THE INTERACTION FORMULAE FOR BEAM-COLUMNS: A NEW STEP OF A YET LONG STORY

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INTRODUCTION

A beam-column is a member subjected to a combination of axial force and either mono-axial or bi-axial bending; bending is due to transverse loads acting between the member ends and/or to end moments. A beam-column provides therefore a link between the axially compressed column, on the one hand, and the beam, which experiences only bending moments and corresponding shear, on the other hand. It is the most common member used in framed structures. Thus columns and beams appear respectively as particular cases of beam-columns where one load component – respectively bending and compression - becomes small enough so as to be negligible.

The behaviour of a beam-column is usually treated as the response of an isolated structural member to a known system of end forces and moments. Then the end moments, also called continuity moments, represent the restraints provided by the surrounding members in rigidly or semi-rigidly framed structures. The effects of a possible sway that results in a translation of one end relative to the other, are normally assessed at the stage of the global frame analysis; therefore the problem is reduced to the consideration of non-sway beam-columns. The structural response of a beam-column is very much dependent on the way this member is loaded and supported. In this respect three basic modes of failure can be identified:

- Failure due to an *excessive deformation in the bending plane*, when the member is subjected to the combination of axial force and minor axis bending or of axial force and major axis bending (under the reservation, in the latter case, that any deflection out of the bending plane is prevented by appropriate bracing or supports): there is thus an interaction between column buckling and mono-axial beam bending;
- Failure involving *spatial deformation* (bending about the minor axis accompanied by twisting), that looks like lateral torsional buckling in beams subjected to bending only: the interaction occurs between column buckling and beam buckling;
- Failure due to *combined bending and twisting*, when the member experiences combined axial force and bi-axial bending: column buckling interacts with bi-axial bending.

The behaviour up to collapse of beam-columns belongs to the most complex problems relative to structural elements. For practice purposes, both the design and check of beam-columns are usually conducted based on the concept of interaction formulae. The latter result from some simplifications and/or approximations. Therefore one could say that there could be as many interaction formulae as there are researchers who look at this topic.

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Much research work has been devoted to this subject. There is no place here for an historical retrospective of the problem. Let us just stress that the basic developments all along the forty years starting from the very first attempts are summarised by Massonnet [1] and that an exhaustive review of the theory of beam-columns is due to Chen and Atsuta [2]. The background is also developed in the successive issues of the guide produced by the Structural Stability Research Council (formerly the Column Research Council) [3]. Those who want to know more are advised to refer to the many bibliographical references listed in the aforementioned publications. At the origin, the theoretical investigations were conducted in the elastic range. The collapse criterion is simply taken as the onset of the yield stress at the most loaded point within the member. The sole difficulty is how to account for the effects of initial imperfections (out-of-straightness) which, offering a lever arm to the axial force, generate additional bending moments and precipitate the onset of the collapse criterion. Despite this fact, the solution is simple. Later, the inelastic range was investigated; the general format established in the elastic range is kept, under the reservation of some amendments aimed at accounting for the effects of both material yielding and residual stresses.

At the time the purely *elastic* design was the rule, rather few divergences existed between the interaction formulae proposed respectively in the North American and European national codes and standards. Simply speaking, these formulae were based on a single basic format: the one used in the 1966 AISC Specifications [4]; they are hardly different in the more recent American specifications [5]. With such a nearly universal format, only minor differences and divergences are observed between the standards; they are more especially relative to:

- The reference curves used for the determination of the *reduction coefficients* for, respectively, column buckling and lateral torsional buckling;
- The *selection* of those of these curves, which are applicable to the structural shape under consideration, for column buckling and lateral torsional buckling respectively;
- The way the *moment distribution* along the member is reflected: through either the loading term, or the resistance term, or both terms;
- The either linear or non linear expression, in terms of end moment ratio, used for the *equivalent moment factor* when bending is produced by end moments;
- The format of the *amplification factor* aimed at increasing the first-order bending moments so as to account for the second-order effects generated by the axial force.

Of course, an *inelastic* design is more realistic when the section allows for plasticity developing significantly. Then, the problem complicates substantially. However, practice purposes impose a pragmatic approach; only rather simple but safe design rules for difficult problems can answer the designer expectations and requirements. Then due allowance for inelastic behaviour shall result in further changes in the design interaction formulae and is the cause of further divergences between the standards. In the Introductory Report to the 2nd International Colloquium on Stability [6], which served as background document to the ECCS Recommendations [7], the derivation of the so-called "plastic design" regarding beam-columns was simply introduced as follows: "This approach [elastic design] is used for plastic design as well, where W_{el} [elastic section modulus] is replaced by W_{pl} [plastic section modulus]". This rough generalisation, which aimed at taking possibly benefit of the progression of plasticity, was thought adequate and appropriate. In [6], both the elastic and the plastic ECCS interaction formulae were compared with test results. For the case of an "elastic design", a quite satisfactory agreement was observed: all the theoretical results were found conservative (see Figure 11 on page 224 of [6]) and sometimes significantly over-conservative. When referring to the "plastic design", it was concluded as follows: "In some cases, the ECCS design equations are optimistic" and "In general, however, there is a good agreement between the design equations and the test results". Despite the first one of these conclusive sentences, little attention was paid at that time to all those results that are on the unsafe side (see Figure 12 on page 224 of [6]).

The comparison of the results got respectively from the existing design interaction formulae, or also between the results of a certain type of interaction formulae and those of experiments or numerical simulations, always exhibit discrepancies. In this respect, it is while reminding an excerpt of what the French mathematician Henri Poincaré (1854-1912) wrote [8] at the beginning of the 20th century: "L'expérience est la source unique de la vérité : elle seule peut nous apprendre quelque chose de nouveau ; elle seule peut nous donner la certitude. Voilà deux points que nul ne peut contester». Accordingly the results of carefully instrumented and documented tests or, presently, those of well-conducted numerical simulations shall prevail against anything else.

The main – and nearly the single - milestone in the improvement of beam-column interaction formulae at the European level during the last twenty years is achieved in the draft of the European pre-standard ENV1993-1-1 (in short Eurocode 3) for the design of steel structures [9]. The basic format of the interaction beam-column formulae is there and similar to the one used in most of the standards. However the handling of the formulae is delicate. Not only there is a lot of intermediate coefficients, the calculation process of which looks like a Chinese box opening, but also these coefficients are either positive or negative, what prevents them frombeing physically understood. Several times the designers pointed out the danger of mistakes and mishandling as well as the lack of transparency of the EC3 formulae. Under the reservation of only slight and minor differences, the interaction formulae of Eurocode 3 [9] are similar to those of the German standard DIN 18800 [10], the latter having been much influenced by [11] and [12]. Unexpectedly, the formal complexity of the Eurocode 3 beam-column interaction formulae [9] does not enable a significantly better agreement with experimental or numerical results, than the earlier formulae; that is especially demonstrated by a comparison work conducted in [13] and [14]. Also, the present ENV 1993-1-1 approach for the design of beam-columns has progressively been recognised by practitioners as few adequate for practice purposes and unduly conservative in several situations. In this context, the problem of beam-columns is definitively considered as worth being revisited. Together with a need for improvement of the design interaction formulae, due attention must be paid to many aspects: simplicity, economy, accuracy, transparency (physical background), generality and consistency.

In the last three years, two major attempts were made in Europe with a view to improve substantially the capability of the EC3 beam-column formulae. Based on a huge amount of numerical simulations [15], an Austrian-German team used intensively the curve fitting technique so as to derive user-friendly interaction formulae. The latter involve indeed a limited number of global factors but each of these factors covers in fact several individual effects at one time. In the Austrian-German approach, simplicity prevails against transparency. Quite independently, a French-Belgian team started the development of formulae where all the physical phenomena are deliberately reflected separately. In contrast with the Austrian-German approach, the French-Belgian one favours transparency and provides a wider range of applicability together with a better accuracy. Incidentally, the works developed by both teams were nearly simultaneously submitted for discussion within ECCS Technical Committee 8 "Structural Stability". Both approaches were appreciated for their respective major quality but were still prone to further improvement. Therefore both teams were requested to join their efforts in order to keep their respective proposals as consistent as possible so as to get expectedly:

- A "Level 1" proposal, due to the Austrian-German team, which, based on the concept of global factors, is the most simple for what regards the general format;
- A "Level 2" proposal, due to the French-Belgian team, which, through a slightly more elaborated format, is more transparent and accurate.

Several working papers relative to both proposals have been produced within ECCS TC8. They are not listed in the list of references because most of them are not published. However it has yet been briefly reported on the status of development of the "Level 2" proposal in [16] and [17]. Present paper aims at presenting a more complete survey on this topic as well as the last improvements achieved in the meantime. For that purpose, it is assumed that the structural shape is fully effective; consequently, in accordance with ENV1993-1-1, it belongs at least to Class 3 (elastic cross-sectional resistance) and possibly to Class 2 (plastic cross-sectional resistance with a limited rotation capacity) or Class 1 (plastic cross-sectional resistance with a substantial rotation capacity). The subject is developed herebelow according to an increase in complexity:

- Behaviour of an *axially compressed imperfect member*;
- Behaviour of a *laterally restrained member subjected to combined axial force and mono-axial bending*;
- Behaviour of a *laterally restrained member subjected to combined axial force and bi-axial bending*;
- Account for possible *lateral torsional buckling.*

Doing so results in some minor duplicates but helps understanding very much.

AXIALLY COMPRESSED MEMBER

Concept of amplification factor. An elastic pin-ended axially compressed prismatic member of length L buckles according to a sine wave mode in the relevant buckling plane. The effects of both initial structural and geometric imperfections, i.e. basically residual stresses and lack of straightness, can be represented by a single so-called sinusoidal *equivalent geometric imperfection* $v_0(x)$ (fig.1):

$$v_o(x) = e_{o.d} \sin\frac{\pi x}{L} \tag{1}$$

where $e_{o.d}$ designates the maximum magnitude of this imperfection, i.e. at $x=0.5L$.

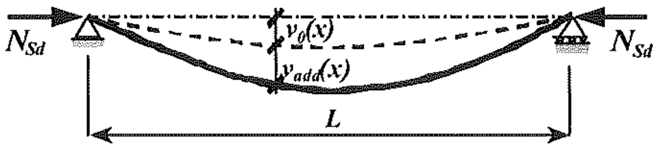

Figure 1 – Deflection of an axially loaded imperfect member

Applying a design axial force N_{Sd} on the imperfect member results in an additional deflection $v_{add}(x)$ (fig.1). The latter can be expressed as a well-known magnification of the initial deflection $v_0(x)$:

$$v_{add}(x) = \frac{\frac{N_{Sd}}{N_{cr}}}{1-\frac{N_{Sd}}{N_{cr}}} v_o(x) \tag{2}$$

where N_{cr} is the Euler buckling load of the member for a buckling length L. Account taken of (1) and (2), the total deflected shape of the member, measured from its chord, is thus:

$$v_{tot}(x) = v_o(x) + v_{add}(x) = \frac{1}{1-\frac{N_{Sd}}{N_{cr}}} e_{o.d} \sin\frac{\pi x}{L} \tag{3}$$

The most loaded section is at mid-length, where the internal design forces, including second-order effects, are:

- A compressive force N_{sd}
- A bending moment $N_{Sd}\, e_{o.d} /(1-\frac{N_{Sd}}{N_{cr}})$

The *first-order bending moment* $N_{Sd}e_{od}$ is thus magnified by the so-called *amplification factor*:

$$K_1 = \frac{1}{1-\frac{N_{Sd}}{N_{cr}}} \tag{4}$$

so as to get the *total non-linear bending moment*, which includes therefore the second-order effects; the wording "non linear" expresses simply the fact that the total bending moment is not proportional to the axial force N_{Sd}.

Design resistance criterion. Designating A and W_{el} as the cross-sectional area and the elastic section modulus respectively, f_y as the material yield stress and γ_M as the partial resistance safety factor, the elastic resistance criterion in the determinative section writes:

$$\frac{N_{Sd}}{A} + \frac{1}{1-\frac{N_{Sd}}{N_{cr}}} \frac{N_{Sd}e_{o.d}}{W_{el}} \le \frac{f_y}{\gamma_M} \tag{5.a}$$

or, alternatively:

$$\frac{N_{Sd}}{N_{pl.Rd}} + \frac{1}{1-\frac{N_{Sd}}{N_{cr}}} \frac{N_{Sd}e_{o.d}}{M_{el.Rd}} \le 1 \tag{5.b}$$

with the design compression resistance $N_{pl.Rd}$ and the design elastic bending resistance $M_{el.Rd}$ of the cross-section:

$$N_{pl.Rd} = A\frac{f_y}{\gamma_M} \quad (6)$$

$$M_{el.Rd} = W_{el}\frac{f_y}{\gamma_M} \quad (7)$$

It is demonstrated elsewhere [18, 19] that this "elastic" format can be used to derive satisfactory analytical expressions for the inelastic column buckling fitting closely with experimental results. A so-called buckling curve provides the *reduction factor* χ for column buckling, i.e. the design buckling load $N_{b.Rd}$ of the member normalised with respect to the design compression resistance $N_{pl.Rd}$ of the cross-sectional area:

$$\chi = \frac{N_{b.Rd}}{N_{pl.Rd}} \quad (8)$$

as a function of the *reduced column slenderness* $\bar{\lambda}$:

$$\bar{\lambda} = \sqrt{\frac{N_{pl}}{N_{cr}}} = \sqrt{\frac{\gamma_M N_{pl.Rd}}{N_{cr}}} \quad (9)$$

The magnitude of the equivalent geometric imperfection $e_{o.d}$ is determined by the limit state criterion: above equation (5.b) is just fulfilled (sign =) when the design axial force N_{Sd} is equal to the design buckling load $N_{b.Rd}$. Therefore:

$$e_{o.d} = (1 - \frac{N_{b.Rd}}{N_{pl.Rd}})(1 - \frac{N_{b.Rd}}{N_{cr}})\frac{M_{el.Rd}}{N_{b.Rd}} = \frac{(1-\chi)\left(1 - \frac{\chi\bar{\lambda}^2}{\gamma_M}\right)}{\chi}\frac{W_{el}}{A} \quad (10)$$

With this expression for $e_{o.d}$, the inelastic resistance criterion (5.b) writes simply:

$$\frac{N_{Sd}}{\chi^* N_{pl.Rd}} \le 1 \quad (11)$$

where:

$$\frac{1}{\chi^*} = 1 + \frac{\left(\frac{1}{\chi} - 1\right)\left(1 - \frac{\chi\bar{\lambda}^2}{\gamma_M}\right)}{1 - \frac{N_{Sd}}{N_{cr}}} \quad (12)$$

It shall be observed that $\chi^* \ge \chi$ and that $\chi^* = \chi$ only at failure, i.e. when $N_{Sd} = N_{b.Rd}$.

ELASTIC MEMBER SUBJECTED TO COINCIDENT AXIAL FORCE AND MONO-AXIAL BENDING MOMENT ACTING IN THE BUCKLING PLANE

Concepts of equivalent moment factor and amplification factor. Let assume that the member under consideration is now subjected to the combination of a design axial force N_{Sd} and a first-order bending due either to end moments M_{Sd} and ψM_{Sd} ($-1 \le \psi \le 1$) or/and transverse loads between the member ends. Referring herein to end moments, the latter can result from appropriate eccentricities e_1 and e_2 of the axial force (fig.2.a). Accordingly, the first-order bending moment distribution varies between the member ends. On its turn, the deflection induced by first-order bending provides a lever arm to the axial force and results in additional second-order bending moments. There is a section, somewhere along the member, where the amplified bending moment, i.e. the total non-linear bending moment (including second-order effects), is maximum and amounts M_{max} (fig.2.a). In order to avoid the determination of the location of this section, the concept of *equivalent uniform moment* M_{equ} is introduced: it is the first-order constant moment distribution which, applied in combination with the specified axial force N_{Sd}, produces the same maximum bending moment M_{max} as the actual moment distribution (fig.2.b) together with this axial force. The equivalent uniform moment is usually expressed as follows:

$$M_{equ} = C_m\, M_{Sd} \quad (13)$$

where M_{Sd} is the maximum design first-order moment within the member. It corresponds to a same eccentricity e_{qu} of the design axial force N_{Sd} at both member ends (fig.2.b).

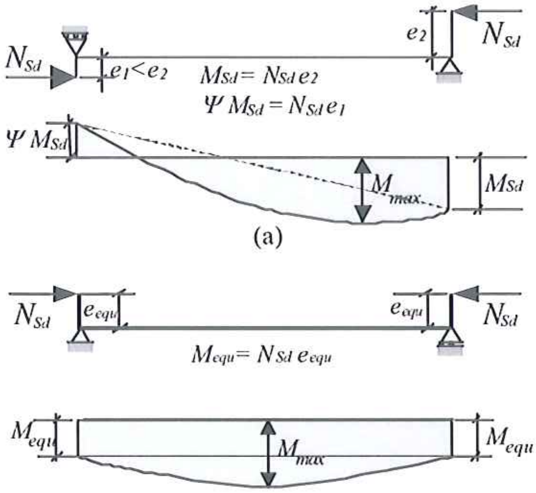

Figure 2 – Equivalent moment concept

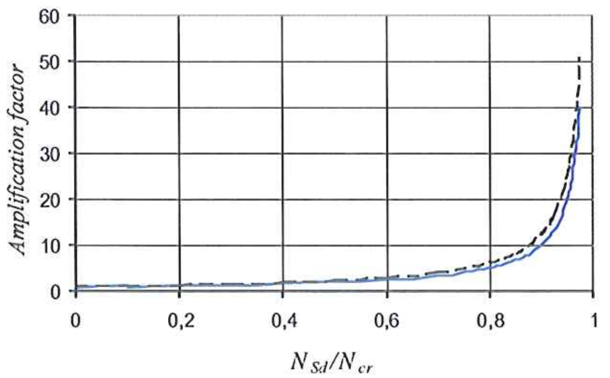

Figure 3 – Amplification factor
a) K_1 : Sinusoidal equivalent geometric imperfection (_____)
b) K_2 : Constant eccentricity of the axial force (-----------)

The presence of the design axial force N_{Sd} makes that the equivalent uniform moment M_{equ} is magnified by second-order effects. It is known that the resulting maximum total non-linear bending moment is given as:

$$M_{max} = \frac{1}{\cos\left(\frac{\pi}{2}\sqrt{\frac{N_{Sd}}{N_{cr}}}\right)} M_{equ} \quad (14)$$

The *amplification factor* for a perfect member loaded by N_{Sd} with a constant eccentricity is different from the one (4) obtained in the case of the axially loaded imperfect member; it writes:

$$K_2 = \frac{1}{\cos\left(\frac{\pi}{2}\sqrt{\frac{N_{Sd}}{N_{cr}}}\right)} \tag{15}$$

As far as the design axial force is not too close to the critical load, the respective magnitudes of both expressions (4) and (15) are not substantially different (fig.3). However a significant discrepancy is observable for very large magnitudes of the relative axial force N_{Sd}/N_{cr}. For practice purposes, K_1 is generally substituted for K_2 in the bending term so as to keep a single format of the amplification factor in the interaction formulae. In order to compensate for the consequences of this substitution (see equ. 21.a), it is suggested to pre-multiply the theoretically based expression of $C_m(\psi)$, represented by the underlined factor of the right hand term of (16), by the correction factor $(1 - N_{Sd}/N_{cr})$:

$$C_m(\psi) = \left(1 - \frac{N_{Sd}}{N_{cr}}\right)\left(\underline{\sqrt{1 - 2\psi\cos\left(\pi\sqrt{\frac{N_{Sd}}{N_{cr}}}\right) + \psi^2} \Big/ \sin\left(\pi\sqrt{\frac{N_{Sd}}{N_{cr}}}\right)}\right) \tag{16}$$

if $N_{Sd} \geq N_{lim}$, and:

$$C_m(\psi) = 1 - (arc\cos\psi/\pi)^2 \tag{17}$$

if $N_{Sd} < N_{lim}$.

where ψ designates the end moment ratio (fig.2) such that $-1 \leq \psi \leq 1$. The limit value N_{lim} corresponds to the exhaustion of the resistance in the most loaded end section of the member:

$$N_{lim} = N_{cr}(arc\cos\psi/\pi)^2 \tag{18}$$

In many standards and codes, the influence of the relative axial force N_{Sd}/N_{cr} on the equivalent moment factor is disregarded. Then, reference is most often made to the so-called either Campus-Massonnet or Austin expression. The non-linear Campus-Massonnet format [20]:

$$C_m(\psi) = \sqrt{0{,}3\left(1 + \psi^2\right) + 0{,}4\,\psi} \geq \frac{1}{2.3} \tag{19.a}$$

prevails for long in Europe; however, for sake of simplicity, ENV1993-1-1 [9] refers to the linear Austin format [21]:

$$C_m(\psi) = 0{,}6 + 0{,}4\,\psi \geq 0{,}4 \tag{19.b}$$

Expressions (19.a) and (19.b) are only approximates of the the sole underlined factor of the right hand term of (16), which are in addition made free from the influence of axial force.

Recently, Villette [22] suggested to approach the complete expression (16) of $C_m(\psi)$ by means of a rather simple but more accurate formula:

$$C_m(\psi) = 0{,}79 + 0{,}21\,\psi + 0{,}36\,(\psi - 0{,}33)\,\frac{N_{Sd}}{N_{cr}} \tag{19.c}$$

This expression enable to fulfil with some requested continuities, as it will be shown later.
When bending is produced by transverse loads M_Q or by combined end moments and transverse loads $M_{\psi+Q}$, an appropriate expression of C_m results as follows from theoretical considerations:

$$C_m(\psi + Q) = 1 + \left(\frac{\pi^2\,EI\,v^*}{M^*\,L^2} - 1\right)\frac{N_{Sd}}{N_{cr}} \tag{20}$$

where M^* and v^* represent respectively the maximum first-order bending moment and the maximum first-order bending deflection wherever they occur. For a point load acting at mid-span, one get:

$C_m = 1 - 0.18\, N_{Sd}/N_{cr}$ while, for a unifom load applied over the whole member length, $C_m = 1 + 0.03\, N_{Sd}/N_{cr}$.

Elastic design resistance criterion for in-plane behaviour. When the member is subjected to the combination of an axial force and a mono-axial bending in the plane of column buckling, the elastic design resistance criterion is simply obtained by implementing equation (5.b) with the relevant term relative to the first-order bending. With due allowance made, as said above, for a single format of the amplification factor, this criterion writes:

$$\frac{N_{Sd}}{N_{pl.Rd}} + \frac{1}{1 - \frac{N_{Sd}}{N_{cr}}} \frac{N_{Sd}\, e_{o,d}}{M_{el.Rd}} + \frac{1}{1 - \frac{N_{Sd}}{N_{cr}}} \frac{C_m\, M_{Sd}}{M_{el.Rd}} \leq 1 \qquad (21.a)$$

or:

$$\frac{N_{Sd}}{\chi^* N_{pl.Rd}} + \frac{C_m\, M_{Sd}}{\left(1 - \frac{N_{Sd}}{N_{cr}}\right) M_{el.Rd}} \leq 1 \qquad (21.b)$$

Some tedious transformations enable another format, where the first term coincides with the specification of ENV1993-1-1 [9] regarding column buckling only. It is:

$$\frac{N_{Sd}}{\chi\, N_{pl.Rd}} + \mu \frac{C_m\, M_{Sd}}{\left(1 - \frac{N_{Sd}}{N_{cr}}\right) M_{el.Rd}} \leq 1 \qquad (21.c)$$

where:

$$\mu = \frac{1 - \frac{N_{Sd}}{N_{cr}}}{1 - \chi \frac{N_{Sd}}{N_{cr}}} \qquad (22)$$

It is referred to this format in the following.

This *elastic* design resistance criterion has the following characteristics:

- It is theoretically based and it is obtained accordingly;
- It is fully consistent with the elastic cross-sectional resistance checks;
- It reduces to the column buckling check in the absence of additional first-order bending moment (due to either end moments or transverse loads);
- Appropriate expressions of the equivalent uniform moment factor are available.

No allowance for plasticity effects is the basic criticism that can be addressed to.

MEMBER SUBJECTED TO COINCIDENT AXIAL FORCE AND BI-AXIAL FIRST-ORDER BENDING

Basic principle for generalisation to bi-axial bending. When bi-axial bending, the axial force amplifies actually both moments about the principal axes *y-y* and *z-z* of the section. That results in a complex coupling between the instabilities in both planes. However, such a coupling is generally disregarded for practice purposes; it is indeed the case in ENV1993-1-1 [9] and in most of the national standards. A quite similar simplification is made herein, being understood that it results in a slight divergence from a fully theoretical approach. Accordingly, the instability is controlled in either bending plane and bi-axial bending is accounted for by simply adding a second bending term, which looks similar to the first one. Then, the simplified expression for the design resistance criterion reads:

$$\frac{N_{Sd}}{\chi_i N_{pl.Rd}} + \mu_i \left[\frac{C_{m.y} M_{y.Sd}}{\left(1 - \frac{N_{Sd}}{N_{cr,y}}\right) M_{el.y.Rd}} \right] + \mu_i \left[\frac{C_{m.z} M_{z.Sd}}{\left(1 - \frac{N_{Sd}}{N_{cr,z}}\right) M_{el.z.Rd}} \right] \leq 1 \tag{23}$$

where the index y (resp. z) is relative to bending about the y-y (resp. z-z) axis of the cross-section.

Elastic design resistance criterion for beam-columns in bi-axial bending. The index i indicates the plane in which failure is likely to occur. As this plane is not known beforehand, one has to check a doublet of formulae, which is likely to govern the uncoupled phenomena:

$$\frac{N_{Sd}}{\chi_y N_{pl.Rd}} + \mu_y \left[\frac{C_{m.y} M_{y.Sd}}{\left(1 - \frac{N_{Sd}}{N_{cr,y}}\right) M_{el.y.Rd}} \right] + \mu_y \left[\frac{C_{m.z} M_{z.Sd}}{\left(1 - \frac{N_{Sd}}{N_{cr,z}}\right) M_{el.z.Rd}} \right] \leq 1 \tag{24.a}$$

$$\frac{N_{Sd}}{\chi_z N_{pl.Rd}} + \mu_z \left[\frac{C_{m.y} M_{y.Sd}}{\left(1 - \frac{N_{Sd}}{N_{cr,y}}\right) M_{el.y.Rd}} \right] + \mu_z \left[\frac{C_{m.z} M_{z.Sd}}{\left(1 - \frac{N_{Sd}}{N_{cr,z}}\right) M_{el.z.Rd}} \right] \leq 1 \tag{24.b}$$

where:

$$\mu_y = \frac{1 - \frac{N_{Sd}}{N_{cr,y}}}{1 - \chi_y \frac{N_{Sd}}{N_{cr,y}}} \tag{25.a}$$

and:

$$\mu_z = \frac{1 - \frac{N_{Sd}}{N_{cr,z}}}{1 - \chi_z \frac{N_{Sd}}{N_{cr,z}}} \tag{25.b}$$

The characteristics of above design formulae, which control respectively the instability about either principal axis, are the same as in the case of in-plane behaviour, except that this doublet of expressions is not derived from a fully rigorous theoretical reasoning.

Plastic design resistance criterion for beam-columns in bi-axial bending. Rigorously speaking, the beam-column interaction formulae established in the previous section hold as far as the behaviour is elastic. More especially, it is valid for Class 3 sections in accordance with the designation of ENV1993-1-1 [9], i.e. those requiring a complete elastic design check. When sections of Class 1 or Class 2 are of concern, the question arises of how to account for the inelastic effects on the member resistance.

In order to keep the governing equation as simple as possible, it is decided:

- To keep the general format established in the case of elastic behaviour;
- To substitute the plastic cross-sectional resistance $M_{pl.Rd}$ to the elastic one $M_{el.Rd}$;
- To generalise implicitly, in the inelastic range, the convenient elastic concepts of buckling length, equivalent moment factor and amplification factor;

- To alleviate the effects of above simplifications and assumptions by the introduction of additional factors, the expressions of which are partly theoretically based but often need some calibration against results of tests or numerical simulations.

In the latter respect, due allowance shall be made for the effects of yielding on:

- The interaction between mono-axial bending and axial force, with due consideration of the effects of the member slenderness, by means of four factors k_{yy}, k_{yz}, k_{zy} and k_{zz};
- The interaction, for the cross-sectional resistance, between bending moments M_y and M_z by means of two factors α^* and β^*.

In the inelastic range, the design criteria (24) become:

$$\frac{N_{Sd}}{\chi_y\, N_{pl.Rd}} + \mu_y \left[\frac{C_{m.y}\, M_{y.Sd}}{\left(1-\frac{N_{Sd}}{N_{cr,y}}\right) k_{yy}\, M_{pl.y.Rd}} \right] + \alpha^* \mu_y \left[\frac{C_{m.z}\, M_{z.Sd}}{\left(1-\frac{N_{Sd}}{N_{cr,z}}\right) k_{yz}\, M_{pl\,z.Rd}} \right] \leq 1 \qquad (26.a)$$

$$\frac{N_{Sd}}{\chi_z\, N_{pl.Rd}} + \beta^* \mu_z \left[\frac{C_{m.y}\, M_{y.Sd}}{\left(1-\frac{N_{Sd}}{N_{cr,y}}\right) k_{zy}\, M_{pl.y.Rd}} \right] + \mu_z \left[\frac{C_{m.z}\, M_{z.Sd}}{\left(1-\frac{N_{Sd}}{N_{cr,z}}\right) k_{zz}\, M_{pl\,z.Rd}} \right] \leq 1 \qquad (26.b)$$

In order to enable the degeneration of these expressions into the (24) ones, all the above factors k_{yy}, k_{yz}, k_{zy} and k_{zz}, on the one hand, and α^* and β^*, on the other hand, need to be taken equal to unity when an elastic design check is requested.

DETERMINATION OF THE ADDITIONAL FACTORS INVOLVED IN THE INELASTIC DESIGN CRITERIA

Determination of the α^* and β^* factors. Basically, the α^* and β^* factors handle the effects of material yielding on the cross-sectional resistance when combined bending moments M_y and M_z only. In other words, they aim at enabling to approach the M_y-M_z interaction curve as closely as possible. The latter governs the member resistance when stocky members (low slenderness) and low axial force. According to ENV1993-1-1 [9], the cross-sectional resistance to bi-axial bending is given as:

$$\left(\frac{M_{y.Sd}}{M_{b.y.Rd}}\right)^{\alpha} + \left(\frac{M_{z.Sd}}{M_{b.z.Rd}}\right)^{\beta} \leq 1 \qquad (27)$$

where $M_{b.y.Rd}$ and $M_{b.z.Rd}$ are the design bending resistances about respectively y-y and z-z. A quite similar expression is used in the North-American standards. The exponents α and β - to be distinguished from above α^* and β^* - are constants, the values of which depend on both the type of design check – elastic or plastic - and the cross-sectional shape. For an elastic cross-sectional resistance check, one has of course $\alpha = \beta = 1$ (Class 3) because the absence of yielding makes the interaction linear. In contrast, $\alpha = 2$ and $\beta = 1$ are realistic values for the plastic cross-sectional resistance (Class 1 or Class 2) of a I or H structural shape, with the result that the admissible domain is convex (fig.4); other values are available when some other types of structural shapes, such as circular and hollow sections, are considered.

The factors α^* and β^* have the same aim as the factors α and β. However, they are introduced in (26) as multipliers, and not as exponents, of the bending moments. Should α^* and β^* be taken respectively as constants, then they make the interaction at the ultimate limit state no more continuously convex but bi-linear (fig.4). When both α^* and β^* are equal to unity, the interaction is simply linear; this situation is appropriate for an elastic design check. Values lower than 1 for these factors enable an increase in cross-sectional resistance due to some plasticity strength reserve; appropriate values are therefore expected suitable for a plastic design resistance check. For sake of simplicity, a single value for both α^* and β^* can be contemplated; for instance, $\alpha^* = \beta^* = 0.6$ seems an appropriate choice for I and H sections. Alternatively, in a more refined way, the following non-equal values can also be considered for Class 1 or Class 2 sections:

$$\alpha^* = 0{,}6\sqrt{\frac{w_z}{w_y}} \tag{28.a}$$

$$\beta^* = 0{,}6\sqrt{\frac{w_y}{w_z}} \tag{28.b}$$

where:

$$w_y = \frac{W_{pl.y}}{W_{el.y}} \leq 1{,}5 \tag{29.a}$$

$$w_z = \frac{W_{pl.z}}{W_{el.z}} \leq 1{,}5 \tag{29.b}$$

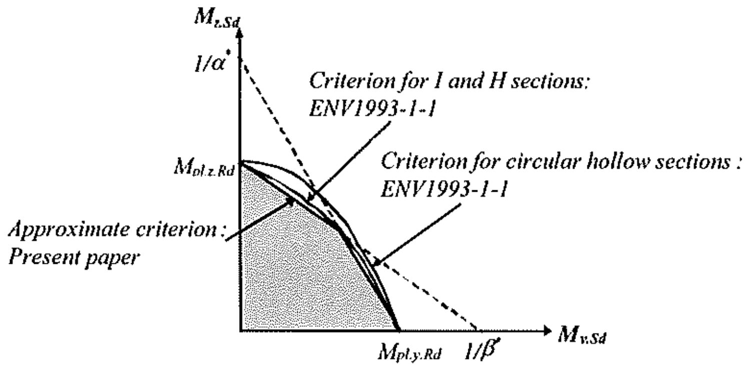

Figure 4 – Plastic cross-sectional resistance to bi-axial bending

Clearly the w_y and w_z factors are larger than unity so that their reciprocals are smaller than unity. The upper boundaries of 1,5 are simply introduced because of the domain where the investigations were carried out. In fact, they do not constitute severe constraints; indeed they enable to cover most of the steel structural shapes.
Let us remind that factors α^* and β^* must be taken equal to 1 when an elastic design check is requested, i.e. when Class 3 sections are of concern. Also w_y and w_z shall be considered equal to 1 in that situation.
At this stage, it is while stressing that no continuity does exist between the classes of sections. The cross-sectional resistance either is based on a purely elastic criterion, and thus represented by the straigth line ($\alpha^* = \beta^* = 1$) , or is the plastic resistance, and then given by the approximate bi-linear interaction (α^* and $\beta^* < 1$). In other words, there is no possibility for a progressive transition, within Class 3, from the elastic resistance (section located just at the boundary between Class 3 and Class 4) up to the plastic resistance (section located just at the boundary between Class 2 and class 3). That results simply from the specifications of ENV1993-1-1 regarding the cross-sectional resistance of sections subjected to mono-axial bending. That observation goes against the physical sense; there is surely a place for improving possibly the specifications in this respect. This disputable question is revisited in the section "*Consistency and Continuity of the Proposal*" at the end of present paper.

Determination of the k_{yy}, k_{yz}, k_{zy} and k_{zz} factors. The k factors aim at handling the plasticity effects in the interaction between mono-axial bending and axial force; in this respect, it shall be kept in mind that, due to the

presence of an axial force, also the member slenderness plays a role on the extent of yielding at the ultimate limit state. Therefore, the interaction that is of concern herein, applies the member and not the cross-section. Factors k_{yy} and k_{zz} rule the *M-N* interaction when the buckling plane is coincident with the plane of bending while factors k_{yz} and k_{zy} do similarly when the buckling direction is perpendicular to the plane of bending.
When the member slenderness vanishes, the k factors must degenerate so as to approach the usual cross-sectional *M-N* interaction curves (fig.5). Accordingly, a design elasto-plastic resistance $k_{ii}M_{pl.Rd,i}$ is substituted for the elastic one, $M_{el.Rd,i}$, being understood that i means y or z according to the buckling plane under consideration. Anyone of the factors k_{ii} shall be such that:

- It is 1 for Class 1 or Class 2 sections so as to take full profit from material yielding;
- It is $W_{el,i}/W_{pl,i}$ for Class 3 sections so as to reduce the elasto-plactic resistance to the elastic one.

The major part of the k factor expressions is physically built so as to comply with these requirements.

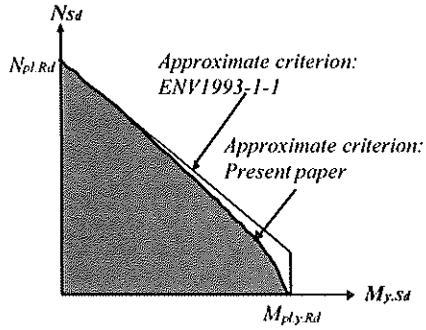

Figure 5 – Plastic cross-sectional resistance to combined mono-axial bending and axial force

In addition, at the ultimate limit state of the member, the extent of yielding within the most loaded section (including second-order effects) decreases when the member slenderness $\bar{\lambda}$ and the reduced axial force $N_{Sd}/N_{pl.Rd}$ increase. Also, it depends on the moment distribution, which is characterised by the equivalent moment factor C_m; this distribution is likely to change the position of the most loaded section indeed. When bi-axial bending is combined to axial force, the determinative bending axis, when the member buckles, governs the proneness of the member to yield and the extent of yielding. In order to avoid the search for this axis, a safe approach is adopted for sake of simplicity; it consists in referring to a single column slenderness:

$$\bar{\lambda}_{max} = max\ imum\left[\bar{\lambda}_y, \bar{\lambda}_z\right] \tag{30}$$

i.e. to the largest of the column slendernesses relative to both possible buckling directions. The effects of both the slenderness and moment distribution cannot be derived easily on a purely physically based reasoning; they are accounted for by means of additional terms, which were obtained by calibration. These terms are those underlined in the expressions given below.
When Class 1 or Class 2 sections are of concern, the following expressions for k_{yy}, and k_{zz} are found adequate to fulfil all the above requirements:

$$k_{yy} = 1 + \left(w_y - 1\right)\left[2 - \underline{\frac{1,6}{w_y} C_{m,y}^{\ 2} \left(1 + \bar{\lambda}_{max}\right)\bar{\lambda}_{max}}\right]\frac{N_{Sd}}{N_{pl.Rd}} \geq \frac{1}{w_y} \tag{31}$$

$$k_{zz} = 1 + \left(w_z - 1\right)\left[2 - \underline{\frac{1,6}{w_z} C_{m,z}^{\ 2} \left(1 + \bar{\lambda}_{max}\right)\bar{\lambda}_{max}}\right]\frac{N_{Sd}}{N_{pl.Rd}} \geq \frac{1}{w_z} \tag{32}$$

The boundaries assigned to the k_{yy} and k_{zz} factors prevent $kM_{pl.Rd}$ from being smaller than $M_{el.Rd}$.

The expressions adopted for k_{yz} and k_{zy} are slightly different. That is due to the lesser effects of plasticity on the *M-N* interaction when the plane of bending and the plane considered for buckling are not coincident.

$$k_{yz} = 1 + (w_z - 1)\left[2 - 14\frac{C_{m.z}^{2}\,\bar{\lambda}_{max}^{2}}{w_z^{5}}\right]\frac{N_{Sd}}{N_{pl.Rd}} \geq \frac{0,6}{\sqrt{w_y w_z}} \tag{33}$$

$$k_{zy} = 1 + (w_y - 1)\left[2 - 14\frac{C_{m.y}^{2}\,\bar{\lambda}_{max}^{2}}{w_y^{5}}\right]\frac{N_{Sd}}{N_{pl.Rd}} \geq \frac{0,6}{\sqrt{w_y w_z}} \tag{34}$$

The boundaries specified for the k_{yz} and k_{zy} factors are necessary in order to prevent $k_{yz}M_{pl.Rd}/\alpha^*$ and $k_{zy}M_{pl.Rd}/\beta^*$ from becoming smaller than, respectively, $M_{el.z.Rd}$ and $M_{el.z.Rd}$. It shall be noticed that they are smaller than unity. Let us remind that factors k_{yy}, k_{yz}, k_{zy} and k_{zz} must be taken equal to 1 when an elastic design check is requested, i.e. when Class 3 sections are of concern.

GENERALISATION TO POSSIBLE LATERAL TORSIONAL BUCKLING

For consistency, the general format of the formulae established above is kept. Only some amendments are brought, which aim at accounting for possible lateral torsional buckling:

- A reduction factor χ_{LT} is applied on the cross-sectional bending resistance about the strong axis;
- The expressions of the k_{yz} and k_{zy} factors are slightly modified;
- An additional factor k_{LT} allows for a smooth transition between the respective responses of open and hollow sections and for the influence of the axial force N_{Sd} on the lateral torsional buckling phenomenon.

Accordingly, the expressions (26) are generalised as follows:

$$\frac{N_{Sd}}{\chi_y N_{pl.Rd}} + \mu_y\left[\frac{k_{LT}}{\chi_{LT}}\frac{1}{\left(1 - \frac{N_{Sd}}{N_{cr,y}}\right)}\frac{C^*_{my} M_{y.Sd}}{k_{yy.mod}\cdot M_{y.Rd}} + \alpha^*\frac{1}{\left(1 - \frac{N_{Sd}}{N_{cr,z}}\right)}\frac{C^*_{mz} M_{z.Sd}}{k_{yz.mod} M_{z.Rd}}\right] \leq 1 \tag{35.a}$$

$$\frac{N_{Sd}}{\chi_z N_{pl.Rd}} + \mu_z\left[\beta^*\frac{k_{LT}}{\chi_{LT}}\frac{1}{\left(1 - \frac{N_{Sd}}{N_{cr,y}}\right)}\frac{C^*_{my} M_{y.Sd}}{k_{zy.mod} M_{y.Rd}} + \frac{1}{\left(1 - \frac{N_{Sd}}{N_{cr,z}}\right)}\frac{C^*_{mz} M_{z.Sd}}{k_{zz.mod} M_{z.Rd}}\right] \leq 1 \tag{35.b}$$

Factor k_{LT} writes:

$$k_{LT} = C^{*2}_{my}\frac{1}{\sqrt{(1 - \frac{N_{Sd}}{N_{cr.z}})(1 - \frac{N_{Sd}}{N_{cr.T}})}} \geq 1 \tag{36}$$

where $N_{cr.T}$ is the critical elastic torsional buckling resistance of the member. The denominator can be deduced theoretically; it reflects the detrimental influence of a possible axial force on the lateral torsional buckling. In the particular case where $N_{Sd} = M_{z.Sd} = 0$, above generalised formulae (35) must degenerate into the single check of a member against lateral torsional buckling, i.e. $M_{y.Rd} \leq \chi_{LT} M_{y.Rd}$, in which case $C_{m.y}$ must vanish. Therefore the following expressions of C^*_m derived from results of experiments and numerical simulations:

$$C^*_{m.z} = C_{m.z} \tag{37}$$

$$C^*_{m.y} = C_{m.y} + (1 - C_{m.y})\frac{a_{LT}\sqrt{e_y}}{1 + a_{LT}\sqrt{e_y}} \tag{38}$$

with, for doubly symmetrical sections:

$$e_y = \frac{M_{y.Sd}}{N_{Sd}} \frac{A}{W_{el.y}} \quad (39)$$

$$a_{LT} = 1 - \frac{I_t}{I_y} \geq 0 \quad (40)$$

I_t and I_y are respectively the torsional inertia about the x-x axis and the flexural inertia about the y-y axis of the section. Of course a_{LT} may be conservatively taken equal to unity. When $C_{m.y}$ is larger than unity, the assumption of $C^*_{m.y} = C_{m.y}$ is only slightly conservative.

The modified $k_{ij\cdot mod}$ factors must degenerate into the k_{ij} factors when lateral torsional buckling (LTB) needs not to be considered, i.e. when $\chi_{LT} = 1$. The changes aim at representing the effects of both lateral-torsional buckling and $M_{z.Sd}$ moments, which contribute an amplification of LTB effects, on the extent of yielding at the ultimate limit state. For Class 1 and Class 2 sections, the modified k_{ij} factors write:

$$k_{yy.mod} = 1 + (w_y - 1)\left[\left(2 - \frac{1,6}{w_y} C_{m.y}^2 \bar{\lambda}_{max} - \frac{1,6}{w_y} C_{m.y}^2 \bar{\lambda}_{max}^2\right) n_{pl} - b_{LT}\right] \geq \frac{1}{w_y} \quad (41)$$

$$k_{yz.mod} = 1 + (w_z - 1)\left[\left(2 - 14 \frac{C_{m.z}^2 \bar{\lambda}_{max}^2}{w_z^5}\right) n_{pl} - c_{LT}\right] \geq \frac{0,6}{\sqrt{w_y w_z}} \quad (42)$$

$$k_{zy.mod} = 1 + (w_y - 1)\left[\left(2 - 14 \frac{C_{m.y}^2 \bar{\lambda}_{max}^2}{w_y^5}\right) n_{pl} - d_{LT}\right] \geq \frac{0,6}{\sqrt{w_y w_z}} \quad (43)$$

$$k_{zz.mod} = 1 + (w_z - 1)\left(2 - \frac{1,6}{w_z^2} C_{m.z}^2 \bar{\lambda}_{max} - \frac{1,6}{w_z^2} C_{m.z}^2 \bar{\lambda}_{max}^2 - e_{LT}\right) n_{pl} \geq \frac{1}{w_z} \quad (44)$$

where:

$$b_{LT} = 0,5 a_{LT} \bar{\lambda}_{LT.0}^2 \frac{M_{y.Sd}}{\chi_{LT} M_{ply.Rd}} \frac{M_{z.Sd}}{M_{plz.Rd}} \quad (45)$$

$$c_{LT} = 10 a_{LT} \frac{\bar{\lambda}_{LT.0}^2}{5 + \bar{\lambda}_z^4} \frac{M_{y.Sd}}{C_{m.y} \chi_{LT} M_{ply.Rd}} \quad (46)$$

$$d_{LT} = 2 a_{LT} \frac{\bar{\lambda}_{LT.0}}{0,1 + \bar{\lambda}_z^4} \frac{M_{y.Sd}}{C_{m.y} \chi_{LT} M_{ply.Rd}} \frac{M_{z.Sd}}{C_{m.z} M_{plz.Rd}} \quad (47)$$

$$e_{LT} = 1,7 a_{LT} \frac{\bar{\lambda}_{LT.0}}{0,1 + \bar{\lambda}_z^4} \frac{M_{y.Sd}}{C_{m.y} \chi_{LT} M_{ply.Rd}} \quad (48)$$

In the definition of factors b_{LT}, c_{LT}, d_{LT} and e_{LT}, $\bar{\lambda}_{LT.0}$ is the reduced slenderness for lateral torsional buckling relative to a constant moment over the member length.
For Class 3 sections, similar expressions as above are used but, for reasons explained earlier, reference shall be made to $M_{Rd.3}$ and W_3 instead of $M_{pl.Rd}$ and W_{pl} respectively.

CONSISTENCY AND CONTINUITY OF THE PROPOSAL

All along the development of above proposal for the design of beam-columns, there was sake for ensuring:

- Continuity between the most general format and the one used in more simple loading situations;
- Continuity between member design checks (stability) and section design checks (cross-sectional resistance);

- Smooth transition for the resistance in the elasto-plastic range, so as to remove the paradoxical situation of ENV1993-1-1, where there is a sudden drop in resistance when passing from Class 2 to Class 3.

Present proposal fulfils with above continuities and enables a better continuity between section classes.

Member subjected to the combination of axial force and strong axis bending. In that case, $M_{z.Sd}$ vanishes and, provided lateral torsional buckling is prevented, above expressions(26) reduce to:

$$\frac{N_{Sd}}{\chi_y N_{pl.Rd}} + \frac{C_{m.y} M_{y.Sd}}{\left(1 - \chi_y \frac{N_{Sd}}{N_{cr,y}}\right) k_{yy} M_{pl.y.Rd}} \leq 1$$

$$\frac{N_{Sd}}{\chi_z N_{pl.Rd}} + \beta^* \mu_z \left[\frac{C_{m.y} M_{y.Sd}}{\left(1 - \frac{N_{Sd}}{N_{cr,y}}\right) k_{zy} M_{pl.y.Rd}}\right] \leq 1$$

Two verifications are thus necessary. The second one represents the effect of the strong axis bending on the buckling about the weak axis. This effect is usually disregarded in the existing codes and standards.

Member subjected to the combination of axial force and weak axis bending. A similar conclusion as above can be drawn.

Member subjected to axial compression only. In that case, both bending moments $M_{y.Sd}$ and $M_{z.Sd}$ vanish. Each equation (26) reduces to the first term only. The largest one governs the design resistance. Therefore:

$$\frac{N_{Sd}}{minimum\left(\chi_y, \chi_z\right) N_{pl.Rd}} \leq 1$$

Cross-sectional resistance . The so-called stability criteria (26) must degenerate into cross-sectional resistance criteria when the member slenderness $\bar{\lambda}_y$ and $\bar{\lambda}_z$ approach zero. Then both reduction factors χ_y and χ_z are then equal to unity, so that $\mu_y = \mu_z = 1$, and $N_{cr.y}$ and $N_{cr.z}$ are infinitely large. Also the equivalent moment reduces to the bending moment in the section under consideration ($C_m = 1$).

In such conditions, it is clear that one get the elementary checks for axial compression or strong axis bending or weak axis bending.

Also the resistance check for a section subjected to bi-axial bending only is obtained. For a purely elastic check, on has $w_y = w_z = 1$ so that k_{yy}, k_{yz}, k_{zy} and k_{zz} are all equal to 1 and $\alpha^* = \beta^* = \alpha = \beta = 1.0$. That results in the single well-known elastic additive criterion:

$$\left[\frac{M_{y.Sd}}{M_{el.y.Rd}}\right] + \left[\frac{M_{z.Sd}}{M_{el.z.Rd}}\right] \leq 1$$

For a plastic cross-sectional check, the design equations reduce to:

$$\left[\frac{M_{y.Sd}}{M_{pl.y.Rd}}\right] + \left[\frac{\alpha^* M_{z.Sd}}{M_{pl.z.Rd}}\right] \leq 1 \quad \text{and} \quad \left[\frac{\beta^* M_{y.Sd}}{M_{pl.y.Rd}}\right] + \left[\frac{M_{z.Sd}}{M_{pl.z.Rd}}\right] \leq 1$$

These expressions are nothing else than the bi-linear criterion approaching the actual one (fig.4). Indeed, in the absence of axial force, the k_{ij} coefficients write:

$$k_{yy} = k_{zy} = 1 \geq \frac{1}{w_y} \quad \text{and} \quad k_{zz} = k_{yz} = 1 \geq \frac{1}{w_z}$$

They are thus to be taken equal to unity.
The plastic cross-sectional resistance of the section subjected to the combination of an axial force and a mono-axial bending, for instance a strong axis bending, reduces to the following doublet:

$$\frac{N_{Sd}}{N_{pl.Rd}} + \frac{M_{y.Sd}}{k_{yy} M_{pl.y.Rd}} \leq 1 \qquad \text{and} \qquad \frac{N_{Sd}}{N_{pl.Rd}} + \beta^* \left[\frac{M_{y.Sd}}{k_{zy} M_{pl.y.Rd}}\right] \leq 1$$

Indeed, the k_{yy} and k_{zy} factors reduce respectively to:

$$k_{yy} = 1 + 2\left(w_y - 1\right) \frac{N_{Sd}}{N_{pl.Rd}} \geq \frac{1}{w_y} \qquad \text{and} \qquad k_{zy} = 1 + 2\left(w_y - 1\right) \frac{N_{Sd}}{N_{pl.Rd}} \geq \frac{\beta^*}{w_y}$$

so that k_{yy} and k_{zy} are equal and must be taken as $[1 + 2\left(w_y - 1\right) N_{Sd}/N_{pl.Rd}]$. Then, the first equation is governing (because $\beta^*<1$) and is the one approaching the *M-N* interaction (fig.5).

Better continuity between Class 2 and Class 3

Above proposal was established based on the specifications of ENV1993-1-1. Accordingly they do not alleviate the physically non-acceptable sudden step in bending resistance when a section is respectively on both sides of the borderline between Class 2 and Class 3. A progressive transition, within the domain of Class 3 sections, between the cross-sectional elastic resistance and the plastic one could however be contemplated (fig.6). Accordingly a Class 3 section would be characterised by elasto-plastic moduli W_3, comprised between W_{el} and the plastic section modulus W_{pl}, which would be governed by the *b/t* ratios of the compression plate elements composing the section. A similar approach is adopted in the Australian standard [23], where the elasto-plastic modulus W_3 is expressed as follows:

$$W_3 = W_{el} + \left(W_{pl} - W_{el}\right) minimum \left[\frac{(b/t)_3 - (b/t)}{(b/t)_3 - (b/t)_2}\right]_i$$

where $(b/t)_{2.i}$ and $(b/t)_{3.i}$ are the relevant (b/t) limit ratios of a given compression plate element *i* of respectively to Class 2 and Class 3. With such a definition, the design cross-sectional bending resistance of Class 3 cross-sections would vary linearly between the elastic and the plastic design bending resistances (fig. 6).
Should this proposal for the definition of an elasto-plastic bending resistance be adopted within Class 3, then W_3 should be substituted to W_{pl} in all the above formulae involving W_{pl}, as far as Class 3 sections are of concern. A similar smooth transition should also be restored in the α^* and β^* so as to enable a progressive change in the interaction curve for bi-axial bending from the elastic linear one to the plastic one. That could be done for instance by introducing:

$$\alpha^* = 1-(1-0,6\sqrt{\frac{w_z}{w_y}})\ min\left[\frac{(b/t)_{3.i}-(b/t)_i}{(b/t)_{3.i} \quad (b/t)2._i}\right]$$

$$\beta^* = 1-(1-0,6\sqrt{\frac{w_y}{w_z}})\ min\left[\frac{(b/t)_{3i} \quad (b/t)_i}{(b/t)_{3.i} \quad (b/t)2._i}\right]$$

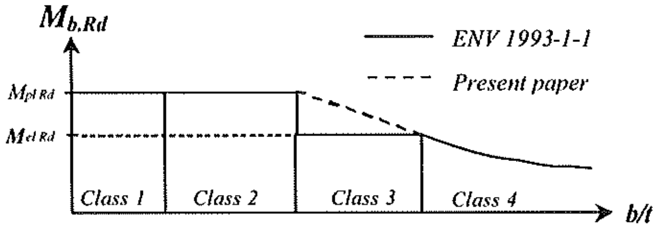

Figure 6 – Continuity within Class 3 sections

COMPARISONS OF INTERACTION FORMULAE WITH RESULTS OF NUMERICAL SIMULATIONS

The results provided by above "Level 2" formulae were compared with those of more than fourteen thousands results of numerical simulations. For that purpose, several structural shapes were considered (HEB 300, IPE 200, RHS 200, IPE 500) and the reduced slenderness was varied from *0,5* up to *3,0*. These numerical simulations have been performed basically in Graz (see for instance [15] and implemented by the junior author during a stay at the University of Liège [24]. The comparison is summarised in four different tables dealing respectively with: i) In-plane instability in the y plane, ii) In-plane instability in the z plane, iii) Spatial instability without lateral torsional buckling, and iv) Spatial instability including lateral torsional buckling. Each table provides the following information:

- The reduced slenderness $\bar{\lambda}$;
- The mean value m of the so-called R value (R_{mean}), defined as the ratio between experimental and analytical (design formulae) results ($R>1$ means that the design formulae are conservative);
- The standard deviation of the R values;
- The maximum individual value of R (R_{max});
- The minimum individual value of R (R_{min});
- The number of simulations Σ_{tests} used for the comparison;
- The number of R values lower than $1,0$ ($\Sigma_{tests<1}$);
- The number of R values lower than $0,97$ ($\Sigma_{tests<0,97}$).

These comparisons are aimed at examining the accuracy of the interaction formulae. Therefore it is essential to prevent as far as possible the discrepancies between the formulae and the results of numerical simulations from being due to other sources than the interaction formulae. These sources are especially: the reduction factors χ_y and χ_z for column buckling and the reduction factor χ_{LT} for lateral torsional buckling. Reference is therefore made not to the relevant specifications of codes in this respect but well to more realistic values as those obtained by numerical simulation of the individual phenomena values for the section and the member under consideration. In addition, the resistance of the member ends may prevail so that the appropriate formulae must be added to above interaction formulae. The format of the cross-sectional resistance formulae for Class 3 sections is not disputable. In contrast, the format of formulae aimed at controlling the resistance of Class 1 or Class 2 sections and provided by codes and standards is more questionable; indeed these formulae are just very simple and usually conservative approaches. For the comparison, reference is therefore made to more refined and theoretically based expressions; those developed by Lescouar'ch [25] are adopted for that purpose.
Above proposal is found safe and accurate.

CONCLUSION

The interaction formulae for beam-columns, which are developped above, constitute a substantial improvement compared to those existing in the standards till now. Because they are safe, accurate and consistent with all the individual stability and resistance checks, they enable a much better assessment of the carrying capacity of such members and are likely to design more economical structural elements. Of course, their format is not fully derived from theoretical considerations only. Indeed, it looks like the one derived in the purely elastic case but with additional factors, which aim at accounting for the inelastic behaviour at the ultimate limit state. Doing so permits the general format to be kept rather simple and provides a physical background to all of its terms and to all the parameters involved in these terms. The resulting transparency can help very much at the didactical point: the set of the individual phenomena is easily identified and their respective effects are clearly visible and assessable. At the practical point, the designer, who wants to handle the design checks in another way than a black box, can get a better understanding of the respective influences of the individual loads. Also, the opportunity is given of a better coverage of Class 3 sections by means of a section bending resistance varying progressively from the elastic resistance up to the plastic one according to the slenderness of the compression plate elements. The generality of the format and the complete continuity and consistency through the classes of sections are additional valuable capabilities of the proposal. Of course, the expressions of the factors involved in the general format (all being positive) can appear complex at first glance. That impression cannot hold if it is agreed that the use of programmable pocket calculators and personal computers prevails henceforth the pure by-hand calculation.

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y-y plane behaviour

	HEB 300			I PE 200			RHS 200			IPE 500			IPE 200 FINELG		
$\bar{\lambda}$	0,5	1	1,5	0,5	1	1,5	0,5	1	1,5	0,5	1	1,5	0,5	1	3
m	1,018	1,052	1,055	1,022	1,040	1,043	1,028	1,053	1,050	1,026	1,044	1,046	1,047	1,043	1,006
s	0,020	0,035	0,040	0,026	0,030	0,032	0,028	0,039	0,039	0,027	0,030	0,032	0,030	0,029	0,010
max	1,073	1,149	1,167	1,107	1,099	1,109	1,110	1,119	1,113	1,114	1,105	1,113	1,097	1,101	1,020
min	0,985	0,990	0,987	0,988	0,979	0,978	0,987	0,976	0,965	0,990	0,983	0,977	0,999	1,000	0,975
$\Sigma tests$	110	106	109	104	110	104	111	110	113	104	105	108	32	33	13
$\Sigma_{tests<1}$	19	6	5	16	8	7	16	10	13	9	6	5	3	0	1
$\Sigma_{tests<0,97}$	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0

z-z plane behaviour

	HEB 300			IPE 200			RHS 200			IPE 500			IPE 200 FINELG		
$\bar{\lambda}$	0,5	1	1,5	0,5	1	1,5	0,5	1	1,5	0,5	1	1,5	0,5	1	1,5
m	1,025	1,023	1,031	1,069	1,038	1,044	0,993	1,020	1,031	1,067	1,036	1,033	1,037	1,016	1,025
s	0,032	0,033	0,032	0,054	0,040	0,037	0,013	0,025	0,030	0,056	0,038	0,038	0,037	0,031	0,031
max	1,099	1,090	1,082	1,176	1,115	1,108	1,032	1,073	1,092	1,190	1,102	1,098	1,132	1,075	1,100
min	0,980	0,969	0,971	0,982	0,970	0,969	0,975	0,974	0,978	0,982	0,968	0,969	0,970	0,967	0,978
$\Sigma tests$	62	62	59	101	105	110	69	68	69	57	59	60	32	32	34
$\Sigma_{tests<1}$	15	18	12	5	22	13	46	12	10	5	12	14	1	9	7
$\Sigma_{tests<0,97}$	0	1	0	0	1	2	0	0	0	0	2	2	1	2	0

Spatial behaviour without lateral torsional buckling

	HEB 300				IPE 200				IPE 200 FINEL G			RHS 200				IPE 500			
$\bar{\lambda}$	0,5	1	1,5	3	0,5	1	1,5	3	0.5	1	1.5	0,5	1	1,5	3	0,5	1	1,5	3
m	1,070	1,070	1,063	1,023	1,093	1,084	1,074	1,032	1,064	1,056	1,047	1,066	1,084	1,093	1,051	1,0976	1,0901	1,0739	1,0326
s	0,057	0,047	0,039	0,033	0,069	0,054	0,045	0,034	0,044	0,040	0,034	0,059	0,053	0,058	0,045	0,0773	0,0572	0,0479	0,0365
max	1,215	1,182	1,145	1,102	1,237	1,196	1,167	1,118	1,161	1,139	1,124	1,204	1,225	1,248	1,183	1,2483	1,2033	1,1726	1,1339
min	0,983	0,965	0,976	0,962	0,972	0,971	0,976	0,961	1,000	0,966	0,974	0,974	0,985	0,988	0,975	0,9633	0,9718	0,9753	0,9576
$\Sigma\, tests$	558	582	592	236	531	551	580	229	76	77	77	498	542	567	232	405	562	581	230
$\Sigma_{tests < 1}$	60	26	25	53	38	20	9	34	0	4	1	50	15	6	15	35	16	18	35
$\Sigma_{tests < 0,97}$	0	1	0	10	0	0	0	8	0	1	0	0	0	0	0	2	0	0	8

Spatial behaviour with LTB

	HEB 300				IPE 200				IPE 200 FINELG			IPE 500			
$\bar{\lambda}$	0,5	1	1,5	3	0,5	1	1,5	3	0,5	1	1,5	0,5	1	1,5	3
m	1,054	1,086	1,095	1,090	1,100	1,094	1,088	1,050	1,077	1,090	1,103	1,097	1,061	1,040	1,020
s	0,059	0,051	0,053	0,056	0,069	0,048	0,044	0,037	0,046	0,051	0,059	0,071	0,052	0,044	0,033
max	1,193	1,197	1,243	1,212	1,262	1,196	1,192	1,154	1,167	1,189	1,238	1,285	1,206	1,154	1,125
min	0,932	0,967	0,976	0,962	0,977	0,971	0,976	0,961	0,995	0,988	0,974	0,978	0,931	0,948	0,962
$\Sigma\, tests$	361	355	351	123	464	475	472	122	100	89	122	359	355	359	110
$\Sigma_{tests < 1}$	59	13	7	8	33	14	9	5	1	4	2	23	37	67	30
$\Sigma_{tests < 0,97}$	14	1	0	2	0	0	0	1	0	0	0	0	8	24	6