

## SCANTLING OPTIMIZATION OF DOUBLE-HULL STRUCTURES TO MINIMIZE THEIR PRODUCTION COST

T. Richir<sup>1,2</sup>, D. G. Karr<sup>3</sup> and P. Rigo<sup>1,4</sup>

### ABSTRACT

Because double-hull structures are more and more used in response to the international conventions for safety and pollution prevention, the LBR-5 optimization software has been improved to properly analyze double-hull structures. The developments of LBR-5 are explained in this paper. Structural analysis and optimization results of a double-hull structure are also shown.

### KEY WORDS

Double-hull; LBR-5 software; scantling optimization; structural analysis; stiffened plate; preliminary design; production cost

### INTRODUCTION

The structural design of ships is always defined during the earliest phases of the project. That is, the preliminary design stage or the first draft that corresponds in most cases to the offer. At this time, few parameters (dimensions) have been definitively fixed and standard finite element modeling is often unusable, particularly for design offices and modest-sized shipyards. An optimization tool at this stage can thus provide precious help to designers. This is precisely the way the LBR-5 optimization software for stiffened structures was conceptualized (Rigo 2001a).

LBR-5 is an integrated package to perform cost and/or weight optimization of stiffened ship structures allowing:

- linear 3D analysis of prismatic structures (cargo holds);
- including all the relevant limit states of the structure in an analysis of the structure based on general solid-mechanics;
- optimization of the scantlings (profile sizes, dimensions and spacing); and
- including the unit construction costs and the production sequences in the optimization process (through a production-oriented cost objective function).

Only basic characteristics such as L, B, T, C<sub>B</sub>, the global structure layout, and applied loads are required. It is not necessary to provide a feasible initial scantling.

LBR-5 does not have the capability of a finite element analysis and is restricted to prismatic structures and linear 3D analysis. On the other hand, LBR-5 uses explicit exact first-order sensitivities (derivatives of the constraints and the objective function by the hundreds of design variables). Heavy and time consuming numerical procedures are then not required. Sensitivities are directly available as the method is based on an analytic solution of the differential equations of the stiffened plates using Fourier series expansion. So the required sensitivity formulations are known analytically. Due to the efficient CONLIN (convex linearization and dual approach) mathematical optimization algorithm (Feuery 1989, Rigo and Feuery 2001), optimization of the whole structure can be performed with hundreds of design variables and constraints using less than 10-15 global structure re-analysis (iterations).

For ships, the application domain of LBR-5 is clearly the ship's central part (the prismatic zone of cargo ships, passenger vessels, etc). The LBR-5 optimization is all the more efficient when this zone is long (tankers, LNG, FSO, etc). For this kind

<sup>1</sup> Naval Architecture Department (ANAST), University of Liege, Liege, Belgium

<sup>2</sup> Fund for Training in Research in Industry and Agriculture of Belgium (F.R.I.A.)

<sup>3</sup> Naval Architecture and Marine Engineering, University of Michigan, Ann Arbor, MI, USA

<sup>4</sup> National Fund of Scientific Research of Belgium (F.N.R.S.)

of ship, double-hull structures are usually used in response to the international conventions for the safety and the pollution prevention. However the double-hull transverse bulkheads were not properly taken into account in the structural analysis with the previous LBR-3 software.

LBR-5 has thus been improved to analyze double-hull structures. The stiffened plate remains the reference element in the new LBR-5 software. Indeed the differential equations of stiffened plates are well known and can be solved using Fourier series expansion so that the sensitivity formulations are still known analytically. The monolithic behavior of the double-hull is then ensured by additional loads applied on each stiffened plates of the double-hull element. The way to determine these additional loads is explained in the first part of this paper. The second part is devoted to applications to the structural analysis and the scantling optimization of a double-hull oil tanker.

## DEVELOPMENTS OF LBR-5 TO ANALYZE DOUBLE-HULL STRUCTURES

### LBR-5 Double-Hull Element

One LBR-5 double-hull element is constituted by two parallel stiffened plates which have the same width and are joined by transverse bulkheads (Figure 1).

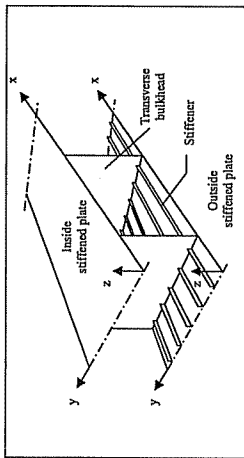


Figure 1: LBR-5 Double-Hull Element

### General Procedure

To take into account the double-hull elements in the structural analysis, the following procedure is adopted (Figure 2):

- each double-hull element is divided into two stiffened plates with a frame web height equal to the half of the transverse bulkheads height;
- additional loads are applied on the two stiffened plates of each double-hull element to ensure the monolithic behavior of the double-hull element.

The model is then constituted by only stiffened plates, but the internal loads have to be added to the external loads.

### Detailed Procedure

Differential Equations of Stiffened Plates without the Flange Shear Contributing. The proposed governing differential equations are known as the Donnell, Von Karman and Jenkins (D.K.V.) differential equations:

$$\begin{cases} (D + \Omega_x) y'' + D \left( \frac{1-v}{2} \right) y'''' + D \left( \frac{1+v}{2} \right) y'' - H_x y'' = -X \\ D \left( \frac{1+v}{2} \right) y'' + (D + \Omega_y) y'''' + D \left( \frac{1-v}{2} \right) y'' - H_y y'' = -Y \\ -H_x u'' - H_y v'' + (K + R_y) y'''' + (2K + T_y + T_x) y'' + (K + R_x) v'' = Z \end{cases} \quad [1]$$

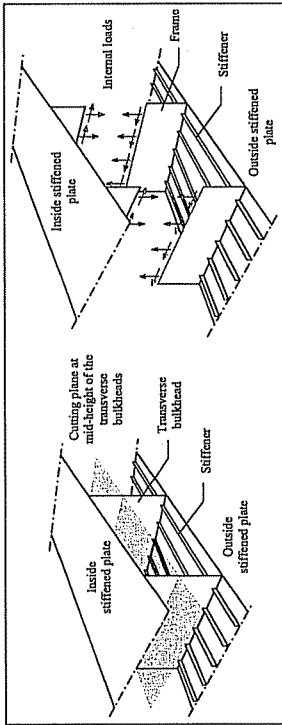


Figure 2: General Procedure

with  $D = \frac{E\delta}{1-D^2}$  and  $K = \frac{E\delta^3}{I^2(1-D^2)}$  [2]

$$\Omega_{x,y} = \frac{E\omega_{x,y}}{A_{x,y}}; H_{x,y} = \frac{EI_{x,y}}{A_{x,y}}; R_{x,y} = \frac{EI_{x,y}}{A_{x,y}}; T_{x,y} = \frac{GK_{x,y}}{A_{x,y}} \quad \text{(stiffeners and frames distribution)} \quad [3]$$

- $\delta$  plate thickness;
- $\omega_{x,y}$  transversal section of a stiffener (frame) without plate;
- $b_{x,y}$  first sectional moment of  $\omega_{x,y}$  to the neutral axis;
- $I_{x,y}$  second sectional moment (inertia) of  $\omega_{x,y}$  to the neutral axis;
- $K_{x,y}$  twisting rigidity of a stiffener (frame);
- $\Delta_{x,y}$  spacing between stiffeners (frames);
- E Young modulus;
- G shear modulus;
- v Poisson coefficient;
- $X, Y, Z$  external loads (N/m<sup>2</sup>).

Note: the derivation of  $u, v$  function according to the  $x$  and  $y$  variables are noted:  $\frac{\partial}{\partial x} = f'$  and  $\frac{\partial}{\partial y} = f''$ .

They are based on the Love-Kirchoff hypotheses which are the following (Figure 3):

- thin plate theory;
- small deformation and linear analysis;
- the points that are on a perpendicular line to the mid-plate surface ( $z = 0$ ) before deformation remain on the same perpendicular after deformation, so  $\gamma_{xz}$  and  $\gamma_{yz} = 0$ ;
- $\sigma_z$  and its effects are negligible; and
- no deformation along  $z$  ( $\epsilon_z = 0$ ).

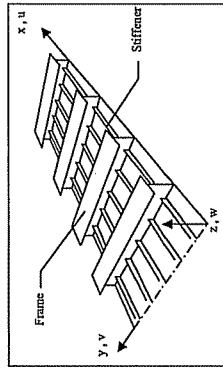


Figure 3: LBR-5 Stiffened Plate Element - Element Coordinate System and Associated Displacements

**Homogeneous Solution.** In the governing differential equations (equation 1) the  $u, v$  and  $w$  displacements are coupled. The three equations have thus to be solved simultaneously. The principle to solve any of these governing differential equations is the same. These equations can be written on the following way:

$$\begin{cases} a_1 u + b_1 v + c_1 w = X \\ a_2 u + b_2 v + c_2 w = Y \\ a_3 u + b_3 v + c_3 w = -Z \end{cases} \quad [4]$$

with  $u(x,y), v(x,y)$  and  $w(x,y)$  the displacements;  $x$  and  $y$  the coordinates of a point on the mid-plane of the plate. The  $z$  coordinate does not appear as the displacements  $u, v$  and  $w$  are only considered at the mid-plate thickness where  $z = 0$  (linear thin plate theory);  $X(x,y), Y(x,y)$  and  $Z(x,y)$  the surface loads; and  $a_1, b_1, \dots$ , and  $c_1$  the derivative operators.

To obtain the homogeneous solution of the governing differential equations (equation 1 and equation 4), the system defined by equation 5 has to be solved:

$$\begin{cases} a_1 u + b_1 v + c_1 w = 0 \\ a_2 u + b_2 v + c_2 w = 0 \\ a_3 u + b_3 v + c_3 w = 0 \end{cases} \quad [5]$$

This problem (equation 5) has a solution if the determinant of the matrix constituted by the derivative operators ( $a_1, b_1, \dots, c_1$ ) is zero (equation 6a and equation 6b).

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0 \quad [6a]$$

$$\text{or } a_1(b_2c_3 - b_3c_2) + a_2(b_3c_1 - b_1c_3) + a_3(b_1c_2 - b_2c_1) = 0 \quad [6b]$$

If we apply this operator (equation 6b) to the  $w(x,y)$  displacement, we obtain:

$$A w_{gg} + B v_{gg} + C u_{gg} + D w_{gg} + E v_{gg} + F u_{gg} + \dots + J v_{gg} + K u_{gg} = 0 \quad [7]$$

This is an 8<sup>th</sup> order differential equation with two coupled variables ( $x$  and  $y$ ). Note that  $w_{ij}$  means the  $i^{\text{th}}$  order derivative of  $w$  by  $x$  and the  $j^{\text{th}}$  order derivative by  $y$ . For instance  $w_{13}$  has the same meaning that  $w^{13}$ .

**Fourier Series Expansion.** To solve this 8<sup>th</sup> order differential equation we have to make an assumption on the shape of the  $u, v$  and  $w$  displacements. This assumption is required to obtain an 8<sup>th</sup> order differential equation with two separable variables:  $w(x,y) = w_1(y) \cdot w_2(x)$ . This assumption is satisfied if we use the "Fourier series expansion theory", i.e. if we assume that:

$$\begin{cases} u(x,y) = u(y) \cdot \cos \lambda x \\ v(x,y) = v(y) \cdot \sin \lambda x \\ w(x,y) = w(y) \cdot \sin \lambda x \end{cases} \quad [8]$$

with  $\lambda = n\pi/L$   
 $n$  = term number of the Fourier series expansion  
 $L$  = span of the structure along  $x, L$  is the same for each stiffened plate

The shape of the displacements used by the Fourier series expansion imposes some limitations on the boundary conditions. The two edges ( $x = 0$  and  $x = L$ ) must behave as simply supported edges, i.e.:  $w = v = M_x = N_x = 0$ .

If we plug the assumed displacement forms (equation 8) in the considered governing differential equations (equation 1), equation 7 becomes an 8<sup>th</sup> order polynomial differential equation with one single variable  $y$  instead of an 8<sup>th</sup> order differential equation with two variables,  $x$  and  $y$ .

From the solution of the 8<sup>th</sup> order polynomial differential equation with a single variable  $y$  (equation 7) and keeping in mind that  $w(x,y) = w(y) \sin \lambda x$  (Equation 8), we obtain:

$$w(x,y) = \sum_{i=1}^8 \frac{e^{u_i y}}{A_i} \cos \beta_i y + B_i \sin \beta_i y + \dots + \sin \lambda x \quad [9]$$

with  $i = 1$  to 2, 3 or 4.

If  $\beta_1$  and  $\beta_2 \neq 0$  then  $i = 1$  to 2 i.e.  $(\alpha_1, \pm\beta_1), (\alpha_2, \pm\beta_2)$  2 complex solutions  
 If  $\beta_1 \neq 0$  and  $\beta_2 = 0$  or  $\beta_1 = 0$  and  $\beta_2 \neq 0$  then  $i = 1$  to 3 i.e.  $(\alpha_1, \pm\beta_1), (\alpha_2, 0), (\alpha_3, 0)$  1 complex and 2 real solutions  
 If  $\beta_1 = \beta_2 = 0$  then  $i = 1$  to 4 i.e.  $(\alpha_1, 0), (\alpha_2, 0), (\alpha_3, 0), (\alpha_4, 0)$  4 real solutions

$A_i, B_i$  are the 8 integration constants included in equation 9. These constants are determined through the boundary conditions. For  $u(y)$  and  $v(y)$  equations similar to equation 9 can also be written. The  $u(y)$  and  $v(y)$  equations contain other integration constants that are directly dependent of the eight integration constants of  $w$  ( $A_i, B_i$ ). This means that once these eight constants are fixed for  $w$ , the equations for  $u$  and  $v$  are also completely defined. Equation 9 for  $w(y)$  and those for  $u(y)$  and  $v(y)$  are defined as to be the "homogeneous solution" of the differential equations. In fact, due to the superposition principle it is not necessary to find a particular solution. The "homogeneous solution" is our basic solution. From this, the solution of the actual panel can be found.

For the stiffened plates included in a double-hull element, frames with a web height equals to the half of the transverse bulkheads height (Figure 2) have to be considered in the  $A_i, B_i$  expressions of these panels.

**Definition of the Four "Basic Unitary Load Lines."** The principle is to find the eight integration constants for the four "basic unitary load lines" (Figure 4) applied on the complete panel (infinite width). This means that four sets of integration constants will be determined (one per unitary load line). Based on these four basic cases (unitary load lines and complete panel) and using the superposition principle it is possible to find the solution ( $u, v, w$ ) for the actual stiffened plates [actual width  $y_0$  and loads (pressure, deadweight, axial compression,...)] that compose the structure.

The four "basic unitary load lines" (Figure 4) applied on the complete panel (infinite width) are the following:

$$\begin{aligned} DISA &= 10000 \cos \lambda x & (N/m) \\ DISB &= 10000 \sin \lambda x & (N/m) \\ DISC &= 10000 \sin \lambda z & (N/m) \\ DISD &= 10000 \sin \lambda z & (N/m/m) \end{aligned} \quad [10]$$

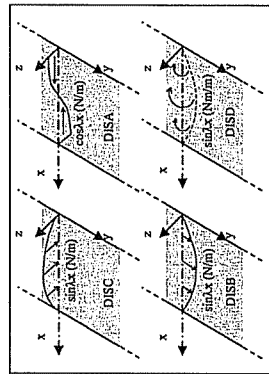


Figure 4: The Four "Basic Unitary Load Lines"

Their forms are compatible with the Fourier series expansions of the actual loads. These unitary load lines are applied at  $y = 0$ . For each of these "unitary load lines," a set of eight integration constants are obtained through the boundary conditions at  $y = 0$  and  $y = \infty$ . To satisfy the boundary conditions, we can define four equations (equilibrium and/or compatibility). In addition,

the symmetry or the anti-symmetry of the resultants and displacements induced by the load line provides another four equations.

**External Loads.** It is also possible to consider the effects of the lateral pressure (varying along  $y$ ), the deadweight and the longitudinal axial compression (induced by the primary bending moment) by integrating the solution(s) obtained for the basic unitary load lines. The unitary load lines are assumed to be applied on a small surface ( $L \cdot \lambda y$ ) at  $z = 0$  ( $L$  is the panel length along  $x$ ). Integrating the solutions obtained for the basic load lines according to the actual load distribution, we get the solutions ( $u, v$  and  $w$ ) for a complete panel (infinite width) submitted to the real load conditions.

**Definition of the Nine "Basic Unitary Surface Loads."** Because the double-hull elements are divided into two stiffened plates, additional loads must be applied on each of these stiffened plates to ensure the monolithic behavior of the double-hull elements (Figure 2 and Figure 5).

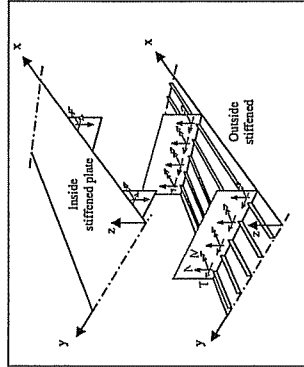


Figure 5: Additional Loads Applied on the Stiffened Plates from a Double-Hull Element

The distribution of the additional loads  $N, T$  and  $M$  (Figure 5) is supposed parabolic (second-degree) along  $y$ . Their expressions are then the following (equation 11):

$$\begin{aligned} N &= \sum_{i=1}^3 n_i N_i = \sum_{i=1}^3 n_i \left[ \frac{10000}{1000} \cdot y^2 + n_2 \frac{10000}{1000} \cdot y + n_3 \frac{10000}{1000} \cdot y^2 \right] \\ T &= \sum_{i=1}^3 t_i T_i = \sum_{i=1}^3 t_i \left[ \frac{10000}{1000} \cdot y^2 + t_2 \frac{10000}{1000} \cdot y + t_3 \frac{10000}{1000} \cdot y^2 \right] \\ M &= \sum_{i=1}^3 m_i M_i = \sum_{i=1}^3 m_i \left[ \frac{10000}{1000} \cdot y^2 + m_2 \frac{10000}{1000} \cdot y + m_3 \frac{10000}{1000} \cdot y^2 \right] \end{aligned} \quad [11]$$

Nine new "basic unitary surface loads" (Figure 6) are applied between  $y = 0$  and  $y = y_0$  on each stiffened plate from a double-hull element. The solutions ( $u, v$  and  $w$ ) for a complete panel (infinite width) submitted to the "basic unitary surface loads" are found by integrating the solutions obtained for the "basic load lines" (equation 10 and Figure 4) according to the "basic surface loads" distributions. It will remain then to find the coefficients  $n, t$  and  $m$  from equation 11.

The additional loads  $N, T$  and  $M$  are uniformly distributed along  $x$  because the frame properties are also uniformly distributed along  $x$  (equation 3) in the Equation 1.

**Actual Panels.** In order to get the solution of the real panel (for the actual panel width  $y_0$ ) we have to consider the actual boundary conditions imposed along the two longitudinal edges ( $y = 0$  and  $y = y_0$ ). Even if we look for the solution for the actual panel, the analyzed panel will remain a panel with an infinite width that has to satisfy the boundary conditions of the actual panel along the  $y = 0$  and  $y = y_0$  edges.

To satisfy these two-edge boundary conditions the idea is to apply along each edge ( $y = 0$  and  $y = y_0$ ) a set of four "basic load lines" (Equation 10 and Figure 4). The problem is to find the amplitude of these load lines. For each panel, the unknowns are the "edge amplification factors" of these load lines (four acting along the  $y = 0$  edge and four along the  $y = y_0$  edge). Then, there are eight unknowns per panel.

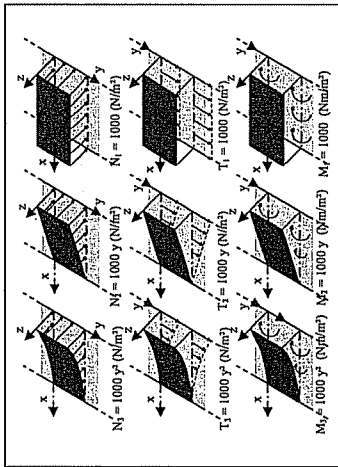


Figure 6: The Nine "Basic Unitary Surface Loads" Applied Between  $y = 0$  and  $y = y_0$

In order to determine these "edge amplification factors", conditions related to the boundary conditions must be imposed:

- for a free edge, the four conditions are:  $M_y = N_y = N_{xy} = R_y = 0$ ;
- for a clamped edge, the four conditions are:  $w = v = u = dw/dy = 0$ ;
- for a simply supported edge, the four conditions are:  $w = u = M_y = N_y = 0$ ;
- for an edge (node) corresponding to the junction between two panels, we impose four compatibility conditions between the displacements of the two panels and four equilibrium equations; and
- for an edge (node) corresponding to the junction between three panels, we impose eight compatibility conditions between the displacements of the three panels (four between panels #1 and #2 and four between panel #2 and #3) and four equilibrium equations.

For a structure composed by N panels, there are 8N unknowns corresponding to the "eight edge multiplication factors" per panel. They are determined by solving a linear problem of 8N equations and 8N unknowns.

In order to establish the equations (compatibility or equilibrium) at the panel edges, it is necessary to know the displacements ( $u, v, w$ ) and the resultants ( $M_y, N_y, N_{xy}, R_y$ ) acting along the edge ( $y = 0$ ) and the edge ( $y = y_0$ ). This work must be done for the "18 standard loading cases" that are:

- The actual external loads:
  - pressures (quasi-static); Z type;
  - gravity loads (deadweight, cargo ... ) that have component along y and along z; Y and Z types;
  - axial compression (induced by the primary bending moment); X type;
- the four basic unitary load lines (equation 10 and Figure 4) acting at  $y = 0$ ;
- the four basic unitary load lines (equation 10 and Figure 4) acting at  $y = y_0$ ; and
- the nine basic unitary surface loads (equation 11 and Figure 6) acting between  $y = 0$  and  $y = y_0$ .

All these displacements and forces are calculated from the solutions of the homogeneous differential equations for the four basic load lines applied on the complete panel (infinite width).

**Edge Amplification Factors.** For each panel we know the displacements ( $u, v, w$ ) and the resultants ( $M_y, N_y, N_{xy}, R_y$ ) along their two boundary edges ( $y = 0$  and  $y = y_0$ ) for the "18 standard loading cases". To satisfy at the actual boundary conditions of each panel, we determine the "amplification factors" of the four "unitary load lines" (equation 10 and Figure 4)

applied at  $y = 0$  and the four "unitary load lines" applied at  $y = y_0$  for the actual external loads and the  $2K \times 9$  "unitary surface loads" applied on each stiffened plate from a double-hull element (K number of double-hull elements). This is done through the compatibility and the equilibrium equations between panels. By solving the global system including all these equations (eight per panel) we get the "edge amplification factors".

**Additional Loads for the Stiffened Plates from a Double-Hull Element.** The solutions ( $u, v, w$ ) of a panel of the structure (with a finite width  $y_0$ ) for the actual external loads or each of the  $2K \times 9$  "basic unitary surface loads" are obtained by adding the following solutions of the same complete stiffened plate (with an infinite width):

- the complete panel under the external loads or one of the nine "basic unitary surface loads";
- at  $y = 0$ , the complete panel under the four "basic unitary load lines" multiplied by their respective "edge amplification factor" corresponding to the external loads or one of the  $2K \times 9$  "basic unitary surface loads";
- at  $y = y_0$ , the complete panel under the four "basic unitary load lines" multiplied by their respective "edge amplification factor" corresponding to the external loads or one of the  $2K \times 9$  "basic unitary surface loads."

Then these solutions ( $u, v, w$ ) are used to determine the "amplification factors"  $m_i, k_i$  and  $n_i$  of the "basic unitary surface loads" in equation 11. On the basis of six compatibility conditions and twelve equilibrium equations, the "amplification factors"  $m_i, k_i$  and  $n_i$  are found for each stiffened plate from one double-hull element. For a structure composed by K double-hull elements, there are  $2 \times 9K$  unknowns corresponding to the nine "amplification factors"  $m_i, k_i$  and  $n_i$  of each stiffened plate from one double-hull element. They are determined by solving a linear problem of  $18K$  equations of  $18K$  unknowns.

For one double-hull element; ( $i = 1, K$ ), the conditions imposed at  $x = L/2$  are the following (Figure 7):

- Equality of the displacement along z (w):

$$\sum_{i=1}^{2K} [m_i \cdot w_{WH,i}(y) + \dots + n_i \cdot w_{WH,i}(y) + \dots + m_i \cdot w_{WH,i}(y) + \dots + w_{ext,i}(y)] + w_{ext,i}(y) \quad [12]$$

at  $y = 1/6 y_0, 3/6 y_0$  and  $5/6 y_0$

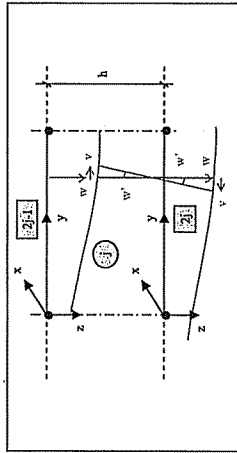


Figure 7: Compatibility of Displacements between the Two Stiffened Plates of a Double-Hull Element

with  $m_i, \dots, n_i$  amplification factors of the basic unitary surface loads applied on the stiffened plate  $i$  ( $i = 1, 2K$ ) from the double-hull element modulo  $(i+1)/2$ ;

$w_{WH,i}(y), \dots, w_{WH,i}(y)$  displacement along z at the abscissa y of the stiffened plate n from the double-hull element modulo  $(i+1)/2$  under the basic unitary surface loads applied on the stiffened plate  $i$  ( $i = 1, 2K$ ) from the double-hull element modulo  $(i+1)/2$ ;

$w_{ext,i}(y)$  displacement along z at the abscissa y of the stiffened plate n from the double-hull element modulo  $(i+1)/2$  under the actual external loads.

- Equality of the displacements along y ( $v$ ) at the neutral axis of the double-hull element:

$$\sum_{i=1}^{2K} [m_{1i} \cdot w_{1i,2}(y) + \dots + m_{3i} \cdot v_{1i,2}(y) + w_{ext,2}(y)] + v_{ext,2}(y) \quad [13]$$

$$\sum_{i=1}^{2K} [m_{1i} \cdot w_{1i,1}(y) + \dots + m_{3i} \cdot v_{1i,1}(y) + w_{ext,1}(y)] + v_{ext,1}(y) =$$

$$\sum_{i=1}^{2K} [m_{1i} \cdot w_{1i,2}(y) + \dots + m_{3i} \cdot v_{1i,2}(y) + v_{ext,2}(y)] +$$

$$\sum_{i=1}^{2K} [m_{1i} \cdot w_{1i,1}(y) + \dots + m_{3i} \cdot v_{1i,1}(y) + w_{ext,1}(y)] + v_{ext,1}(y) \cdot (-z_{m,i,2})$$

at  $y = 1/6 y_0, 3/6 y_0$  and  $5/6 y_0$

with  $z_{m,i}$  distance between the neutral axis of the double-hull element modulo  $(t+1)/2$  and the mid-plate of the panel  $i$  ( $i = 1, 2K$ );

- Translation equilibrium along z:  $m_{1,2,1} = -m_{1,2}$  for  $i = 1, 3$  [14]
- Translation equilibrium along y:  $t_{1,2,1} = -t_{1,2}$  for  $i = 1, 3$  [15]
- Global rotational equilibrium along x:  $m_{1,2,1} = m_{1,2}$  for  $i = 1, 3$  [16]
- Local rotational equilibrium along x:  $m_{1,2,1} = -t_{1,2} \cdot h/2$  for  $i = 1, 3$  [17]

This constitutes a linear problem of 18K equations of 18 K unknowns for K double-hull elements.

**Final Solution.** Then, the final solutions ( $u, v, w$ ) of a panel of the structure (with a finite width  $y_0$ ) are obtained by adding different solutions of the same complete stiffened plate (with an infinite width):

- the complete panel under the external loads;
- the complete panel under the nine basic unitary surface loads multiplied by their respective amplification factor  $n_i, t_i$  or  $m_i$ ;
- at  $y = 0$ , the complete panel under the four "basic unitary load lines" multiplied by their respective "edge amplification factor" corresponding to the external loads and to the  $2K \times 9$  basic unitary surface loads multiplied by their respective amplification factor  $n_i, t_i$  or  $m_i$ ; and
- at  $y = y_0$ , the complete panel under the four "basic unitary load lines" multiplied by their respective "amplification factor" corresponding to the external loads and to the  $2K \times 9$  basic unitary surface loads multiplied by their respective amplification factor  $n_i, t_i$  or  $m_i$ .

The detailed procedure is summarized in Figure 8.

### Shear Deflection

According to the third hypothesis ( $v_x$  and  $v_y = 0$ ) on which the differential equations of stiffened plates (equation 1) are based, the shear deflection is not considered in the previous developments. However, the ratio of height to span for the double-hull bulkheads is quite considerable so that the shear deflection is not negligible in the double-hull elements.

The shear deflection changes the stresses and the internal loads if the structure is hyperstatic. Then the amplification factors  $n_i, t_i$  and  $m_i$  of the basic unitary surface loads have to be modified if the shear deflection is considered. However, the equations 12 to 17 can not take into account the shear deflection because the displacement along z,  $w_{shear}(y)$ , is the same for the stiffened plates 2i-1 and 2j of the double-hull element j and the displacement along y,  $v_{shear}$ , is equal to zero (Figure 9). The solution would be to improve the differential equations of stiffened plates (equation 1) to consider the shear deflection. This task is difficult and could be another topic of research. Thus the shear deflection in double-hull elements is neglected in this paper.

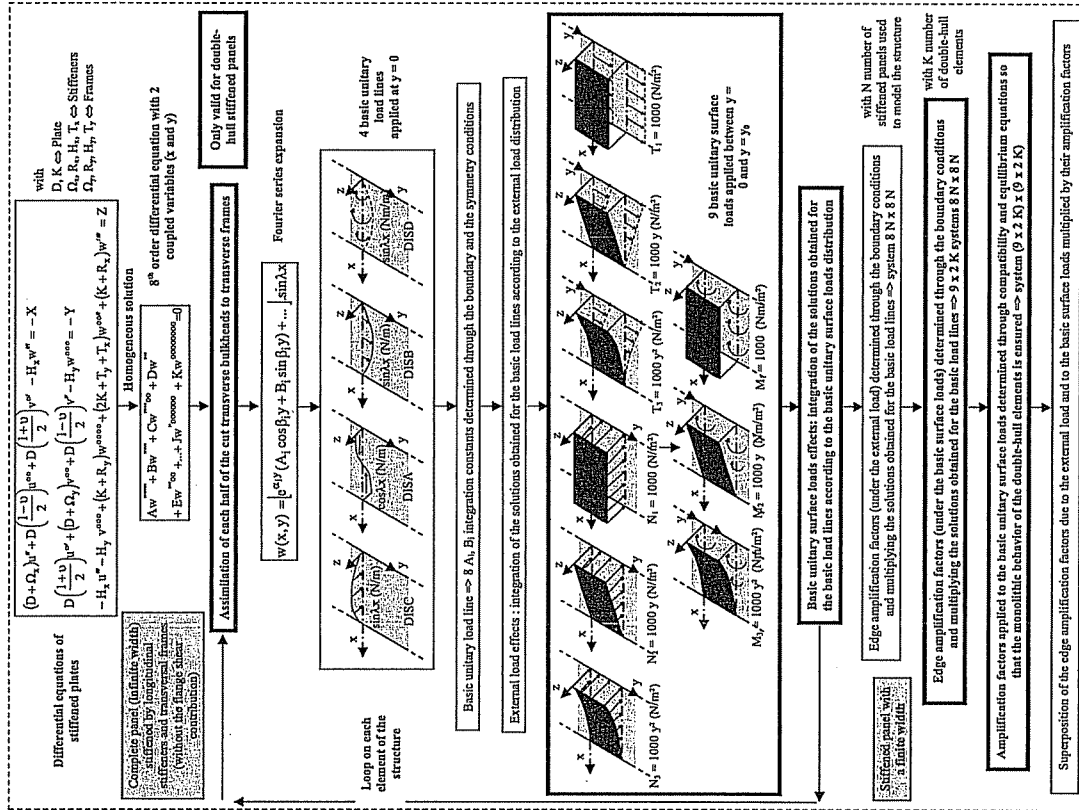


Figure 8: Detailed Procedure

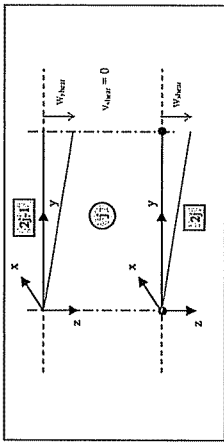


Figure 9: Shear Deflection of a Double-Hull Element  $j$

### Optimization

The integration of the double-hull element in the scantling optimization process consists in determining the analytic sensitivity formulations, i.e. the first derivative by the design variables, of:

- the effects of the nine basic unitary surface loads applied on the complete panel;
- the “edge amplification factors” under the  $2k \times 9$  basic unitary surface loads applied on each stiffened plate from a double-hull element; and
- the amplification factors  $b_3$ ,  $t_1$  and  $m_1$  of the basic unitary surface loads.

### APPLICATIONS

In this part, structural analysis and least cost optimization of the central hold of a 300,000 DWT very large crude carrier (VLCC) are studied. The midship section and general view of the existing ship are shown in Figure 10 and Figure 11, respectively. The six loading conditions required by DNV classification are given in Figure 12. The maximum total bending moments of sagging and hogging conditions are  $1.688 \times 10^6$  ton-m (Load case 7) and  $1.694 \times 10^6$  ton-m (Load case 8), respectively (Karr et al. 2002).

### Structural Analysis

To validate the previous developments implemented in LBR-5, the structural analysis of the VLCC central hold was performed using LBR-5 and ANSYS. At the midship section, Von Mises stresses in plates and displacements are compared for the first loading case.

**LBR-5 Mesh Model.** The LBR-5 mesh model (Figure 13) of the VLCC central hold is composed of 44 stiffened panels including eight double-hull elements. The model length is 51.2 m.

**ANSYS Mesh Model.** The central hold and the half of both adjacent holds are modeled so that the results in the midship section are not influenced by the inaccurate modeling of the boundary conditions. Using structure and load symmetry, only half of the structure is modeled. The ANSYS mesh model (Figure 14) includes 10,820 BEAM44 elements and 33,689 SHELL43 elements. Symmetry conditions are applied at both ends of the model and the vertical displacements of both transverse bulkheads are not allowed.

**Comparison between LBR-5 and ANSYS Results.** Von Mises stresses in plates at the midship section are given in Figure 15. Except for a few points, both the results are similar and the difference between the maxima is only 3.5 %.

The deformed shapes of the midship section are shown in Figure 16. Both results are similar and the difference between the maxima is 12.1 %. Nevertheless the maxima are not located at the same place and the structure seems stiffer using LBR-5.

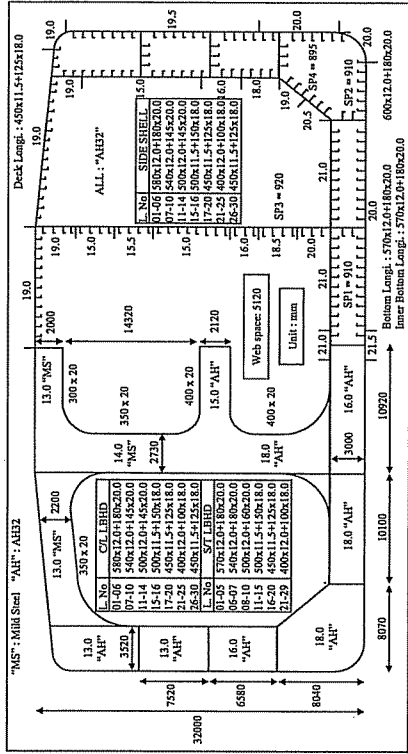


Figure 10: Midship Section of the VLCC

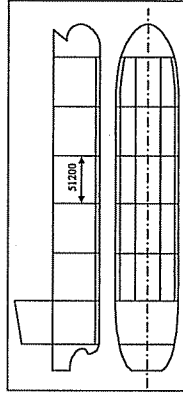


Figure 11: General View of the VLCC ( $L = 320$  m,  $B = 58$  m,  $D = 31$  m,  $T = 22$  m and  $C_B = 0.83$ )

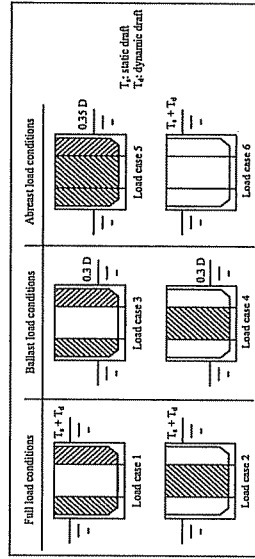


Figure 12: DNV Classification Loading Conditions

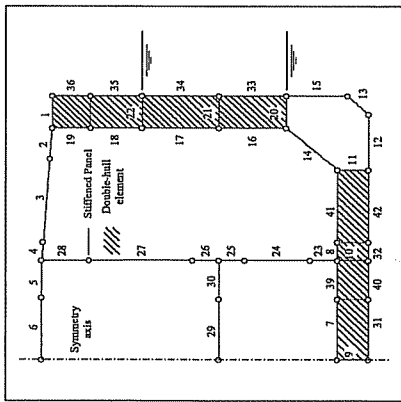


Figure 13: LBR-5 Mesh Model of the VLCC Central Hold

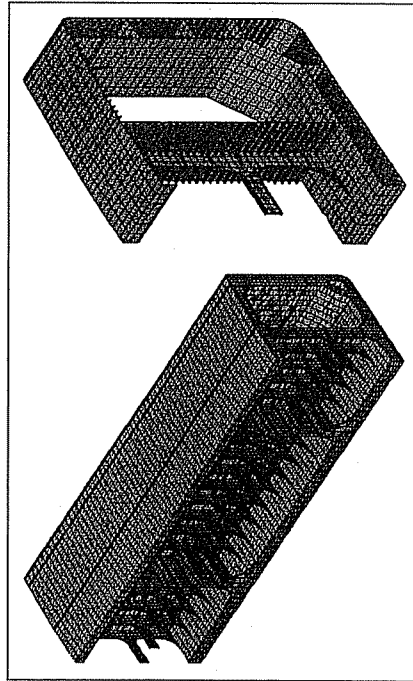


Figure 14: ANSYS Mesh Model of the VLCC Central Hold

Figures 15 and 16 validate the new developments implemented in LBR-5 to analyze double-hull structures for optimization in early design even if a few differences occur between LBR-5 and ANSYS results. Comparing LBR-5 (44 stiffened panels) and ANSYS (10,820 BEAM 44 and 33,689 SHELL43 elements) mesh models, one of the main advantages of LBR-5 is shown: the simplicity of the mesh model.

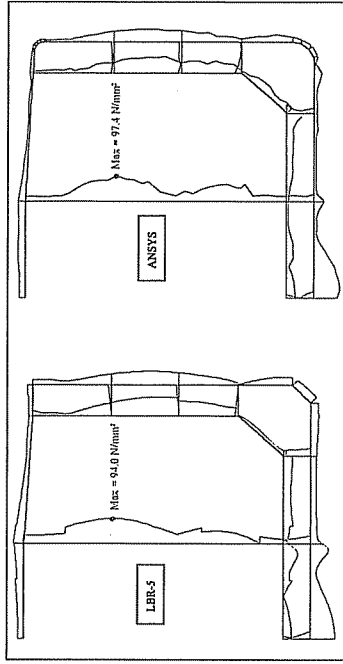


Figure 15: Von Mises Stresses in Plates at the Midship Section

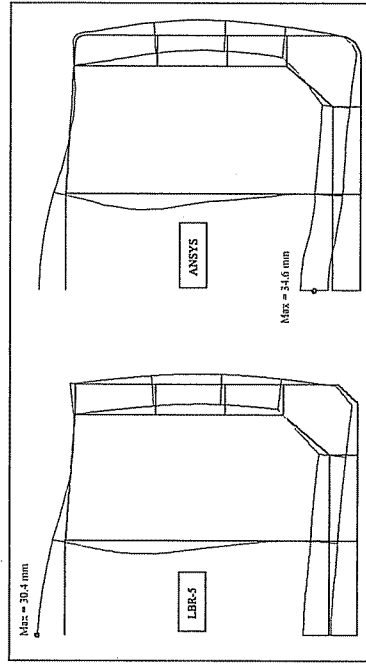


Figure 16: Deformed Shape of the Midship Section

### Least Cost Optimization

The least cost optimization of the VLCC central hold was carried out using LBR-5. The optimum scantlings are given in this section.

*Optimization Data.* The eight loading cases shown above (Figure 12) were considered.

The optimum cost was calculated using the following cost and productivity data (Rigo 2001b, Rigo 2003):

- Reference plate thickness: 20 mm
- Unitary labor cost (\$/m-h) / material cost (\$/t):  $k = 0.08$
- Unitary price of steel:  $C1 = 0.50$  \$/kg

- Unitary price of welding (materials only):
- Unitary working load (labor):
- Plate assembling:
- Welding stiffeners on the panel:
- Welding members on the panel:
- Built the members:
- Slot for stiffeners:
- Bracket or web stiffener:

C8 = 2.00 \$/kg

- P10 = 0.25 m-h/m<sup>2</sup>
- P4 = 0.5 m-h/m
- P5 = 1.5 m-h/m
- P9 = 0.0 m-h/m (not built on site)
- P6 = 0.6 m-h/piece
- P7 = 0.6 m-h/piece

These values are only given as an example. They vary from shipyard and from country to country.

The mesh model of the VLCC central hold includes:

- 44 stiffened panels (including 8 double-hull elements) with 9 design variables each (some are not considered as variables)
- 243 design variables (on average 5 to 7 design variables per panel)
- 92 equality constraints between design variables are used, e.g. to impose uniform frame spacing
- 174 geometrical constraints (about 4 to 6 per panel), dealing with:
  - o Stenciness of the web stiffeners
  - o ratio between web height and flange width (for stiffener only)
  - o ratio between plate and web thickness (for stiffeners and frames)
  - o ratio between flange and web thickness (for stiffeners and frames)
- 2040 structural constraints (255 per loading case), concerning mainly:
  - o Plate yielding (Von Mises) and plate buckling
  - o Stiffener yielding (web and flange)
  - o Frame yielding (web and flange)
  - o Stiffener ultimate strength
- 243 constraints are imposed on the design variables, e.g.:
  - o 8 mm ≤ Plate thickness ≤ 30 mm
  - o 3660 mm ≤ Frames spacing ≤ 10240 mm
  - o 700 mm ≤ Stiffeners spacing ≤ 1500 mm
  - o 8 mm ≤ Frames web thickness ≤ 30 mm
  - o 300 mm ≤ Stiffeners web height ≤ 900 mm
  - o 8 mm ≤ Stiffeners web thickness ≤ 30 mm
  - o 100 mm ≤ Stiffeners flange wide ≤ 600 mm

Note that with the new LBR-5 software it is not yet possible to impose constraints concerning the double-hull transverse bulkheads. Their thickness is then not modified during the optimization process.

**Optimum Scantlings.** Optimum frames spacing is 5534 mm, i.e. an 8.1 % of increase compared with the initial frame spacing. Other optimum scantlings are presented in Figures 17-22. The "raw" scantlings shown in these figures are not "ready to use". To establish execution plans and for practical and construction reasons, standardization is of course required.

Analysis of Figures 17-22 shows that the way to reduce the production cost of the example VLCC central hold is to increase the frame and stiffener spacings and to decrease the frame and stiffener transversal sections, which are accompanied by an increase in the plate thicknesses.

Figure 23 gives the convergence of the optimization process. The cost saving is about 6.3 %.

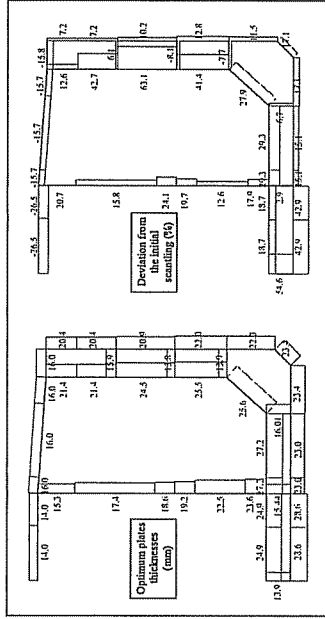


Figure 17: Optimum Plate Thicknesses

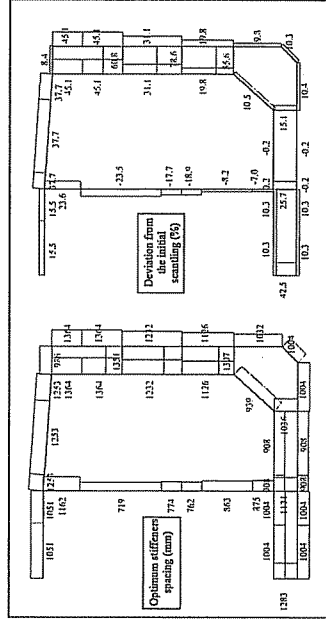


Figure 18: Optimum Stiffener Spacing

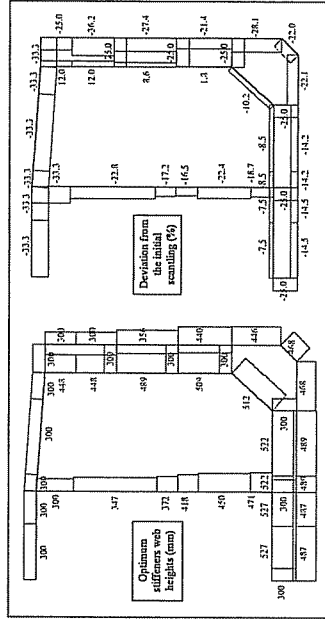


Figure 19: Optimum Stiffener Web Heights



