ARGUMENTS AGAINST THE FLATNESS PROBLEM

P. HELBIG

Thomas-Mann-Str. 9, 63477 Maintal, Germany

Since I have been at most moderately successful in convincing the community of the lack of existence of the flatness problem, I highlight some similar claims from various authors better known than myself.

Here, I consider only ideal Friedmann–Robertson–Walker (FRW) models, because historically fine-tuning claims have been discussed within the context of those models, and the issues remain even in more-realistic models. I use notation such that $\Omega = \frac{8\pi G\rho}{3H^2}$ refers to the density of matter ('dust') and $\lambda = \frac{\Lambda}{3H^2}$ is the normalized cosmological constant (with dimension time^{-2} so that Λ has the same dimension as $G\rho$); the subscript 0 refers to the current value of a time-dependent parameter. $K = \Omega + \lambda - 1$ and $k = \mathrm{sign}(K)$. The two most common formulations of the flatness are referred to by Holman [1] as the *fine-tuning problem* ("there must be some reason why $\Omega = 1$ to very high precision in the early universe") and the *instability problem* ("even given that $\Omega = 1$ to very high precision in the early universe, if Ω is not exactly 1, then it would be unlikely to observe $\Omega \approx 1$").

The first argument in the literature against the flatness problem in FRW models appears to have been by Cho & Kantowski [2]. Putting "The Flatness Problem" in scare quotes makes their point already in the title. The last sentence of their abstract sums up their argument well: **"It is a distorted distribution of Ω values that sometimes misleads the casual observer to conclude that Ω must be exactly equal to 1."** Coles & Ellis [3] state clearly that **"*there is no flatness problem* in a purely classical cosmological model"** [emphasis in the original]. Kirchner & Ellis [4] also use Jaynes's principle to **"solve the flatness problem"** (direct quotation). Carroll [5], describing his work with collaborators [6,7,8], notes that **"flatness isn't a problem at all"**; **"[t]he flatness problem, meanwhile, turns out to be simply a misunderstanding"**; **"the flatness problem really isn't a problem at all; it was simply a mistake, brought about by considering an informal measure rather than one derived from the dynamics"**; **"The flatness problem, as conventionally understood, does not exist; it is an artifact of informally assuming a flat measure on the space of initial cosmological parameters"**; and **"is not intrinsic to the standard Big Bang model"**.

Rindler[9] points out that **"the so-called 'flatness problem'—the alleged improbability of finding the value of Ω_0 even within a factor of 10 of unity"** seems unproblematic for two reasons, first that **"at the big bang ($R = 0$), Ω always starts at one and then wanders away from that value unless $k = \Lambda = 0$"** (thus disputing the fine-tuning problem) and second that, in FRW models with $\lambda = 0$ and $\Omega > 1$, **"$\Omega < 10$ . . . is true for fully 60 per cent of the entire time interval"**.

It appears that our Universe has a positive cosmological constant and will expand forever. For such models with $k = +1$, Lake[10] demonstrates that the instability argument does not hold because λ and Ω are large and the universe significantly non-flat only in the case that they are fine-tuned in the sense that $\alpha = k(27\Omega^2\lambda)/(4K^3) \approx 1$. Lake suggests that α, which has a fixed value throughout the life of the universe, is what should be used to characterize model universes. Adler & Overduin[11] discuss various definitions of 'nearly flat', using essentially using the same parameter as α used by Lake[10], and arriving at the same conclusion, namely that a significantly non-flat universe implies fine-tuning in α.

Arguments against the flatness problem and their history are discussed in much more detail by Helbig[12,13,14] and Holman[1]. See also Brawer[15] for an interesting historical perspective. The complete poster and some supplementary material can be found at http://www.astro.multivax.de:8000/helbig/research/publications/info/moriond2022_1.html .

This research has made use of NASA's Astrophysics Data System Bibliographic Services.

References

1. M. Holman, *Found. Phys.* **48**, 1617 (2018).
2. H. T. Cho and R. Kantowski, *Phys. Rev. D* **50**, 6144 (1994).
3. P. Coles and G. F. R. Ellis, *Is the Universe Open or Closed?* (Cambridge University Press, Cambridge, 1997).
4. U. Kirchner and G. F. R. Ellis, *Class. Quant. Grav.* **20**, 1199 (2003).
5. S. M. Carroll, arXiv:1406.3057 (2014).
6. S. M. Carroll and H. Tam. arXiv:1007.1417 (2010).
7. G. N. Remmen and S. M. Carroll, *Phys. Rev. D* **88**, 083518 (2013).
8. G. N. Remmen and S. M. Carroll, *Phys. Rev. D* **90**, 063517 (2014).
9. W. Rindler, *Relativity: Special, General, and Cosmological* (Oxford University Press, Oxford, 2001).
10. K. Lake. *Phys. Rev. Lett.* **94**, 201102 (2005).
11. R. J. Adler and J. M. Overduin. *Gen. Rel. Grav.* **37**, 1491 (2005).
12. P. Helbig, *MNRAS*, **421** 561 (2012).
13. P. Helbig, *MNRAS*, **495** 3571 (2020).
14. P. Helbig, *Eur. Phys. J. H* **46**, 10 (2021).
15. R. Brawer, *Inflationary cosmology and horizon and flatness problems: the mutual constitution of explanation and questions*, master's thesis (MIT, Boston, 1996), http://hdl.handle.net/1721.1/38370.