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ON THE MODELLING OF COMPRESSED LONG PLATES RELATED TO SHEET-SECTION BEHAVIOUR

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Abstract: For the behaviour of thin-walled sheet-sections without stiffeners, often a square part of the compressed flange is modelled. However, once the sheet-section includes stiffeners, the flange can only be studied by modelling a long plate, preferentially with a prescribed displacement function along the length. For this long plate, even without stiffeners, convergence problems occur if a quasi-static solution method is used. This has been solved by a dynamic (explicit) solution procedure with which a parameter study was carried out. For small imperfections, so-called mode-jumping occurs always whereas for large imperfections this occurs less often. During mode-jumping, the wave-length and location of the buckles change. As mode-jumping occurs before plasticity starts, this leads to the question whether existing failure predicting models should be used in their current form. Finally, it is shown that the same conclusions are valid for plates with stiffeners.

Keywords: *Sheet-section, Mode-jumping, Stiffeners, Displacement function, Concentrated load, Bending moment.*

1. INTRODUCTION

For first generation sheet-sections, which are sheet-sections without stiffeners, the so-called ultimate failure model enables the prediction of the sheet-section's ultimate load for combined concentrated load and bending moment, figure 1, Hofmeyer et al. (2001). It first determines the elastic indentation of the sheet-section's cross-section (defined as web-crippling deformation) due to the concentrated load using an energy or beam-on-elastic-foundation approach. This web-crippling deformation is then used to calculate the out-of-plane deflection of a square part of the compressed flange. This out-of plane deflection is used as imperfection in Marguerre's plate equations, which can be used to predict the stresses in the square part of the compressed flange. If these stresses reach the yield strength at a specific location, the sheet-section is assumed to fail.

Recently, an alternative for Marguerre's equations within the ultimate failure model was developed, namely the two-strip model. In this model, the nonlinear elastic behaviour of a compressed plate is described by dividing the plate into two strips, Bakker et al. (2006). The edge strip behaves linear elastically whereas the centre strip can buckle similar to an Euler column. The initial imperfection and the maximum displacement of the centre strip were scaled to the corresponding values of the real plate using FE-simulations. The nonlinear behaviour can then be used to predict plate failure by using specific elasto-plastic criteria.

Because the two-strip model provides insight in the failure behaviour of plates without stiffeners and, used within the ultimate failure model, predicts the ultimate load of sheet-sections at least equally correctly as the currently used design rules, it is a candidate for future design rules. However, the question is whether the model is suitable for predicting the behaviour of plates with stiffeners. Therefore the nonlinear elastic behaviour and the failure behaviour of plates with stiffeners should be studied. Like for plates without stiffeners, this study should be carried out with the finite element method because experiments are not accurate enough to control exactly the boundary conditions and initial imperfections of a plate. However, Lint (2006) showed that even when using a finite element method the boundary conditions cannot be modelled correctly. To explain this, first the standard boundary conditions for a plate without stiffeners as used for the two-strip model are presented (figure 2a). All edges are simply supported, that means $w = 0$. The unloaded edges are free in y-direction (thus the edges are able to "wave" in-plane) or are forced to remain straight but are still able to move along the y-direction. The loaded edges are kept straight and are loaded using displacement control.

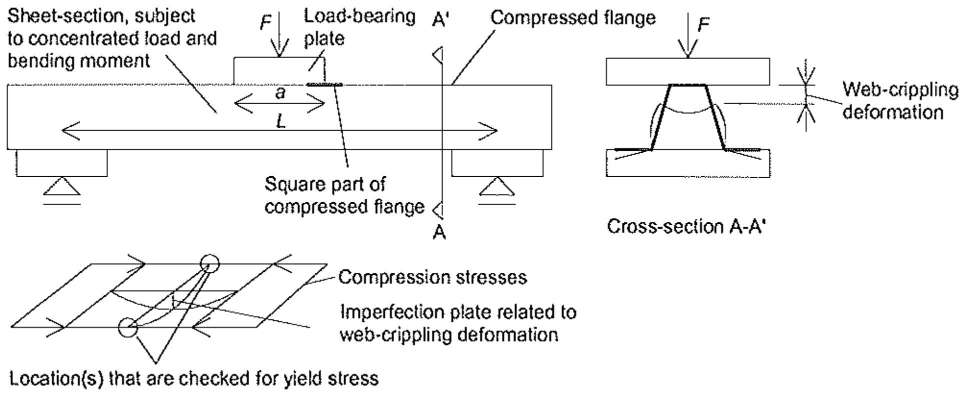

Figure 1: Ultimate failure model.

If these boundary conditions are used for a plate with an eccentric stiffener and the prescribed displacement along the loaded edge is also applied to the stiffener, the plate bends due to the fact that the load resultant of the prescribed displacement is not in line with the neutral axis, even for a linear elastic simulation, figure 2b. But if only the flat parts of the loaded edges are loaded, the stiffener is not loaded, which does not resembles the situation in practice. A solution for this problem is to model the plate for one half of the convex curved geometry and one half for the concave curved part, figure 2c. Now, the loaded edges of these parts do not rotate in practice and thus no clamping stresses can occur.

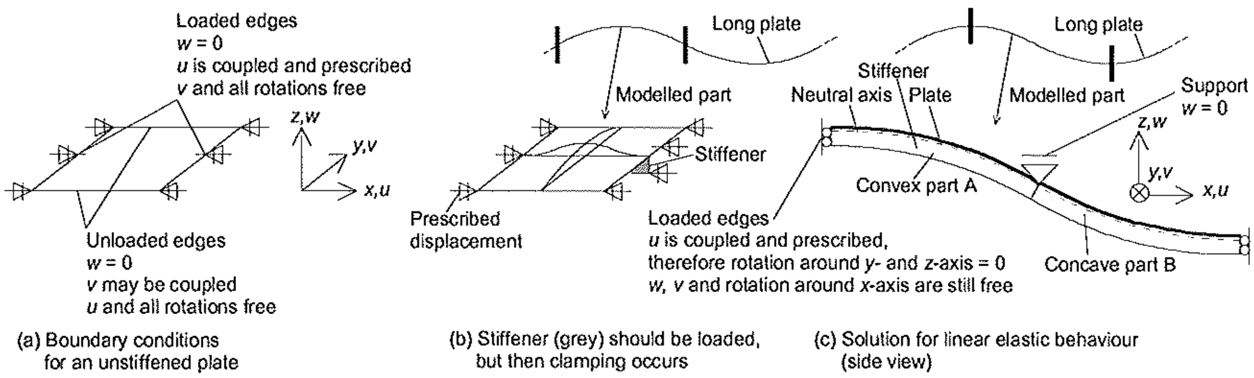

Figure 2: (a) boundary conditions, (b) stresses due to clamping of the stiffener, (c) solution for linear elastic simulations.

For an elastic nonlinear simulation, even this solution is not adequate. This because for the convex part (part A in figure 2c) the largest part (neutral axis will be only slightly below the plate surface) of the stiffener is under compression, whereas for the concave part B, the largest part of the stiffener is under tension. This means that for part A the compressed stiffener will amplify the bending of the plate and part B experiences a reduced bending. For this case, it is unlikely that the wave lengths of convex and concave parts are still the same, and the model as shown in figure 2c may show to be erroneous in using the same lengths for both parts. The only solution for these problems is the modelling of the complete compressed flange and applying boundary conditions such that the actions of the webs, the concentrated load and the supports as shown in figure 1 are taken into account.

2. LONG PLATE BEHAVIOUR

If a complete compressed flange has to be simulated, several modelling issues have to be considered. First of all, symmetry can be used for the (nonlinear) elastic simulations, but as soon as elasto-plastic simulations are carried out (presented in section 5), asymmetrical yield-line patterns may occur. Secondly, if a linear varying strain is used as loading, it can be chosen to incorporate the constant strain zone beneath the load-bearing plate (see figure 1 and 3) or to neglect this relatively small zone (width a). It is chosen here to incorporate the constant strain zone because in this way, results of the plate in this zone can be compared with the results of simulations of a square plate, for which also a constant strain was applied. Thirdly, for a sheet-section the webs can buckle and thus the strain will not vary linearly over the compressed flange but for now, first plate behaviour will be investigated for pure linearly varying strain. And finally, the length of the finite element model L_{mod} may be reduced to the area where the strain is larger than the critical strain. Giving all these considerations, the loading of the plate is defined by a quadratic varying prescribed displacement u along the edges, resulting in a linear varying strain ε similar to the loading of a compressed flange in a sheet-section, figure 3.

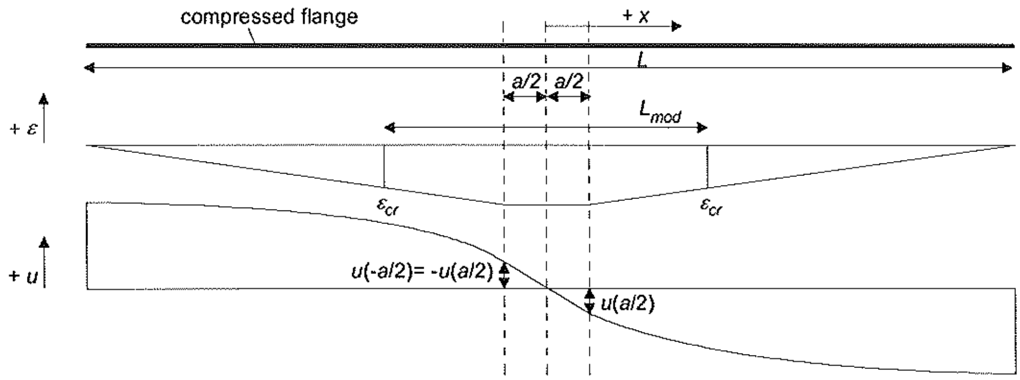

Figure 3: Displacement and strain distribution.

For the determination of the displacement function along the longitudinal edges, the following can be derived (see figure 3 for variable definitions):

$$-\frac{L}{2} \leq x \leq -\frac{a}{2}: \; u(x) = \frac{2u(a/2)}{a(L-a)}x^2 + \frac{2Lu(a/2)}{a(L-a)}x + \frac{au(a/2)}{2(L-a)} \tag{1}$$

$$-\frac{a}{2} < x < \frac{a}{2}: \quad u(x) = \frac{2u(a/2)}{a}x \tag{2}$$

$$\frac{a}{2} \leq x \leq \frac{L}{2}: \; u(x) = -\frac{2u(a/2)}{a(L-a)}x^2 + \frac{2Lu(a/2)}{a(L-a)}x - \frac{au(a/2)}{2(L-a)} \tag{3}$$

Using the displacement function, a finite element simulation was carried out as shown in figure 4. Dimensions were taken from previous simulations on square plates, Bakker et al. (2006), and the total length of the compressed flange L was chosen to be equal to the average of values used in practically designed experiments, Hofmeyer et al. (2001), which is 1500 mm. Edges were simply supported and additionally, the displacement in length direction (u) was coupled for two interior lines (defined as the square edges) as shown in figure 4 in order to compare the simulation with the previous simulations on square plates.

For the previous simulations on square plates, Bakker et al. (2006), the maximum $u(a/2)$ taken was 0.25 mm and the critical strain ε_{cr} was 0.000178. For the simulation presented here, with a length L equal to 1500 mm, a square plate length of 99.8 mm, and plate thickness t = 0.7 mm, the strain was predicted using equation 1 and 3. The results then showed that the flange has a strain larger than the critical strain over the whole flange length. This means the whole flange was modelled. For verification reasons, the finite element model was first used to make a comparison: Only the square part, given a sinusoidal imperfection, was loaded and was compared with the previous simulations on a square part. A good agreement between the simulations was found and differences could be explained, Hofmeyer (2007). Secondly, the whole edge was loaded by the prescribed displacement function (defined as line loaded) but the square edges were still forced to remain straight. The first eigenmode was used as an imperfection (with maximum $0.01t$ or $2t$) and scaled to the same maximum value as the previous simulations. Results are shown in figure 5. It can be seen that the results are correct qualitatively; only the line loaded simulations predict a higher load. This is presumably due to redistribution of forces between the square part and the two remaining parts on the left and right.

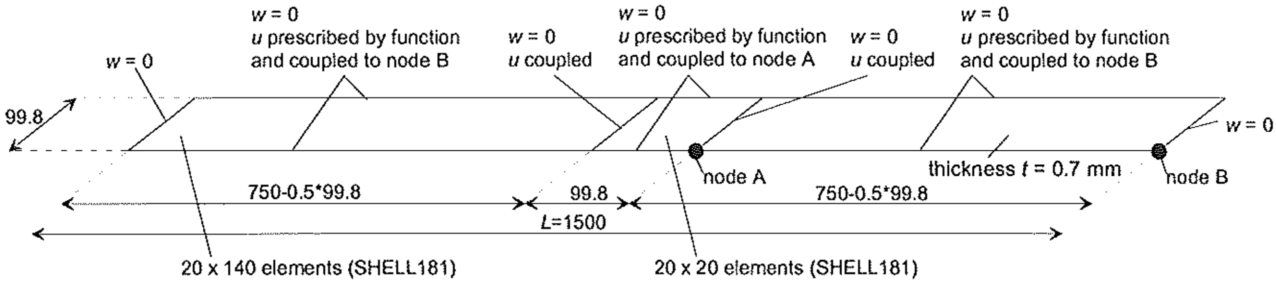

Figure 4: Set-up finite element model.

A simulation was carried out for the situation where the constraints for the square edges were removed. Because new eigenmodes occurred they were determined first. Then for both the first and second eigenmode (used as imperfection) at a load time 0.02 (plate shortening equal to 0.04 mm) convergence problems occurred, which could not be solved by taking smaller time steps. It is assumed that mode-jumping phenomena cause the convergence problems in a quasi-static solution procedure as presented by Riks et al. (1996). This can be solved by using a transient procedure, to be presented in the next section, in which the dynamical (transient) effects have been taken into account. However, to introduce the initial imperfections, the simulation will start with a quasi-static part.

3. IMPLICIT-TO-EXPLICIT SIMULATIONS

3.1 Quasi-static part

The set-up of the quasi-static part is similar to the simulation discussed in section 2. Figure 4 can be used for the geometrical set-up, however the constraints in the plate itself (i.e. that the left and right lines of the square part are coupled) have been removed. SHELL181, Ansys (2007), elements were used instead of SHELL43 because these elements can be converted into elements for the transient response part later on. An eigen-mode analysis was carried out. The first mode was used to introduce a stress-less imperfection equal to $0.01t$. Hereafter, a very small nonlinear calculation was carried out with a total load equal to 0.0001 mm shortening of the square part of the plate.

3.2 Transient response part

For the transient response part loading, a program "genlist.lgw" have been written in APDL (ANSYS Parametric Design Language) that automatically scans the longitudinal edges of the plate and generates an output file with a list of all nodes along the edges and their displacement. Then a Pascal program changes this output file into commands for the transient response part, for which LS-Dyna version 970 revision 3858 is used. Instead of describing displacements, it was found that linear increasing velocities are better suited to load the structure. For the real loading time (100 ms), research on simply supported square plates revealed that a small value of alpha damping (1000) is useful for damping out oscillations in the reaction forces without compromising the static behaviour of the structure, Courage (2007). Default isotropic elastic material was used, with $\rho = 7.83*10^{-9}$ ton/mm^2, $E = 210000$ N/mm^2, and $\upsilon = 0.3$. Shell elements SHELL163 were used with default (key-) options and 3 integration points over the thickness. The solution procedure used explicit time integration and as such no convergence problems occurred.

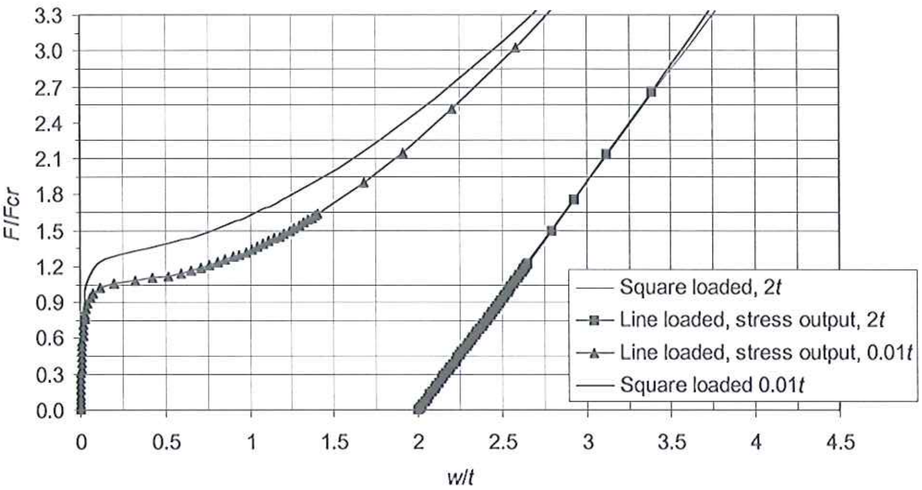

Figure 5: Long strip, comparison between square loaded and line loaded, left and right square edges remain straight.

3.3 Results

The resulting load-plate shortening behaviour is shown in figure 6. Note that in figure 5 the out-of-plane deflection w was used on the horizontal axis because this presents optimally the buckling phenomenon, however in figure 6 the plate shortening is used because the buckling shape will change and thus the out-of-plane deflection cannot be determined objectively. The load on the plate was determined using two different post-processors (A: LS-PrePost and B: Ansys) giving the same results for the critical deformation range around mode-jumping. The figure shows this mode-jumping by a load irregularity, at a shortening equal to 0.025 mm. This is a similar value to the plate shortening at which the quasi-static solution showed non-convergence. During the load irregularity a non-stable equilibrium exists that jumps from one equilibrium path to another path and that this cannot be described using a quasi-static path-following solution procedure alone. Also contour plots of equal out-of-plane displacements (on the left of the figure) show clearly that the number of buckles (and thus the wavelength) changes.

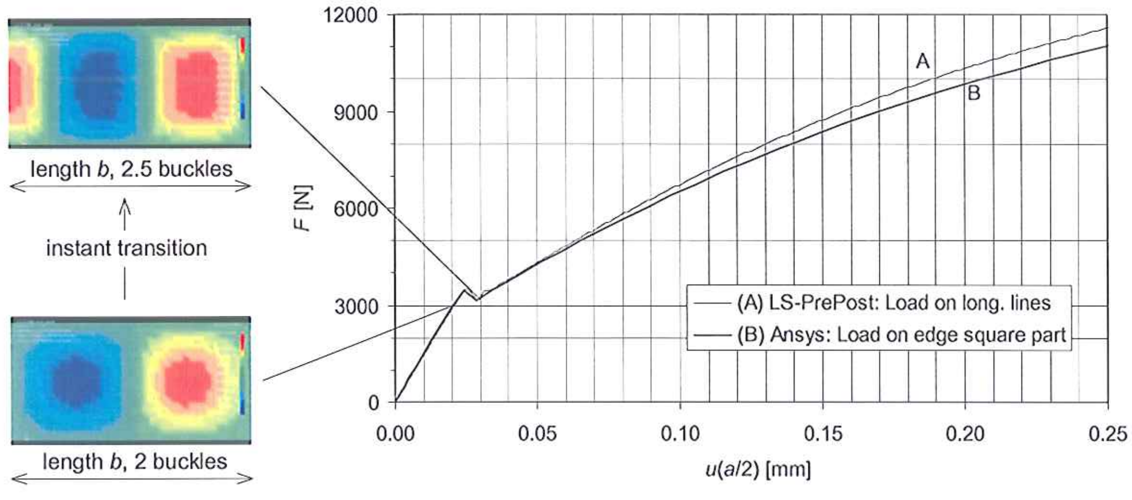

Figure 6: Load-plate shortening behaviour, load determined by reaction loads on edge and by average membrane stresses in the plate.

4. PARAMETER STUDY

It can be expected that the shape and magnitude of the initial imperfection influences the results of the previous presented simulation. More specifically, Riks et al. (1996) mentioned "... it is not possible to guarantee that such arbitrary, but nevertheless particular choice of the initial disturbance will result in the expected jumping motion ... the jumping motion can only occur along predetermined directions. The initial (or transition) conditions should therefore be chosen accordingly". To investigate this problem for the simulation presented here, a parameter study was carried out as shown in table 1.

Table 1: Imperfection parameter study.

Imperfection	Wave length before mode-jumping [# elements]	# mode-jumps	# buckles after mode-jumping along length	Wave length after mode-jumping [# elements]
(A) 0.01*t*, mode I	40	1	18	33
(B) 0.01*t*, mode II	40	1	18	32
(C) 0.01*t*, mode III	13	1	18	31
(D) 0.01*t*, 1/3 mode I, 1/3 mode II, 1/3 mode III	mixed	1	18	33
(E) 2*t*, mode I	40	0	16	38
(F) 2*t*, mode II*	40	1 (at 3/4 of time)	17	37
(G) 2*t*, mode III	13	0	18	26
(H) 2*t*, 1/3 mode I, 1/3 mode II, 1/3 mode III*	mixed	0	16	42

* does not converge for implicit nonlinear solution prior to explicit part.

Different eigen-modes were used, and especially an imperfection pattern was chosen (1/3 mode I, 1/3 mode II, 1/3 mode III) that breaks the symmetrical pattern of imperfections as was shown by Riks et al. (1996) in a similar problem to give different results. The load versus plate shortening is shown in figure 7.

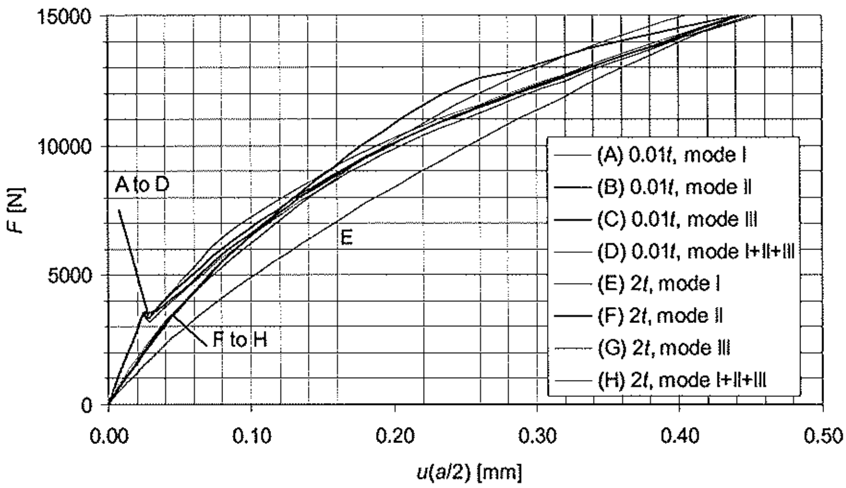

Figure 7: Load-plate shortening behaviour for several initial imperfection geometries and sizes.

For small imperfections, regardless the geometry of the imperfection, mode jumping occurs at around 1/3 of the maximum applied displacement, as shown in figure 7 by the load irregularities. The higher the mode number (thus the higher the corresponding buckling load) the less severe the load irregularity is. After mode-jumping, 18 buckles exist over the plate length. The wave length at the centre of the plate varies slightly (31 to 33 elements) but is regarded as constant. It can thus be concluded that for a small imperfection, the imperfection geometry does hardly influence the results and mode-jumping occurs for all cases.

For the large imperfection (2*t*) however, mode jumping occurs only for mode II and this much later in the load-plate deflection diagram than for small imperfections. In the plate centre, mode I and mode III more or less resemble the (anti-symmetrical) geometry of the plate after mode-jumping, and this may be a reason that no mode-jumping occurs for these imperfections; the imperfection's geometry already resembles the geometry that normally would occur after mode-jumping. Figure 7 also shows that larger imperfections make the plate less stiff. For simulation E this is very strongly the case in comparison with the other simulations.

5. PLASTIC BEHAVIOUR

The previous presented simulations were also modelled with elastic-plastic material behaviour. Because for the small imperfections no significant differences in elastic behaviour occurred, only simulation A (see table 1) was carried out. For the large imperfections, all previous simulations were repeated. For the material, no time-dependent behaviour was used, and a simple bilinear stress-strain curve was defined with a yield stress equal to 300 N/mm^2. For all other aspects, the simulations were the same as those presented in section 4. Load-plate shortening behaviour is shown in figure 8(a).

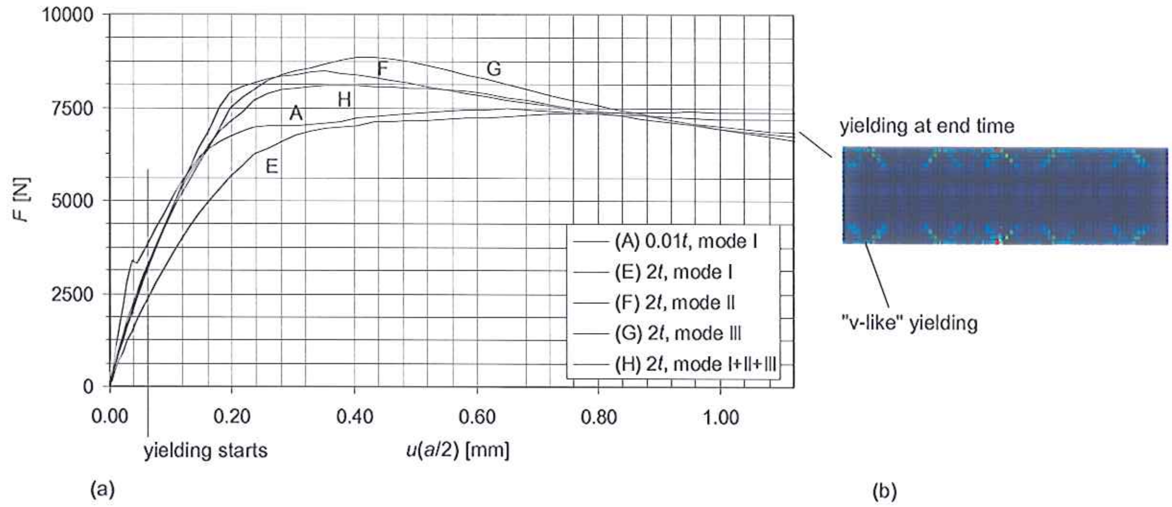

Figure 8: (a) Load-plate shortening behaviour for several initial imperfection geometries and sizes, (b) plastic behaviour.

The elastic behaviour is comparable to the previous simulations, figure 4: mode-jumping for (A) is just before first yielding starts whereas the jumping for (F) is not visible anymore, probably due to the plastic deformations. All imperfection geometries and sizes result in yielding that starts at approximately $u(a/2)$ = 0.065 mm, simultaneously for the middle and outer layers of the shell at the exact middle of the plate edge. After some further loading, "v-like" yielding shapes occur as shown in figure 8(b). Mostly, these yielding shapes are distanced equally, but for case (F) and (H) some mixed geometries occur. It can be concluded that for long plates first yielding is not a good prediction for the ultimate load and the ultimate load differs significantly for different imperfection modes and sizes.

6. PLATES WITH STIFFENERS

For square plates with one central intermediate stiffener, a two-strip model has been developed and was verified against nonlinear elastic, quasi-static FE-simulations, Bakker et al. (2008). In these simulations no convergence problems occurred. This is not a surprise, taken the previous information into account, because the square plate is a very highly constrained problem. In this section, the simulation of a stiffened long plate will be presented. The properties of this plate are related to the simulations already carried out. This means a = 99.8 mm, t = 1 mm, E = 210000 N/mm^2, υ = 0.3, and the initial imperfection w_0 = 0.005t. The stiffener, modelled by BEAM4 elements (Ansys (2007)) has properties A = 100 mm^2 and I = 100 mm^4. First a buckling analysis was carried out. For the first mode, the plate did not buckle in square buckles, due to the stiffener. The half wave length was more than two times the plate width (20 elements) and the buckling load (u_{cr} = 0.0521) was about five times higher than the first buckling load for a plate without stiffeners (u_{cr} = 0.0097). For the first three modes, the cross-section deformed smoothly without observing a real effect of the stiffener.

The fourth mode was the first mode in which the cross-sectional deformation shows the effect of the stiffener by a reduction of the out-of-plane displacement. This fourth mode was then used to apply an imperfection of $0.005t$ to the plate, where after an explicit dynamic calculation was carried out similar to the previous simulations. The load-plate deflection curve and deformed geometries showed clearly that mode-jumping occurred.

7. CONCLUSION

The compressed flange behaviour of sheet-sections with stiffeners cannot be modelled by a square plate. Therefore long plates should be modelled with a prescribed displacement function along the length.

Long plates without stiffeners already show mode-jumping phenomena for the prescribed displacement function and a transient procedure should be used to simulate this behaviour.

A parameter study shows that for small imperfections, mode-jumping occurs always and before plasticity starts whereas for large imperfections it occurs less often and after plasticity starts, which possibly hides the mode-jumping behaviour.

During mode-jumping, the wave-length and location of the buckled shapes change and this is also the case for plates with stiffeners. Furthermore, significant differences in ultimate strength exist for different imperfection sizes and geometries and first yields is not a good prediction for the ultimate load. Current failure predicting models disregard most of these effects.

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Please find enclosed a copy of the submitted paper. As soon as I have the proceedings you will also receive a final version.

Hope to see you then or earlier if we have a Liege-Eindhoven meeting again.

Best Regards and thank you for all your help,

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