RESISTANCE OF JOINTS SUBMITTED TO COMBINED AXIAL FORCE AND BENDING

Analytical procedures and comparison with laboratory tests

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ABSTRACT

Present paper deals with the behaviour of structural bolted joints between I or H steel profiles. In the first part, analytical procedures for the prediction of the resistance of joints subjected to combined bending moments and axial forces are introduced. Ductile and brittle responses of the end-plate connections are considered as well as the resistance of the column web panels in shear. In a second part, these methods are compared to experimental evidence. Finally the correlation between the experimental and analytically predicted resistances is studied and needs for further investigations are expressed.

1. INTRODUCTION

1.1 The component method

Nowadays the component method is a widely recognised procedure for the evaluation of the design properties of structural joints. It is used as a reference in Eurocode 3 [1] and Eurocode 4 [2], respectively for joints in steel and composite constructions, but it may also apply to many other joint configuration and connection types [3].

In the component method, any joint is seen as a set of elements (called components). The mechanical properties of these components, in terms of elastic deformation, design resistance and deformation capacity are evaluated through appropriate design models; then the component properties are "assembled" so as to finally derive the mechanical properties of the full joint, i.e. its rotational stiffness, its moment and shear design resistances, its failure mode and its level of rotation capacity.

So the characterisation of the joint properties through the component method implies three successive steps: (i) identification of the constitutive components, (ii) evaluation of the mechanical properties of the components and (iii) assembly of the components.
In Eurocode 3 Part 1.8 [4], simple analytical calculation procedures are provided; they mainly allow to derive the design moment resistance and the elastic rotational stiffness (called "initial stiffness") of steel joints subjected to bending moments and shear forces.

1.2 Structural joints subjected to bending moment *M* and axial force N

In most of the cases, beam-to-column joints and beam splices are subjected to compression or tension axial forces in addition to bending moments and shear forces. These ones have an influence on the rotational stiffness, moment resistance and rotation capacity of the joints. And that is why in Part 1.8 of Eurocode 3 the proposed field of application is limited to joints in which the axial force N_{Ed} (noted N in the paper for sake of simplicity - and the same applies to M_{Ed}, noted M -) acting in the joint remains lower than 5% of the axial design resistance of the connected beam ($N_{pl,Rd}$):

$$\left|\frac{N_{Sd}}{N_{pl,Rd}}\right| \leq 0{,}05 \tag{1}$$

Under this limit it is considered that the rotational response of the joints is not significantly influenced by the axial forces. It has however to be stated that this value is a fully arbitrary one and is not at all scientifically justified.

The 5% rule applies to most of the beam-to-column joints and beam splices in multi-storey building frames, but usually not to similar joints in pitched-roof industrial portal frames. Similarly column bases and column splices transfer high axial forces and therefore do not fulfil the limiting criterion prescribed by Part 1.8.

When the 5% rule is not satisfied, Part 1.8 considers that the interaction resistance diagram is defined by the polygon assembling the 4 points corresponding respectively to the hogging and sagging bending resistances in absence of axial force and to the tension and compression axial resistances in absence of bending.

These provisions are seen to be quite questionable [5]. And in order to develop an improved design procedure according to the component method – which is still valid as the behaviour of the components is independent on the type of loading applied to the whole joint –, a new assembly procedure is required to cover the combined action of bending moments and axial forces. The main difficulty results from the modification of the list of active components within the joints according to the relative importance of the bending moment and axial force, and obviously according to the respective signs of the applied forces. These items are addressed in the present paper, as far as resistance is concerned.

2 MECHANICAL MODEL AND PARTICULAR ASPECTS

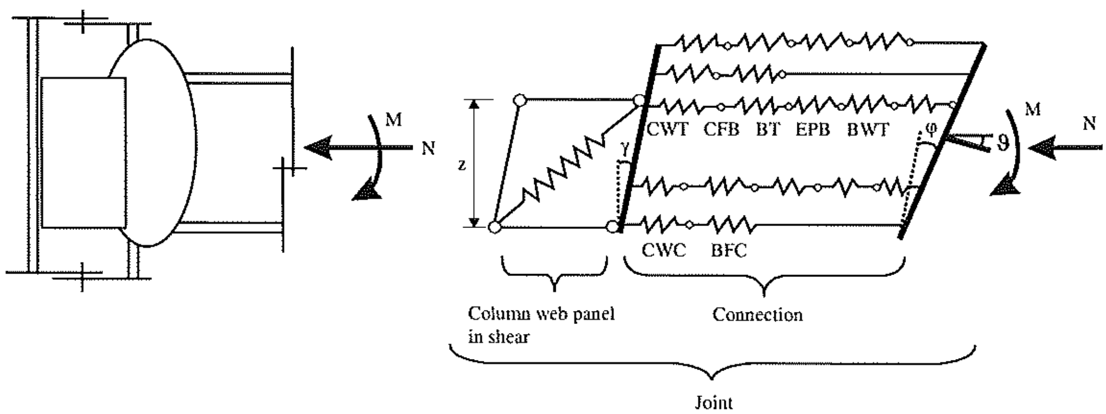

Fig. 1 – Mechanical model used in the proposed analytical procedures

The analytical investigations presented later on consider the mechanical model shown in Fig. 1 as a reference to represent the behaviour of a joint submitted to both bending and axial forces. This mechanical model is also used in the software ASCON - developed at Liège University (see [5]) - which allows to predict in a numerical way the response of structural joints under so-called M-N interaction. In this model, each constitutive component of the joint is represented by a extensional spring characterised by a non-linear F-Δ curve, where F and Δ

represent respectively the force acting in the component and the related displacement. According to the definitions given in Eurocode 3 Part 1.8, the joint is seen to be constituted of a *connection* subjected to bending moment and axial force and a *column web panel* in shear.

Two particular features of the component method have also to be carefully considered:

- "Group effects" : these effects are likely to occur in plate components subjected to transverse bolt forces (endplates in bending – EPB -, column flanges in bending - CFB, …in Fig. 1). Where a bolt force is applied (BT), a yield plastic mechanism may develop in the plate component; if the bolt distances are high, separate yield lines will form in the plate component around the bolts (*individual bolt mechanisms*), while a single yield plastic mechanism common to several bolts may develop when the distance between the latter decreases (*bolt group mechanisms*). Group effects also affect the resistance of the following components (Fig. 1): column web in tension – CWT - and beam web in tension - BWT -.
- "Component interactions": these ones may occur in "column components" where three types of stresses interact: shear stresses in the web panel, longitudinal stresses due to axial and bending forces in the column and transversal stresses due to the load-introduction in the joint area (column web in tension - CWT -, column web in compression - CWC - and column web panel in shear).

3 DUCTILE INTERACTION DIAGRAM FOR CONNECTIONS

3.1 Definition

The behaviour of each of the constitutive joint components is here assumed to be infinitely ductile. As a result, a full plastic redistribution of the internal forces in the joint under M and N carried out on the basis of the so-called static theorem and to which it is referred in Eurocode 3 Part 1.8 may be contemplated. How to achieve this goal is extensively described in [5] and reported in Section 3.5. But before, the equilibrium equations to satisfy and the resistance criteria to fulfil are expressed in sections 3.3 and 3.4.
The so-derived ductile resistance interaction diagram corresponds to a plastic resistance surface; the actual applied bending moment and axial force in a connection define a couple of values which should remain inside the interaction diagram so as to ensure the sufficient resistance of the studied connection.

3.2 Conventions

Developments are presented for the general case of a bolted endplate connection with N_b bolt rows in which only tension forces may be transferred; in addition, two compression zones located at mid-thickness of the upper and lower beam flanges may be identified (respectively noted "*upper*" or "*up*" and "*lower*" or "*lo*" and constituted, as seen in Fig. 1, of two components: beam flange and web in compression - BFC - and column flange in compression - CWC -). This leads to a total of $N_b+2=n$ rows where internal forces may be transferred from the beam to the column. Conventionally the tension forces are assumed to be positive or equal to zero while a compression force has a negative or zero value. All the rows are numbered from *1* to n by starting from the upper row. As an example, for an extended endplate connection with one external bolt row, the compression row "up" is the row n° *2* while it is the row n° 1 for a flush endplate connection.

This is illustrated in Fig. 2 for a joint with an extended endplate connection including 5 ($=N_b$) bolt rows. The kinematics of the problem is such that, for instance: (i) the force in row n° *2* ("upper") is equal to zero when the forces in rows n° *1* and *3* are different from zero and (ii) the bolt group mechanism noted *[1,4]* will only involve rows in tension, n° *1*, *3* and *4*.

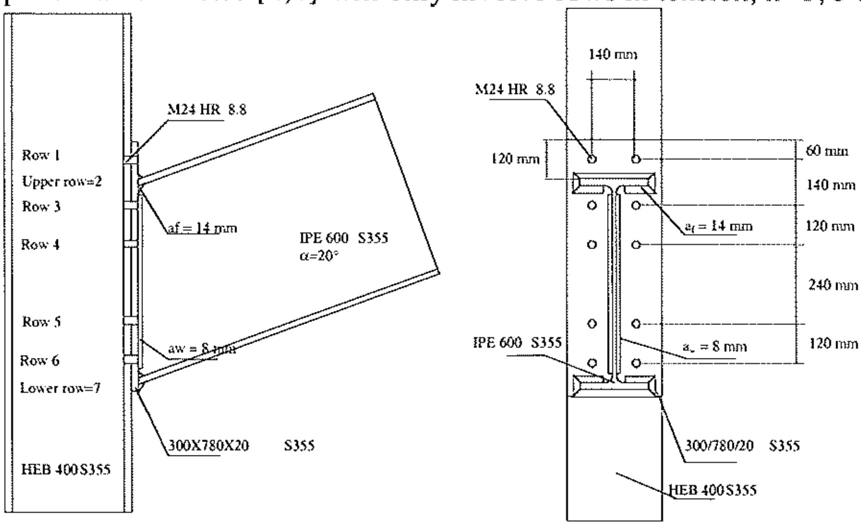

Fig. 2 – Bolted joint with numbering of force transfer rows

3.3 Equilibrium equations for the connection and load eccentricity

The evaluation of the resistance of the connection based on the static theorem requires at failure a equilibrium between the distribution of internal forces and the external applied loads. For a connection subjected to M and N, the equilibrium criteria write:

$$M=\sum_{i=1}^{n} h_i.F_i \qquad N=\sum_{i=1}^{n} F_i \tag{2}$$

where F_i designates the force in row i and h_i the corresponding lever arm; this one is defined as the vertical distance between the reference beam axis where M and N are applied and the row in itself (h_i values are positive for rows located on the upper side of the reference axis).
The applied bending moment and axial force are linked through the concept of load eccentricity e as follows (the positive values of M and N are defined as indicated in Fig.1):

$$M = e.N \tag{3}$$

3.4 Resistance criteria

According to the static theorem, the resistance of each row - which is equal to the resistance of the weakest component in the row - should never be exceeded. At first sight it looks easy as long as the individual resistances of bolt-rows are concerned but it is much more questionable when group effects develop in the connections.
In the present study, any group of rows *[m, p]* in which group effects appear is considered as an equivalent fictitious row with an equivalent lever arm and a group resistance equal to that of the weakest group component. Therefore the resistance criteria for each of the rows belonging to the *[m, p]* group may write, for any constitutive component α:

$$\sum_{i=m}^{p} F_i \le F_{mp}^{Rd\,\alpha} \qquad m = 1,...p;\ p = m, m+1, ..., n \tag{4}$$

$F_{mp}^{Rd\,\alpha}$ is the resistance of the component α for the group of rows m to p. When m equals p, $F_{mp}^{Rd\,\alpha}$ designates the individual resistance of the component α for row m. Such a resistance criterion may be derived for each of the constitutive row components and the final resistance of the group of rows *[m, p]*, noted F_{mp}^{Rd} may be defined as the smallest of the $F_{mp}^{Rd\alpha}$ values.

This situation is illustrated in Fig. 3 for a connection with three bolt rows, *1*, *2* and *3*, but more generally covers the case of any connection with *n* rows in which group effects would develop in three bolt rows numbered *r*, *s* and *t*.

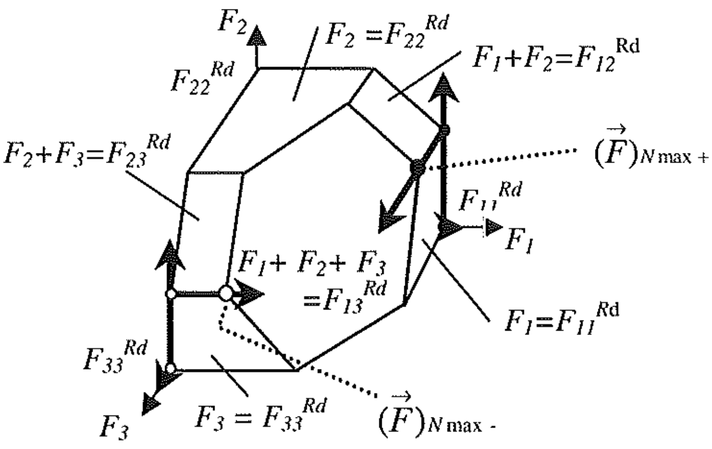

Fig. 3 - Interaction between three bolt rows and definition of F_j^{Rd}

3.5 Definition of the failure criterion for the whole connection

Details about the application of the static theorem to a connection with *n* rows are given in reference [5] that can be afforded to any interested reader. This application leads to the following definition and writing of the *M-N* resistance interaction diagram:

The interaction criterion between the bending moment (M) and the axial force (N) at failure is described by a set of 2 n parallel straight line segments; the slope of each of the 2 n parallel segments is equal to the value of the lever arm (h_k) and along these segments, the force (F_k) varies between 0 at one end and the maximum resistance row resistance at the other end.

$$
\begin{aligned}
& M = h_k.N + \sum_{i=1}^{n}(h_i - h_k).F_i^c \qquad k = 1,2...n \\
& either \quad F_i^c = max(F_i^{Rd+},0) \ \ if \quad i < k \\
& \qquad\quad F_i^c = min(F_i^{Rd+},0) \ \ if \quad i > k \\
& or \quad F_i^c = min(F_i^{Rd-},0) \ \ if \quad i < k \\
& \qquad\quad F_i^c = max(F_i^{Rd-},0) \ \ if \quad i > k \\
& with \\
& \qquad F_i^{Rd+} = min(F_{mi}^{Rd} - \sum_{\substack{j=m \\ i\neq up,lo}}^{i-1} F_j^{Rd+}, m = 1,..i) \qquad i < k \\
& \qquad F_i^{Rd-} = min(F_{im}^{Rd} - \sum_{\substack{j=i+1 \\ \neq up,lo}}^{m} F_j^{Rd-}, m = i,...n) \qquad i > k
\end{aligned}
\tag{5}
$$

The resistance of the rows *i* (F_i^{Rd+} and F_i^{Rd-}) differs when *i* is lower than *k* (F_i^{Rd+}) or higher than *k* (F_i^{Rd-}). The evaluation procedure of the F_i^{Rd+} and F_i^{Rd-} values is illustrated in Fig. 3 for a connection with three bolt rows where the black and white dots respectively show the successive steps for the evaluation of F_j^{Rd+} and F_j^{Rd-}.

Application rules are also proposed in [5] which allow a direct evaluation of the connection resistance for a specific value of the load eccentricity; this situation is the one to which the designer is likely to be faced in the design practice.

3.6 Ductile resistance of the connection and stress interaction between components phenomenon

Eurocode 3 Part 1.8 considers that stress interaction phenomena can affect the resistances of the following column components: column web in compression (CWC), column web in traction (CWT) and column flange in bending (CFB).
The resistance of these components is affected by a reduction factor depending of the ratio between shear and transverse stresses (CWC and CWT) and/or the ratio between longitudinal and transverse stresses (CFB and CWC). Details on how these stress interactions have been integrated in the analytical model given by formulae (5) may be found in [5].

4 NON DUCTILE INTERACTION DIAGRAM FOR CONNECTIONS

4.1 Non ductile components

The ductility of some components is sometimes not sufficient to allow for a full plastic redistribution of the internal forces in the connections. When a non ductile component reaches its deformation capacity, any additional deformation of the connection causes the brittle failure of that component and consequently of the whole joint.

Besides welds, Eurocode 3 Part 1.8 only considers bolts in tension as non ductile components. It is assumed that the deformation capacity around a bolt is sufficient if design resistance F^{Rd} of the "plate-bolt assembly" is lower or equal to *95%* of the tension bolt resistance.
Moreover, the "beam flange and web in compression" component (BFC) may also be considered as non very ductile when the beam cross-section becomes slender and its resistance is limited by buckling phenomena (class 4 sections).

4.2 Analytical evaluation of the non ductile resistance of the connection

The resistance of a non ductile connection is reached once the deformation of a non ductile component is equal to its deformation capacity. By assuming that the connection cross-section remains un-deformed, equation (7) is found where Δ and φ are respectively the displacement corresponding to the zero-lever arm ($h_\Delta=0$) and the rotation of the connection.

$$\Delta_i=\Delta+h_i.\varphi \tag{7}$$

If the displacement of two rows is known then the entire deformation of the joint may be defined .When a component of a row k reaches its limited deformation capacity, the only row whose deformation is known is that row k (Δ_k^{Rd}). The deformation of any other row is then needed to define the deformation of the connection. Here it is assumed that the position of the zero displacement point (h_0, defined by equation (8)) is known.

$$\begin{aligned} &\Delta_0 = 0 = \Delta + h_0.\varphi \Rightarrow h_0 = -\frac{\Delta}{\varphi} \\ &\Delta_i = (h_i - h_0).\varphi \quad \forall i \end{aligned} \tag{8}$$

By varying the value of h_0 from $-\infty$ to $+\infty$, all possible states of deformation of the connection corresponding to the limited deformation capacity of row k are obtained. As a result the proposed method applies as follows for each non ductile row k:

- evaluation of the deformation capacity Δ_k^{Rd} of row k presenting a non ductile behaviour and selection of one value of zero point displacement h_0;
- evaluation of the displacement Δ_i for all rows and of the corresponding internal force F_i (by means of the F_i - Δ_i relationship);
- calculation of the axial and bending forces corresponding to these internal forces;
- re-application of the procedure until the whole domain of h_0 values has been examined.

4.3 Application of the analytical evaluation procedure

The procedure is applied on Fig. 4 to the connection shown in Fig. 2, in which rows 3 and 6 exhibit a non ductile failure mode (BT).

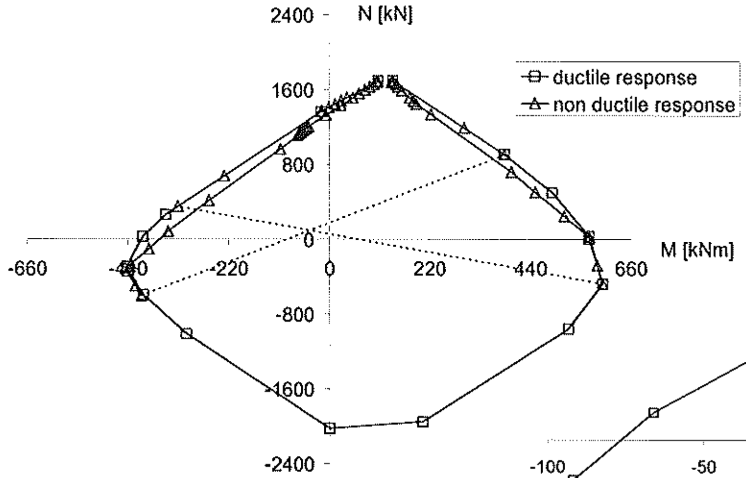

Fig. 4 - Ductile and non ductile *M-N* interaction diagrams (including stress interactions)

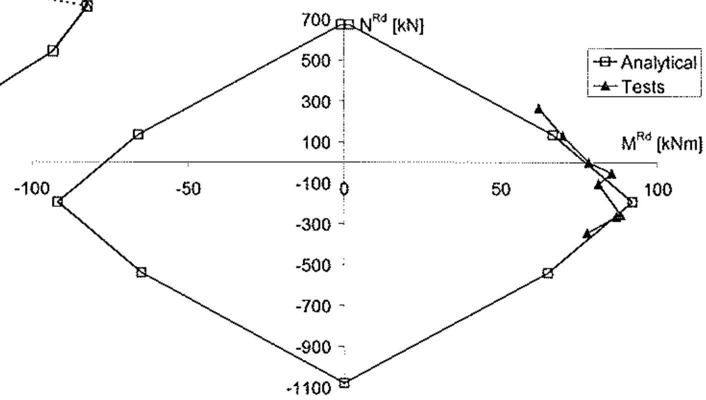

Fig.5 - Tests-model comparison - Coimbra tests with flush endplates

5 INTERACTION DIAGRAM FOR WEB PANELS IN SHEAR

The procedure on how to evaluate the *M-N* interaction diagram for web panels in shear are not provided in the present paper due to the limited number of pages. However the interested reader will find all relevant details in [5].

6 EXPERIMENTAL VALIDATION

Laboratory tests on joints subjected to combined bending and axial have been recently carried out at the University of Prague [6] and at the University of Coimbra [7].

Two types of joints have been tested in Prague: (i) beam-to-beam joints with flush endplates (3 tests) and (ii) beam-to-column joints with extended endplates (2 tests). Eccentricity remains unchanged during each test but differs from one test to the other.

In Coimbra, two types of configuration have also been tested: (i) beam-to-column joints with flush endplates (8 tests) and beam-to-column joints with extended endplates (7 tests). For these tests, the axial load on the joint remains unchanged while bending moment is increasing until collapse.

In [5] the rather good agreement between the tests and the analytical method is shown; differences are rarely higher than 15%. Furthermore:

- the actual shape of the interaction curves is well reproduced by the models;
- the many different failure modes observed in tests are well predicted;
- for the only available test significantly influenced by stress interaction phenomena, the accuracy of the model including these effects is similar to the other tests;

It has however to be indicated that the limited domain covered by the Prague and Coimbra tests is not at all sufficient (axial compression forces, low values of the *N/M* ratio) so as to fully validate the proposed model. Some further experimental investigations should therefore be performed:

- tests covering the entire interaction diagram between axial force and bending and particularly tests with high tension axial forces; in such tests, bolt group phenomena would become significant;
- tests presenting a large number of bolt rows in order again to develop bolt group effects;
- tests exhibiting brittle failure modes;
- tests where high stress interaction phenomena develop.

7 CONCLUSIONS

Analytical procedures for the evaluation of the design resistance of steel joints with bolted endplates subjected to combined bending moment and axial force are proposed in the present paper. These ones respect the design principles given in Eurocode 3 Part 1.8. Comparisons with four available series of laboratory tests have been achieved; a quite reasonable agreement is obtained but the need for other test results covering a wider range of relative variation of *M* and *N* values has been pointed out. Parameters influencing the *M-N* interaction have been identified; this indicates the direction in which new tests should be performed to achieve the full validation of the proposed analytical approach.

Finally it has to be underlined that the presented study may similarly be applied to column bases and column splices.

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KEYWORDS

Structural joints, resistance interaction, bending moment and axial force, analytical model