).

2.2 Model updating using natural frequency data

Natural frequencies constitute data obtained with great confidence so we try introducing them into the present FRF updating procedure in order to improve the precision and convergence. A simple description is presented here and a complete theoretical development may be found in [7]. Starting from the dynamic equilibrium equation and pre-multiplying it by r-th FE modal shape $\phi_r \in \Re^{N_{fe} \times 1}$

$$\phi_r^{\mathrm{T}}\left[\mathbf{K} - \omega_r^2 \mathbf{M}\right]\phi_r = 0 \tag{13}$$

Differentiating (13) with respect to updating parameter $\mathbf{p}$

$$\frac{\partial \phi_r^{\mathrm{T}}}{\partial \mathbf{p}}\left[\mathbf{K}-\omega_r^2\mathbf{M}\right]\phi_r + \phi_r^{\mathrm{T}}\frac{\partial}{\partial \mathbf{p}}\left[\mathbf{K}-\omega_r^2\mathbf{M}\right]\phi_r + \phi_r^{\mathrm{T}}\left[\mathbf{K}-\omega_r^2\mathbf{M}\right]\frac{\partial \phi_r}{\partial \mathbf{p}} = 0 \tag{14}$$

Due to (13) the first and third terms of (14) are zero and the term in the middle gives

$$\frac{\partial}{\partial \mathbf{p}}\left[\mathbf{K}-\omega_r^2\mathbf{M}\right] = \frac{\partial \mathbf{K}}{\partial \mathbf{p}} - \frac{\partial \omega_r^2}{\partial \mathbf{p}}\mathbf{M} - \omega_r^2\frac{\partial \mathbf{M}}{\partial \mathbf{p}} = 0 \tag{15}$$

Finally one finds

$$\frac{\partial \omega_r^2}{\partial \mathbf{p}} = \phi_r^{\mathrm{T}}\left[\frac{\partial \mathbf{K}}{\partial \mathbf{p}} - \omega_r^2\frac{\partial \mathbf{M}}{\partial \mathbf{p}}\right]\phi_r \Big/ \left[\phi_r^{\mathrm{T}}\mathbf{M}\phi_r\right] = \frac{1}{\phi_r^{\mathrm{T}}\mathbf{M}\ \phi_r}\left[\phi_r^{\mathrm{T}}\frac{\partial \mathbf{Z}^r}{\partial \mathbf{p}}\phi_r\right] \tag{16}$$

Similarly, the experimental natural frequency term $\overline{\omega}_r^2$ may be expressed as a Taylor expansion about the FE solution in term of the updating parameter **p** (remaining only the first order) :

$$\overline{\omega}_r^2 = \omega_r^2 + \frac{\partial \omega_r^2}{\partial \mathbf{p}}\mathbf{p} \tag{17}$$

By substituting (16) into (17), one may constitute a system of linear equations in natural frequency data analogue to the previous one (7) in the FRF data:

$$\mathbf{A}_r\mathbf{p} = \mathbf{b}_r \tag{18}$$

with

$$\mathbf{A}_r = -\frac{1}{\phi_r^{\mathrm{T}}\mathbf{M}\ \phi_r}\left[\phi_r^{\mathrm{T}}\frac{\partial \mathbf{Z}^r}{\partial \mathbf{p}}\phi_r\right] = -\frac{1}{\phi_r^{\mathrm{T}}\mathbf{M}\ \phi_r}\left[\phi_r^{\mathrm{T}}\frac{\partial \mathbf{Z}_r}{\partial p_1}\phi_r\ ,\ \phi_r^{\mathrm{T}}\frac{\partial \mathbf{Z}_r}{\partial p_2}\phi_r\ , \ldots,\ \phi_r^{\mathrm{T}}\frac{\partial \mathbf{Z}_r}{\partial p_{Np}}\phi_r\right] \tag{19}$$

$$\mathbf{b}_r = \omega_r^2 - \overline{\omega}_r^2 \tag{20}$$

This may be repeated for N_ω chosen modes to form $\mathbf{A}_\omega\mathbf{p} = \mathbf{b}_\omega$, where $\mathbf{A}_\omega = [\mathbf{A}_1 \quad \mathbf{A}_2 \quad \ldots \quad \mathbf{A}_{N_\omega}]^{\mathrm{T}}$ and $\mathbf{b}_\omega = [\mathbf{b}_1 \quad \mathbf{b}_2 \quad \ldots \quad \mathbf{b}_{N_\omega}]^{\mathrm{T}}$.

The linear equations of (18) may be inserted in the previous FRF updating equation (12) to form an enhanced updating system.

$$\mathbf{A}\mathbf{p} = \mathbf{b} \tag{21}$$

with

$$\mathbf{A} = \begin{bmatrix}\mathbf{A}_h \\ \mathbf{A}_\omega\end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix}\mathbf{b}_h \\ \mathbf{b}_\omega\end{bmatrix} \tag{22}$$

However it should be pointed out that such a procedure may lead to numerical problem when solving the equation. It is due to the fact that the matrices $\mathbf{A}_h$, $\mathbf{A}_\omega$ result from different types of data so that they may be of very different order of magnitude, forcing some equations to dominate. Therefore a numerical normalization may be necessary.

The problem defined by (21) is generally over-determined: the number of equations ($N_{fr}{\times}N_{fe}$+ N_ω) is usually much larger than that of updating parameters (N_p). It can be solved simultaneously in a least-squares sense by application of SVD (singular value decomposition) technique to give a set of updated *p*-parameters

$$\mathbf{p} = \mathbf{A}^{+}\mathbf{b} \tag{23}$$

This technique will be simply presented in §3.4. Since the FRF and modal data are non-linear with respect to the updating parameters, an iterative process is required to lead to a convergent solution.

3. Numerical implementation

3.1 Iteration of updating parameters

The presented method is based on the assumption that the dynamic matrix $\mathbf{Z}(\mathbf{K}, \mathbf{M})$ is a function of the chosen updating parameters **p** by means of

$$\mathbf{K}^{n+1} = \mathbf{K}^n + \Delta\mathbf{K} \approx \mathbf{K}^n + \sum_{j=1}^{N_{pk}}\frac{\partial \mathbf{K}_j^n}{\partial p_j}p_j \tag{24}$$

$$\mathbf{M}^{n+1} = \mathbf{M}^n + \Delta\mathbf{M} \approx \mathbf{M}^n + \sum_{j=1}^{N_{pm}}\frac{\partial \mathbf{M}_j^n}{\partial p_j}p_j \tag{25}$$

where $\mathbf{K}_j$ and $\mathbf{M}_j$ are, respectively, the stiffness and mass matrices of the *j*-th substructure. In an iteration calculation, the updating parameters p_j^{n+1}, j=1,…,N_p, (N_p = N_{pk}+N_{pm}), are defined as the relative variation between *n*+1 and *n* iteration on physical parameters such as Young's modulus, area moments of inertia of beam, mass density, etc. Taking the stiffening matrix updating problem as example, one can write for the first iteration (assuming that $\mathbf{K}_j$ be the updating physical parameters):

$$\mathbf{K}_j^1 = (1+\overline{p}_j^1)\mathbf{K}_j^0 \tag{26}$$

and then for the second iteration

$$\mathbf{K}_j^2 = (1+p_j^2)\mathbf{K}_j^1 = (1+p_j^2)(1+\overline{p}_j^1)\mathbf{K}_j^0 = (1+\overline{p}_j^2)\mathbf{K}_j^0 \tag{27}$$

Therefore, the obtained updating parameter with respect to the initial FE model is

$$\overline{p}_j^2 = \overline{p}_j^1 + p_j^2 + \overline{p}_j^1 p_j^2 \tag{28}$$

Generally in *n*+1 iteration, the updating solution is presented as follows:

$$\overline{p}_j^{n+1} = \overline{p}_j^n + p_j^{n+1} + \overline{p}_j^n p_j^{n+1} \tag{29}$$

3.2 Sensitivity calculation

In the case of a linear relation between the structure matrice and the updating parameters, the differentiation with respect to parameter **p** in (8) and (19) is simply an assembly of the stiffness (or mass) matrices of the concerned elements:

$$\frac{\partial \mathbf{K}_j}{\partial p_j} = \sum_{k=1}^{N_j} \mathbf{K}_k \;\; , \;\; \frac{\partial \mathbf{M}_j}{\partial p_j} = \sum_{k=1}^{N_j} \mathbf{M}_k \tag{30}$$

where N_j is the element number related to p_j. However, when the updating parameters involve only some coefficients of the element matrix, for example the area moment of inertia about one particular axis for a beam element, we re-write (30) as follows:

$$\frac{\partial \mathbf{K}_j}{\partial p_j} = \sum_{k=1}^{N_j} \mathbf{K}_k \mathbf{L}_k \;\; , \;\; \frac{\partial \mathbf{M}_j}{\partial p_j} = \sum_{k=1}^{N_j} \mathbf{M}_k \mathbf{L}_k \tag{31}$$

where $\mathbf{L}_k$ localizes the related updating parameters in matrices $\mathbf{K}$ and $\mathbf{M}$ and rotates the element matrix to the global FE axis.

3.3 Model matching strategy

In the above updating formulation, the model prediction results and the experimental responses are compared assuming that their corresponding DOF are matched. However due to the fact that only a small amount of DOF of the FE model is measured, it is necessary to overcome this information incompleteness.

In the present work, we perform a model reduction throughout a well-known System Equivalent Reduction Expansion Process (SEREP) to reduce the complete FE matrices onto the experimentally measured DOF dimension. The SEREP method may be regarded as a simplified MECE (minimisation of errors on constitutive equations) solution. The reduced model has exactly the same frequencies and mode shapes as the full system for the selected modes of interest [7]. The SEREP uses the following transformation matrix $\mathbf{T} \in \Re^{N_{fe} \times N_{ex}}$, where N_{ex} is the number of experimentally measured DOF:

$$\mathbf{T} = \psi \left[\psi_m^{\mathrm{T}} \psi_m \right]^{-1} \psi_m^{\mathrm{T}} \tag{32}$$

where $\psi \in N_{fe} \times N_{md}$ is the mode shape matrix of the FE model with N_{md} chosen modes; $\psi_m \in N_{ex} \times N_{md}$ is the partition of ψ on the measured DOF. The reduced system matrices are obtained as:

$$\bar{\mathbf{K}} = \mathbf{T}^{\mathrm{T}} \mathbf{K} \; \mathbf{T} \tag{33}$$

$$\bar{\mathbf{M}} = \mathbf{T}^{\mathrm{T}} \mathbf{M} \; \mathbf{T} \tag{34}$$

where $\bar{\mathbf{K}}, \bar{\mathbf{M}} \in \Re^{N_{ex} \times N_{ex}}$. Consequently, the FRF data of the FE model corresponding to the measured DOF will be used in the updating procedure.

However, a numerical problem appears while using this reduction strategy: the reduced system matrices $\bar{\mathbf{K}}, \; \bar{\mathbf{M}}$ by (32-34) may be ill conditioned to lead to a difficulty in inverting the dynamic matrix to construct the FRF sensibility matrix $\mathbf{A}$ by (8). This problem results from the special property of transformation matrix $\mathbf{T}$ that is rank deficient when the number of the used modes is smaller than that of the active (measured) DOF, as already mentioned by Heylen *et al.* [8]. Theoretically, one may avoid this difficulty by taking the number of modes equal to or higher than that of the measured DOF (e.g. $N_{md} \geq N_{ex}$) when constructing reduction matrix $\mathbf{T}$. Unfortunately, this is not always possible and may leads to the dependence of matrix ψ_m in some cases.

In this work, we consider two numerical techniques to overcome the difficulty of rank deficiency of the reduced dynamic matrix $\bar{\mathbf{Z}}(\bar{\mathbf{K}}, \bar{\mathbf{M}})$. The first is a simple diagonal perturbation technique: to add a small perturbation value on diagonal terms of the reduced dynamic matrix. Consequently, the resulting dynamic matrix diverges a little from the original one and is no longer singular. However, it is not always easy to determine a common value of perturbation that is appropriate for extensive problems. The second one consists in applying the singular value decomposition (SVD) technique, which is briefly described in the following section.

3.4 Singular value decomposition (SVD)

The SVD technique is commonly used to invert the reduced dynamic matrix ($\bar{\mathbf{H}} = \bar{\mathbf{Z}}^{-1}$ in §4.3) and to solve the updating equation ($\mathbf{p} = \mathbf{A}^{+}\mathbf{b}$ in §2.2). We give following description in a general case where the system matrix $\mathbf{A}$ is not square. When a direct inverse solution is impossible, the Moore-Penrose pseudo inverse is performed by solving the singular value decomposition of $\mathbf{A}$, i.e.,

$$\mathbf{A}_{M \times N} = \mathbf{U}_{M \times M} \Sigma_{M \times N} \mathbf{V}_{N \times N}^{\mathrm{T}} \tag{35}$$

with

$$\Sigma = \begin{bmatrix} \Sigma_1 & 0 \\ & \Sigma_2 \\ 0 & 0 \end{bmatrix} \tag{36}$$

$$\mathbf{U}^{\mathrm{T}}\mathbf{U} = \mathbf{U}\mathbf{U}^{\mathrm{T}} = \mathbf{I} \tag{37}$$

$$\mathbf{V}^{\mathrm{T}}\mathbf{V} = \mathbf{V}\mathbf{V}^{\mathrm{T}} = \mathbf{I} \tag{38}$$

where M is the number of rows of matrix A (for example $M = N_{fr} \times N_{ex} + N_{\omega}$), $N = N_p$ is the number of updating parameters. $\Sigma_1 = \mathrm{diag}(\sigma_1, \sigma_2, \ldots, \sigma_r)$ and $\Sigma_2 = \mathrm{diag}(\sigma_{r+1}, \sigma_{r+2} \ldots, \sigma_N)$ are non-negative diagonal elements in decreasing order, which is called singular values of matrix $\mathbf{A}$

$$\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_r \gg \sigma_{r+1} \geq \ldots \geq \sigma_N \rightarrow 0 \tag{39}$$

In most practical situations, the considered row might not be completely linearly dependent, the elements in

Σ_2, instead of zero but are small value with respect to the elements in Σ_1. So we have to establish a criterion for the rejection or acceptance of small singular values. This criterion is easy to find in the case of handling the inverse of the reduced dynamic matrix $\bar{\mathbf{Z}}$ since we know that the rank-deficiency of $\bar{\mathbf{Z}}$ is just due to the difference between the number of the chosen modes and that of the measured DOF. In other word, the rank of $\bar{\mathbf{Z}}$ is generally equal to the number of the used modes (e.g. $r=N_{md}$). In the case of solving the updating equation (23), we accept generally a criterion written in following form:

$$\frac{\sigma_{r+1}}{\sigma_r}<\varepsilon \tag{40}$$

where ε is a small value (10^{-4} for example). Once the rank r of the system matrix is determined, the pseudoinverse $\mathbf{A}^+$ (or $\bar{\mathbf{Z}}^+$) is then computed as

$$\mathbf{A}^+ = \mathbf{V}_1 \Sigma_1^{-1}\mathbf{U}_1^{\mathrm{T}} = \mathbf{V}_1 .\mathrm{diag}(\sigma_1^{-1},\sigma_2^{-1},...,\sigma_r^{-1}).\mathbf{U}_1^{\mathrm{T}} \tag{41}$$

This method constitutes in fact a SVD-based truncation technique. The solution provides minimum error for the least-squares problem.

3.5 Updating parameter and frequency selection

It is crucial to select appropriate updating parameters **p** for an efficient updating procedure. Note that each column in FRF sensitivity matrix $\mathbf{A}$, eq.(8), represents the operation deflection shape of the structures under an "external force" associated to p_j element (group). If the elements are too close in space, their effects on the global dynamic behaviour will be similar leading to ill-conditioned updating system. Therefore, we intent to select the updating parameters on level of substructures or element group. From a numerical point of view, the selection of updating parameters may be based on the sensitivity and error localization analyses. The sensitivity analysis provides a list of parameters to which the dynamic behaviour is sensitive. Updating procedure with these parameters may have a rapid convergence. A simple sensitivity analysis concerns a computation of eq. (19) on the basis of FE modal analysis. However, it must be kept in mind that the parameters with large sensitivity do not necessarily concern the defaults of the structures. On the other hand, the error localisation analysis detects directly the elements in the FE model that are responsible for the discrepancy between the FE-calculating and experimental responses. As an extensively used technique, the MECE (Minimisation of Errors on Constitutive Equation) indicator consists in evaluating the residual energy (density) associated to each r mode shape vector at a local level (element or element group) [9], which, however, requires the knowledge of the mode shapes.

The selection of updating frequencies is also an important step in an updating method based on the FRF data. The first criterion may be given by considering that the chosen frequencies should correspond to a larger sensitivity value. From Eq. (8), it is better to choose the frequencies (ω_i) close to the natural ones to have a lager value of $\bar{\mathbf{h}}_i$. This selection is also preferred from the point of view of alleviating the noise effect since this effect becomes relatively important where is away from the natural frequencies. However, it is known that the FRF at the vicinity of natural frequencies changes largely with damping coefficient of structures. When the exact damping coefficients of the structures are not known (this is true in most practical situations), one should not choose the frequencies too close to natural frequencies to avoid the error related to damping effect. This constitutes another criterion. Considering the above two contradictory requirements, we suggest a compromise to take frequencies ω_i in the vicinity of natural frequencies $\bar{\omega}_i$, which is numerically based on "half-power points bandwidth" analysis:

$$\omega = (1 \pm c\xi)\bar{\omega}_i \tag{42}$$

where ξ is a damping ratio coefficient; c is a user-defined correction. In the present work, we usually choose $c\xi$= 5~10%. In this way, the updating frequencies are easily selected.

3.6 A simplified correlation criterion: SFDAC

In order to evaluate the correlation of models in frequency domain, Pascual and Golinval have proposed a so-called Frequency Domain Assurance Criterion (FDAC) [10]:

$$FDAC(\omega_i,\bar{\omega}_j) = \frac{\mathbf{h}_i^{\mathrm{T}}.\bar{\mathbf{h}}_j}{\|\mathbf{h}_i\|.\|\bar{\mathbf{h}}_j\|} \tag{43}$$

where $\mathbf{h}_i,\bar{\mathbf{h}}_j$ are, respectively, the FE-model FRF at frequency ω_i and the measured FRF at frequency $\bar{\omega}_j$. By comparing the FDAC with well-known Modal assurance Criterion (MAC) in modal analysis, we propose a modification of (43) as following:

$$FDAC(\omega_i,\bar{\omega}_j) = \frac{\left(\mathbf{h}_i^{\mathrm{T}}\bar{\mathbf{h}}_j\right)\left|\mathbf{h}_i^{\mathrm{T}}\bar{\mathbf{h}}_j\right|}{\left(\mathbf{h}_i^{\mathrm{T}}\mathbf{h}_i\right)\left(\bar{\mathbf{h}}_j^{\mathrm{T}}\bar{\mathbf{h}}_j\right)} \tag{44}$$

This modification is really to perform a square of (43) in value but remains the sign of the FDAC same as that of $\left(\mathbf{h}_i^{\mathrm{T}}\bar{\mathbf{h}}_j\right)$, which makes the modified FDAC (44) in the frequency domain be equivalent to the MAC in the modal domain. To use (44), we generally transform the complex FRF value, $H=a+ib$, into a real FRF value, $h = \mathrm{sign}(a)\sqrt{a^2+b^2}$. It can be shown that the *FDAC* takes the values in the range (-1,1) and a value *FDAC* >0 means that both shapes are "in phase". In particular, *FDAC*=1 corresponds to a perfectly correlation in the concerning frequencies.

To avoid the model expansion procedure, the *FDAC* may be calculated only on the measured DOF. In the present updating analysis, we propose further a simplified *FDAC*, called here as *SFDAC,* which is calculated at only the commonly chosen frequencies as follows

$$SFDAC_i = \frac{\left(\mathbf{h}_i^{\mathrm{T}}\overline{\mathbf{h}}_i\right)\left|\mathbf{h}_i^{\mathrm{T}}\overline{\mathbf{h}}_i\right|}{\left(\mathbf{h}_i^{\mathrm{T}}\mathbf{h}_i\right)\left(\overline{\mathbf{h}}_i^{\mathrm{T}}\overline{\mathbf{h}}_i\right)} \quad (45)$$

where i=1,…, N_{fr}, N_{fr} is the number of chosen frequencies. While the *FDAC* is a matrix, the *SFDAC* is a vector. This reduces the calculating effort. The *SFDAC* takes values also in the range (-1,1). We use their mean value as an approximate evaluation of the updating model:

$$\overline{SFDAC} = \frac{1}{N_{fr}}\sum_{i}^{N_f r} SFDAC_i \quad (46)$$

$\overline{SFDAC}$ =1 corresponds to a "perfectly" updated FE model. Particularly, the minimum value of $SFDAC_i$ is a more sensitive indicator of correlation. Of course, the value of $\overline{SFDAC}$ depends on the selection of updating frequencies. With the finitely-measured DOF, the $\overline{SFDAC}$, as well as $SFDAC_i$, is only a relative indicator of correlation. In the present work, it serves to indicate if the updating procedure goes in a correct direction. By increasing the numbers of updating parameters, measured DOFs and updating frequencies, the $\overline{SFDAC}$ may intent to stabilise.

Instead of the present explicit updating procedure, the modal updating procedure developed in calculating code BOSS-QUATTRO [11] uses a finite differential calculating method to minimise the objective function consisting of modal data. This calculating procedure may be adopted to use the frequency domain data with the following objective function consisting of the *SFDACs* and the difference of resonance frequencies f_i:

$$\min_{\mathbf{p}}\left(1 - \frac{1}{N_{fr}}\sum_{i}^{N_f r} SFDAC_i + \frac{1}{N_{md}}\sum_{i}^{N_{md}} \frac{\left|f_i - f_i^{ex}\right|}{f_i^{ex}}\right) \quad (47)$$

This may consist of a further development. A large commercial FE code SAMCEF [6] has been used for the implementation of the present updating method for the sake that the extensive industrial problems may be dealt with. Dynamic analysis of FE model is performed with initial updating parameters. The modal analysis solutions are read by the developed tool. Transformation matrix $\mathbf{T}$ and sensitivity matrix $\mathbf{A}$ are constructed, and the FRF solutions are compared to the experimental measures. Solving the updating equations results in a set of updated parameters. Since the FRF data and modal parameters are non-linear with respect to the updating parameters, an iterative process as a cycle calculation of SAMCEF–updating program is required until the arrival of convergence of updating calculation. A simple and crucial convergence criterion is:

$$\overline{SFDAC} \to 1 \quad (48)$$

For numerical convenience, we adopt, instead of (48), the relative variation of updating parameters as a stopping criterion: (basing on the nth iteration)

$$\max_{j=1,Np}\,(p_j^{n+1}) < \varepsilon \quad (49)$$

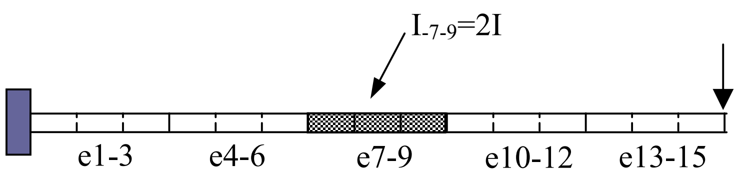
where $\mathbf{p}_{n+1}$ is the current solution of (12) or (21). In the present work, we take ε =0.01.

4. Applications

Some beam and framework structures are first examined as benchmark tests. The solution of a FE model is used as "experimental measure" to update another FE model containing "defects". In this case the exact solutions are known so we can validate the developed updating program. Finally, an airplane model is studied using real experimental FRF data.

4.1 A cantilever beam case

The model is composed of 15 Euler-Bernoulli beam elements and it is excited on the right end of beam (Fig 1). The "experimental" structure differs from the initial FE model by the bending stiffness of the elements 7-9 which has been doubled in the experimental model. The vertical displacements of 15 nodes are measured. We take 12 updating frequencies equal to ±8% of 6 first resonance frequencies of measurements. 5 updating parameters are defined, each of which is related to 3 elements. It is shown in Table 1 that with only 3 iterations almost exact results are obtained. This shows that the problem is well conditioned.



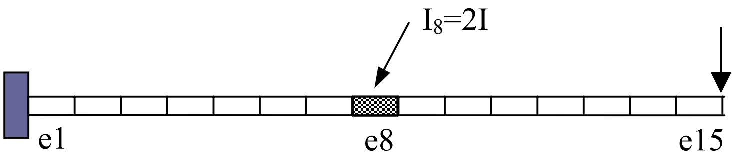
L=1.8m, A=10^{-4}m^2, E=2×10^{11} Pa, ρ=7800 kg/ m^3, I=8.33×10^{-10}m^4, $I_{7\text{-}9}$=2I

Fig.1. FE model of a clamped beam

Table 1. Evolution of updating parameters

Iteration	p_1 (ele.1-3)	p_2 (ele.4-6)	p_3 (ele.7-9)	p_4 (ele.10-12)	p_5 (ele. 13-15)	$\overline{SFDAC}$
0	0	0	0	0	0	0.6409
1	-3.090×10^{-2}	-4.616×10^{-2}	0.80019	2.618×10^{-2}	-8.154×10^{-2}	0.9928
2	1.174×10^{-3}	-9.900×10^{-3}	0.99205	4.429×10^{-3}	-3.485×10^{-3}	0.9999
3	-2.584×10^{-4}	4.588×10^{-4}	0.99998	-2.163×10^{-4}	-2.302×10^{-4}	~1.0000
Exact solution	0	0	1	0	0	1.0

Now we consider a similar structure where only the bending stiffness of elements 8 is doubled in the experimental model, see Fig. 2. We define 15 updating parameters corresponding to 15 elements. As elements are close in space, their similar effects on the global dynamic behaviour make the updating system nearly ill-conditioned. The updating results become sensitive to the calculating parameters, leading to a difficulty in finding the exact solution of **p**. Fig. 3 shows the evolution of updating parameters, where we see that the defect in element 8 is distributed rather on elements 7-9.



L=1.8m, A=10^{-4}m^2, E=2×10^{11} Pa, ρ=7800 kg/ m^3, I=8.33×10^{-10}m^4, I_8=2I

Fig.2. FE model of clamped beam2

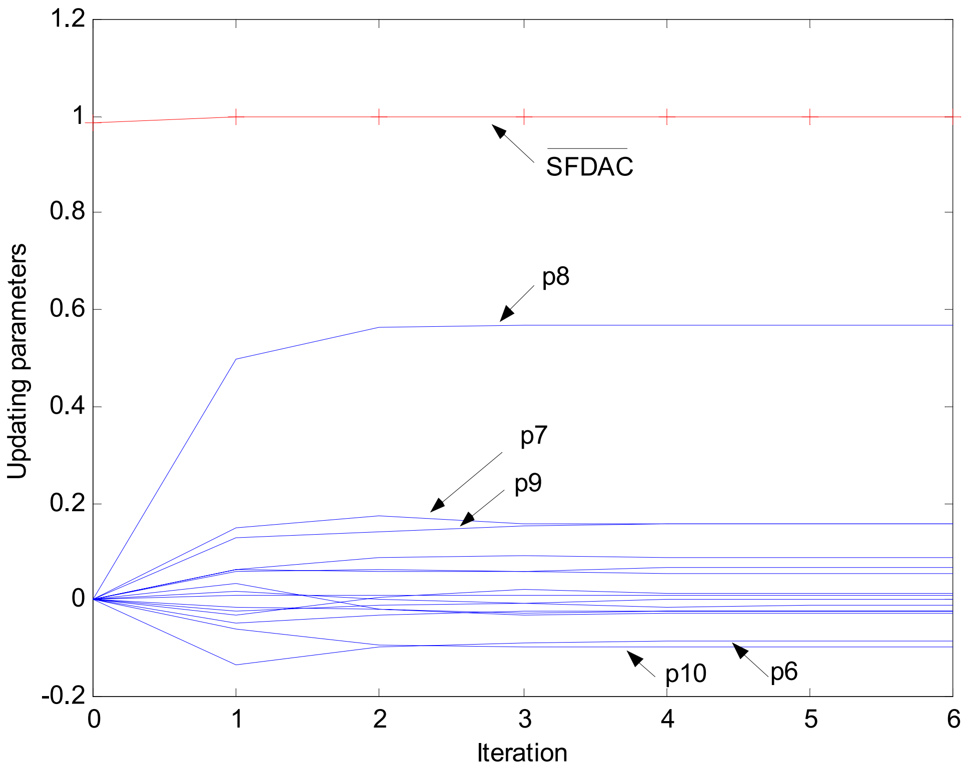
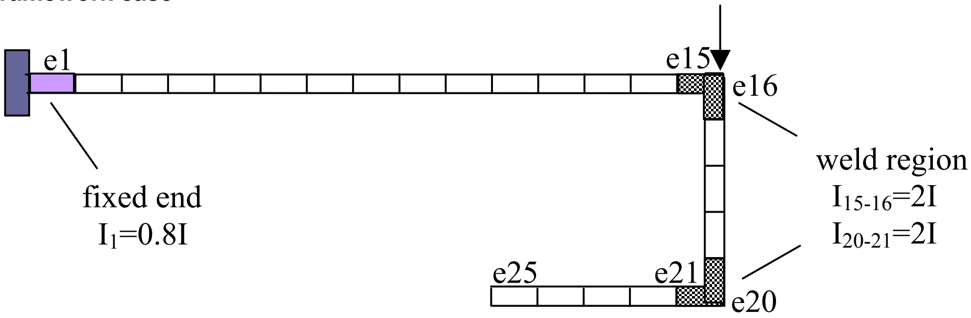


Fig. 3. Convergence of updating parameters

4.2 A cantilever framework case



A=10^{-4}m^2, E=2×10^{11} Pa, ρ=7800 kg/ m^3, I=8.33×10^{-10}m^4

Fig.4 FE model of a welded framework

Consider a framework made of 3 beam pieces by welding (Fig. 4). The following defects exist in experimental model with respect to the FE model (25 beam elements): 1) the imperfect fixation is represented by a reduction of 20% in the bending stiffness in element 1; 2) the stiffening effect in two welding regions are equivalent to an increment of 100% in bending stiffness in elements 15-16 and 20-21. For numerical convenience, measurements are done in the vertical direction of 25 nodes. This selection is obviously not optimal because it ignores the horizontal displacement of vertical beam, which is important for some dynamic modes. This explains why the convergence of the updating computation is relatively slow in comparison with case 1 in §4.1. In fact, more iterations are necessary to decrease the error in vertical beam (parameter $p_{5\text{-}6}$, Fig. 5). The first 8 modes are adopted in constructing the reduction matrix T, corresponding to 16 updating frequencies by taking ±6% of the first 8 resonance frequencies of measurements. Convergence is attained at 8th iteration. The calculating results are presented in Table 2.

Table 2. Updating parameters calculation

Iteration	p_1 ele.1	p_2 ele.2-8	p_3 ele.9-14	p_4 ele.15-16	p_5 ele.17-19	p_6 ele.20-21	p_7 ele.22-25	$\overline{SFDAC}$
0	0	0	0	0	0	0	0	0.8582
1	-2.8564×10^{-1}	3.1420×10^{-2}	-6.9151×10^{-3}	4.1593×10^{-1}	1.1402×10^{-1}	1.6118×10^{-1}	-3.0725×10^{-2}	0.9787
8	-1.9997×10^{-1}	4.2173×10^{-5}	3.7688×10^{-5}	9.9844×10^{-1}	2.5203×10^{-2}	9.9143×10^{-1}	-7.2131×10^{-4}	~1.0000
Exact solution	-0.2	0	0	1.0	0	1.0	0	1.0

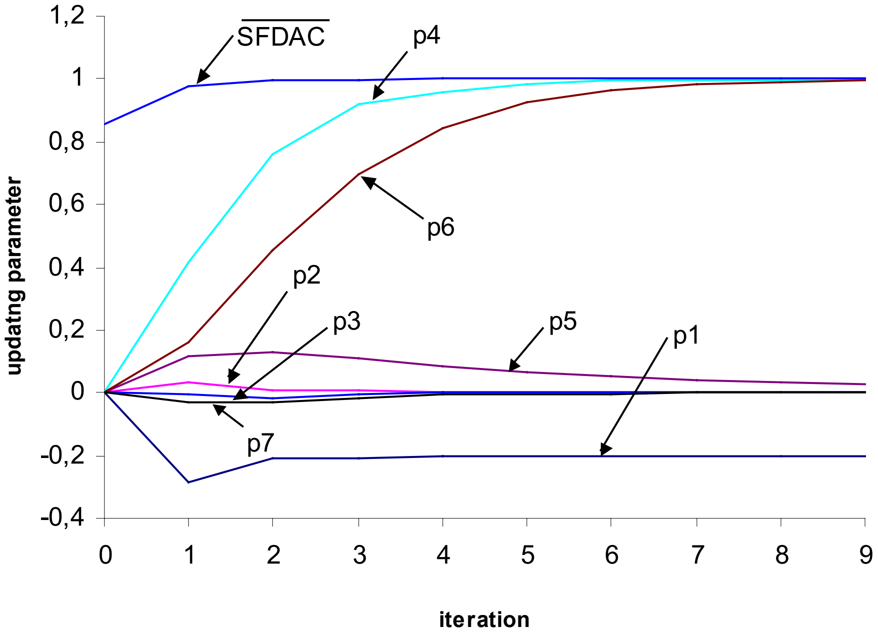
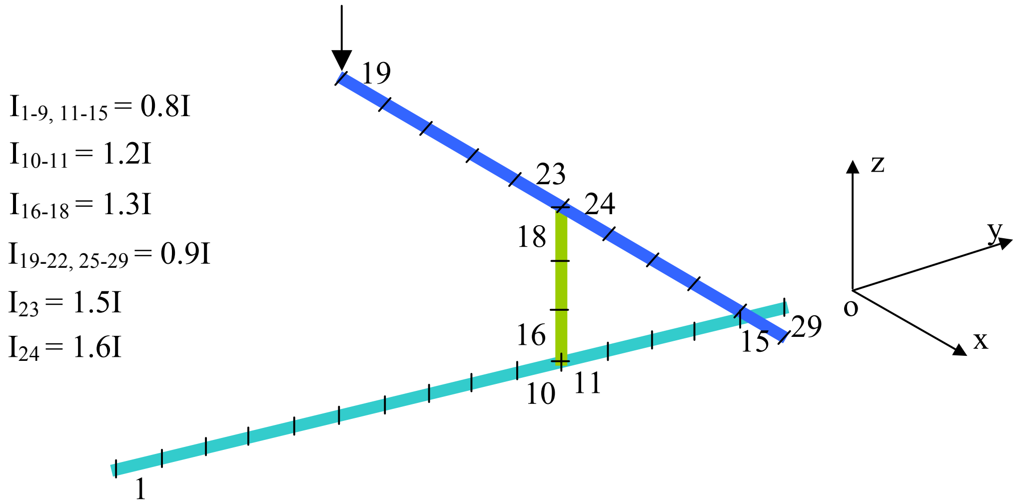


Fig.5 Convergence of updating parameters

4.3 A 3D framework

The structure consists of 3 orthogonal beams of square area and it is modelled by 29 beam elements (Fig. 6). The differences between the FE and experimental models concern only bending stiffness I. For numerical simplicity, the structure is constrained so that beam 1 (ele.1-15) deforms in yoz plane and beam 2-3 (ele. 16-29) deform in xoz plane. Measurements are performed at all 29 nodes of direction z. The noise of three level (0, 1% and 5% of RMS of FRF data in chosen frequencies) are added to the measured FRF to examine the effect of noise.

We choose 8 frequencies for updating calculation presented in the last line of Table 3, according to the following considerations: 1) the chosen frequencies are neither too close to nor far from the resonance frequencies of experimental model (±5~10% of experimental resonance frequencies); 2) avoid choosing the frequencies in the regions where the difference of resonance frequencies between the FE and measured models is large; 3) choose the frequencies in extensive frequency region (concerning modes as many as possible).



$A=10^{-4}m^2$, $E=2\times10^{11}$ Pa, $\rho=7800$ kg/ m^3, $I=8.33\times10^{-10}$ m^4, $I^t=1.40944\times10^{-9}$ m^4

Fig.6 FE model of a 3D framework

Table 3. Resonance frequencies of models and chosen frequencies

Mode	Test freq. of resonance	Initial FE model	Updated FE model (no noise)	Updated FE model (1% noise)	Updated FE model (5% noise)
N°	f_{ex}	f_{fe}^{0}	f_{fe}^{n0}	f_{fe}^{n0}	f_{fe}^{n0}
1	7.360	7.622	7.360	7.357	7.352
2	13.81	12.16	13.81	13.807	13.799
3	26.18	26.53	26.17	26.162	26.171
4	39.84	38.24	39.85	39.864	39.887
5	47.19	42.04	47.19	47.18	47.186
6	54.45	56.45	54.44	54.429	54.421
7	109.16	116.68	109.17	109.12	108.94
8	151.97	158.65	151.96	151.89	151.82
9	161.09	162.40	161.09	161.07	161.03
10	198.99	204.21	198.97	198.90	198.93
11	234.84	236.18	234.85	234.90	234.92
12	269.93	288.43	269.94	269.84	269.62
$\frac{1}{N_f}\sum_{i=1}^{N_f}\left\|1-f_{fe}^{0}(i)/f_{ex}(i)\right\|$		4.8%	0.0096%	0.038%	0.077%
Chosen updating frequencies (Hz):		6, 24, 36, 59, 145, 166, 190, 240			

Table 4. Model updating with different noise levels

Upd. para. elements	P_1 1-9,12-15	p_2 10-11	p_3 16-18	p_4 19-22	p_5 23	p_6 24	$\overline{SFDAC}$
Initial	0	0	0	0	0	0	*
Updated 5% noise	-0.20526	0.21097	0.29585	-0.10030	0.64993	0.49327	0.77494
Updated 1% noise	-0.20064	0.19592	0.29828	-0.10007	0.52250	0.59000	0.97753
Updated no noise	-0.19970	0.19728	0.29990	-0. 09996	0.50072	0.60011	~1.0000
Exact solution	-0.2	0.2	0.3	-0.1	0.5	0.6	1.0

* value depending on the noise level

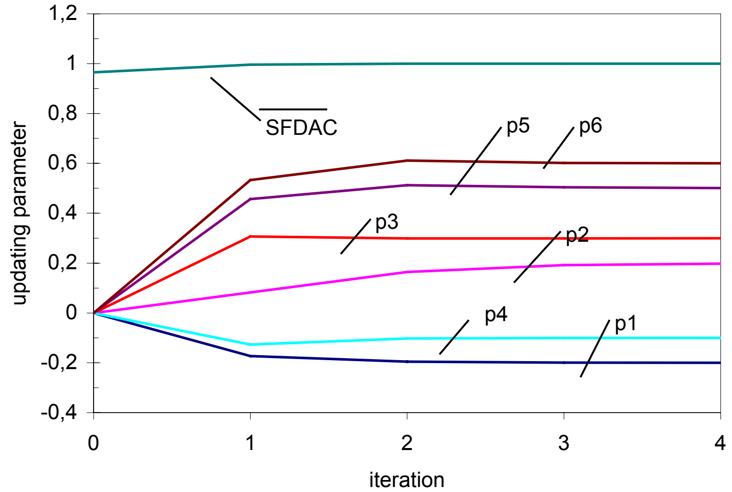


Fig.7 Evolution of updating parameters of a 3d structure (without noise)

The dynamic calculation with initial FE model presents the first 12 natural frequencies in column 3 of Table 3, which is compared with those of experimental model with different levels of noise. It is reasonable that the measurement noises affect the precision of updated models. It is shown in Table 4 that when existing noise one could not obtain the exact updating parameters as do as the case of without noise, see specially the updating parameter p_5 and p_6. However this error may be not important from point of view of predicting the resonance frequencies as shown in Table 3. Fig. 7 give an example of updating parameter evolution in the case of without noise. Note the convergence speed of updating calculation depends on several factors. In most cases the convergence attains in less than ten iterations.

4.4 Application to SM-AG19 GARTEUR structure

A simplified airplane model GARTEUR SM-AG19, Fig. 8, was proposed as benchmark in the framework of European COST F3 Action in structural dynamics, which is described in ref. [12]. In the present work, we present some preliminary results using the test data provided by University of Wales Swansea. There are 26 frequency response functions available due to excitation at the left wing tip (112-z), each of which records 801 frequency measure points up to about 200 Hz. The data are recorded in universal format files.

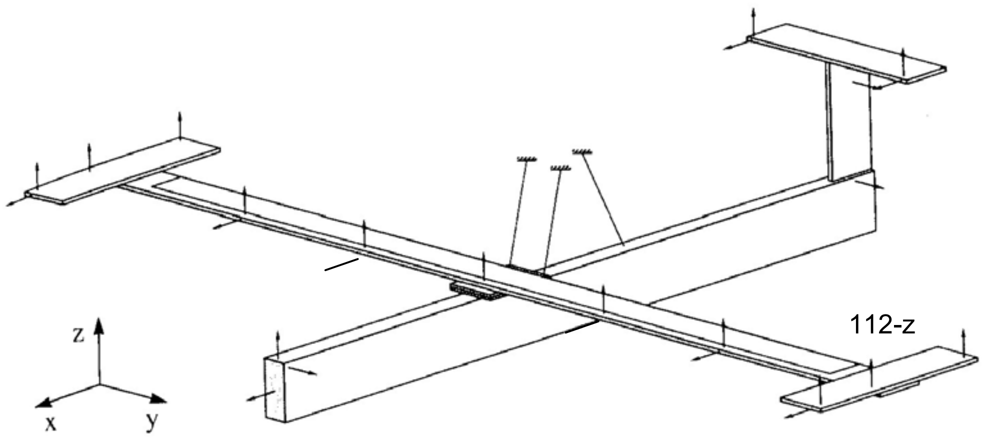


Fig.8 GARTEUR test structure: locations and directions of the 26 common accelerometers (exciting force is at 112-z)

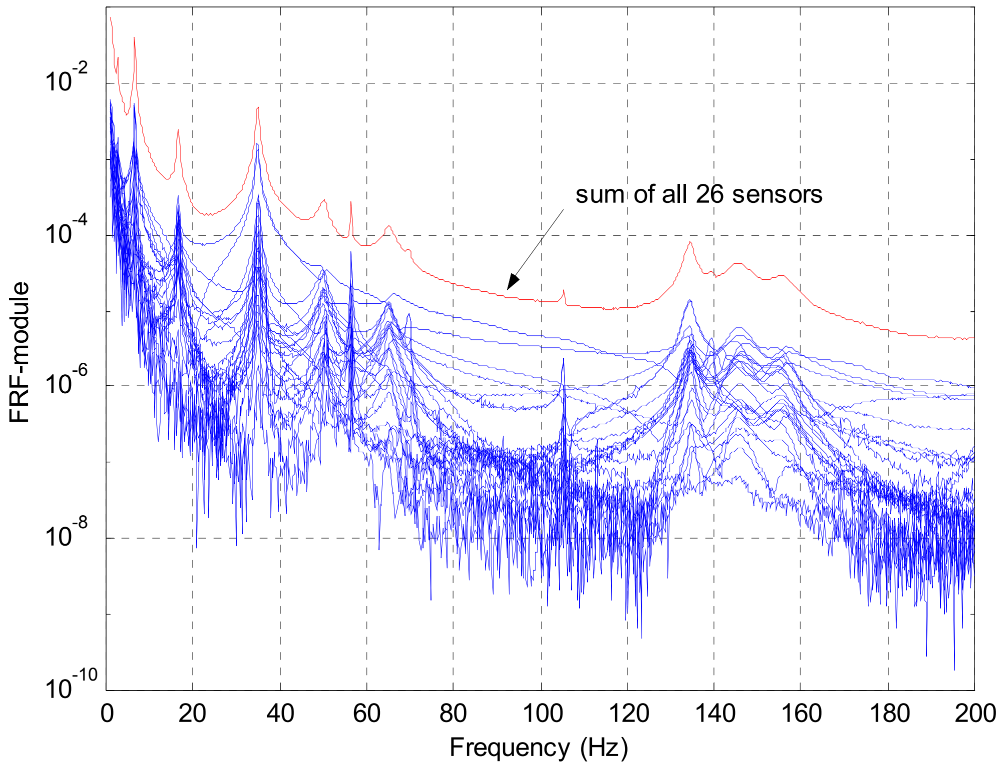


Fig. 9 FRF data of 26 sensors in GARTEUR model

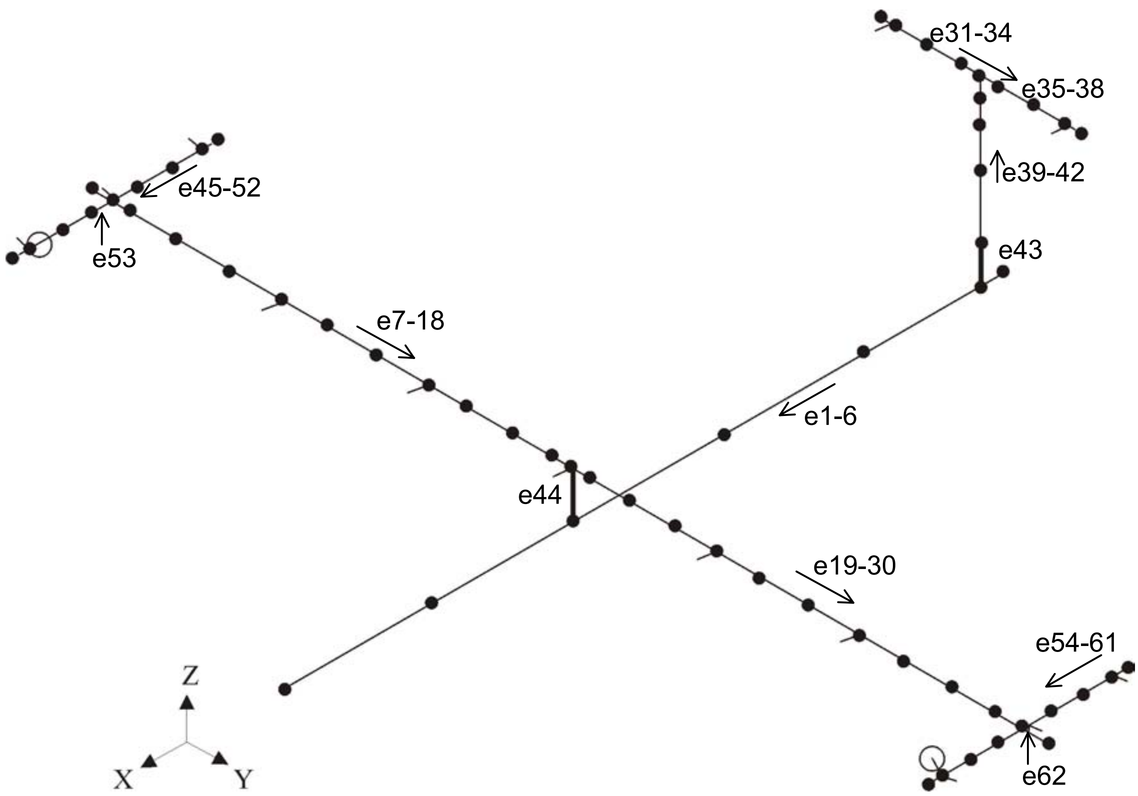


Fig.10 Beam finite element mesh

The measured FRF data in form of complex accelerate ($H=a+ib$) are transformed to a real FRF value:

$$h = -\mathrm{sign}(a)\frac{\sqrt{a^2+b^2}}{\omega_j^2} \quad (50)$$

The FRF data in displacement of 26 sensors are presented in Fig. 9. The FRF updating method takes the experimentally measured FRF as "exact" value to update the FE model. Unfortunately, experimental data are often infected by noise and pose possibly large errors. The FRF curve of each sensor varies strongly depending on the location of sensors. It is noticed that some resonance frequencies are very close in value. Some sensors are sensitive only to certain resonance frequencies of vibration.

The FE model is constructed using 75 beam elements with 456 DOF (76 nodes). Note that only 62 elements are used to model the structure (the fuselage, the wing, the tail and the drums), and other 13 short beam elements (with very high stiffness and very low mass density) are used to reproduce the exact location of the sensors. Besides, additional masses and mass inertia are imposed at tips of two drums. As the structure is suspended in space by three lines, no fixation is imposed. The finite element mesh is illustrated in Fig. 10.

The updating parameters are defined on basis of error localisation analysis and eigenvalue sensibility analysis [9]. In brief, a set of 21 parameters were investigated for the first 9 modes of vibration, the parameters are defined with generally an element group or special element in joint location, see Table 5.

The updating frequencies are simply chosen at two sides of resonance frequencies of mode 1~10, Table 6. Note that some experimental resonance frequencies are too close to be separated. With the frequencies chosen in somewhat simple way, the FRF correlation is quite poor between the initial FE model and the experimental one. After updating calculation, the correlation is largely improved at the chosen frequencies: the $\overline{SFDAC}$ (mean value) change from -0.352 to 0.893 and the $SFDAC_{min}$ (minimum value) from –0.963 to 0.675. The correlation is largely improved. However in finite iterations, the updating convergence is not yet attained (Fig. 11). It seems that when using frequencies concerning fewer modes of vibration the convergence is better than concerning more modes of vibration. However, a stable solution does not necessarily lead to a correct model of the structure. It gives only an optimal estimation from point of view of having the best correlation at the chosen frequencies. In the case of ill-conditioned updating system due to a bad selection of updating parameters, this optimal point may be local but global. Specially when the FRF measurement is infected by noise, the updated model may be also an infected one. In this case, the updating formulation of resonance frequencies (18) is necessary to be introduced into (21) in order to assure also a good prediction of resonance frequencies by the updated model. Unluckily, due to numerical difficulties, this calculation is not yet realised. So it is seen in Table 7 that the prediction of resonance frequencies with the updated FE model is not very consistent with experimental analysis, though the results have been evidently improved in comparison with initial FE model.

Table 5. List of updating parameters and corrections

N°	Parameter - (name in data file)	Area moment of inertia (about axis)	Element N°	Correction %
Fuselage			1-6	
1	I_X-*(FX)*	Torsion (x)		0
Right wing			7-17	
2	I_X-*(ADZ)*	Bending (x)		10.5
3	I_Y-*(ADX)*	Torsion (y)		26.0
4	I_Z-*(ADY)*	Bending (z)		-11.0
Left wing			20-30	
5	I_X-*(AGZ)*	Bending (x)		24.3
6	I_Y-*(AGX)*	Torsion (y)		25.1
7	I_Z-*(AGY)*	Bending (x)		-2.0
Connection on right wing			18	
8	I_X-*(JADZ)*	Bending (x)		-0.16
9	I_Y-*(JADX)*	Torsion (y)		0
10	I_Z-*(JADY)*	Bending (x)		100
Connection on left wing			19	
11	I_X-*(JAGZ)*	Bending (x)		-0.04
12	I_Y-*(JAGX)*	Torsion (y)		0
13	I_Z-*(JADY)*	Bending (x)		30.8
Vertical tail			39-41	
14	I_X-*(VZ)*	Bending (x)		-4.1
15	I_Z-*(VX)*	Torsion (z)		-20
Connection on vertical tail			42	
16	I_X-*(JEVZ)*	Bending (x)		11.1
17	I_Z-*(JEVX)*	Torsion (z)		0.1
Connection on horizontal tailplane			34-35	
18	I_X-*(JEHZ)*	Bending (x)		0.1
19	I_Y-*(JEHX)*	Torsion (y)		0
Fuselage-wings connection			44	
20	I_Z-*(JCAF)*	Torsion (z)		11.0
Fuselage-tail connection			43	
21	I_Z-*(JCGF)*	Torsion (z)		0.1

Table 6. The chosen updating frequencies and corresponding *SFDAC* values

Freq.	6.	7.	16.	17.5	33.5	38.	48.5	52.	55.5	57.5	63.5	67.	70.75	Mean
S_f^i	.932	-.963	.758	-.946	.941	-.952	.204	-.884	-.955	-.952	.024	-.844	-.944	-.352
S_f^u	.974	.980	.947	.930	.967	.961	.882	.838	.675	.904	.943	.681	.921	.893

Note: S_f^i : SFDAC with initial model, S_f^u : SFDAC with updated model

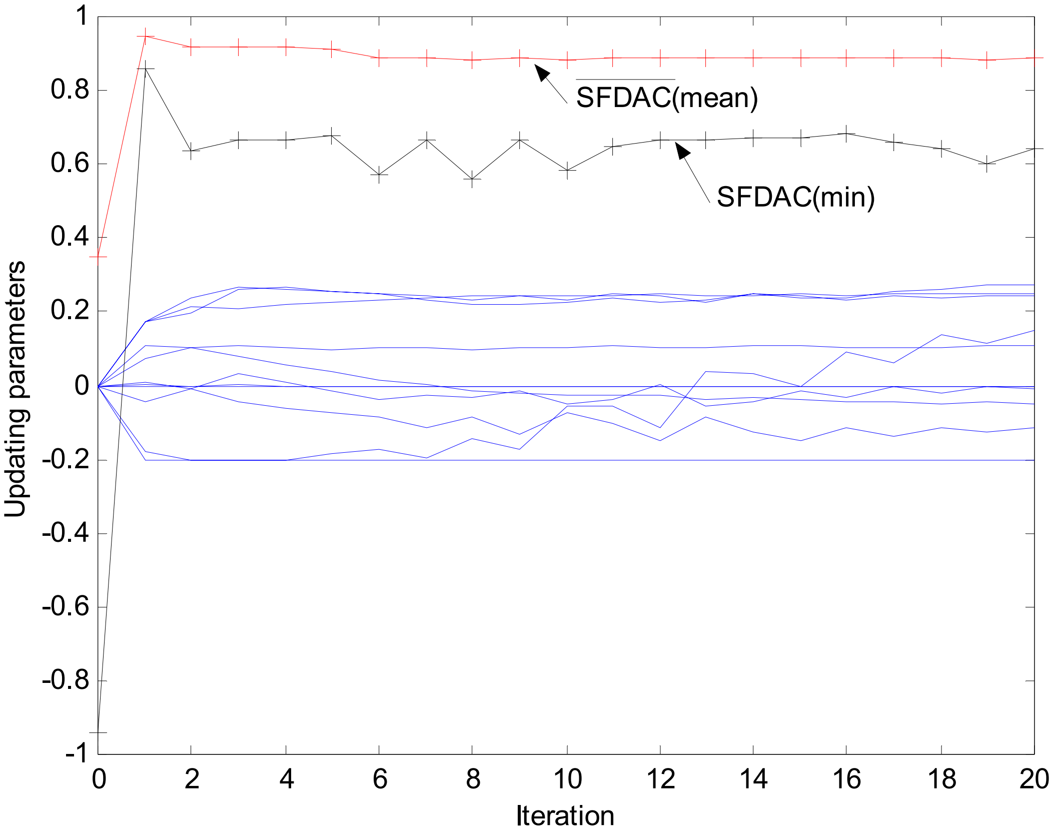


Fig. 11 Evolution of updating parameters with chosen frequencies

Table 7. Resonance frequency deviation

Mode n°	Exp. (Hz)	Initial FE model	*df/f* (%)	Updated FE	*df/f* (%)
1	6.548	5.890	-10.04	6.378	-2.59
2	16.61	15.89	-4.33	16.92	1.87
3	34.88	31.52	-9.63	34.86	-0.01
4	35.36	31.59	-10.66	35.29	-0.02
5	36.71	35.42	-3.51	36.92	0.57
6	50.09	46.38	-7.41	46.73	-6.70
7	50.72	47.83	-5.70	51.75	2.03
8	56.44	56.75	0.55	55.14	-2.30
9	65.14	62.16	-4.57	61.10	-6.20
10	69.64	66.20	-4.94	66.58	-4.39
$\frac{1}{N_{md}}\sum_i^{N_{md}}\frac{\lvert df\rvert}{f_i^{ex}}$			6.134		2.668

5. Conclusions and discussions

In this paper, we have presented an updating technique using the FRF data in the environment of a large commercial finite element code. Communication between the developed updating program and the FE code is established through user-interface to construct an automatic numerical iteration procedure. The numerical implementation techniques are discussed. System equivalent reduction expansion process (SEREP) is adopted to reduce the complete FE eigenvectors onto the experimental measured DOF. The rank-deficiency difficulty with this method is overcome by either of two numerical techniques: diagonal perturbation and singular value decomposition. The modified and simplified *FDAC* have been proposed to evaluate the correlation between the FE model and the measured structures in the chosen frequencies. These constitute the contributions of this paper.

The developed FRF updating program is verified by several numerical benchmarks. The existing defects in initial FE model are accurately identified in few iterations even in the case of existing noise. However if the problem approaches ill-conditioned, the convergence may not be assured. The difficulties appear in obtaining a convergent and satisfying solution when applying the method to an experimental aircraft model, although the correlation between experimental and FE models is obviously improved after model updating. It seems that updating solution with the frequencies concerning fewer modes is easier to be stabilised but not necessarily leading a correct solution. The application shows two possible disadvantages of updating technique using FRF data: 1) only a small part of measured data is used in updating calculation, as the chosen updating frequencies is very limited in comparison with the available frequency range, so the obtained solution may be only locally but globally optimal; 2) the difference of FE and measured FRF is directly used in updating formulation as driving force; It may be difficult to reach convergence when this difference in some chosen frequencies is too large or defected severely by noise. It seems necessary to adopt some numerical techniques to limit this difference. Further work is necessary to make the developed algorithm applicable with more affectivity to practical structures.

Acknowledgement

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REFERENCES

[1] Friswell M.I., Mottershead J.E. (1995) *Finite element model updating in structural dynamics.* Dordrecht: Kluwer Academic Publishers.

[2] Lin R.M., Ewins D.J. (1990): Model updating using FRF data. *Proceedings of the 15th international seminar on modal analysis.* Leuven, Belgium.

[3] D'Ambrogio W., Fregolent A., Salvini P. (1993): Updatability conditions of non-conservative FE models with noise on incomplete input-output data. *Proceedings of structural dynamics modelling test, analysis and correlation,* 29-38

[4] Lammens S., Heylen W. Sas P. (1993): Model updating using experimental frequency response functions: case studies. *Proceedings of structural dynamics modelling test, analysis and correlation,* 195-204

[5] Pascual R. Golinval J.C., Razeto M. (1998): Model updating using operating deflection shaps. *Proceedings of XVI international modal analysis conference.* Santa Barbara, California, 12-18

[6] SAMCEF, User Manuals, release 8.1, Samtech S.A., 1999

[7] Maia M.M., Silva M.M. (1997): Theoretical and experimental modal analysis. Research studies press LTD, John Wiley & Sons INC.

[8] Heylen W., Lammens S., Sas P. (1998): Modal analysis theory and testing. K.U. Leuven, PMA (page A.6.8)

[9] Thonon C., Golinval J.C. (2001): Finite element model updating of the GARTEUR SM-AG19 structure. Research report of LTAS-VIS, University of Liege, Belgium.

[10] Pascual R., Golinval J.C., Razeto M. (1997): A frequency domain correlation technique for model correlation and updating. *Proceedings of XV international modal analysis conference.* Orlando, Florida, 587-592.

[11] BOSS-QUATTRO, User Manuals, release 2.1, Samtech S.A., 1999

[12] Degener M., Hermes. (1996): Ground Vibration Test and Finite Element Analysis of the GARTEUR SM-AG19 Testbed, Deutsche Forschungsanstalt für Luft- und Raumfahrt e. V. Institut für Aeroelastik (23200), October 1996.