Don't take it from me: literature overview of arguments against the flatness problem

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Abstract

Several authors (including myself) have made claims, none of which has been convincingly rebutted, that the flatness problem, as formulated by Dicke and Peebles, is not really a problem but rather a misunderstanding. In particular, we all agree that no fine-tuning in the early Universe is needed in order to explain the fact that there is no strong departure from flatness, neither in the early Universe nor now. Nevertheless, the flatness problem is still widely perceived to be real, since it is still routinely mentioned in papers and books as an outstanding (in both senses) problem in cosmology. Most of the arguments against the idea of a flatness problem are based on the change with time of the density parameter Ω and normalized cosmological constant λ (often assumed to be zero before there was strong evidence that it has a non-negligible positive value) and, since the Hubble constant H is not considered, are independent of time scale. In addition, taking the time scale into account, it is sometimes claimed that fine-tuning is required in order to produce a Universe which neither collapsed after a short time nor expanded so quickly that no structure formation could take place. None of those claims is correct, whether or not the cosmological constant is assumed to be zero. Since I have been at most moderately successful in convincing the community of the lack of existence of the flatness problem, I highlight some similar claims from various authors better known than myself.

Introduction

Here, I consider only ideal Friedmann–Robertson–Walker (FRW) models, because historically fine-tuning claims have been discussed within the context of those models, and the issues remain even in more-realistic models. Note that the flatness problem is different from another problem of classical cosmology, the isotropy or horizon problem. The latter does not exist, by definition, in an ideal FRW universe, while the point of the former is that *even given the fact* that the Universe is described by an FRW model (*why* that is the case is, of course, a different question), there is something puzzling about the *values* of the cosmological parameters which are observed.

I use notation such that $\Omega = \frac{8\pi G\rho}{3H^2}$ refers to the density of matter ('dust') and $\lambda = \frac{\Lambda}{3H^2}$ is the normalized cosmological constant (with dimension time^{-2} so that Λ has the same dimension as $G\rho$); the subscript 0 refers to the current value of a time-dependent parameter. $K = \Omega + \lambda - 1$ and $k = \text{sign}(K)$. There are various notation schemes; mine closely follows that of Harrison (2000).

There are several, somewhat related, formulations of the flatness problem. The two most common are referred to by Holman (2018) as the *fine-tuning problem* ("there must be some reason why $\Omega = 1$ to very high precision in the early universe") and the *instability problem* ("even given that $\Omega = 1$ to very high precision in the early universe, if Ω is not exactly 1, then it would be unlikely to observe $\Omega \approx 1$"). Both are concerned with the change (or lack thereof) of Ω (and λ) with time. However, due to lack of the Hubble constant H, the time scale is irrelevant. Taking the time scale, and thus at least implicitly H, into account, another claim is that (near-)flatness is necessary to prevent the Universe from, after a short time, having collapsed or expanded so quickly that no structure could form; that claim is also incorrect. Note that Dicke (1970) (in total two paragraphs on the flatness problem) mentions the fine-tuning and time-scale problems while Dicke and Peebles (1979) mention all three aspects of the flatness problem (again in two paragraphs).

Arguments against the fine-tuning problem

The first argument in the literature against the flatness problem in FRW models appears to have been by Cho and Kantowski (1994), concentrating on the fine-tuning problem but also touching on the time-scale problem, well after the papers by Dicke and Peebles (1979) and Guth (1981). Putting "The Flatness Problem" in scare quotes makes their point already in the title. The last sentence of their abstract sums up their argument well: **"It is a distorted distribution of Ω values that sometimes misleads the casual observer to conclude that Ω must be exactly equal to 1."** Kantowski is of course a major figure in the fields of general relativity and cosmology and the work was published in *Physical Review D*, hardly an obscure journal. Quite frankly I wonder why that article didn't put an end to the idea of the flatness problem once and for all.

Coles and Ellis (1997) state clearly that **"*there is no flatness problem* in a purely classical cosmological model"** [emphasis in the original]. Following Jaynes (1968), they advocate choosing a prior based on the principle of maximum information entropy, which contradicts the assumption of a constant prior for Ω. Kirchner and Ellis (2003) also use Jaynes's principle to **"solve the flatness problem"** (direct quotation). Carroll (2014), describing his work with collaborators (Carroll and Tam, 2010; Remmen and Carroll, 2013, 2014), notes that **"flatness isn't a problem at all"**, **"[t]he flatness problem, meanwhile, turns out to be simply a misunderstanding"**, **"the flatness problem really isn't a problem at all; it was simply a mistake, brought about by considering an informal measure rather than one derived from the dynamics"**. A conclusion of Carroll and Tam (2010) is a good summary of this section: **"The flatness problem, as conventionally understood, does not exist; it is an artifact of informally assuming a flat measure on the space of initial cosmological parameters"** and **"is not intrinsic to the standard Big Bang model"**.

The fine-tuning argument is wrong basically because Ω is not the appropriate parameter to use (*e.g.* Cho and Kantowski, 1994; Coule, 1995; Evrard and Coles, 1995; Coles and Ellis, 1997; Kirchner and Ellis, 2003; Adler and Overduin, 2005; Gibbons and Turok, 2008; Roukema and Blanlœil, 2010; Helbig, 2012); that is most easily seem by studying the change in λ and Ω during the evolution of the universe as a dynamical system (*e.g.* Stabell and Refsdal, 1966; Ehlers and Rindler, 1989; Goliath and Ellis, 1999; Uzan and Lehoucq, 2001; Coley, 2003; Wainwright and Ellis, 2005), some such studies explicitly pointing out that this point of view demonstrates the lack of a flatness problem in classical cosmology (*e.g.* Kirchner and Ellis, 2003; Lake, 2005; Helbig, 2012).

Arguments against the instability problem

The first suggestion that the flatness problem could be avoided *via* a *relative* time-scale argument seems to be due to Tangherlini (1993), though not in the context of an FRW universe. Rindler (2001) points out that **"the so-called 'flatness problem'—the alleged improbability of finding the value of Ω_0 even within a factor of 10 of unity"** seems unproblematic for two reasons, first that **"at the big bang ($R = 0$), Ω always starts at one and then wanders away from that value unless $k = \Lambda = 0$"** (thus disputing the fine-tuning problem) and second that, in FRW models with $\lambda = 0$ and $\Omega > 1$, **"$\Omega < 10$ . . . is true for fully 60 per cent of the entire time interval"** (thus disputing the instability problem, which has to do with *relative* time scales). The second point is also obvious from figure 5 in Sandage (1968) (keeping in mind that $\Omega = 2q$ for $\Lambda = 0$). Rindler was of course also a major figure in the fields of cosmology and general relativity and surely many have read various editions of his textbooks. But his argument against the flatness problem seems to have, for the most part, fallen on deaf ears. (Although he explicitly discusses the $\lambda = 0$ case, his argument also applies for general FRW models which collapse in the future (Helbig, 2012).)

It appears that our Universe has a positive cosmological constant and will expand forever. For such models with $k = +1$, Lake (2005) demonstrates that the instability argument does not hold because λ and Ω are large and the universe significantly non-flat only in the case that they are fine-tuned in the sense that $\alpha = k(27\Omega^2\lambda)/(4K^3) \approx 1$. (Note that α is a constant of motion, *i.e.* its value is constant along an evolutionary trajectory in the λ–Ω plane; physically it is proportional to the square of the mass of the universe and the cosmological constant Λ. See my other poster for more on that topic.) Note that this is the opposite of the claim that fine-tuning is required in order to have a flat universe (though, as noted above, that claim is false). Lake suggests that α, which has a fixed value throughout the life of the universe, is what should be used to characterize model universes. In the words of Lake (2005) (his Ω is my $\Omega + \lambda$): **". . . it is shown that for the cosmological constant $\Lambda > 0$ there exist non-flat FLRW models for which the total density parameter Ω remains ~ 1 throughout the entire history of the universe. Further, it is shown that in a precise quantitative sense these models are not finely tuned. . . . The flatness problem involves the explanation of [$\Omega_0 \sim 1$] given [$\Omega = \Omega(t)$]. The problem can be viewed in two ways. First, one can take the view that there is a tuning problem in the sense that at early times Ω must be finely tuned to 1 [references to Dicke and Peebles (1979) and Peacock (1999), the latter as an example of a standard argument in then current cosmological texts]. However, this argument is not entirely convincing since *all* standard models necessarily start with Ω exactly 1."** [Emphasis in the original.]

Adler and Overduin (2005) discuss various definitions of 'nearly flat', using essentially using the same parameter as α used by Lake (2005), and arriving at the same conclusion, namely that a significantly non-flat universe implies fine-tuning in α. Their analogy, too long to quote in full here, is particularly convincing: **". . . consider a test particle of mass m with total energy E falling into the Netwonian gravitational field of a mass M. . . . Note that the difference [between the ratio of the kinetic to potential energy and 1] becomes arbitrarily small as one approaches $r \to 0$, in exactly the same way that $\Omega_T - 1$ [their Ω_T is my $\Omega + \lambda$] does in cosmology as $t \to 0$. Yet one would hardly be justified in concluding from this that E 'must be' zero on the grounds of naturalness."**

Arguments against the time-scale problem

There is less literature concerning this problem than concerning the fine-tuning and instability problems, although it is often mentioned in casual discussions. The usual formulation is that if one changed the density (or some other parameter) at early times, then the Universe would have expanded or contracted so quickly that it would be vastly different from that which we live in. The problem here is that it makes no sense to imagine changing just *one* parameter; for the Friedmann equation to remain an equation, at least two parameters have to be changed. However, in general such minimal changes describe universes very different from our own, such as a closed universe with a mass of one kilogram. Yes, such a universe might collapse after a very short time, but that is irrelevant since it is not our Universe nor even a slight perturbation of it in any meaningful sense. (See Helbig, 2020, for more details.)

A red herring

The flatness problem as discussed here was formulated when it was known that Ω_0 is within, say, an order of magnitude of 1, before it was known that $\Omega_0 + \lambda_0 \approx 1$ to within a per cent or better. Not all of the arguments here can explain $\Omega_0 + \lambda_0 \approx 1$ to within a per cent or so (*i.e.* the instability problem), though that of Lake (2005) perhaps can. (There is no such issue with the fine-tuning problem and the time-scale problem. Note that the absence of the fine-tuning problem does not necessarily imply the absence of the instability problem.) With regard to inflation, one should not take the observed flatness as an indication that inflation must have happened. On the other hand, either inflation happened or it didn't, independently of the question whether there is a flatness problem which must be solved. Note also that when the flatness problem was originally formulated, it was assumed (at least by those who formulated it) that $\lambda = 0$. Many of the arguments are the same whether or not $\lambda = 0$ (in particular the discussion of the fine-tuning problem is the same, since $\lambda \approx 0$ at early times). However, the argument of Lake (2005) against the instability problem depends on λ being large enough that the universe will expand forever, but that is not a mark against his argument since we live in such a universe.

Conclusions

Most literature on the flatness problem can be traced back to Dicke and Peebles (1979). Most people today probably connect it with inflation, though it had been discussed long before the idea of inflation arose. It appears that Guth (1981) made an extra effort in his paper to convince the community that the flatness problem is indeed a problem (and thus that inflation offers a solution). In the appendix to his article, Guth writes **"This appendix is added in hope that some skeptics can be convinced that the flatness problem is real."** For almost thirty years, in the leading journals of the field, well known cosmologists have made various arguments against its existence though such arguments seem to have had little impact. As far as I know none of them has been rebutted.

Arguments against the flatness problem and their history are discussed in much more detail by Helbig (2012, 2020, 2021) and Holman (2018) (Marc Holman is also here at the conference). See also Brawer (1996) for an interesting historical perspective, in particular her claim that the flatness problem was not considered to be an important issue until inflation suggested a solution to it.

It might seem strange to some to claim that something which is believed by a majority of the community is wrong. However, there are several examples where the consensus was wrong until the community was convinced otherwise:

- the solution to Olbers's paradox (*e.g.* Harrison, 1964, 1965, 1974, 1977, 1980, 1984, 1986, 1987, 1990a,b, 2000)
- Einstein's rejection of the cosmological constant, now one of the most important topics in cosmology (the fact that Einstein had rejected it probably caused many to see it with too much scepticism, even though Einstein was often wrong in his later years)
- the question whether black holes can form by known astrophysical processes
- the question whether gravitational waves transport energy
- cosmological horizons, a topic which was cleared up in a landmark paper by Rindler (1956)

Still ongoing is the debate as to whether the cosmological-constant problem really exists; my guess is that it is also based on confusion and misunderstanding—at least I'm in good company (Bianchi and Rovelli, 2010a,b; Rovelli, 2021).

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