Cross-efficiency for advanced manufacturing technology selection: a multi-task approach*

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Abstract

Advanced manufacturing technologies (AMTs) are more and more used by firms to perform repetitive tasks in the production processes. As opting for an ATM represents an important investment for firms, several methodologies have been suggested to help firm decision-makers selecting the best one. A popular concept in that context is the cross-efficiency technique. In short, it endogenously selects the best ATM by computing scores using linear programmings. In this paper, we extend the cross-efficiency technique by adding a new feature: we model ATMs as multi-task processes. The multi-task approach presents two main advantages. One, it naturally gives the option to allocate inputs/costs and indicators/attributes to every task, yielding to a more realist modelling of the AMT processes. Two, AMTs can be compared for every task separately, increasing the discriminatory power of the selection process. As a consequence, the overall performances can be better understood, and, in particular, the reasons for declaring a specific AMT to be best can be investigated. We demonstrate the usefulness of our approach by considering a numerical example and two applications. In each case, we demonstrate the practical and managerial usefulnesses of our approach.

Keywords: advanced manufacturing technology (AMT); Data Envelopment Analysis (DEA); efficiency; cross-efficiency; robot selection.

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1 Introduction

An increasing number of production processes are using advanced manufacturing technologies (AMTs). The benefits of opting for AMTs are clear for firms: AMTs can be programmed to keep a constant speed and a predetermined quality when performing one or several tasks repetitively, even if the work conditions are not sustainable for humans. As such, AMTs increase the flexibility, improve product quality, reduce production and delivery delay, save labour costs, and so on; with the ultimate benefits of reducing costs and increasing profit. Examples of AMTs include group technologies, industrial robots, computer numerical control machines, flexible manufacturing systems, intelligent integrated manufacturing technologies, incremental manufacturing technologies, and cellular manufacturing systems.

The popularity of AMTs implies that several types/models are available. Therefore, the natural question for firms is how to decide which one to buy? In other words, how to select the best or the most efficient one? Answering these questions, while of crucial importance for firms since AMTs represent an important investment, is not an easy task in practice. Indeed, AMT selection is generally a multi-criteria decision-making process. That is, AMTs are measured by various quantitative and qualitative performance attributes/indicators representing the different tasks. As such, to rank the different AMTs that are available in order to select the best one, the first step is to aggregate the various attributes by defining weights.

In general, selecting the weights is difficult for firms, and when they can define values for the weights, it induces the presence of subjectivity into the selection process. As a consequence, the best AMT could be different for different firms. Clearly, this is not very attractive. An alternative is to rely on endogenous weights; that is, weights determined by a specific method. Amongst the methods used in this context, Data Envelopment Analysis (DEA) has gained in popularity. DEA, introduced by Charnes, Cooper, and Rhodes (1978), has become popular both as an analytical research instrument and as a practical decision-support tool. In practice, efficiency scores are provided by the DEA approach making the ranking between the ATMs easy. Nevertheless, as it is not guaranteed that only one ATM obtains the highest efficiency score, the concept of DEA cross-efficiency has been introduced by Doyle and Green (1994).

In this paper, we present a new DEA-based multi-task cross-efficiency approach.

The main idea is to keep the advantages of the DEA cross-efficiency while offering a more realistic modelling of the ATM production process. Indeed, to date, AMTs are modelled as overall processes that use inputs/costs to perform one or several tasks. Alternatively, we suggest modelling AMTs by considering each task individually. This offers two main advantages: it increases the realism of the modelling by giving the option to allocate the inputs/costs and the indicators/attributes to each task; and AMTs can be compared for every task. All in all, the multi-task approach does not alter the advantages of the cross-efficiency technique but only add a new useful feature.

The rest of the paper is structured as follows. In Section 2, we present a brief literature review of the models used to classify ATMs. In Section 3, we introduce the multi-task approach for cross-efficiency. In Section 4, we present an illustration and two applications. In Section 5, we present our conclusions.

2 Literature review

The multi-task cross-efficiency approach is directly related to the concept of cross-efficiency and to the multi-output approach for efficiency analysis. Before explaining our contribution, we propose a brief literature review of these two concepts. This allows us to better posit our approach in the relevant literature.

2.1 Cross-efficiency

We can regroup existing methods to classify ATMs into two categories: exogeneous methods that induces the presence of subjectivity into the selection process; and endogenous weights; that is, weights determined by a specific method. Examples of methods include scoring models, analytic hierarchy processes, techniques for order preference by similarity to ideal solution, outranking methods, goal programming models, stochastic methods, fuzzy MCDM methods, and life-cycle cost models. For applications, see, for example, Ghandforoush, Huang, and Taylor (1985), Imany and Schlesinger (1989), Sambasivarao and Deshmukh (1997), Parkan and Wu (1999, 2000), Braglia and Gabbrielli (2000), Karsak (2002), Karsak and Ahiska (2005, 2008), Amin and Emrouznejad (2007), Folgado, Pecas, and Henriques (2010), Kreng, Wu, and Wang (2011), Ghazinoory, Daneshmand-Mehr, and Azadega (2013), Oztaysi (2014), and Ren and Lutzen (2015).

One technique that has gained popularity as an analytical research instrument and as a practical decision-support tool is DEA, introduced by Charnes, Cooper, and Rhodes (1978). The goal of such analysis is to compare the performances of Decision Making Units (DMUs; in our context, the DMUs are the AMTs) by evaluating their efficiency. The increasing attention of DEA for AMT selection contexts could be explained by two main reasons. One, the weights are given by solving mathematical models. Two, DEA does not make any assumptions regarding the relationship between the inputs/costs and the attributes/indicators representing the tasks.

Nevertheless, DEA-based methods are not built to select only one AMT. Indeed, these methods are built to evaluate efficiency of DMUs, but nothing guarantees that only one DMU is set as efficient. In fact, in general, several DMUs are declared as efficient. In the context of selecting the best AMT, this feature of DEA is clearly not attractive. To overcome this drawback, Doyle and Green (1994) suggested using the concept of cross-efficiency, introduced by Sexton, Silkman, and Hogan (1986), in the DEA models. In words, cross-efficiency evaluation uses a peer-evaluated approach for efficiency evaluation of the DMUs. As such, DEA-based cross-efficiency methods guarantee that only one AMT is set as the best one, while DEA-based methods cannot guarantee this feature. This model has received a lot of attention as it is easy to use and to interpret in practice. For applications and extensions, see, for example, Doyle and Green (1995), Green, Doyle, and Cook (1996), Baker and Talluri (1997), Shang and Sueyoshi (1995), Talluri and Yoon (2000), Anderson, Hollingsworth, and Inman (2002), Sun (2002), Ertay and Ruman (2005), Liang et al. (2008a, b), Wu, Liang, and Chen (2009), Wu, Liang, and Yang (2009), Wang and Chin (2010), Jahanshahloo et al (2011), Wu et al (2011), Contreras (2012), Lim (2012), Macro et al (2012), Maddahi et al (2014), Cook and Zhu (2014), Du et al (2014), Lim, Oh, and Zhu (2014), Cui and Li (2015), and Wu et al (2016a, b).

2.2 Multi-output efficiency

The multi-output DEA approach for evaluating the efficiency of production processes considers that each output can be modelled individually. In other words, there exists output-specific production processes. This is different from more standard DEA models that consider an overall productions processes for all outputs and all inputs only. The multi-output modelling dates to Samuelson (1966), Lau (1972), Hall (1973), Kohli (1983, 1985), van den Heuvel (1986), and has been considered, more recently, by Fernandez, Koop, and Steel (2000, 2002, 2005), Ferreira and Steel (2007), Cherchye et al (2013), Cherry et al (2014), Cherchye, De Rock and Walheer (2015, 2016), Walheer (2016a, b, 2018b, c, f), and Walheer and Zhang (2020).

The multi-output approach offers two main advantages. First, it offers a more realistic modelling of the output productions by giving the option to allocate the inputs to the outputs. Different types of inputs have been considered such as joint inputs, private inputs, output-specific inputs, sub-joint inputs, and proportional inputs. See, for example, Salerian and Chan (2005), Despic, Despic, and Paradi (2007), Cherchye, De Rock, and Vermeulen (2008), Färe and Grosskopf (2000), Färe, Grosskopf and Whittaker (2007), Tone and Tsutsui (2009), Cherchye, De Rock, and Walheer (2015), and Walheer (2018e) for more discussion. These authors consider that available information about the production process can be used to allocate the inputs to the outputs.

A second advantage of the multi-output setting is that it naturally provides more detailed information about the (in)efficiency behaviour of the production processes. Contrary to more standard DEA models that only provide an overall efficiency score, multi-output DEA models also provide efficiency score for each output. These output-specific efficiency scores allow us to better understand the overall (in)efficiency behaviour.

2.3 Multi-task cross-efficiency

In this paper, we suggest to consider AMTs as multi-task processes. This offers several advantages. Firstly, it increases the realism of the modelling by giving the option to allocate the inputs/costs and the indicators/attributes for each task. Indeed, while some inputs/costs are used to perform all tasks, some other inputs/costs could be used for specific tasks. Also, it could be that some indicators measure some specific tasks. Next, AMTs can be compared for every task. Indeed, by modelling each task separately, cross-efficiency can be evaluated for each task. As a consequence, the overall performances can be better understood, and, in particular, the reasons of declaring a specific AMT to be best can be investigated. Therefore, more detailed information can be provided to firms when selecting their ideal AMT. This allows them, for example, to select the AMTs given their relative preferences for each task. In other words, the discriminatory power of the technique is improved.

One immediately appreciates the conceptual similarity between the multi-output modelling for efficiency analysis and the multi-task approach for cross-efficiency analysis. The main difference is that the multi-task modelling is more complex, as each task is composed of several indicators/attributes. Moreover, in production contexts, it is not generally required to declare only one DMU as the best one. All in all, these two similarities imply that the multi-task setting is more general than the multi-output setting and that our modelling is not limited to AMT selection problems (see, Section 4.3 for an application to selecting R&D projects).

3 Methodology

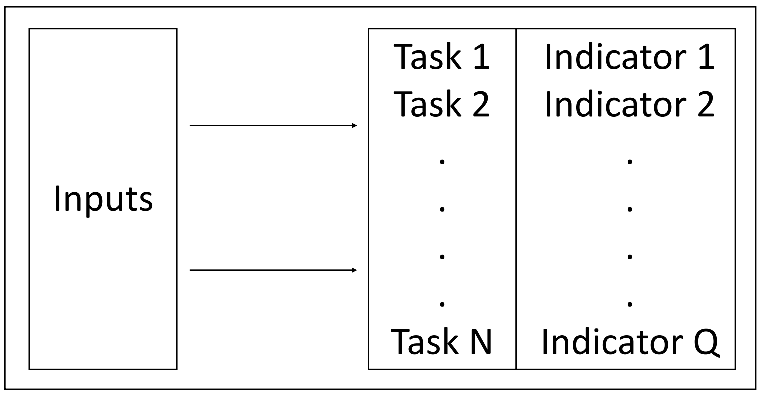
We consider that K AMTs are available, and that the aim is to find the best one/the most efficient one. We asume that AMTs are built to do N tasks. Also, we assume that P inputs/costs, captured by $\mathbf{x} = (x_1, \ldots, x_P)' \in \mathbb{R}^P_+$, are used by the AMTs to perform the tasks, and that the tasks are proxied by Q indicators/attributes, captured by $\mathbf{y} = (y_1, \ldots, y_Q)' \in \mathbb{R}^Q_+$.[1] In the following, we consider the aggressive cross-efficiency model introduced by Doyle and Green (1994). It is straightforward to extend to the beneficent counterpart (in fact, it suffices to change min by max in Step 3), or to alternative secondary goals (as those of, for example, Liang et al (2008), Wang and Chin (2010), Jahanshahloo et al (2011), and Contreras (2012)). We consider the aggressive cross-efficiency model given its popularity for practical applications.

3.1 Multi-task production process

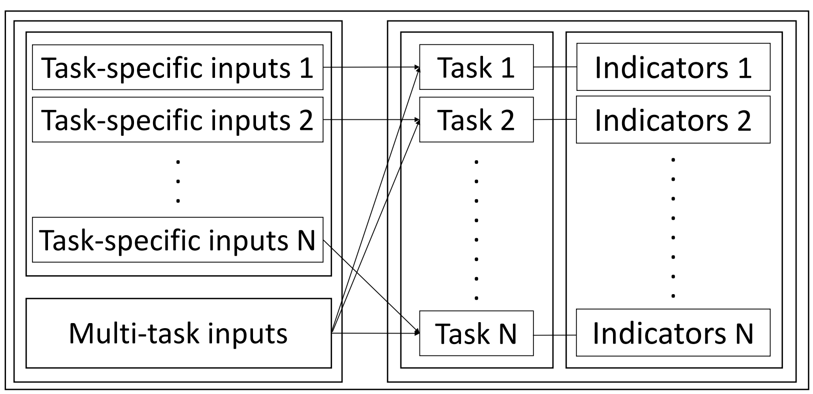
Before presenting the multi-task approach, we briefly explain how the cross-efficiency technique introduced by Doyle and Green (1994), and used in most of the following papers (see the citations in the Introduction), model the AMTs. This allows us to better position the new approach. In Doyle and Green's (1994) approach, the distinction between tasks do not exist. As such, they consider AMTs as overall processes that use

[1]Note that, in general, the number of indicators/attributes exceed the number of tasks, i.e. $Q \geq N$, but this is not required by the multi-task approach.

inputs/costs to perform the tasks, proxied, by the indicators/attributes. This implies that the links between inputs/costs and tasks, and between attributes/indicators and tasks are not taken into account. In other words, the inputs are used simultaneously to perform all the tasks, and the attributes/indicators are not related to the tasks. Doyle and Green's (1994) approach for the AMT process is displayed in Figure 1(a). As shown in this Figure, all the inputs are used simultaneously to perform the tasks, and tasks and indicators are considered together.



(a) Overall approach



(b) Multi-task approach

Figure 1: AMT processes

On the contrary, the multi-task approach does not consider the AMT as an overall process that uses inputs/costs to perform several tasks, but rather, models each task individually. As a consequence, the inputs/costs can be allocated to each task, and the indicators/attributes can be related to every task. The multi-task approach for AMT is displayed in Figure 1(b). Two different types of inputs are considered in the multi-task approach. On the one hand, multi-task inputs are those used to perform all tasks. On the other hand, task-specific inputs are those used for a specific task. Also, the indicators/attributes are related to the task they actually measure. At this point, it should be clear that indicators are vectors (from 1 to N) in Figure 1(b), while they are numbers (from 1 to Q) in Figure 1(a).

To facilitate the presentation of the multi-task approach, we introduce two extra notations: the inputs used to perform a specific task n, denoted by $\mathbf{x}^n = (x_1^n, \ldots, x_P^n)' \in \mathbb{R}_+^P$, and the indicators proxying task n, denoted by $\mathbf{y}^n = (y_1^n, \ldots, y_Q^n)' \in \mathbb{R}_+^Q$. Attractively, these two concepts could be directly connected to the initial definitions of inputs/costs ($\mathbf{x}$), and indicators/attributes ($\mathbf{y}$).

The inputs used to perform task n are defined, for every AMT k, as follows:

$$x_{pk}^n = \begin{cases} x_{pk}, & \text{for } p \text{ a multi-task input,} \\ a_{pk}^n x_{pk}, & \text{for } p \text{ a task-specific input.} \end{cases} \tag{1}$$

a_{pk}^n represents, for each AMT k, the share of task-specific input p used to perform task n. By construction a_{pk}^n are non-negative and sum to 1 over tasks: $\sum_{n=1}^N a_{pk}^n = 1$.

In a similar vein, the indicators that proxy task n can be obtained, for each AMT k, from the initial set of indicators as follows:

$$y_{qk}^n = \begin{cases} y_{qk}, & \text{if indicator } q \text{ measures task } n, \\ 0, & \text{if indicator } q \text{ does not measure task } n. \end{cases} \tag{2}$$

At this point, we emphasize that, in the following, we assume that the allocation of the inputs and of the indicators to tasks are observed (that is $\mathbf{x}_k^n$, and $\mathbf{y}_k^n$ are observed for every n and k). It could be that, in some settings, this information is not or only partially observed, at both the task or AMT levels. In that case, it would only complicate the computational aspect, and not impact the advantages of the proposed approach. In particular, the programs explained in Section 3.2 would be non-linear. In that case, transformations of the variables in the programs or assumptions about the allocation could be used to make the programs linear. One way to proceed is to try to recover the allocating factors. Inspiration could be found, for example, in Beasley (2003), Li, Yang, Liang, and Hua (2009), Yu, Chern, and Hsiao (2013) and

Du, Cook, Liang, and Zhu (2014). Those works have discussed how to recover the allocation of inputs to outputs in production contexts. Another way is to assume that all the inputs are multi-task inputs. For the attributes, we could, for example, rely on correlations to allocate those concepts to the tasks.

Also, it is worth noting that the allocation of inputs/costs and indicators/attributes to tasks is only an advantage of the multi-task approach. Indeed, even when no allocation is assumed, multi-task modelling gives the advantage of providing task-specific efficiency and cross-efficiency measurements. See Section 4.2 for an illustration.

As a final remark, it is important to note that the definitions of our concepts of inputs allocated to task and of indicators/attributes measuring specific tasks could be extended to match applications. For example, we could consider the case of inputs that are used to perform a sub-set of tasks. Next, we could consider the case of shares (conceptually similar to the a_{pk}^{n}'s defined for the inputs) for the indicators/attributes.

3.2 Linear programmings

Our objective is to obtain cross-efficiency scores to evaluate the ATMs while recognizing the multi-task nature of the AMTs. As explained before, we use a DEA-based approach as it does not require a subjective judgment but rather use endogenous weights. The algorithm consists of five main steps. First, efficiency scores for each task of every ATM are computed. This is done by, first, solving DEA-based linear programs (Step 1) and, second, computing task-specific efficiency scores (Step 2). Next, cross-efficiency scores for each task of every ATM are computed. Again, two steps are needed: first, DEA-based linear programs are used (Step 3), and, second, task-specific cross-efficiency scores are computed (Step 4). Note that the task-specific efficiency scores are needed to compute the task-specific cross-efficiency scores (see **(C-3)** of (5)). Note that the task-specific scores are independent: it is possible that an ATM has a high score for a specific task and a low score for another task. Finally, cross-efficiency scores for each ATM are obtained as a weighted sum of the task-specific counterparts (Step 5).

The linear programs for the multi-task approach are very similar to the ones proposed by Doyle and Green (1994) to compute cross-efficiency of AMTs.[2] The main

[2] Linear programmings are easy to solve and deal with. This represents an advantage of the suggested methodology. Note that in these programs the value of ϵ has to be selected by the practitioners. Usually a small enough number is picked (Podinovski and Bouzdine-Chameeva, 2017).

difference is that the constraints are defined for each task individually, while in Doyle and Green's (1994) approach, the constraints are defined for all tasks simultaneously. In fact, both approaches coincide when there is only one task ($N = 1$) for the AMT process. Indeed, in one-task contexts, all the inputs are used for that task, and all indicators also measure that specific task. When there is more than one task, the two approaches no longer coincide. One advantage of the multi-task approach is to provide task-specific performance indicators, while this is not possible when considering Doyle and Green's (1994) approach. Also, the multi-task approach gives the extra advantage of giving the option to allocate inputs, and relate indicators/attributes to every task. All in all, the multi-task approach, while remaining consistent with Doyle and Green's (1994) approach, gives the advantages of providing more detailed results and improving the realism of the modelling of the AMT process.

Step 1. Solve the following linear program for each AMT $k \in \{1, \ldots, K\}$:

$$\begin{aligned}
& \max_{u^n_{qk}\ (q\in\{1,\ldots,Q\},n\in\{1,\ldots,N\}),v^n_{pk}\ (p\in\{1,\ldots,P\},n\in\{1,\ldots,N\})} \sum_{n=1}^{N}\sum_{q=1}^{Q} u^n_{qk} y^n_{qk} \\
\text{s.t.} \quad & \forall n \in \{1,\ldots,N\},\ \text{the following holds:} \\
& \textbf{(C-1)} : \frac{\sum_{q=1}^{Q} u^n_{qk} y^n_{qs}}{\sum_{p=1}^{P} v^n_{pk} x^n_{ps}} \leq 1,\ \text{for } s = 1,\ldots,K, \\
& \textbf{(C-2)} : u^n_{qk} \geq \epsilon,\ \text{for } q = 1,\ldots,Q, \\
& \textbf{(C-3)} : v^n_{pk} \geq \epsilon,\ \text{for } p = 1,\ldots,P, \\
& \textbf{(C-4)} : \sum_{n=1}^{N}\sum_{p=1}^{P} v^n_{pk} x^n_{pk} = 1.
\end{aligned} \tag{3}$$

Step 2. Compute the task-specific efficiency scores for each AMT $k \in \{1, \ldots, K\}$:

$$\theta^n_k = \frac{\sum_{q=1}^{Q} u^{n*}_{qk} y^n_{qk}}{\sum_{p=1}^{P} v^{n*}_{pk} x^n_{pk}},\ \text{for } n = 1,\ldots,N, \tag{4}$$

Finally, it is worth mentioning that, in some cases, multiple optimal solutions could be found in models (3) and (5). In that case, this would lead to several cross-efficiency scores and thus a difficulty to find the best ATM. More discussions and potential solutions are provided, for example, in Appa (2002) and Oral et al. (2015). We also refer to Charnes, Cooper, and Rhodes (1978) for more discussion about the multiplicative DEA models and their features (e.g. feasibility, objective function, linearization).

where u_{qk}^{n*}, for $q = 1, \ldots, Q$, and v_{pk}^{n*}, for $p = 1, \ldots, P$, are the optimal values found after solving (3).

Step 3. Solve the following linear program for each AMT $k \in \{1, \ldots, K\}$:

$$\begin{aligned}
& \min_{\nu_{qk}^n\ (q\in\{1,\ldots,Q\}, n\in\{1,\ldots,N\}), \mu_{pk}^n\ (p\in\{1,\ldots,P\}, n\in\{1,\ldots,N\})} \sum_{n=1}^{N}\sum_{q=1}^{Q} \nu_{qk}^n \left(\sum_{s=1, s\neq k}^{K} y_{qs}^n\right) \\
\text{s.t.} \quad & \forall n \in \{1, \ldots, N\},\ \text{the following holds:} \\
& \textbf{(C-1)} : \frac{\sum_{q=1}^{Q} \nu_{qk}^n y_{qs}^n}{\sum_{p=1}^{P} \mu_{pk}^n x_{ps}^n} \leq 1,\ \text{for } s = 1, \ldots, K, \\
& \textbf{(C-2)} : \sum_{n=1}^{N}\sum_{p=1}^{P} \mu_{pk}^n \left(\sum_{s=1, s\neq k}^{K} x_{ps}^n\right) = 1, \\
& \textbf{(C-3)} : \frac{\sum_{q=1}^{Q} \nu_{qk}^n y_{qk}^n}{\theta_k^n \sum_{p=1}^{P} \mu_{pk}^n x_{pk}^n} = 1, \\
& \textbf{(C-4)} : \nu_{qk}^n \geq \epsilon,\ \text{for } q = 1, \ldots, Q, \\
& \textbf{(C-5)} : \mu_{pk}^n \geq \epsilon,\ \text{for } p = 1, \ldots, P.
\end{aligned} \tag{5}$$

Step 4. Compute the task-specific cross-efficiency scores for each AMT $k \in \{1, \ldots, K\}$ with respect to AMT $s \in \{1, \ldots, K\}$:

$$\gamma_{ks}^n = \frac{\sum_{q=1}^{Q} \nu_{qs}^{n*} y_{qk}^n}{\sum_{p=1}^{P} \mu_{ps}^{n*} x_{pk}^n},\ \text{for } n = 1, \ldots, N, \tag{6}$$

where ν_{qs}^{n*}, for $q = 1, \ldots, Q$, and μ_{ps}^{n*}, for $p = 1, \ldots, P$, are the optimal values found after solving (5) for every AMT $s \in \{1, \ldots, K\}$.

Step 5. Compute the cross-efficiency scores for each AMT $k \in \{1, \ldots, K\}$:

$$\gamma_k^n = \frac{1}{K}\sum_{s=1}^{K} \gamma_{ks}^n,\ \text{for } n = 1, \ldots, N. \tag{7}$$

3.3 Overall efficiency and cross-efficiency measurements

When selecting the best AMT, it is also important to provide efficiency and cross-efficiency measurements when considering all tasks together. Those measurements allow us to compare AMTs overall, while the task-specific efficiency and cross-efficiency allow us to rank AMTs for each task individually. Given the task-specific efficiency and cross-efficiency measurements defined previously in (4), (6) and (7), natural counterparts at the overall level are given for every AMT $k \in \{1, \ldots, K\}$ by:

$$\theta_k = \frac{\sum_{n=1}^{N} \sum_{q=1}^{Q} u_{qk}^{n*} y_{qk}^{n}}{\sum_{n=1}^{N} \sum_{p=1}^{P} v_{pk}^{n*} x_{pk}^{n}} = \sum_{n=1}^{N} \sum_{q=1}^{Q} u_{qk}^{n*} y_{qk}^{n}. \tag{8}$$

$$\gamma_{ks} = \frac{\sum_{n=1}^{N} \sum_{q=1}^{Q} \nu_{qs}^{n*} y_{qk}^{n}}{\sum_{n=1}^{N} \sum_{p=1}^{P} \mu_{ps}^{n*} x_{pk}^{n}}, \text{ for } s = 1, \ldots, K. \tag{9}$$

$$\gamma_k = \frac{1}{K} \sum_{s=1}^{K} \gamma_{ks}. \tag{10}$$

Attractively, those overall efficiency and cross-efficiency measurements are directly related to the task-level definitions. We obtain the following relationship for the efficiency measurements:

$$\begin{aligned}
\theta_k &= \frac{\sum_{n=1}^{N} \sum_{q=1}^{Q} u_{qk}^{n*} y_{qk}^{n}}{\sum_{n=1}^{N} \sum_{p=1}^{P} v_{pk}^{n*} x_{pk}^{n}}, \\
&= \frac{\sum_{n=1}^{N} \sum_{q=1}^{Q} u_{qk}^{n*} y_{qk}^{n}}{\sum_{n=1}^{N} \sum_{p=1}^{P} v_{pk}^{n*} x_{pk}^{n}} \times \frac{\sum_{p=1}^{P} v_{pk}^{n*} x_{pk}^{n}}{\sum_{p=1}^{P} v_{pk}^{n*} x_{pk}^{n}}, \\
&= \sum_{n=1}^{N} \frac{\sum_{p=1}^{P} v_{pk}^{n*} x_{pk}^{n}}{\sum_{n=1}^{N} \sum_{p=1}^{P} v_{pk}^{n*} x_{pk}^{n}} \times \frac{\sum_{q=1}^{Q} u_{qk}^{n*} y_{qk}^{n}}{\sum_{p=1}^{P} v_{pk}^{n*} x_{pk}^{n}}, \\
&= \sum_{n=1}^{N} \omega_k^n \times \theta_k^n.
\end{aligned} \tag{11}$$

In a similar vein, we obtain the following relationship for the cross-efficiency measurements:

$$\begin{aligned}
\gamma_{ks} &= \frac{\sum_{n=1}^{N}\sum_{q=1}^{Q}\nu_{qs}^{n*}y_{qk}^{n}}{\sum_{n=1}^{N}\sum_{p=1}^{P}\mu_{ps}^{n*}x_{pk}^{n}}, \\
&= \frac{\sum_{n=1}^{N}\sum_{q=1}^{Q}\nu_{qs}^{n*}y_{qk}^{n}}{\sum_{n=1}^{N}\sum_{p=1}^{P}\mu_{ps}^{n*}x_{pk}^{n}} \times \frac{\sum_{p=1}^{P}\mu_{ps}^{n*}x_{pk}^{n}}{\sum_{p=1}^{P}\mu_{ps}^{n*}x_{pk}^{n}}, \\
&= \sum_{n=1}^{N} \frac{\sum_{p=1}^{P}\mu_{ps}^{n*}x_{pk}^{n}}{\sum_{n=1}^{N}\sum_{p=1}^{P}\mu_{ps}^{n*}x_{pk}^{n}} \times \frac{\sum_{q=1}^{Q}\nu_{qs}^{n*}y_{qk}^{n}}{\sum_{p=1}^{P}\mu_{ps}^{n*}x_{pk}^{n}}, \\
&= \sum_{n=1}^{N}\omega_{ks}^{n} \times \gamma_{ks}^{n}. \qquad (12)
\end{aligned}$$

In words, the weights ω_k^n and ω_{ks}^n reveal how the efficiency (θ_k^n) and cross-efficiency (γ_{ks}^n) measurements of each task n contribute, respectively, to the overall efficiency (θ_k) and cross-efficiency measurements (γ_{ks}). As such, those weights allow us to investigate how each task contributes to the overall performances. Putting differently, (11) and (12) provide a decomposition of the overall measurements into task-specific counterparts.

These weights are attractive as they are natural, given by the model, and fit with economic intuition. Also, they are consistent with recent works that have considered the disaggregation of overall efficiency (or efficiency-related) measurements into output-specific measurements (see, for example, Cherchye et al (2013), Cherchye, De Rock, and Walheer (2016), and Walheer (2016a, b, 2017)), and also with works that have considered the aggregation of DMU-specific efficiency (or efficiency-related) measurements to obtain group counterparts (see, for example, Färe and Zelenyuk (2003), Zelenyuk (2006, 2016), Mayer and Zelenyuk (2014), Färe and Karagiannis (2017), and Walheer (2018a, 2019)).

We end this part with two remarks. Firstly, other weights could be used at this stage to obtain the overall efficiency and cross-efficiency measurements. For example, weights taking the preferences of the firms for every task into account. Next, weight restrictions could be used to improve the realism of the computed weights, and to avoid extreme weights (see, for example, Allen et al (1997), Pedraja-Chaparro, Salinas-Jimenez, and Smith (1997), and Kuosmanen, Cherchye, and Sipilainen (2006)). Refer to Section 4.1 for more discussion about extreme weights.

4 Illustrations

We illustrate how the multi-task approach works in practice by considering a numerical example, an application to a robot selection problem, and an application of selecting the best R&D project. This last application demonstrates that the multi-task approach is not limited to AMT selection, but could be used in various contexts.

Robot selection problem is a typical example of ATM section problem. Robots present several advantages over humans such as increasing the flexibility, improving product quality, reducing production and delivery delay, and saving labour costs. For the firm decision makers, opting for a robot represents an important investment while the profit increase is not guaranteed (Karande, Zavadskas and Chakraborty, 2016). Also, it is important to be sure to select the best robot when several are available. The main question is therefore: how to select the best robot ? Usually, robots are defined in terms of several quantitative and qualitative performance attributes/indicators representing the different tasks (Mondal and Chakraborty, 2013).

R&D project section problem is, in nature, complicated by many factors, interrelated selection criteria and multiple interrelated resources (Danila, 1989; Schmidt and Freeland, 1992). As robot selection problem, R&D project problem implies limited resources to a plethora of candidate projects (Henriksen and Traynor, 1999). Selecting the best R&D project is crucial for the firm decision makers as it determines the technology strategies and impact the long-term success of the firms. In some cases, selecting the best R&D project has a high impact on the probability to stay in business as innovation is directly related to competitiveness (Meade and Presley, 2002).

For each case, we compare our measurements θ_k and γ_k, respectively, with the efficiency measurement of Charnes, Cooper, and Rhodes (1978) and with the (aggressive) cross-efficiency measurement of Doyle and Green (1994). We refer to these models as *CCR* and *Cross-CCR* . Our task-specific measurements θ_k^n and γ_k^n, for every task n, cannot be compared, for the simple reason that those measurements are not provided in previous works. As such, we rather focus our discussion about what they add to the investigation of the best AMT.

4.1 Numerical example

We consider that a firm plans to buy a new AMT amongst six available on the market. Also, we assume that the AMTs are designed to perform two tasks ($N = 2$). Four attributes/indicators are measured for each AMT ($Q = 4$), denoted by y_1, y_2, y_3, and y_4, respectively. We assume that y_1 and y_2 measure task 1, while y_3 and y_4 measure task 2. Two inputs ($P = 2$), denoted by x_1 and x_2, are involved in the process. The fictional data are displayed in Table 6. We consider two different settings. In setting 1, the two inputs are both multi-task. In setting 2, we assume that the first input is a multi-task input, while the second input is a task-specific input. We assume that the allocation factors are observed for all ATMs. As such, setting 1 is probably less extreme for the comparison with existing methods than setting 2. Using the notation of Section 3.1, we obtain, for every AMT $k = 1, \ldots, 6$, when considering setting 1:

$$\mathbf{y}_k^1 = \begin{bmatrix} y_{1k} \\ y_{2k} \\ 0 \\ 0 \end{bmatrix}; \mathbf{y}_k^2 = \begin{bmatrix} 0 \\ 0 \\ y_{3k} \\ y_{4k} \end{bmatrix}; \text{ and } \mathbf{x}_k^1 = \begin{bmatrix} x_{1k} \\ x_{2k} \end{bmatrix}; \mathbf{x}_k^2 = \begin{bmatrix} x_{1k} \\ x_{2k} \end{bmatrix}. \tag{13}$$

Setting 2 is very similar to setting 1. In fact, only the input side differs:

$$\mathbf{x}_k^1 = \begin{bmatrix} x_{1k} \\ a_{2k}^1 x_{2k} \end{bmatrix} \text{ and } \mathbf{x}_k^2 = \begin{bmatrix} x_{1k} \\ a_{2k}^2 x_{2k} \end{bmatrix}. \tag{14}$$

By definition, we have that $a_{2k}^1 + a_{2k}^2 = 1$, for every AMT k (see our discussion of (1)). The allocation factors, different for each AMT, are given in Table 7. The efficiency measurements for all specifications are given in Table 1.

Table 1: Numerical example: efficiency

AMT	*CCR*	*Setting 1*			*Setting 2*		
		θ_k	θ_k^1	θ_k^2	θ_k	θ_k^1	θ_k^2
1	1	0.8883	0.8885	0.7386	0.9991	1	0.5909
2	0.9844	0.9827	0.9840	0.3343	0.9980	1	0.2555
3	1	0.9994	0.6176	1	0.9992	1	0.6466
4	0.8601	0.6908	0.6912	0.5062	0.6241	0.5474	0.6243
5	1	0.9995	1	0.7059	0.9992	1	0.6000
6	1	0.9990	0.4118	1	0.9987	0.3500	1

The *CCR* model declares four of the six AMTs as efficient. The multi-task approach reveals, when considering setting 1, AMT 5 as the best for task 1, and AMTs 3 and 6 as the best for task 2. These results demonstrate that there is no specific link between the performances of the ATMs on the different tasks. When considering setting 2, AMTs 1, 2, 3, and 5 are efficient for task 1, while only AMT 6 is efficient for task 2. The overall scores show that AMT 5 is the best for setting 1, and AMTs 3 and 5 are the best for setting 2. As a consequence, for setting 1, AMT 5 should be selected. For setting 2, it is unclear.

Two important remarks have to be made at this point. Firstly, when computing the efficiency scores, we give full freedom to the program. That is, we do not impose any restriction for the weights. As a result, the weights, displayed in Table 2, are very extreme, i.e. close to 1 for one of the two tasks. Extreme weight issue is not specific to our method, but holds true for most of the DEA-based techniques. As explained in Section 3.2, this could be avoid by imposing weight restrictions. Next, it could seem counterintuitive that no AMT is set as overall efficient. This is a direct consequence of the high discriminatory power of the technique. To be fully efficient, AMTs have to be efficient on all tasks.

Table 2: Numerical example: weights

AMT	*Setting 1*		*Setting 2*	
	ω_k^1	ω_k^2	ω_k^1	ω_k^2
1	0.9982	0.0018	0.9978	0.0022
2	0.9979	0.0021	0.9973	0.0027
3	0.0015	0.9985	0.9977	0.0023
4	0.9976	0.0024	0.0027	0.9973
5	0.9983	0.0017	0.9980	0.0020
6	0.0017	0.9983	0.0020	0.9980

The results for cross-efficiency are given in Table 3.

Cross-CCR reveals that AMT 5 is the best one. This is confirmed by the multi-task approach when considering setting 1. Note that it is not always the case that the multi-task approach and *Cross-CCR* give the same conclusion (see Section 4.3). Also, note that cross-efficiency is not necessary to come with that conclusion, as, it was clear from Table 1 (AMT 5 has the highest efficiency score). AMT 5 is the best for task 1, while, for task 2, it is AMT 3. Cross-efficiency is needed for the latter conclusion, as in Table 1, we found that both AMTs 3 and 6 are efficient for task 2.

Table 3: Numerical example: cross-efficiency

AMT	*Cross-CCR*	*Setting 1*			*Setting 2*		
		γ_k	γ_k^1	γ_k^2	γ_k	γ_k^1	γ_k^2
1	0.68	0.60	0.65	0.64	0.66	0.85	0.50
2	0.53	0.49	0.79	0.29	0.55	0.78	0.21
3	0.77	0.74	0.66	0.95	0.70	0.85	0.56
4	0.60	0.57	0.63	0.48	0.37	0.48	0.36
5	0.82	0.78	0.95	0.63	0.68	0.90	0.49
6	0.74	0.51	0.40	0.91	0.48	0.24	1

As a results, no AMT is the best for both tasks. Therefore, a natural question is why AMT 5 is declared as the best? In a sense, the method makes a balance between the cross-efficiency scores of both tasks to declare AMT 5 as the best. Clearly, a different choice could be made by the firm. For example, if task 2 is seen as more important, then AMT 3 should be chosen. As such, by providing task-specific efficiency and cross-efficiency scores, our method gives the option to better understand why an AMT is declared as the best, and, thus, give the possibility to firms to select the AMTs given their relative preferences for the tasks. For setting 2, it is revealed that AMT 5 is the best for task 1, and AMT 6 is the best for task 2 (this was known from Table 1). Overall, AMT 3 is the best in that case (this was unknown from Table 1). The same remarks made for setting 1 apply here. Of course, the allocation factors have an important impact on the decision.

4.2 Robot selection

The problem consists of selecting the best robot amongst 12 possible robots. Four indicators/attributes are observed for the robots (i.e. $Q = 4$): the handling coefficient (y_1); the ability of the robot to return to the same point (y_2), the maximum transportable weight (y_3), and the maximum attainable speed (y_4). Also, the total cost is observed (i.e. $P = 1$, denoted by x). This problem has been first considered by Braglia and Petroni (1999), and studied, in, for example, Karsak and Ahiska (2005, 2008). The data are given in Table 8.

In our multi-task approach, we make a clear distinction between tasks. We could consider that the robots are designed to perform four tasks ($N = 4$): handle capacity (task 1), repeatability (task 2), load capacity (task 3), and velocity (task 4). As such, each task is measured by one indicator. Also, the only input is a multi-task input as it is used to perform all tasks. At this point, it is worth noting that, while the distinction between the two approaches seem small in this context, the multi-task approach gives the advantage of providing task-specific results. Using the notation of Section 3.1, we have, for every robot $k = 1, \ldots, 12$:

$$\mathbf{y}_k^1 = \begin{bmatrix} y_{1k} \\ 0 \\ 0 \\ 0 \end{bmatrix}; \mathbf{y}_k^2 = \begin{bmatrix} 0 \\ y_{2k} \\ 0 \\ 0 \end{bmatrix}; \mathbf{y}_k^3 = \begin{bmatrix} 0 \\ 0 \\ y_{3k} \\ 0 \end{bmatrix}; \mathbf{y}_k^4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ y_{4k} \end{bmatrix}; \text{ and } x_k^1 = x_k^2 = x_k^3 = x_k^4 = x_k. \tag{15}$$

The results, for both the efficiency and cross-efficiency scores, are given in Table 4 (weights are provided in Table 9). *CCR* shows that AMTs 5, 8, and 12 are efficient, and *Cross-CCR* shows that the best one is 12. This is confirmed by both the multi-task efficiency and cross-efficiency scores. Also, the task-specific efficiency scores reveal that AMT 12 is the best for tasks 2 and 4, AMT 5 is the best for task 1, and AMT 8 is the best for task 3. In that context, the task-specific cross-efficiency scores are not needed as the best AMTs are found for each task only with the efficiency scores. This reveals, once more, the higher discriminatory power of the model. Finally, the theoretical relationships described in Section 3.3 hold true for the results in Table 4. For example, let us illustrate how the relationship in (11) works for ATM 2: $\theta_2 = 0.65 = \omega_2^1 \times \theta_2^1 + \omega_2^2 \times \theta_2^2 + \omega_2^3 \times \theta_2^3 + \omega_2^4 \times \theta_2^4 = 0.76 \times 0.71 + 0.07 \times 0.38 + 0.09 \times 0.23 + 0.08 \times 0.70$.

While simple this robot selection problem reveals the managerial usefulness of the multi-task approach. Indeed, by offering more detailed results our approach gives the option to take a better decision. For instance, it make sense to select ATM 12 as it is the best performer as declared by all approaches in Table 4. Nevertheless, the other approaches are silent about the reasons why this robot is the best one. As shown by the task-specific scores, ATM 12 is the best performer for tasks 2 and 4. If for any reasons tasks 1 and/or 3 that are the most important tasks for the firm decision-makers, it is a better idea to select ATM 5 or 8. Such options are not offered by the *CCR* and *Cross-CCR* techniques.

Table 4: Robot selection: efficiency and cross-efficiency

AMT	*CCR*	θ_k	θ_k^1	θ_k^2	θ_k^3	θ_k^4	*Cross-CCR*	γ_k
1	0.65	0.51	0.57	0.53	0.12	0.44	0.52	0.41
2	0.82	0.65	0.71	0.38	0.23	0.70	0.62	0.59
3	0.95	0.82	0.89	0.20	0.63	0.57	0.75	0.67
4	0.95	0.81	0.83	0.36	0.43	0.78	0.74	0.68
5	1	0.92	1	0.27	0.75	0.34	0.83	0.74
6	0.56	0.45	0.49	0.48	0.23	0.25	0.46	0.44
7	0.68	0.56	0.58	0.64	0.16	0.23	0.54	0.53
8	1	0.91	0.64	0.11	1	0.65	0.68	0.54
9	0.77	0.63	0.67	0.28	0.50	0.65	0.62	0.55
10	0.71	0.62	0.49	0.71	0.16	0.42	0.52	0.51
11	0.91	0.82	0.73	0.91	0.41	0.32	0.75	0.72
12	1	0.98	0.83	1	0.80	1	0.98	0.88

4.3 R&D projects

Our last illustration considers the case of selecting the best R&D projects amongst 32 projects. We select that example to show that the multi-task approach could also be used in other contexts that select the best AMT. Each project is measured by five indicators: indirect economic contribution (y_1), direct economic contribution (y_2), technical contribution (y_3), social contribution (y_4), and scientific contribution (y_5). The only input is total budget (x). This example has been introduced by Oral, Kettani, and Lang, (1991), and used, in, for example, Green, Doyle, and Cook (1996), Liang et al (2008b) and Wu et al (2016a). The data are shown in Table 10.

We can consider that the R&D project consist of 4 tasks ($N = 4$): creating economic (task 1), technical (task 2), social (task 3), and scientific (task 4) contributions. The first task consists of 2 indicators: indirect and direct economic contributions, the second task of 1 indicator: technical contribution, the third task of 1 indicator: social contribution, and the fourth task of 1 indicator: scientific contribution. The budget is used for all tasks. As there is no detailed information on how the budget is used for each task (i.e. to contribute to every domain), we assume that the budget is a multi-task input. If such information was available, it could be incorporated in the model as task-specific inputs.[3] Using the notation of Section 3.1, we have, for every

[3]Our model could be extended to recover the share of the budget used for each task. Refer to our discussion at the end of Section 3.1.

project $k = 1, \ldots, 37$:

$$\mathbf{y}_k^1 = \begin{bmatrix} y_{1k} \\ y_{2k} \\ 0 \\ 0 \\ 0 \end{bmatrix}; \mathbf{y}_k^2 = \begin{bmatrix} 0 \\ 0 \\ y_{3k} \\ 0 \\ 0 \end{bmatrix}; \mathbf{y}_k^3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ y_{4k} \\ 0 \end{bmatrix}; \mathbf{y}_k^4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ y_{5k} \end{bmatrix}; \text{ and } x_k^1 = x_k^2 = x_k^3 = x_k^4 = x_k. \tag{16}$$

The results, for both the efficiency and cross-efficiency scores, are given in Table 5 (weights are provided in Table 11). The best project, based on *Cross-CCR*, is number 38. Note that *CCR* points to two projects as efficient: 17 and 38. The conclusion of the multi-task approach differs from the one of *Cross-CCR*. Indeed, project 17 is the one that presents the highest efficiency (and cross-efficiency). This is explained as this project presents the highest efficiency scores for tasks 2, 3, and 4. Project 35 has the highest efficiency score for task 1 only. As such, it shows that, by allocating the indicators/attributes to task, the conclusion could differ from *Cross-CCR* . We believe that selecting project 17 is more accurate as it has the highest scores for 3 of the 4 tasks. Of course, as discussed previously for the robot selection problem in Section 4.2, the choice could be different if the most important task is the economic contribution (task 1). In that case, project 35 should be picked. The multi-task approach gives the option to better understand how the projects perform for every task, and to make the choice given this extra valuable information. Finally, let us illustrate how the relationship in (11) works for ATM 2: $\theta_2 = 0.48275 = \omega_2^1 \times \theta_2^1 + \omega_2^2 \times \theta_2^2 + \omega_2^3 \times \theta_2^3 + \omega_2^4 \times \theta_2^4 = 0.96 \times 0.48 + 0.01 \times 0.40 + 0.02 \times 0.26 + 0.01 \times 0.33$. This is just one example, clearly all the theoretical relationships described in Section 3.3 hold true for the results in Table 5.

All in all, we see two important managerial implications of our results. First, by correctly allocating the indicators/outputs and the costs/inputs to each task (here objective) of the project, a better decision can be made. Indeed, it is often the case that inputs/costs are allocated to the tasks and that indicators measure some specific tasks. In a sense, we correctly take all the aspects of R&D projects into account when using the multi-task approach. It is not the case for the *CCR* and *Cross-CCR* techniques that may give bias or wrong conclusions as they ignore the connections between the inputs, the outputs, and the tasks. Second, by providing

Table 5: R&D projects: efficiency and cross-efficiency

AMT	*CCR*	θ_k	θ_k^1	θ_k^2	θ_k^3	θ_k^4	*Cross-CCR*	γ_k
1	0.65	0.59119	0.59	0.47	0.29	0.36	0.60	0.38
2	0.55	0.48275	0.48	0.40	0.26	0.33	0.51	0.32
3	0.34	0.32699	0.33	0.08	0.12	0.08	0.17	0.15
4	0.53	0.51672	0.52	0.20	0.17	0.19	0.44	0.27
5	0.51	0.47863	0.48	0.29	0.23	0.23	0.45	0.30
6	0.61	0.57769	0.58	0.25	0.17	0.19	0.51	0.28
7	0.51	0.44873	0.45	0.26	0.20	0.14	0.42	0.27
8	0.42	0.39396	0.39	0.33	0.15	0.27	0.40	0.22
9	0.52	0.42928	0.43	0.25	0.27	0.20	0.43	0.32
10	0.54	0.49840	0.50	0.43	0.14	0.40	0.51	0.24
11	0.56	0.53122	0.53	0.45	0.16	0.40	0.53	0.27
12	0.55	0.49803	0.50	0.44	0.12	0.41	0.52	0.23
13	0.50	0.44930	0.45	0.37	0.20	0.29	0.45	0.26
14	0.65	0.59875	0.60	0.40	0.23	0.36	0.60	0.34
15	0.65	0.63325	0.63	0.19	0.24	0.20	0.52	0.34
16	0.85	0.82757	0.83	0.67	0.24	0.61	0.76	0.40
17	1	0.99998	0.74	1	1	1	0.97	0.93
18	0.76	0.68089	0.68	0.54	0.12	0.55	0.69	0.28
19	0.52	0.49722	0.50	0.34	0.12	0.39	0.47	0.23
20	0.35	0.32860	0.25	0.30	0.09	0.33	0.30	0.14
21	0.60	0.50493	0.50	0.47	0.22	0.48	0.55	0.30
22	0.51	0.47910	0.48	0.38	0.24	0.40	0.46	0.30
23	0.68	0.60065	0.60	0.54	0.28	0.44	0.64	0.38
24	0.50	0.41978	0.42	0.39	0.23	0.40	0.47	0.29
25	0.40	0.35271	0.31	0.30	0.10	0.35	0.35	0.17
26	0.66	0.58123	0.58	0.54	0.24	0.44	0.62	0.34
27	0.74	0.65069	0.65	0.58	0.25	0.56	0.69	0.37
28	0.35	0.28716	0.28	0.29	0.17	0.29	0.32	0.20
29	0.58	0.51454	0.51	0.45	0.17	0.42	0.55	0.27
30	0.55	0.53064	0.53	0.41	0.10	0.35	0.52	0.23
31	0.95	0.83988	0.84	0.75	0.09	0.54	0.84	0.31
32	0.64	0.63917	0.64	0.40	0.05	0.36	0.59	0.21
33	0.43	0.38008	0.38	0.34	0.09	0.29	0.39	0.17
34	0.80	0.79720	0.80	0.42	0.08	0.22	0.67	0.26
35	1	0.99991	1	0.71	0.16	0.62	0.98	0.41
36	0.77	0.73827	0.74	0.57	0.16	0.51	0.74	0.33
37	0.74	0.73173	0.73	0.41	0.13	0.31	0.66	0.30

detailed information about the performances of the R&D projects, the multi-task approach gives the option to better understand why a project is declared as the best one (here project 17), but also lets the decision-makers adjust their decision in light of their own preferences about the tasks. For example, it may make sense to select project 35 if the economic task is set as the most important one. Again, this option is not possible when using the *CCR* or *Cross-CCR* techniques.

5 Conclusion

Selecting the AMTs is of crucial importance for firms as buying an AMT represent an important investment. In practice, the concept of cross-efficiency has been wide used. In this paper, we suggest considering AMTs as multi-task processes. That is, we model AMTs by considering each task individually. This offers several advantages. On the one hand, it increases the realism of the modelling by giving the option of allocating the inputs/costs and the indicators/attributes to each task. On the other hand, AMTs can be compared for every task. Indeed, by modelling each task separately, cross-efficiency can be evaluated for each task. As a consequence, the overall performances can be better understood, and, in particular, the reasons for declaring a specific AMT as the best can be investigated. Therefore, more detailed information can be provided to firms when selecting the best AMT. Overall, our methodology allows firms to take better decision.

We have applied our methodology to two selection problems faced by many firm decision makers: robots and R&D projects. In both cases, taking the correct decision is crucial as it involves allowing a limited firm resources while several candidates are available. For the application to robots, the decision given by the multi-task approach does not differ from the one of the more standard models. The added value of the multi-task approach is, in that case, to give more details about the efficiency behaviour of each task (this is, in fact, not possible when relying on more standard models). For the application to the R&D projects, the multi-task approach allows us to consider a more realistic modelling and it provides a more detailed efficiency analysis. In that case, the decision differs from the one given by more standard models.

Our methodology can be extended in several directions. First, a natural extension is to allow for more complex modelling. In a sense, our setting can be seen as a first step in the direction of improving cross-efficiency. Indeed, our multi-task setting can, in principle, be applied to other cross-efficiency techniques (as those cited in Section 2). Next, giving the option to consider subjective or additional information about the weights in the model is also a natural extension. Finally, considering a dynamic setting, i.e. when firms have to take multiple decisions over time. As a final remark, we once more emphasize that our multi-task approach could also be used in alternative methods for selecting the best AMTs. There are thus several potential extensions.

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Appendix

Table 6: Illustrative example: data

AMT	*Indicator* 1	*Indicator* 2	*Indicator* 3	*Indicator* 4	*Input* 1	*Input* 2
1	4	8	6	7	5	8
2	9	7	8	1	7	7
3	4	5	1	2	6	5
4	7	8	4	8	8	9
5	8	3	5	4	5	7
6	3	4	9	8	5	7

Table 7: Allocation factors for input 2

AMT	*Task 1*	*Task 2*
1	0.25	0.75
2	0.23	0.72
3	0.20	0.80
4	0.56	0.44
5	0.43	0.57
6	0.86	0.14

Table 8: Robot selection: data

Robot	Cost (US \$)	Handling coefficient (index)	Load capacity (kg)	Repeatability (mm^{-1})	Velocity (m/s)
1	100000	1.00	85.00	1.70	3.00
2	75000	0.93	45.00	2.50	3.60
3	56250	0.88	18.00	5.00	2.20
4	28125	0.41	16.00	1.70	1.50
5	46875	0.82	20.00	5.00	1.10
6	78125	0.66	60.00	2.50	1.35
7	87500	0.88	90.00	2.00	1.40
8	56250	0.63	10.00	8.00	2.50
9	56250	0.65	25.00	4.00	2.50
10	87500	0.75	100.00	2.00	2.50
11	68750	0.88	100.00	4.00	1.50
12	43750	0.63	70.00	5.00	3.00

Table 9: Robot selection: weights

Robot	Handling coefficient	Load capacity	Repeatability	Velocity
1	0.70	0.10	0.10	0.10
2	0.78	0.08	0.08	0.08
3	0.83	0.06	0.06	0.06
4	0.92	0.03	0.03	0.03
5	0.86	0.05	0.05	0.05
6	0.77	0.08	0.08	0.08
7	0.09	0.74	0.09	0.09
8	0.06	0.06	0.83	0.06
9	0.83	0.06	0.06	0.06
10	0.09	0.74	0.09	0.09
11	0.07	0.79	0.07	0.07
12	0.04	0.04	0.04	0.87

Table 10: R&D projects: data

Project	Budget	Indirect economic contribution	Direct economic contribution	Technical contribution	Social contribution	Scientific contribution
1	84.2	67.53	70.82	62.64	44.91	46.28
2	90.00	58.94	62.86	57.47	42.84	45.64
3	50.2	22.27	9.68	6.73	10.99	5.92
4	67.5	47.32	47.05	21.75	20.82	19.64
5	75.4	48.96	48.48	34.9	32.73	26.21
6	90.00	58.88	77.16	35.42	29.11	26.08
7	87.4	50.1	58.2	36.12	32.46	18.9
8	88.8	47.46	49.54	46.89	24.54	36.35
9	95.9	55.26	61.09	38.93	47.71	29.47
10	77.5	52.4	55.09	53.45	19.52	46.57
11	76.5	55.13	55.54	55.13	23.36	46.31
12	47.5	32.09	34.04	33.57	10.6	29.36
13	58.5	27.49	39.00	34.51	21.25	25.74
14	95.00	77.17	83.35	60.01	41.37	51.91
15	83.8	72.00	68.32	25.84	36.64	25.84
16	35.4	39.74	34.54	38.01	15.79	33.06
17	32.1	38.5	28.65	51.18	59.59	48.82
18	46.7	41.23	47.18	40.01	10.18	38.86
19	78.6	53.02	51.34	42.48	17.42	46.3
20	54.1	19.91	18.98	25.49	8.66	27.04
21	74.4	50.96	53.56	55.47	30.23	54.72
22	82.1	53.36	46.47	49.72	36.53	50.44
23	75.6	61.6	66.59	64.54	39.1	51.12
24	92.3	52.56	55.11	57.58	39.69	56.49
25	68.5	31.22	29.84	33.08	13.27	36.75
26	69.3	54.64	58.05	60.03	31.16	46.71
27	57.1	50.4	53.58	53.06	26.68	48.85
28	80.00	30.76	32.45	36.63	25.45	34.79
29	72.00	48.97	54.97	51.52	23.02	45.75
30	82.9	59.68	63.78	54.8	15.94	44.04
31	44.6	48.28	55.58	53.3	7.61	36.74
32	54.5	39.78	51.69	35.1	5.3	29.57
33	52.7	24.93	29.72	28.72	8.38	23.45
34	28.00	22.32	33.12	18.94	4.03	9.58
35	36.00	48.83	53.41	40.82	10.45	33.72
36	64.1	61.45	70.22	58.26	19.53	49.33
37	66.4	57.78	72.1	43.83	16.14	31.32

Table 11: R&D projects: weights

Project	Economic contribution	Technical contribution	Social contribution	Scientific contribution
1	0.96	0.01	0.02	0.01
2	0.96	0.01	0.02	0.01
3	0.98	0.01	0.01	0.01
4	0.97	0.01	0.01	0.01
5	0.96	0.01	0.01	0.01
6	0.96	0.01	0.02	0.01
7	0.96	0.01	0.02	0.01
8	0.96	0.01	0.02	0.01
9	0.95	0.02	0.02	0.01
10	0.96	0.01	0.01	0.01
11	0.96	0.01	0.01	0.01
12	0.98	0.01	0.01	0.01
13	0.97	0.01	0.01	0.01
14	0.95	0.02	0.02	0.01
15	0.96	0.01	0.02	0.01
16	0.98	0.01	0.01	0.01
17	0.01	0.98	0.01	0.00
18	0.98	0.01	0.01	0.01
19	0.96	0.01	0.01	0.01
20	0.02	0.01	0.01	0.97
21	0.96	0.01	0.01	0.01
22	0.96	0.01	0.02	0.01
23	0.96	0.01	0.01	0.01
24	0.95	0.01	0.02	0.01
25	0.02	0.01	0.01	0.96
26	0.97	0.01	0.01	0.01
27	0.97	0.01	0.01	0.01
28	0.02	0.95	0.01	0.01
29	0.96	0.01	0.01	0.01
30	0.96	0.01	0.02	0.01
31	0.98	0.01	0.01	0.01
32	0.97	0.01	0.01	0.01
33	0.97	0.01	0.01	0.01
34	0.99	0.00	0.01	0.00
35	0.98	0.01	0.01	0.01
36	0.97	0.01	0.01	0.01
37	0.97	0.01	0.01	0.01