# Redshift drift and strong gravitational lensing

Phillip Helbig[⋆]
*Thomas-Mann-Str. 9, D-63477 Maintal, Germany*

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**ABSTRACT**

In general, the cosmological redshift of an object changes with time, a phenomenon known as redshift drift. Although long known theoretically, recently interest has been renewed because of the possibility of measuring it on a reasonable time-scale. Strong gravitational lensing offers a possibility to measure it on a much shorter time-scale, by making use of a time delay of $n$ years rather than making observations separated by $n$ years, but, perhaps at least in part because of the expectation that the signal would be swamped by a larger change in redshift due to transverse motion of the lens, that has not attracted much interest. I present a method to extract the small signal, making use of the fact that the light-travel time through different parts of an Einstein ring is the same (and hence the difference in redshift due to redshift drift vanishes), thus enabling the measurement of redshift drift on a much shorter time-scale, and show how that can help in the measurement of the distribution of (dark) matter.

**Key words:** gravitational lensing: strong – galaxies: clusters: general – galaxies: distances and redshifts – dark matter – cosmology: miscellaneous – cosmology: theory

## 1 INTRODUCTION

Redshift drift, the change in the redshift $z$ with time, is interesting for several reasons, such as a consistency check for standard cosmology (many alternative explanations for the cosmological redshift do not predict redshift drift), a method of determining cosmological parameters independently of corrections for evolutionary effects, and, as discussed below, a shortcut to using strong–gravitational-lens systems to map the distribution of (dark) matter. Long considered too small to be observable on interesting time-scales, recent developments have changed the situation, and there have been several feasibility studies on the topic, also considering variations of the basic idea (e.g. Esteves et al. 2021, and references therein).

The plan of this paper is as follows. After briefly discussing the history of the theory of redshift drift, I discuss its use in strong–gravitational-lens systems before mentioning future prospects after my summary and conclusions.

## 2 THEORY

For a homogeneous and isotropic ('Robertson–Walker') universe consisting of non-relativistic matter ('dust') of density $\rho$ and the cosmological constant $\Lambda$ (with dimension time$^{-2}$)[1], the change in scale factor with time is described by the Friedmann equation

$$\dot{R}^2 = \frac{8\pi G\rho R^2}{3} + \frac{\Lambda R^2}{3} - kc^2\,, \qquad (1)$$

with the dimensionless constant $k$ equal to $-1$, 0, $+1$ depending on spatial curvature (negative, vanishing, or positive, respectively); $R$ is the scale factor (with dimension length) of the universe, $G$ the gravitational constant, and $c$ the speed of light. It is useful to define the following terms:

$$\begin{array}{rcllll} H &:=& \frac{\dot{R}}{R} & & & \\ \lambda &:=& \frac{\Lambda}{3H^2} & & & \\ \Omega &:=& \frac{\rho}{\rho_{\mathrm{crit}}} & \equiv & \frac{8\pi G\rho}{3H^2} & \\ K &:=& \Omega+\lambda-1 & & & \\ k &:=& \mathrm{sign}(K) & & & \\ q &:=& \frac{-\ddot{R}R}{\dot{R}^2} & \equiv & \frac{-\ddot{R}}{RH^2} & \equiv \frac{\Omega}{2}-\lambda \end{array} .$$

The Hubble constant $H$ has the dimension time$^{-1}$; all other quantities defined above are dimensionless: the normalized cosmological constant $\lambda$, the density parameter $\Omega$, the curvature parameter $K$, and the deceleration parameter $q$. $\rho_{\mathrm{crit}}$ is known as the critical density.

The basic cosmological observable is the redshift $z$, which is related to the scale factor:

$$z := \frac{R_0}{R} - 1\,. \qquad (2)$$

In general, like the other parameters defined above except $k$, $R$ is a function of time; the subscript 0 denotes the present time. For most applications, the time-scale of the expansion of the Universe is so long compared to the intervals between observations that both $R$ (the time of emission of the radiation which is observed with redshift $z$) and $R_0$ (the time of observation) can be regarded as constant. In general, however, an observation at time $t_{\mathrm{A}}$ will measure a redshift

$$z_{\mathrm{A}} := \frac{R_A}{R_1} - 1 \qquad (3)$$

while one at $t_{\mathrm{B}}$ will measure a redshift

$$z_{\mathrm{B}} := \frac{R_B}{R_2} - 1\,, \qquad (4)$$

⋆ Email: helbig@astro.multivax.de

[1] Such models are often known as Friedmann–Robertson–Walker (FRW) models (Friedmann 1922, 1924; Robertson 1935, 1936; Walker 1935, 1937, the latter paper by Walker is very often incorrectly cited as having been published in 1936).

where $t_1 < t_2 \ll t_A < t_B$ (the current time considered to be the latest time, i.e., $t$ increases with increasing $R$ as the Universe expands). In other words, $t_A - t_2 \gg t_2 - t_1$ and $t_A - t_2 \gg t_B - t_A$. In general, $z_A \neq z_B$.

Although implicitly clear from the earliest days of relativistic cosmology (e.g. Friedmann 1922, 1924; Lemaître 1927, 1931), redshift drift was first discussed explicitly by Sandage (1962) for the $\lambda = 0$ cases (all possible values of $k$) and also for the steady-state model (still a viable model at the time). An appendix by McVittie (1962) to that paper expands the discussion to non-zero values of $\lambda$, obtaining a general expression for the instantaneous change in $z$ at the present time in terms of $\sigma_0$ ($\sigma = \Omega/2$) and $q_0$, which are parameters in common use at the time (e.g. Stabell & Refsdal 1966). Lake (1981) generalizes the discussion to include radiation (found to be unimportant) as well as dust[2], noting that a measurement involving absorption lines in QSO spectra might be feasible in the (then) not too distant future. He also gives an overview of the literature at that time. Lake (2007) returned to the topic almost thirty years later, exploring the basic equation

$$\dot{z} = H_0 \left(1 + z - \frac{H(z)}{H_0}\right) = H_0 Z(z)\,, \tag{5}$$

which is simple, but also very general, applicable to any $H(z)$, and can thus include components with various equations of state, possibly varying with time. For models with dust, a cosmological constant, both, or neither,

$$H(z) = H_0 \sqrt{\Omega_0 (1+z)^3 - K_0 (1+z)^2 + \lambda_0}\,. \tag{6}$$

Eq. (5) is sufficient for many purposes; most recent literature is more concerned with applications (e.g. Balbi & Quercellini 2007; Zhang et al. 2007; Quartin & Amendola 2010). The paper by Sandage (1962) has, at the time of writing, 263 citations according to ADS, 19 before 2006 and 244 since then. Interestingly, it is not cited by Loeb (1998), who revived interest in the topic when it became clear that a measurement might be feasible soon. Now, a precision of the order of 1 cm/s is possible (e.g. Li et al. 2008) and such measurements are one reason for building high-resolution spectrographs for extremely large telescopes (see e.g. Liske et al. 2008, for a survey of the literature and overview of future prospects).

Redshift drift is a particular case of real-time cosmology (e.g. Quercellini et al. 2012, and references therein). Apart from measuring the redshift drift per se, it is also possible to measure the relative drift between two unrelated sources at different cosmological redshifts (e.g. Cooke 2019).

## 3 STRONG GRAVITATIONAL LENSING

A common definition, and the one used here, for strong gravitational lensing is a gravitational-lens system in which there is more than one image of each source. Of course, like other objects at cosmological distances, the redshift drift should be detectable in such images, but such systems offer more possibilities due to the fact that the light-travel time is in general different for different images. For example, a system such as Supernova Refsdal (Kelly et al. 2015)[3] with one clearly defined time of emission (the explosion of the supernova) but different times of observation, separated by a year or so, offers a variant on the conventional redshift-drift scenario. However, considering that the time delay between the images is known, the difference between the redshifts of the images depending on the time the explosion is seen serves mainly as a consistency check for our understanding of cosmology, though of course as with other measurements a combination of cosmological parameters can be measured.

More interesting is perhaps the conventional idea of a gravitational-lens system, where one observes several images at a given time which, due to the difference in light-travel time, correspond to different times of emission (though the different times of emission normally play no role). By measuring the difference in redshift between two images, one effectively measures the time delay even if the object is not variable or the time delay is hundreds or thousands of years, and does so of course within a much shorter time (see also Kim et al. 2015, regarding strongly lensed quasars). Lensing by clusters rather than individual galaxies is more interesting in that regard, as the redshift difference in the case of a single lensing galaxy is much smaller, and if the source is variable the time delay could be measured via conventional means. For the typical velocity dispersion of a rich cluster (1000–1500 $\mathrm{km\,s^{-1}}$), the typical time delay (which of course depends on the exact configuration) is in the range of several hundred to a few thousand years. Loeb (1998), however, pointed out that, in practice, that might not be possible, because in general a gravitational lens will have a velocity component perpendicular to the observer, which will lead to a difference in redshift between the images of the lensed source (Birkinshaw & Gull 1983) which in general is larger than that due to redshift drift; for a velocity of 300 $\mathrm{km\,s^{-1}}$ and a separation of 20 arcsec, the difference in redshift is about $1 \times 10^{-7}$, corresponding to about 30 $\mathrm{m\,s^{-1}}$ (Wang, Bolejko & Lewis 2022) (see also Wucknitz, Spitler & Pen 2021), about an order of magnitude larger than the difference due to redshift drift (Wang et al. 2022). Peculiar velocities and peculiar accelerations due to cosmological perturbations, i.e. taking into account the fact that the Universe is not perfectly smooth, however, are not a significant source of uncertainty, being less than 1 per cent of the drift signal (e.g. Uzan, Bernardeau & Mellier 2008).

In addition to multiple images of background sources, many strong–gravitational-lens systems include at least a partial Einstein ring, the ring-shaped image of the source occurring when there is a (nearly) perfect alignment of source, lens, and observer; since such a situation is axially symmetric, there is a ring instead of multiple images. Due to symmetry, the light-travel time between source and observer is the same for all portions of the ring. There is therefore no redshift-drift effect between different portions of the ring. Any redshift difference must be due to the moving-lens effect.[4] Thus, by measuring such a difference, one could determine the transverse velocity of the lens and thus calculate the expected redshift difference between a pair of images of a background source due to the moving-lens effect. Subtracting this difference from the observed difference then gives the difference due to the redshift drift.[5] Wucknitz et al. (2021) discuss a similar scheme involving a quadruply imaged source rather than an Einstein ring; Wang et al. (2022) point out the advantage of having more than one lensed source to correct for the same effect.

Unfortunately, the relevant quantity is the relative velocity of

[2] As noted by Lake (1981), similar results had been obtained by Rüdiger (1980).

[3] Note that, in a series of papers, Refsdal (1964a,b, 1966a,b, 1970) essentially single-handedly founded the modern field of gravitational lensing.

[4] Or to smaller, higher-order effects not considered here (e.g. Wucknitz et al. 2021; Wang et al. 2022).

[5] Of course, the measurement of the transverse velocities of galaxy clusters is interesting in itself.

source, lens, and observer. In general, different sources will have different transverse velocities. Thus, to use the method as described above, the Einstein ring and the multiple images used to measure a difference in redshift and hence time delay must be from the *same* source, which is possible in a more complicated lens system. Thus, the number of systems in which this method can be used is reduced, but, considering planned surveys which will find many lens systems, hopefully not to zero.

Another approach is to make use of the fact that there can be more than one source which is multiply imaged. Wang et al. (2022) have shown that if one has ten or more sources then it is possible to remove the signal due to the transverse velocity (see their fig. 4), concluding that 'the effects of peculiar velocity are either negligible or can be subtracted with sufficient precision'.

Another caveat is that in the case of a multiply imaged source or Einstein ring one will see the source from slightly different angles; in the former case in general also at slightly different times. Although the sources are complex objects with internal dynamics and so on, even on time-scales of hundreds of years, at least for galaxies [as opposed to active galactic nuclei (AGN)], one would not expect the redshift (measured over the entire object, not just a part of it) to change appreciably. Since the angles involved are small, less than 1 arcmin, and again considering that one measures the redshift integrated over the entire source, the slightly different line of sight will probably not lead to an appreciable difference in redshift.

Of course, the method can work only if there is sufficiently high spectral resolution (which is within the reach of instruments planned or being built now), but also sufficiently sharp spectral features. Wang et al. (2022) give an example of a cluster lens (velocity dispersion 1500 km/s, $z_\mathrm{l} = 0.4$, $z_\mathrm{s} = 4$ and eccentricity parameter $\varepsilon = 0.6$) with $\Delta z = 2.67 \times 10^{-8}$, corresponding to a wavelength difference of $\Delta\lambda = 4.45 \times 10^{-5}$ Å. Thus, one might need an AGN (for the sharp emission lines) for the source, but in that case the precision is within the reach of next-generation optical telescopes (and similar precision should be possible in the radio (Lu et al. 2022).)

This topic is relatively new. Hopefully further investigation will remove any showstoppers (in the negative meaning of the word). Note that Sandage (1962) was rather pessimistic (about improvements in observational techniques, not about the future of humanity): '… a precision redshift catalogue must be stored away for the order of $10^7$ years …' and '… data for extragalactic astronomy must be collected from ancient literature.'

Traditionally, measurement of the time delay has been used to scale a dimensionless model of a gravitational-lens system which allows one to infer the Hubble constant (Refsdal 1964b) and/or other cosmological parameters (Refsdal 1966a), given a lens model. However, despite the current 'Hubble tension', the Hubble constant is probably already known to a precision greater than that measurable via gravitational-lens time delays, and the precision of the values of of $\Omega_0$ and $\lambda_0$ is even more certain (e.g. *Planck* Collaboration 2020). It is thus perhaps more interesting to take the cosmological parameters as given and use measured time delays to constrain mass models for strong–gravitational-lens systems and thus determine the distribution of (dark) matter in such systems. The method discussed above allows one to determine such time delays even if the source is not variable and/or if the time delay is too long to be measured by conventional methods (in fact, the longer the delay, the easier it is to infer via the redshift-drift technique).

A cluster lens can have a large number of multiply imaged sources and hence differences in light-travel time. By using all of them, the mass model can be constrained much better than by using only those with measurable time delays (which depend on the source being variable and the time delays reasonably short). Of course, one must know which observed images map to the same source. An approximate model (to be refined via the relative time delays) could help, but more practical would be to measure the redshift of all background sources. The difference in redshift due to redshift drift between two images of the same source will almost always be much smaller than the difference in the conventional cosmological redshift between different sources, making it obvious which images correspond to the same source.

Wucknitz et al. (2021) discuss similar ideas in the context of fast radio bursts. In their particularly thorough paper, they also give a good overview of the theory involved and how various effects can be differently interpreted and disentangled.

For completeness, I note that it is in principle possible to measure the *change* in the redshift difference, as well as changes in image positions, distortions, and amplifications; also, a pair of images could suddenly (dis)appear if located close to a moving caustic (e.g. Broadhurst & Oliver 1991; Martins et al. 2016; Piattella & Giani 2017; Korzyński & Kopiński 2018; Wucknitz et al. 2021; Covone & Sereno 2022). The difference itself corresponds to the time delay; the change in the difference corresponds to the change in the time delay. Such a change can be caused by the change in the distances involved (which depend on the changing redshift) or by transverse motion of the lens relative to the source and observer so that the time delay changes because the corresponding mass distribution affecting the time delay changes; the latter is important in cases in which a short time delay is measured (presumably by conventional means) then later the time delay is measured again (Zitrin & Eichler 2018).

## 4 SUMMARY, CONCLUSIONS, AND OUTLOOK

Measuring the change of cosmological redshift on a time-scale of a few years is now within reach. Exploiting strong–gravitational-lens time delays, and using an Einstein ring to correct for the change in redshift due to the moving-lens effect, such a measurement can be done within a very short time, the change in time being supplied by the time delay rather than waiting for years between observations. Apart from being a model-independent consistency check on standard cosmology, such a technique allows the measurement of time delays much longer than can be measured in the conventional manner and/or for non-variable sources; such information can help constrain models of strong–gravitational-lens systems and thus give information about the distribution of (dark) matter. Measuring such redshift differences for known systems with multiple images, and for new ones as they are found, offers an efficient method of significantly improving lens models; the information about the transverse velocity of the (galaxy-cluster) lenses is also useful in its own right.

## NOTE

After the draft of this paper had been written, I became aware of the papers by Wucknitz et al. (2021) and Wang et al. (2022), which have much overlap with the present paper. I have left the text essentially the same as in the original draft, amending it to include additional points discussed by Wucknitz et al. (2021) and Wang et al. (2022) which are relevant to this paper (and of course citing them in the relevant passages). There is of course much overlap regarding the basic idea (and historians might want to investigate why three papers on the same topic appeared independently within such a short time), and of course I acknowledge their priority in that respect.

My emphasis is on known strong–gravitational-lens systems with longer time delays and/or with non-variable sources, the time delays themselves rather than their change in time, the use of the Einstein ring to correct for the moving-lens effect, the possibility to constrain the mass model from the measured redshift differences, and using the redshifts to match images to a source.

## DATA AVAILABILITY

There are no new data associated with this article.

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