A NEW DESIGN MODEL FOR THE RESISTANCE OF STEEL SEMI-COMPACT CROSS-SECTIONS

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1 INTRODUCTION

The new EN version of Eurocode 3 Part 1–1 design rules [1], which has now a normative status, introduces some rather different beam-columns formulae, in comparison with their ENV counterparts. Two different sets of formulae are now available, according to either Method 1 or Method 2. Based on second-order principles, both methods provide a high level of accuracy and emphasize a clear physical background, which is of prime importance in the perspective of a safe daily application.

Amongst them, Method 1 offers a high level of generality, consistency and continuity with the other formulae of the code, in addition to a significantly improved accuracy. Whenever considerable efforts have been made for the new formulae to provide a maximum of continuities, a gap still exists in the determination of cross-section bending resistance (see Fig. 1), due to the cross-section classification concept used in the code that defines a discontinuous level of resistance along the cross-section class ranges.

Present paper presents a new set of design rules that aims at filling this gap. It details a mechanical model offering fully continuous solutions along the different cross-section classes for both sections and members and proposes the results of parametric studies led towards the validation of the proposed design formulae.

2 DERIVATION OF A DESIGN MODEL FOR CLASS 3 CROSS-SECTION MEMBERS

2.1 Preliminary remarks

Cross-section resistance in Eurocode 3 is based on the concept of cross-section classes; 4 classes of resistance are defined, from Class 1 (plastic) to Class 4 (slender). The limits between classes reflect the influence of premature or late occurrence of local buckling in the section, and therefore depend on the b/t ratios of the different walls composing the cross-section and on the actual yield stress. In this respect, a convenient dimensionless measure of the sensitivity of a plate to local buckling is the so-called plate relative slenderness:

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} = \sqrt{f_y \frac{12\left(1-\nu^2\right)}{k_\sigma \pi^2 E}}\,\frac{b}{t} \tag{1}$$

Eq. (1) k_σ factor represents the "plate buckling factor", which depends on the aspect ratio, support conditions and state of stress in the considered plate. Well-known values vary from 0.43 for flange outstands under compression, to 4.0 for a simply supported plate under pure compression and to 23.9 for a simply supported plate in pure bending. Eurocode 3 makes use of the plate relative slenderness concept to define the limits between cross-section classes:

- $\lambda_p = 0.5$ for the limit between Class 1 and Class 2 (plate in compression);
- $\lambda_p = 0.6$ for the limit between Class 2 and Class 3 (plate in compression);
- $\lambda_p = 0.74$ for the limit between Class 3 and Class 4 (plate in compression);
- $\lambda_p = 0.9$ for the limit between Class 3 and Class 4 (plate in bending).

By means of Eq. (1), the corresponding b/t values may be determined; Table 1 summarises the (rounded) values proposed by Eurocode 3 for the different k_σ situations, as a function of $\varepsilon = \sqrt{235/f_y}$:

Table 1. EN 1993-1-1 limit values

k_σ	$\lambda_p = 0.5$	$\lambda_p = 0.6$	$\lambda_p = 0.74$ (0.724)	$\lambda_p = 0.9$ (0.887)
0.43	9ε	10ε	14ε	/
4.0	33ε	38ε	42ε	/
23.9	72ε	83ε	/	124ε

It is to be noted here that whenever the 0.5 and 0.6 limits have been defined arbitrarily, the 0.74 and 0.9 values at the Class 3 – 4 border arise from analytical considerations. They are based on the so-called "Winter formula" (see Eq. (2)) that provides plate effective resistance, at the exact limit where the plate resistance starts decreasing due to buckling. A strict application of Eq. (2) left part respectively leads to 0.724 and 0.887 values, to be compared to the 0.74 and 0.9 ones.

$$\rho = \frac{\bar{\lambda}_p - 0.05(3+\psi)}{\bar{\lambda}_p^{\,2}} \leq 1 \text{ (Winter)} \quad \text{and} \quad \rho = \frac{\bar{\lambda}_p - 0.055(3+\psi)}{\bar{\lambda}_p^{\,2}} \leq 1 \text{ (modified Winter)} \qquad (2)$$

Besides this, it may be noted that Eurocode 3 Part 1.5 "Plate design" [2] proposes a slightly modified Winter formula, see Eq. (2), which leads to 0.673 and 0.874 limit values for λ_p. Table 2 summarises the corresponding b/t ratios, where the impact of both an accurate calculation of the b/t limits and the use of the modified formulae is seen to be quite important (see bold values). In particular, it is seen that the whole Class 3 range for a simply supported plate in compression ($k_\sigma = 4.0$) is shifted: it becomes comprised between 34ε to 38ε instead of extending from 38ε to 42ε. This feature is essential here, since the aim of the research undertaken is to increase the knowledge within the Class 3 field. For sake of accuracy and consistency, Table 2 (rounded) values have been kept as a reference for further developments.

Table 2. Limit values based on the "modified Winter formula"

k_σ	$\lambda_p = 0.5$	$\lambda_p = 0.6$	$\lambda_p = 0.673$	$\lambda_p = 0.874$
0.43	$9.32\varepsilon \approx 9\varepsilon$	$11.18\varepsilon \approx 11\varepsilon$	$12.54\varepsilon \approx 13\varepsilon$	/
4.0	**$28.42\varepsilon \approx 28\varepsilon$**	**$34.10\varepsilon \approx 34\varepsilon$**	**$38.25\varepsilon \approx 38\varepsilon$**	/
23.9	$69.47\varepsilon \approx 69\varepsilon$	$83.36\varepsilon \approx 83\varepsilon$	/	$121.43\varepsilon \approx 121\varepsilon$

2.2 Elastic-plastic cross-section resistance

According to the principles of cross-section classification in Eurocode 3, the maximum bending resistance that a Class 3 cross-section can experience consists in its *elastic* resistance. This results in a lack of continuity at the Class 2 – 3 border, as Fig. 1 shows. Indeed, the cross-section resistance for pure bending drops from $M_{pl,Rd}$ to $M_{el,Rd}$ when the cross-section passes from Class 2 to Class 3. More than 50% of resistance may be lost due to the presence of this gap, which is physically not acceptable; the loss of resistance may rise up to 65% when biaxial bending is of concern.

In order to propose a solution to this problem, an RFCS research project named "Semi-Comp" has been initiated [3], where a mechanical model for a continuous cross-section and member buckling resistance along the Class 3 field has been developed. The basic idea on which the model relies on consists in offering an intermediate "elastic-plastic" resistance of the semi-compact cross-section in bending, between pure plastic and pure elastic behaviour (see Fig. 1). In accordance, the distribution of stresses of Fig. 2 is adopted.

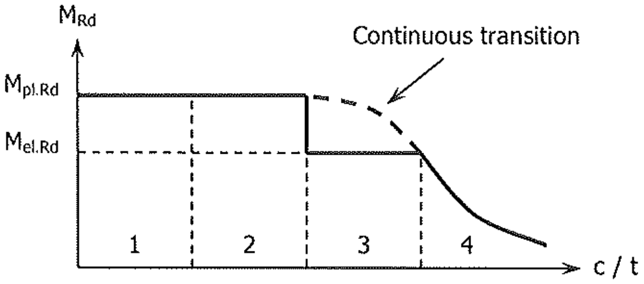

Fig. 1. Proposed continuous distribution of bending resistance along cross-section class ranges

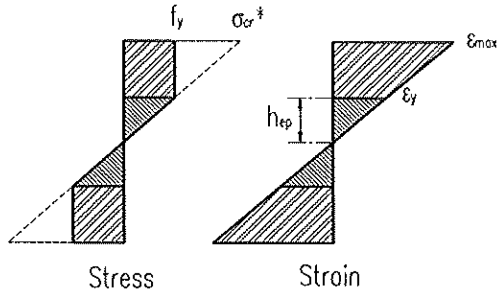

Fig. 2. Elastic-plastic distribution of stresses

The total bending resistance then consists of two distinctive contributions (Eq. (3)):

- a plastic contribution $W_{3,pl} f_y$ that corresponds to the yielded fibres of the section;
- an elastic contribution $W_{3,el} f_y$ arising from the other fibres of the section that have not reached the yield stress (fibres within h_{ep}, see Fig. 2).

$$M_{3,Rd} = \left(W_{3,pl} + W_{3,el}\right) f_y \tag{3}$$

Obviously, the key aspect here lies in the correct determination of h_{ep}, that must be so that the cross-section reaches a full plastic resistance at the Class 2 – 3 border (i.e. $h_{ep} \to 0$), and that the cross-section exhibits its sole elastic resistance at the Class 3 – 4 border (i.e. $h_{ep} = h$). This is achieved in assuming that whenever the stress distribution was still linear beyond f_y, the maximum stress reached would be equal to σ_{cr}[1], at a strain level $\varepsilon = \varepsilon_{max} > \varepsilon_y$, ε_y being the maximum elastic strain. Moreover, in order to fulfil the continuity aspects at the ends of the Class 3 field, it is necessary to bring modifications to the original definition of σ_{cr}. Indeed, in accordance with Fig. 2, σ_{cr} must be so that:

- $\sigma_{cr} = \infty$ at the limit between Class 2 and Class 3;
- $\sigma_{cr} = f_y$ at the limit between Class 3 and Class 4.

In the particular case of a simply supported plate in compression, such a modified σ_{cr}^* writes:

$$\sigma_{cr}^* = 1.616\, E \left(\frac{1}{9.5\, c/t - 323\varepsilon} \right)^2 \tag{4}$$

2.3 Extension to combined loading and member behaviour

The proposed design model simply makes use of the plastic cross-section design checks of Eurocode 3 for combined loading, adequately modified to allow for continuous transitions along the Class 3 range. For *monoaxial* bending with axial force, the proposed modification is as follows:

$$M_{N,3,i,Rd} = M_{3,i,Rd} \frac{1-n}{1-0.5a\left(1-\dfrac{f_y}{\sigma_{cr}^*}\right)} \le M_{3,i,Rd} \tag{5}$$

where i designates either the y or z axis. For *biaxial* bending and axial force, the criterion becomes:

$$\left(\frac{M_{y,Ed}}{M_{N,3,y,Rd}}\right)^{\alpha^*} + \left(\frac{M_{z,Ed}}{M_{N,3,z,Rd}}\right)^{\beta^*} \le 1 \text{ where } \alpha^* = \beta^* = \frac{1.66}{1-1.13n^2}\left(1-\frac{f_y}{\sigma_{cr}^*}\right) + \frac{f_y}{\sigma_{cr}^*} \le 6 \text{ (hollow)} \tag{6}$$

[1] The critical plate stress σ_{cr} appears as a convenient stress measure here, since the local (plate) instability effects play the key role.

These equations ensure full continuity with the simple loading cases and within the Class 3 range.
The implementation of this cross-section model is straightforward in the Method 1 set of beam-column design formulae [4], since it is only required to replace $M_{pl,y,Rd}$ and $M_{pl,z,Rd}$ by $M_{3,y,Rd}$ and $M_{3,z,Rd}$ in the different factors where they appear:

$$\frac{N_{Ed}}{\chi_y\, N_{pl,Rd}} + \mu_y \left[\frac{C_{my}\, M_{z,Ed}}{\left(1 - N_{Ed}/N_{cr,y}\right) C_{yy}\, M_{3,y,Rd}} + \gamma^* \frac{C_{mz}\, M_{z,Ed}}{\left(1 - N_{Ed}/N_{cr,z}\right) C_{yz}\, M_{3,z,Rd}} \right] \leq 1 \qquad (7)$$

$$\frac{N_{Ed}}{\chi_z\, N_{pl,Rd}} + \mu_z \left[\delta^* \frac{C_{my}\, M_{z,Ed}}{\left(1 - N_{Ed}/N_{cr,y}\right) C_{zy}\, M_{3,y,Rd}} + \frac{C_{mz}\, M_{z,Ed}}{\left(1 - N_{Ed}/N_{cr,z}\right) C_{zz}\, M_{3,z,Rd}} \right] \leq 1 \qquad (8)$$

In addition, one needs to adjust the γ^* and δ^* factors as in Eq. (9):

$$\gamma^* = 0.6\sqrt{\frac{w_z}{w_y}} \left(1 + \frac{1 - 0.6\sqrt{w_z/w_y}}{0.6\sqrt{w_z/w_y}} \frac{f_y}{\sigma_{cr}^*} \right) \text{ and } \delta^* = 0.6\sqrt{\frac{w_y}{w_z}} \left(1 + \frac{1 - 0.6\sqrt{w_y/w_z}}{0.6\sqrt{w_y/w_z}} \frac{f_y}{\sigma_{cr}^*} \right) \qquad (9)$$

3 VALIDATION OF PROPOSED MODEL – PARAMETRIC STUDIES

3.1 Development and assessment of FEM numerical models

In order to validate the proposed beam-column formulae, systematic comparisons with reference results have been performed (see Section 3.2). Preference has been given here to numerical FEM results, regarding the high number of parameters involved in the behaviour of Class 3 members; indeed, not only member *global* behaviour but also *local* behaviour has to be characterised here.
Accordingly, suitable GMNIA shell[1] FEM models have been developed for H-shaped and hollow section profiles [3]. They have been prepared to account for both *global* and *local* initial geometrical imperfections, and special attention was paid to the modelling of the radius and corner zones. For full details on the development of these models, one may refer to reference [3].
In order to assess the results provided by the FEM models, 24 buckling tests on open and hollow section members have been performed at the University of Liège. The tests consisted in either monoaxial or biaxial bending with compression tests, where first order bending was applied through eccentrically applied thrust. Detailed results and measurements are available in references [3], [5], and [6]. In comparison with the tests, the FEM models showed an excellent level of accuracy, within 5% of the experimental maximum load in average.

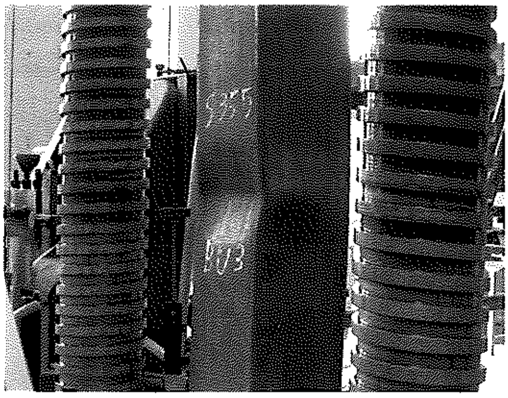

Fig. 3. SHS section after test

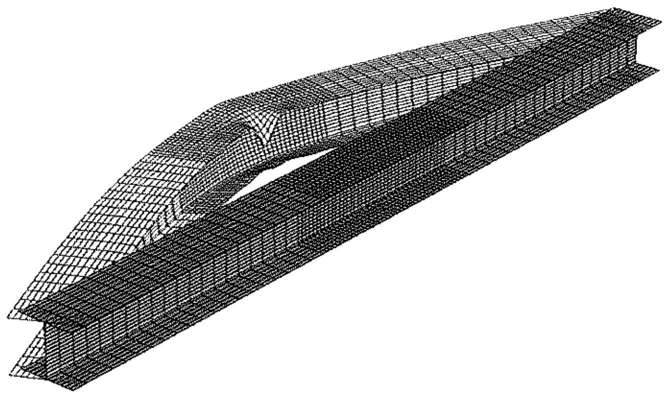

Fig. 4. FEM modelling of buckling test

[1] Indeed, the key aspect here consists in characterising the early or late occurrence of local buckling on the behaviour of the semi-compact cross-section member, thus the need for *shell* elements.

3.2 Parametric studies

The numerical models being shown to be accurate and appropriate, extensive parametric studies have been performed. The following parameters have been taken into account:

- 4 different cross-section shapes: HEAA 220, HEAA 300, RHS 250x150x6 and SHS 180x5;
- 2 steel grades: $f_y = 235\ N/mm^2$ and $f_y = 355\ N/mm^2$;
- 2 primary linear bending moments distributions: $\psi_y = \psi_z = 1$ and $\psi_y = \psi_z = 0$ ($M_{y,Ed}$ and $M_{z,Ed}$ end-moments applied on the same side);
- 2 relative slenderness $\lambda_z = 0.5$ and $\lambda_z = 1.0$;
- 4 different values of relative axial compression $n = N_{Ed} / N_{b,Rd}$: n = 0, 0.30, 0.50 and 0.70, where $N_{b,Rd}$ represents the flexural buckling load under pure compression;
- For each fixed values of the previous parameters, 9 combinations of $M_{y,Ed}$ and $M_{z,Ed}$ values have been investigated, so that to allow the determination of the biaxial bending interaction.

In total, 1152 non-linear FEM-shell calculations have been performed, on so-called simply supported members (fork conditions). Due allowance for material imperfections and initial *local* and *global* defaults have been made, as explained before.

Table 3. Summary of results for H-shaped and hollow section specimens

	HEAA220 – $\psi = 1$	HEAA220 – $\psi = 0$	HEAA300 – $\psi = 1$	HEAA300 – $\psi = 0$
m	1.141	1.155	1.294	1.348
s	0.115	0.127	0.271	0.318
min	0.856	0.900	0.860	0.868
max	1.434	1.518	2.131	2.572
Σ *tests*	144	144	144	144
	RHS – $\psi = 1$	RHS – $\psi = 0$	SHS – $\psi = 1$	SHS – $\psi = 0$
m	1.263	1.316	1.275	1.340
s	0.141	0.183	0.229	0.307
min	0.916	0.907	0.903	0.917
max	1.552	1.756	1.820	2.092
Σ *tests*	144	144	144	144

Table 3 proposes a summary of results on the comparison between numerical and analytical results. The values reported refer to so-called R_{simul} values, which represent the ratio of the $M_{y,Ed} - M_{z,Ed}$ loading leading to failure according to the FEM result to the *proportional* $M_{y,Ed} - M_{z,Ed}$ loading leading to failure according to the proposed formulae. Consequently, a value higher than unity means safety, while a value lower than one indicates an unsafe result. The R_{simul} values also form a good indicator of the level of accuracy of the proposal: a value of 1.10 indicates a 10% resistance reserve. As can be seen in Table 3, the mean *m* and standard deviation *s* values of the R_{simul} ratios are quite satisfactory, highlighting the good accuracy of the proposal.
In addition to this "accuracy" parametric study, a second "safety" parametric study has been undertaken, aiming at the determination of so-called γ_{M1} safety factors. In this respect, some 1152 additional cases have been studied, where all relevant geometrical and material data have been generated by random (using the Monte-Carlo simulation principles) and introduced in both the FEM models and the analytical calculations. A rigorous and strict application of EN 1990 Annex D principles [7] respectively led to 1.10 and 1.03 γ_{M1} values for open and hollow section members, which comply with the recommended value of 1.10 in Eurocode 3.

Fig. 5 further illustrates the ability of the proposed model to lead to intermediate results between full plastic behaviour and pure elastic behaviour. The tendency of exhibiting a relatively significant level of plastic reserve has been generally observed on the cases studied, even in situations where the cross-section is classified as Class 4, as Fig. 6 shows.

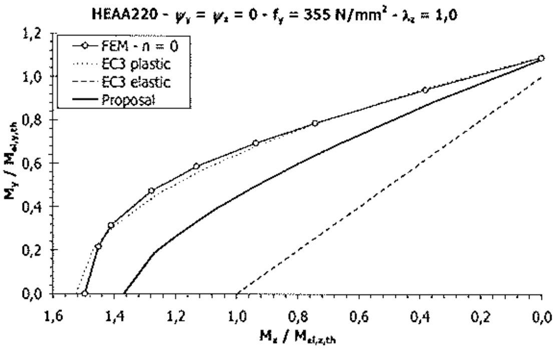

Fig. 5. Illustration of an intermediate elastic-plastic behaviour

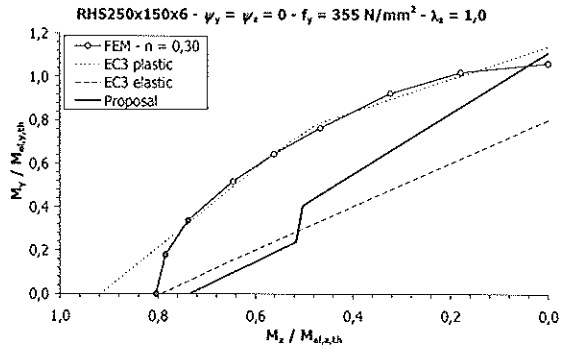

*Fig. 6.*Situation where plastic reserve is observed while cross-section is Class 4 or nearly (first 3 points only are Class 4)

4 CONCLUSIONS

This paper presented a new design model for the bending resistance of Class 3 cross-sections based on an elastic-plastic distribution of stresses. The use of a modified critical stress allows a fully continuous bending resistance along the Class 3 field, between full plastic and pure elastic capacity. Extensions of the model to combined loading and member behaviour have also been detailed.
Through an extensive validation study comparing the results of the proposed model with more than 1100 FEM results, the formulae have been shown to be adequate and accurate. An additional parametric study oriented towards the determination of the level of safety of the formulae led to γ_{M1} values complying with Eurocode 3 requirements.

5 ACKNOWLEDGEMENTS

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