ANALYTICAL STUDY OF THE INTERACTION BETWEEN BENDING AND AXIAL FORCE IN BOLTED JOINTS

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ABSTRACT

The paper presents a part of the developments carried out these last years in Liège in the field of structural steel joints subjected to combined bending moments and axial forces. In particular an analytical procedure for the evaluation of the design resistance and the initial elastic rotational stiffness of such joints is introduced and commented. Finally some of the limitations of the procedure are raised; these ones are now being progressively removed and a final complete calculation model should be soon made available by the first author in the form of a forthcoming Ph.D. Thesis.

Key words: *Structural joints, bending moment and axial force, resistance, rotational stiffness*

1. INTRODUCTION

1.1 The component method

The component method is a nowadays well-known and widely recognised procedure for the evaluation of the design properties of structural joints. It is used as a reference method in Eurocode 3 [1] and Eurocode 4 [2], respectively for joints in steel and composite construction, but it may apply to many other joint configuration and connection types.

In the component method, any joint is seen as a set of elements (called components). The mechanical properties of these components, in terms of elastic deformation, design resistance and deformation capacity are evaluated through appropriate design models; then the components properties are "assembled" to finally derive the mechanical properties of the full joint, i.e. its rotational stiffness, its moment and shear design resistances, its collapse mode and its level of rotational capacity. The interested reader will find more information about the component method and its field of application in [3].

So the characterisation of the joint properties through the component method implies three successive steps:

- identification of the constitutive components;
- evaluation of the mechanical properties of the components;

- "assembling" of the components to derive the mechanical properties of the whole joint.

In Revised Annex J of Eurocode 3 [4], simple analytical calculation procedures are provided; they mainly allow to derive the design moment resistance and the elastic rotational stiffness (called "initial stiffness") of steel joints subjected to bending moments and shear forces.

1.2 Structural joints subjected to bending moment M and axial force N

In most of the cases, beam-to-column joints and beam splices are subjected to compression or tension axial forces in addition to bending moments and shear forces. These ones have an influence on the rotational stiffness, moment resistance and rotational capacity of the joints. And that is why in Revised Annex J of Eurocode 3 the proposed field of application is limited to joints in which the force (N_{Sd}, simply noted N in the paper for sake of simplicity – and the same applies to M_{Sd}, noted M -) acting in the joint remains lower than 10% of the axial design resistance of the connected beam ($N_{pl,Rd}$):

$$\left|\frac{N_{Sd}}{N_{pl,Rd}}\right| \leq 0{,}1 \tag{1}$$

Under this limit it is considered that the rotational response of the joints is not significantly influenced by the axial forces. It has however to be stated that this value is a fully arbitrary one and is not at all scientifically justified.

The 10% rule applies to most of the beam-to-column joints and beam splices in multi-storey building frames, but usually not to similar joints in pitched-roof industrial portal frames. Similarly column bases and column splices transfer high axial forces and therefore do not fulfil the limiting criterion prescribed by Revised Annex J.

For such joints, the principles of the component method is still valid – as the behaviour of the components is independent on the type of loading applied to the whole joint – but a new assembly procedure is required to cover the combined action of bending moments and axial forces. Another difficulty results from the variation of the active components in the joints according to the relative importance of the bending moment and axial force, and obviously according to the respective signs of the applied forces. These items are addressed in the present paper.

1.3 Short survey of existing research works

In the last years several research works (Hoffmann [5], Hermann [6], Slivers [7], Da Silva and al [8], SCI [9], Steenhuis and al [10]) have been carried out; these were aimed at deriving appropriate assembly procedures for the distribution of internal forces within bolted joints with endplate connections subjected to combined bending and axial tension or compression. An extensive analysis of these research works is included in [11].

At the University of Liège, Finet [12], Jaspart [3] and then Cerfontaine have developed a software called ASCON to study the so-called M-N action in structural joints. ASCON refers to the principles of the component method and is designed to follow the progressive loading of the joint until failure. One of its originalities is to deal with rotational stiffness, resistance and ductility aspects. Fig. 1 illustrates the mechanical model used as a basis for the development of ASCON. Each constitutive component of the joint is represented by a extensional spring characterised by a non-linear F-Δ curve, where F and Δ represent respectively the force acting in the component and the related displacement. According to the definitions given in Revised Annex J of Eurocode 3, the joint is seen to be constituted of a *connection* subjected to bending moment and axial force and a *column web panel* in shear. More detailed information on ASCON may be found in [11].

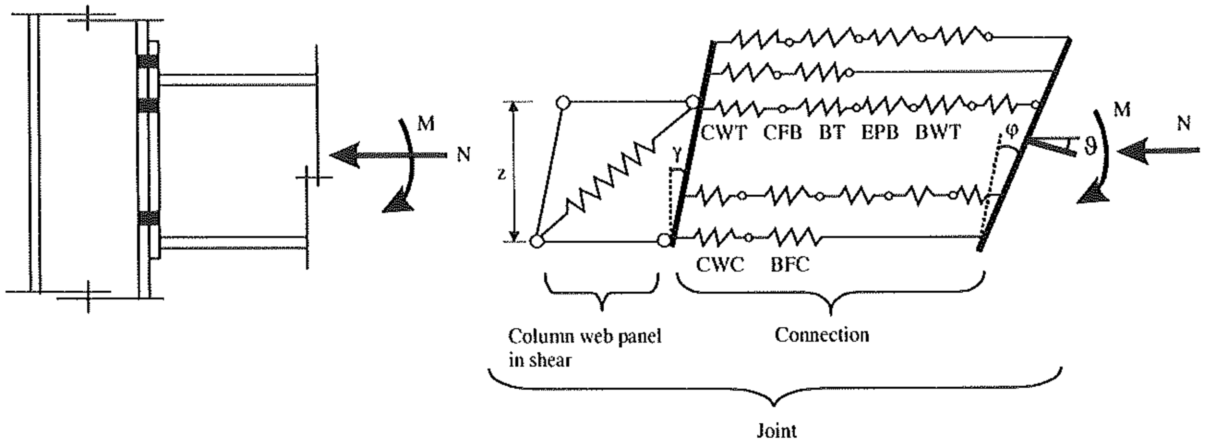

Fig. 1 – Mechanical model used in ASCON and in the proposed analytical procedures

Some preliminary conclusions may be drawn from these various studies:

- No model for the prediction of the joint rotational stiffness exists.
- The available procedures for strength calculation are not fully satisfactory as they disregard the ductility aspects and the group effects (see below) or cover them through rough assumptions.
- The iterative procedure implemented in ASCON brings some first solutions to the aforementioned stiffness and ductility aspects as it allows to predict a value of the elastic rotational stiffness under M-N interaction and to limit the plastic redistribution of internal forces within the joint as soon as the deformation capacity (ductility) of one of the active components in the joint is reached. But, on the other hand:
  - The consideration of the "group effects" is not yet optimum in ASCON. "Group effects" may occur in bolted connections and more especially (Fig. 1) in constitutive plate components subjected to transverse bolt forces (endplates in bending – EPB -, column flanges in bending - CFB, ...). There where a bolt force is applied, a yield plastic mechanism may develop in the plate component; if the distance between bolts is high, separate yield lines will form in the plate component around the bolts (*individual bolt mechanisms*), while a single yield plastic mechanism common to several bolts may develop when the distance between the latter decreases (*bolt group mechanisms*). Group effects also affect the resistance of following components (Fig. 1): column webs in tension – CWT – and beam web in tension – BWT -.
  - ASCON is not in line with designer's expectations as it requires, for each loading situation, to activate a long iterative calculation procedure.

In the present paper, the principles for the development of analytical models for the evaluation of M-N interaction diagrams for design resistance and rotational stiffness of bolted connections are introduced. Similar works on web panel and joints as well as further developments that allow an easy prediction of the mechanical properties of a joint for specific loading condition have been achieved, but these ones are not presented here.

2 DUCTILE RESISTANCE INTERACTION DIAGRAM FOR CONNECTIONS

2.1 Definition

The ductile bending moment – axial force interaction diagram of the connection defines a plastic resistance surface; the actual applied bending moment and axial force in a connection define a couple of values which should remain inside the interaction diagram so as to ensure the sufficient resistance of the studied connection.

2.2 Conventions

The plasticity surface is defined in a general way here below for a bolted endplate connection with N bolt rows where only tension forces may be transferred and 2 compression zones located at mid-thickness of the upper and lower beam flanges (respectively noted "upper" or "up" and "lower" or "lo" and constituted, as seen in Fig. 1, of two components: beam flange and web in compression – BFC – and column flange in compression – CWC -). This leads to a total of N+2=n rows where internal forces may be developed. By convention, the tension forces are assumed to be positive, or equal to zero while a compression force has a negative, or zero, value. All the rows are numbered from 1 to n by starting from the upper row. As an example, for an extended endplate connection with one external bolt row, the compression row "up" is the row n° 2 while it is the row n° 1 for a flush endplate connection.

This is illustrated in Fig. 2 for a joint with an extended endplate connection including 5 (=N) bolt rows. The kinematics of the problem is such that, for instance:

- the force in row n° 2 ("upper") is equal to zero when the forces in rows n° 1 and 3 are different from zero;
- the bolt group mechanism noted (1,4) will only involve rows in tension, n° 1, 3 and 4.

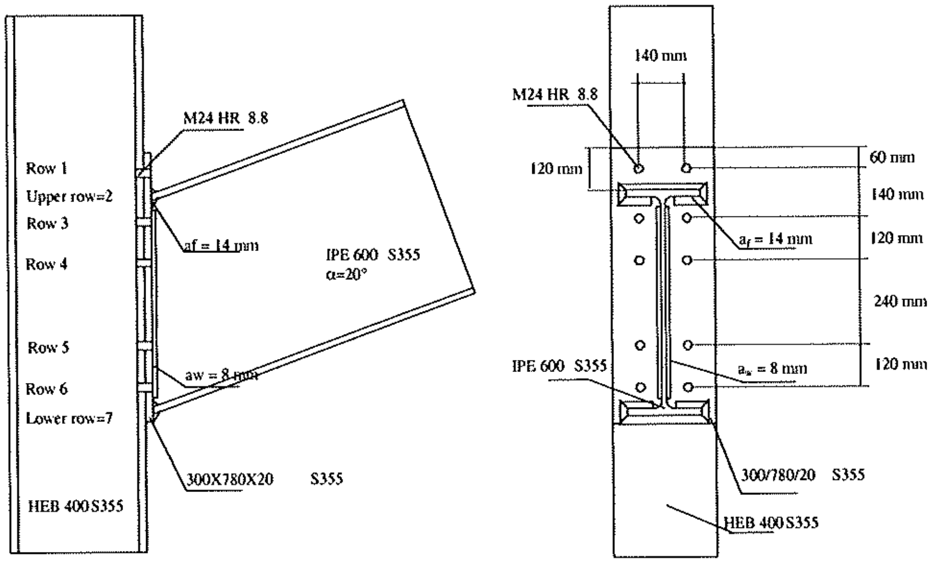

Fig. 2 – Bolted joint with numbering of force transfer rows

2.3 Assumptions and bases for the definition of the interaction diagram

The behaviour of each of the constitutive joint components, and therefore of all the load transfer rows, is assumed to be infinitely ductile. As a result, a full plastic redistribution of the internal forces in the joint carried out on the basis of the so-called static theorem defined in [13] and to which it is referred in Eurocode 3 Revised Annex J may be contemplated. In other words, for combination of bending moment and axial force, a full plastic distribution of internal forces within the joints in equilibrium with the externally applied forces will be defined and the related load factor level will be computed.

How to achieve this goal is extensively described in [11] and reported in Section 2.6. But before, the equilibrium and resistance to fulfil are expressed in the two following sections.

2.4 Equilibrium equations for the connection and load eccentricity

The evaluation of the resistance of the connection based on the static theorem requires at failure a equilibrium between the distribution of internal forces and the external applied loads. For a connection subjected to M and N, the equilibrium criteria write:

$$M = \sum_{i=1}^{n} h_i . F_i \qquad N = \sum_{i=1}^{n} F_i \tag{2}$$

where F_i designates the force in row i and h_i the corresponding lever arm; this one is defined as the vertical distance between the reference beam axis where M and N are applied and the row in itself (h_i values are positive for rows located on the upper side of the reference axis).

The applied bending moment and axial force are linked through the concept of load eccentricity e as follows (the positive values of M and N are defined as indicated in Fig.1):

$$M = e.N \tag{3}$$

2.5 Resistance criteria

According to the static theorem, the resistance of each row - which is equal to the resistance of the weakest component in the row - should never be exceeded. This looks easy as long as the individual resistances of bolt-rows are concerned but is more questionable when group effects develop in the connections (see Section 1.3).

In the present study, any group of rows [m, p] in which group effects appear is considered as an equivalent fictitious row with an equivalent lever arm and a group resistance equal to that of the weakest component. Therefore the resistance criteria for each of the rows belonging to the [m, p] group may write, for any constitutive component α:

$$\sum_{i=m}^{p} F_i \le F_{mp}^{Rd\,\alpha} \qquad m = 1,...p;\ p = m, m+1, ..., n \tag{4}$$

$F_{mp}^{Rd\,\alpha}$ is the resistance of the component α for the group of rows m to p. When m equals p, $F_{mp}^{Rd\,\alpha}$ designates the individual resistance of the component α for row m. Such a resistance criterion may be derived for each of the constitutive row components and the final resistance of the group of rows [m, p], noted F_{mp}^{Rd} may be defined as the smallest of the $F_{mp}^{Rd\alpha}$ values.

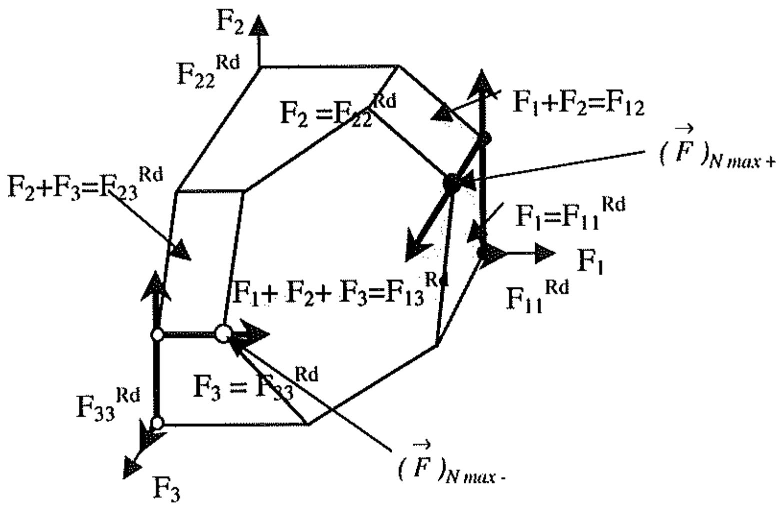

Fig. 3 - Interaction between three bolt rows and definition of F_j^{Rd}

This situation is illustrated in Fig. 3 for a connection with three bolt rows, 1, 2 and 3, but more generally covers the case of any connection with n rows in which group effects would develop in three bolt rows numbered r, s and t.

2.6 Definition of the failure criterion for the whole connection

Details about the application of the static theorem to a connection with n rows are given in reference [11] that can be afforded to any interested reader. This application leads to the following definition of the M-N resistance interaction diagram:
The interaction criterion between the bending moment (M) and the axial force (N) at failure is described by a set of 2 n parallel straight line segments; the slope of each of the 2 n parallel segments is equal to the value of the lever arm (h_k) and along these segments, the force (F_k) varies between 0 at one end and the maximum resistance row resistance at the other end.
This criterion may write:

$$
\begin{aligned}
& M = h_k.N + \sum_{i=1}^{n}(h_i - h_k).F_i^c \qquad k = 1,2...n \\
& \textit{either} \quad F_i^c = max(F_i^{Rd+},0) \ \textit{if} \quad i < k \\
& \qquad F_i^c = min(F_i^{Rd+},0) \ \textit{if} \quad i > k \\
& \textit{or} \quad F_i^c = min(F_i^{Rd-},0) \ \textit{if} \quad i < k \\
& \qquad F_i^c = max(F_i^{Rd-},0) \ \textit{if} \quad i > k \\
& \textit{with} \\
& \qquad F_i^{Rd+} = min(F_{mi}^{Rd} - \sum_{\substack{j=m \\ i \neq up,lo}}^{i-1} F_j^{Rd+}, m = 1,..i) \qquad i < k \\
& \qquad F_i^{Rd-} = min(F_{im}^{Rd} - \sum_{\substack{j=i+1 \\ \neq up,lo}}^{m} F_j^{Rd-}, m = i,...n) \qquad i > k
\end{aligned}
\tag{5}
$$

The resistance of the rows i (F_i^{Rd+} et F_i^{Rd-}) differs when i is lower than k (F_i^{Rd+}) or higher than k (F_i^{Rd-}). The evaluation procedure of the F_i^{Rd+} and F_i^{Rd-} values is illustrated in Fig. 3 for a connection with three bolt rows where the black and white dots respectively show the successive steps for the evaluation of F_j^{Rd+} and F_j^{Rd-}. Fig. 4 shows the M-N interaction resistance diagram obtained for the connection given in Fig. 2. In this figure, the distribution of internal forces within the connection at failure is indicated for four different load eccentricities.

Practical application methods have also been developed which allow a direct evaluation of the connection resistance for a specific value of the load eccentricity; this situation is the one to which the designer is likely to be faced in the design practice.

3 INITIAL CONNECTION STIFFNESS

3.1 Initial stiffness of load transfer rows

If the stiffness of all the constitutive components α in a row i is known, the initial stiffness of this row is simply derived by considering the row as a series of extensional springs:

$$K_{i,ini} = \frac{1}{\sum_{\alpha} \frac{1}{K_{i,ini}^{\alpha}}} \tag{6}$$

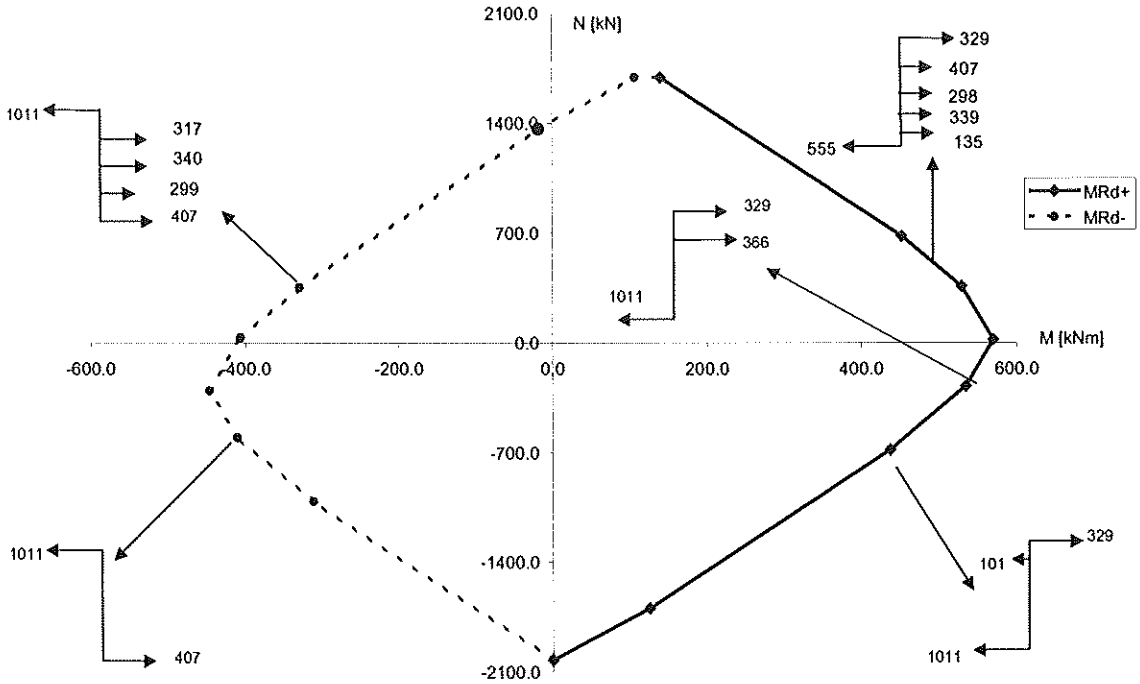

Fig. 4 – Resistance interaction diagram at failure

3.2 Assumption and definition of a centre of rotation

In Fig. 1, the connection cross-section is assumed to remain un-deformed; therefore a linear relationship exists between all the displacements (elongation or shortening) of the rows in the connection ; if Δ is the displacement of a reference point - chosen in ASCON as the displacement corresponding to the zero-lever arm (h_Δ=0) -, the displacement Δ_i of row i writes as follows:

$$\Delta_i = \Delta + h_i.\varphi \tag{7}$$

where φ is the section rotation and h_i the lever arm of row i.

It is also interesting to introduce the zero-displacement point (Δ_0 = 0 with h_0, the corresponding lever arm). As soon as this point is known, the definition, for the considered load situation, of the rows which are loaded and those which are not becomes obvious :

$$\begin{aligned} &\Delta_0 = 0 = \Delta + h_0.\varphi \Rightarrow h_0 = -\frac{\Delta}{\varphi} \\ &\Delta_i = (h_i - h_0).\varphi \quad \forall i \end{aligned} \tag{8}$$

3.3 Derivation of expressions for the initial stiffness

On the basis of :

- the equations (6) for the elastic stiffness of the rows;
- the linear relationship (7) between the respective displacements of the rows ;
- the equilibrium equations (2) ;
- the definition of the eccentricity (3) and of the zero-displacement point (8) in the elastic domain h_0^{el} ;

two different expressions may be obtained to characterise the extensional K_N^{el} stiffness and rotational K_M^{el} stiffness of the connection [11]:

$$K_N^{el} = \left(\frac{N}{\Delta}\right)^{el} = \frac{\sum K_{i,ini}.(h_0^{el} - h_i)}{h_0^{el}} = \frac{\sum K_{i,ini}.h_i.(h_0^{el} - h_i)}{e.h_0^{el}}$$
$$K_M^{el} = \left(\frac{M}{\varphi}\right)^{el} = \sum K_{i,ini}.h_i.(h_i - h_0^{el}) = e.\sum K_{i,ini}.(h_i - h_0^{el}) \quad (9)$$

From these expressions, the eccentricity expressed as a function of the lever am h_0^{el} and the reciprocal relationship may be derived:

$$e = \frac{\sum K_{i,ini}.h_i.(h_i - h_0^{el})}{\sum K_{i,ini}.(h_i - h_0^{el})}$$
$$h_0^{el} = \frac{\sum K_{i,ini}.h_i.(h_i - e)}{\sum K_{i,ini}.(h_i - e)} \quad (10)$$

The following remarkable relationship exists between K_M^{el} and K_N^{el} :

$$K_M^{el} = -e.h_0^{el}.K_N^{el} \quad (11)$$

K_N^{el} and K_M^{el} formulae may be expressed independently of h_0^{el}, by simply introducing Equ. (10) into Equ. (9), but these expressions are not reported here as the use of Equ. (9) is of higher practical interest. As a matter of fact, the difficulty is to determine, for a particular load case (in other words, for a particular value of e), the rows which are activated in tension or compression. So, in practice, h_0^{el} is unknown and an preliminary assumption on its value has to be made, and then checked through the use of Equ. (10). Then Equ. (9) will be used to derive the value of K_N^{el} and K_M^{el}. It is suggested to select a first value of h_0^{el} according to the value of e : h_0^{el} is small for high eccentricities and rather large for low eccentricities.

The variation of h_0^{el} as well as h_0^{Rd} (at ductile failure) versus e is reported in Fig. 5 for the connection given in Fig. 2.

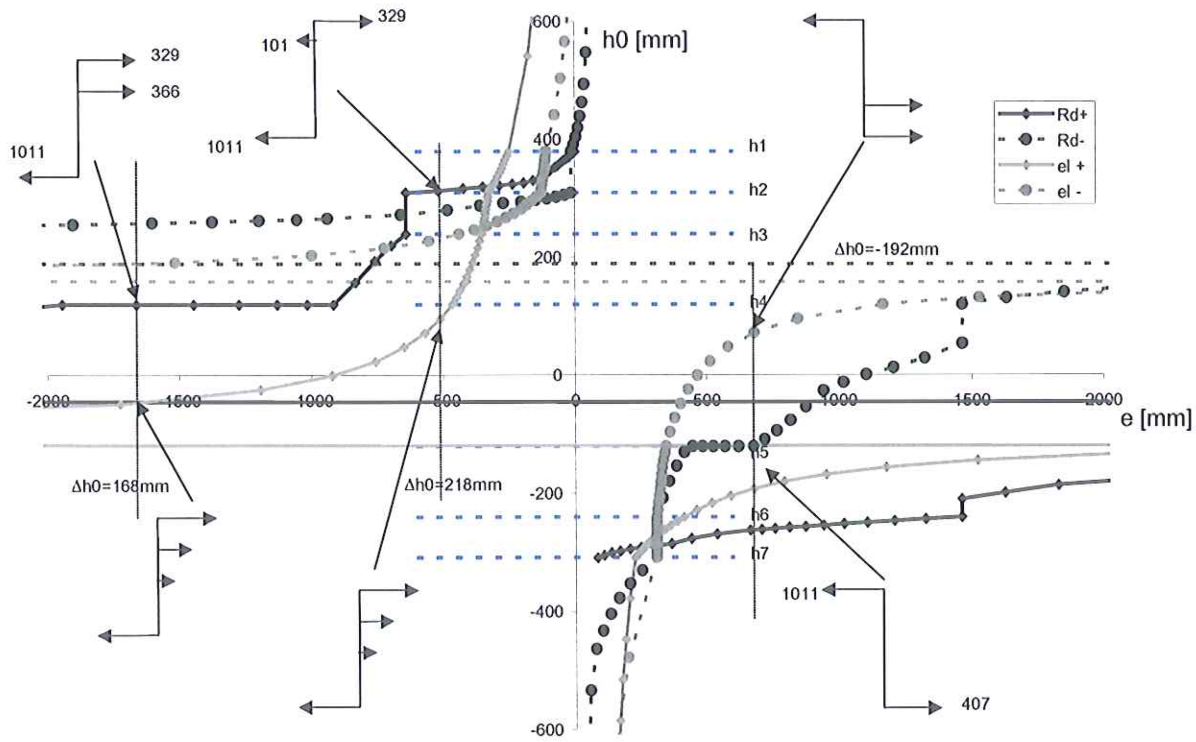

Fig. 5 - Position of the zero-displacement point (h_0) in the elastic domain (^{el}) and at ductile failure (Rd)

4 CONNECTION MOMENT-ROTATION CURVES

In Eurocode 3 revised Annex J, a procedure to derive the whole moment-rotation curve on the basis of the elastic rotational stiffness and moment resistance of the joint is proposed. A similar approach may be followed here.

5 LIMITATIONS OF THE METHOD, FURTHER DEVELOPMENTS

5.1 Ductility of the components

In the above-presented developments, all the components, and therefore all the load transfer rows, are assumed to exhibit a highly ductile behaviour. However it is known that some components have a low ductile behaviour, or even a fully brittle one.

As a consequence, the interaction resistance diagrams such as the one shown in Fig. 4 are possibly no more valid, as they are based on a full plastic distribution of internal forces within the connection.

When the resistance of a non-ductile component is reached during the connection loading, any further redistribution of plasticity within the connection is prohibited and the resistance level reached at that moment has to be considered as the maximum one. This results in a reduction of the design resistance of the comparison in comparison to the one obtained in Fig. 4.

Analytical solutions with different degrees of complexity have been developed to deal with this reduction of resistance but they are not presented in this paper.

5.2 Coupling effects and groups

For joints subjected to bending, Annex J and ASCON lead to different resistances when group effects play a role in the definition of the resistance of the bolt rows. A close examination of the problem shows that the coupling effects characterising group resistances should also be considered for stiffness calculation, when bolt rows are in the plastic domain. This effect, which is not considered at present in ASCON, leads to different final rotation and displacements and also influence the resistance when ductility problems occur.

This effect is rather limited but its understanding allows to establish a link between the ASCON and Eurocode 3 approaches. More detailed information on this topic should be presented in the forthcoming Ph.D. Thesis of the first author.

5.3 Stress interactions and influence on the resistance of the components

In a connection, stress interactions occur between some of the constitutive components and the design resistance of the latter has to be reduced accordingly. Eurocode 3 Revised Annex J provides rules for the evaluation of the corresponding reduction factors, but their strict application leads to an iterative calculation procedure as the respective level of stresses in the relevant components is initially unknown.

For connections subjected in bending only, practical recommendations for a simple and non-iterative calculation are available. Similar ones should be produced for joints under combined bending moment and axial force.

6 CONCLUSIONS

Analytical procedures for the evaluation of the :

- initial elastic extensional stiffness and rotational stiffness;
- design resistance ;

of connections subjected to combined bending moment and axial force are proposed in the present paper. These ones are fully in agreement with the principles contained in Eurocode 3 Revised Annex J. From these main properties, the moment-rotation curve characterising the behaviour of the whole connection may be built for any loading eccentricity e (e = M/N).

One of the next development steps to achieve is now the validation of this analytical approach through intensive comparisons with the available experimental test results from the literature.

Once this is done, the analytical model will be used to perform parametrical results aiming at defining practical rules for the conceptual design and the calculation of connections under bending and axial forces and, in particular, for column bases, column splices or connections in industrial pitched-roof portal frames.

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