

Uncertainty Analysis of a Lumped Physically Based Numerical Model of Dam Breaching

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Abstract

Failures of dams and dikes often lead to devastating consequences in protected areas. Numerical models are crucial tools to assess flood risk and guide emergency plans, but numerous sources of uncertainty exist. Therefore, identifying input variables uncertainties that induce critical uncertainties in model outputs is essential. Using Monte Carlo procedure, we conducted a sensitivity analysis based on Sobol indices of total order on 28 reference configurations. In each case, it allowed ranking input variables as a function of the significance of their uncertainty on the output variability and, for the first time, the dependency between reference configurations and sensitivity analysis results was highlighted. Uncertainty propagation was performed based on our implementation of the lumped fully coupled hydro-morphodynamic model developed by Wu (2016). Also, two new methods were developed to generate samples of dependent input variables.

Keywords: Uncertainty analysis; Dam breaching; Sobol indices; Sampling method; Numerical model

1. INTRODUCTION

With the surge of extreme meteorological events and intensification of urbanisation downstream of hydraulic structures, the need for predicting failure of dams and dikes has become of paramount importance to establish emergency response procedures (Zhong et al., 2021). To this end, numerical models used for embankment breach modelling are crucial tools, but input parameters are subject to uncertainties.

Different approaches exist to quantify uncertainties, depending on data availability and input type. When the data set is large, the maximum likelihood method is often used to define parameter distributions (Froehlich, 2008; Kalinina et al., 2020; Sattar, 2014). In contrast, when data is scarce, Bayesian inference is preferred (Peter et al., 2018). When information on the parameters of interest are too limited or unavailable, arbitrary parameter distributions might be defined based on modeler expertise (Abdedou et al., 2020; Bellos et al., 2020; Froehlich & Goodell, 2012; Tsai et al., 2019). Except in rare cases (Froehlich & Goodell, 2012; Kalinina et al., 2020), dependency between input parameters is not considered, although it might influence input probability distributions significantly.

The uncertainty in input parameters should be propagated through the numerical model to assess its impact on the model outputs. It is most often implemented using Monte Carlo sampling method, which is sometimes coupled to stochastic expansion methods (Arnst & Ponthot, 2014). In Monte Carlo procedure, many simulations are run. Each time, the model is fed with a different set of input variable values generated according to their probability distributions. As the number of runs increases, approximations of the model outputs statistics converge (Janssen, 2013). The classical Monte Carlo sampling method is computationally expensive, making it suitable for fast models only (Abdedou et al., 2020; Bellos et al., 2020; Froehlich, 2008; Sattar, 2014). In contrast, Latin Hypercube sampling method does not select input values purely based on their probability distributions but it fosters sets of input values located in underrepresented areas of the input space (Kalinina et al., 2020; Peter et al., 2018; Pheulpin, Bacchi, & Bertrand, 2020). When considering detailed physically based models, i.e. time consuming models, a surrogate model (or metamodel) is usually fit to the original numerical model to mimic the relationship between input and output variables while improving computational efficiency models (Abdedou et al., 2020; Kalinina et al., 2020; Pheulpin et al., 2020). Output statistics may then be deduced from metamodel parameters and input statistical moments (Arnst & Ponthot, 2014). Alternatively, the point

estimate method is based on a limited number of model evaluations and aims at computing output statistical moments using statistical moment of the input variables (Froehlich & Goodell, 2012; Tsai et al., 2019).

Uncertainty propagation results can be used in two different ways. On one hand, the focus is set on the output variability characterization, including output main statistical moments or complete probability distribution (Abdedou et al., 2020; Peter et al., 2018; Westoby et al., 2015). On the other hand, many sensitivity methods highlight the relationships between uncertainties on specific input and output variables. Among them, global variance-based sensitivity analysis is shown to be particularly general in its applicability and in its capacity to reflect nonlinear processes and the effects of interactions among variables (Hall et al., 2009). The general goal of variance-based methods is to rank inputs or subsets of inputs as a function of the significance of their uncertainty on the output variability. Within this context, various indicators and techniques exist, including screening techniques (Alhasan et al., 2016; Bellos et al., 2020), least square linearization technique (Ahmadisharaf et al., 2016; Sattar, 2014), or coefficients of variation (Tsai et al., 2019). Sobol indices are also often used (Sobol, 2001). There exist two main ways to handle them: Sobol indices of first order (Kalinina et al., 2020; Pheulpin et al., 2020) or total order (Pheulpin et al., 2020). The Sobol index of first order of a given input subset illustrates the portion of the global output variance caused by the uncertainty on this specific subset. In contrast, Sobol index of total order also includes the interactions of this subset with any other input subsets. Both indices aim at assessing the relative significance of input variables and setting the focus on the most critical ones (Iooss & Lemaître, 2015).

Dam breaching is computed here using our implementation of the physics-based lumped fully coupled hydro-morphodynamic model developed by Wu (2016). It contains three modules: hydrodynamic module, a sediment transport module and a morphodynamic module. In this work, modelling of homogeneous dams made of non-cohesive material is considered. The numerical model involves 20 independent input variables and 2 sets of dependent input variables. The probability density functions (PDF) associated to each input were generated based on limited information and expert knowledge, with a special care for dependent input variables. Thanks to the model computation efficiency, the Monte Carlo sampling method was adopted. Sobol indices of total order were then computed for each input subset in 28 embankment configurations.

While previous works were often limited to the analysis of a single case study, we applied a global sensitivity based on Sobol indices of total order to 28 embankment configurations, including 27 laboratory-scale tests and one field-scale test. The main contribution of this paper is to highlight the dependency between global sensitivity analysis results and test configuration while generating samples of dependent input parameters based on observational data without postulating any specific multivariate distribution profile.

2. DATA AND METHODS

2.1 Computational model

Based on initial and boundary conditions, the model hydrodynamic module evaluates the water level in the main channel and on the different parts of the embankment, the breach discharge, and the resulting shear stress using two main assumptions: critical flow on the dam crest and uniform flow on the downstream face. Those values are then introduced in the sediment transport module to compute both suspended-load and bed-load through the breach assuming non-equilibrium sediment transport. Based on the embankment eroded volume, the new dam geometry is generated by the morphodynamic module. The procedure is repeated by feeding the hydrodynamic module with the updated dam geometry. Wu (2016) provides a thorough description of the model. The numerical model was coded using *Matlab* software and explicit resolution schemes were used. The time step was 0.5 [s] for laboratory-scale tests and 5 [s] for the field-scale test.

2.2 Uncertainty quantification and sensitivity analysis

Input variables considered as uncertain are gathered in Table 1. They are divided into two types, namely model parameters that need to be set based on experimental observations and input parameters, e.g. material characteristics and dam geometry parameters. Samples of input variables were then generated. The approach differs for variables whose values are independent from each other and variables with dependent values, e.g. empirical regression coefficients in the present case.

A beta distribution was assigned as probability distribution function (PDF) to all independent variables due to its high versatility, its wide-spread use in statistics and its finite support. Due to the approximate knowledge of input distributions, it was decided to fix the PDF maximum at the input reference value. Its variance was selected as a function of the degree of uncertainty on each input variable.

Two different methods were developed to generate sets of regression coefficients. Based on available empirical data, Method 1 generates new data clouds by bootstrapping and computes associated regression coefficients for each of them. This method exhibits an optimal accuracy but poor computational performance because a regression procedure is performed for each input sample. Method 2 uses variable changes and

Principal Component Analysis (PCA) to decouple input variables and defines independent marginal probability distributions. Although being less accurate, this method significantly reduces the time consumption of the sampling procedure compared to Method 1. However, in the current study this gain was deemed negligible compared to the time required to run simulations using Monte Carlo procedure. To maximize sensitivity analyses results accuracy, Method 1 was considered here.

Input uncertainties are finally propagated to model outputs through our implementation of the numerical model of Wu (2016). In this work, a global sensitivity analysis is led using Sobol indices of total order to highlight the most influential input uncertainties. Using Monte Carlo approach, 4,000 runs were required to converge for each tested configuration.

Table 1. Summary of parameters involved in the numerical model (see Wu (2016) for more details).

	Symbol	Description
Input parameters	S_u, S_d	Up- and downstream slopes of the dam
	L_k	Dam crest length
	h_d	Dam height
	φ_r	Sediment repose angle
	ρ_s	Sediment density
	d_{50}	Sediment median size
	p	Dam material porosity
	Q_{in}	Inflow discharge
Model parameters	c_{eff}	Weir efficiency coefficient involved in breach discharge evaluation
	λ_{loss}	Sum of local head loss coefficients at breach inlet and outlet
	A_n	Parameters in Strickler formula: $n = \max\left(\frac{d_{50}^{1/6}}{A_n}; n_{min}\right); n' = \max\left(\frac{d_{50}^{1/6}}{A_n'}; n_{min}\right),$ with n and n' total and effective Manning's coefficients, respectively
	A_n'	
	n_{min}	
	θ_{cr}	Critical Shields parameter
	$\lambda_{0,a}, \lambda_{0,b}$	Coefficient and exponent involved in effective shear stress computation (Damgaard et al., 1997)
	S_p	Corey shape factor
	$C_{*a}, C_{*b}, C_{*c}, C_{*d}$	Regression coefficients involved in the suspended load concentration
	$q_{b,a}^*, q_{b,b}^*$	Regression coefficients involved in bed load transport capacity formula
	λ	Empirical coefficient involved in mixing length computation
	$c_{b,coef}$	Artificial breach widening limitation coefficient

2.3 Test cases

The sensitivity analysis was applied to various experimental tests to assess how test configuration influences the relation between uncertainties in input and output variables. 27 laboratory experiments led by Frank (2016) were first considered. Additionally, the uncertainty analysis was applied to one field test of the IMPACT project (Morris & Hassan, 2008).

3. RESULTS AND DISCUSSION

Sobol indices of total order were computed for the peak breach discharge, $Q_{b,peak}$. They are presented in Figure 1 for the considered configurations. Only parameters with a Sobol index larger than 5% for at least one configuration are presented. Configurations related to a same series are separated by vertical dotted lines.

Seven particularly influential input uncertainties are identified: c_{eff} , A_n , n_{min} , θ_{cr} , λ , h_d and Q_{in} . Uncertainties in model parameters are the most influential, though physical parameter Q_{in} becomes important in some specific configurations. In most cases, erosion related parameters A_n , θ_{cr} and λ influence most the model outcomes due to their strong relation with erosion intensity and resulting breach expansion. When the median grain size, d_{50} , is small, the influence of n_{min} rises, as illustrated in Frank's test 15. The breach discharge is directly related to parameters Q_{in} and c_{eff} , which explains the considerable impact of their uncertainty. Finally, the relative significance of uncertainties in the dam height, h_d , remains limited in all cases.

Figure 1 also shows that modifications in the reference configuration may alter the relative importance of uncertainties in input variables. When the reference inflow discharge, Q_{in} , rises (Frank's tests 18, 19, 20 and 21), the relative impact of its uncertainty slightly grows, as does the influence of variations in c_{eff} and λ . This reduces the relative influence of parameters used to compute bed-load concentration, i.e. A_n and θ_{cr} . Similar trends are observed when the initial breach width, B_{mi} , is increased (Frank's tests 20, 24, 25, 26 and 27), except for c_{eff} , whose relative influence vanishes.

In contrast, varying the reference median grain size, d_{50} , does not influence the sensitivity analysis results significantly (Frank's tests 20, 22 and 23). This is consistent with the low value of the Sobol indices related to this parameter uncertainty.

The slight increase in the dam crest width, L_k , between Frank's tests 28 and 20 leads to a smaller impact of uncertainties in λ and a larger relative impact of other erosion parameters, i.e. A_n and θ_{cr} in this case. In test 29, the crest width becomes very large and the embankment gets much more resistant. The influence of Q_{in} blows up, at the expense of erosion parameters A_n , θ_{cr} and λ .

When the reservoir area, A_r , grows and no inflow discharge is injected (Frank's tests 40, 36, 41, 42, 43, and 44), the water level decreases more slowly and the water level above the breach rises. In this case, the relative significance of c_{eff} rises. Those observations can be extrapolated to the IMPACT Field test 2, whose reservoir area is even larger in relative terms, and the inflow discharge is negligible.

No clear trend can be observed in Frank's tests corresponding to scale families (tests 11, 10, 8, 12, 13, 14, 15, 16 and 17). Though, most influential input uncertainties are the same as the ones identified for all other tests presenting a non-negligible inflow discharge, i.e. all tests except tests 40, 36, 41, 42, 43, and 44.

4. CONCLUSIONS

This paper highlighted the need to perform sensitivity analysis to identify most critical input variables uncertainties in embankment breaching numerical models. It also demonstrated the dependency between sensitivity analysis results and test configuration. Uncertainty propagation was performed using our implementation of the model of Wu (2016), in which 22 sets of input variables were deemed uncertain. Also, two new sampling methods of dependent inputs were developed.

Sobol indices of total order were computed for each input variable in 28 reference configurations. It allowed us to assess the relative influence of input uncertainties on the variability of the peak breach discharge and its dependency with test configurations. Uncertainties in three erosion related parameters (A_n , θ_{cr} and λ) appeared to be the most influential in most configurations. The influence of the inflow discharge uncertainty grew with the inflow discharge reference value, the initial breach width and the dam crest length. When no inflow discharge was injected, the impact of the weir coefficient uncertainty on the peak discharge rose with the reservoir area.

The present work allows identifying parameters whose uncertainty is critical for model outputs accuracy, depending on the configuration considered. The relevance of using highly uncertain model parameters which strongly influence the results may be questioned. Also, the reliability of the numerical model may be partly assessed by looking at the amplitude of the output uncertainties related to a given configuration. Finally, the procedure described here may valuably be used to determine if a modification in the model structure brings a notable improvement in results accuracy but also a reduction of the output uncertainty range. In future work, sensitivity analysis should be led for different input probability distributions and compared to assess how strong their dependency is.

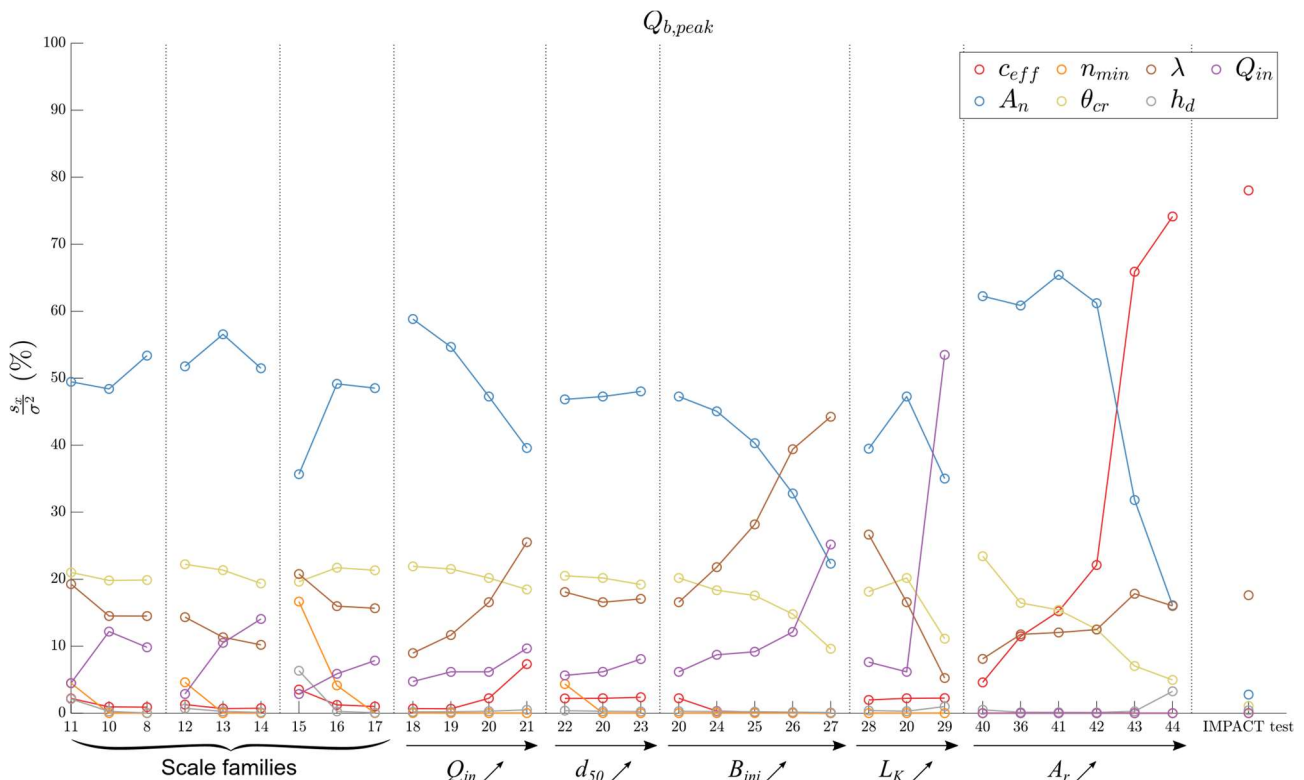


Figure 1. Sobol indices of total order for peak breach discharge in considered configurations (numbers = Frank's (2016) tests; IMPACT test = IMPACT field test 2).

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