Data-driven wall shear stress model for Large Eddy Simulations applied to flow separation

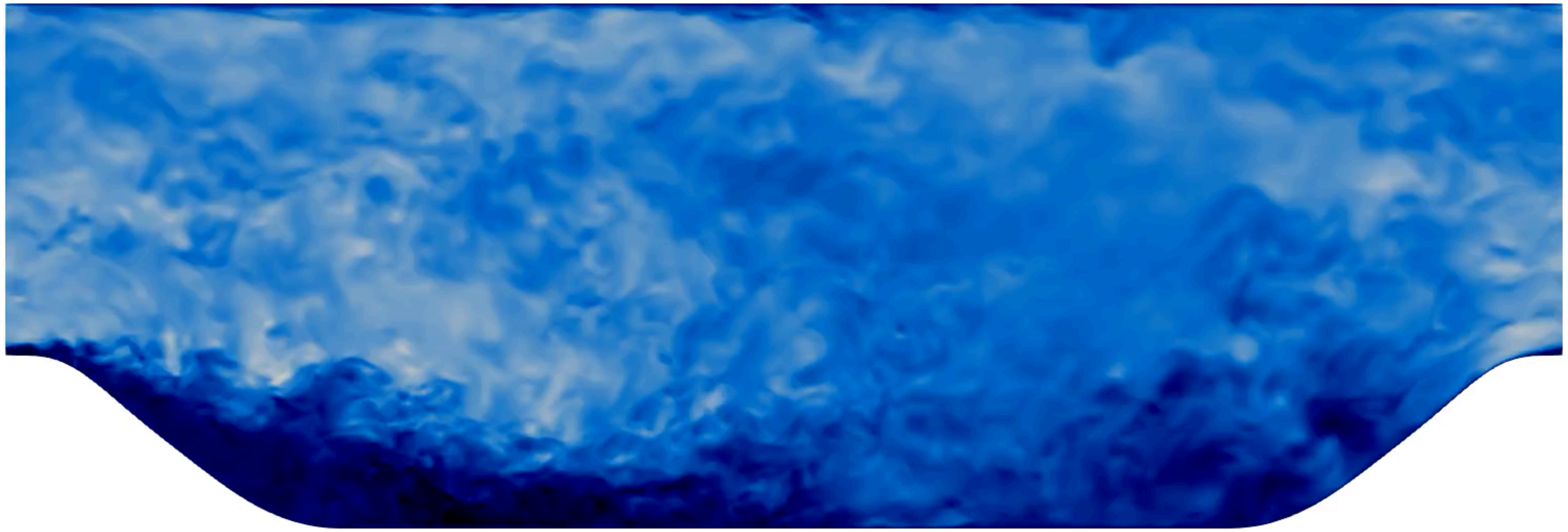
DLES13, Direct and Large-Eddy Simulation 13, October 26th-29th 2022, Udine, Italy

M. Boxho, M. Rasquin, T. Toulorge, G. Dergham,
G. Winckelmans and K. Hillewaert
ULiege, UCLouvain, Cenaero, Safran Tech
Contact: margaux.boxho@cenaero.be

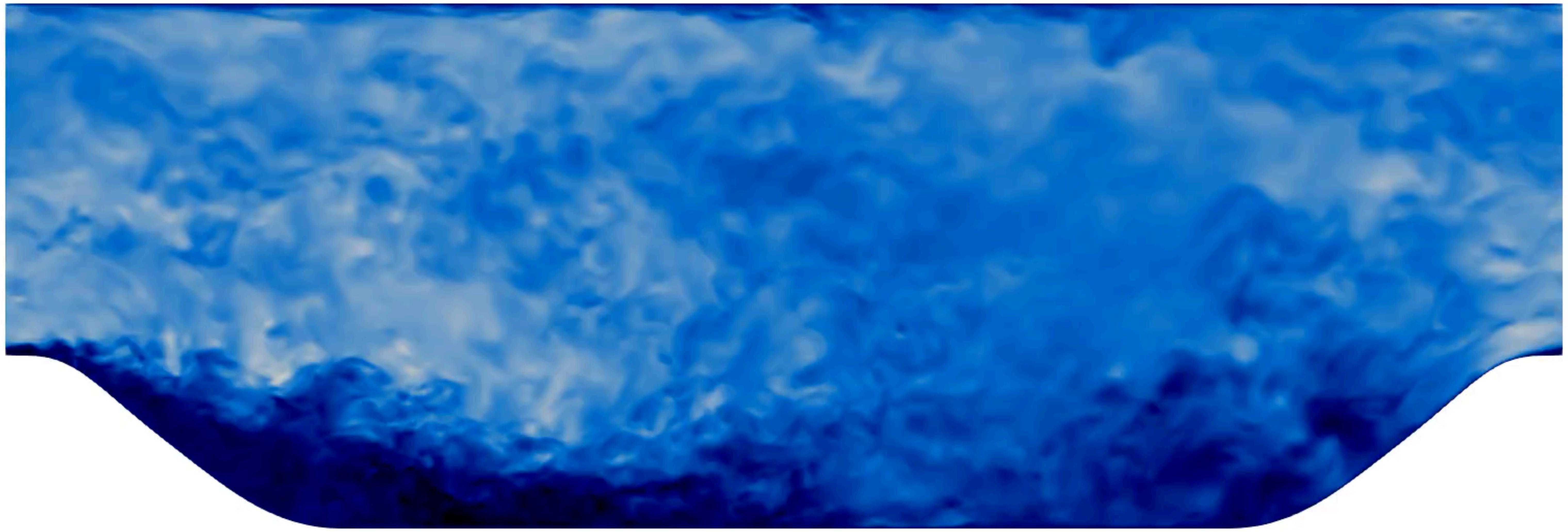
Doc. ref.: 2019030-THESELDP-SAFRAN

PROD-F-015-02

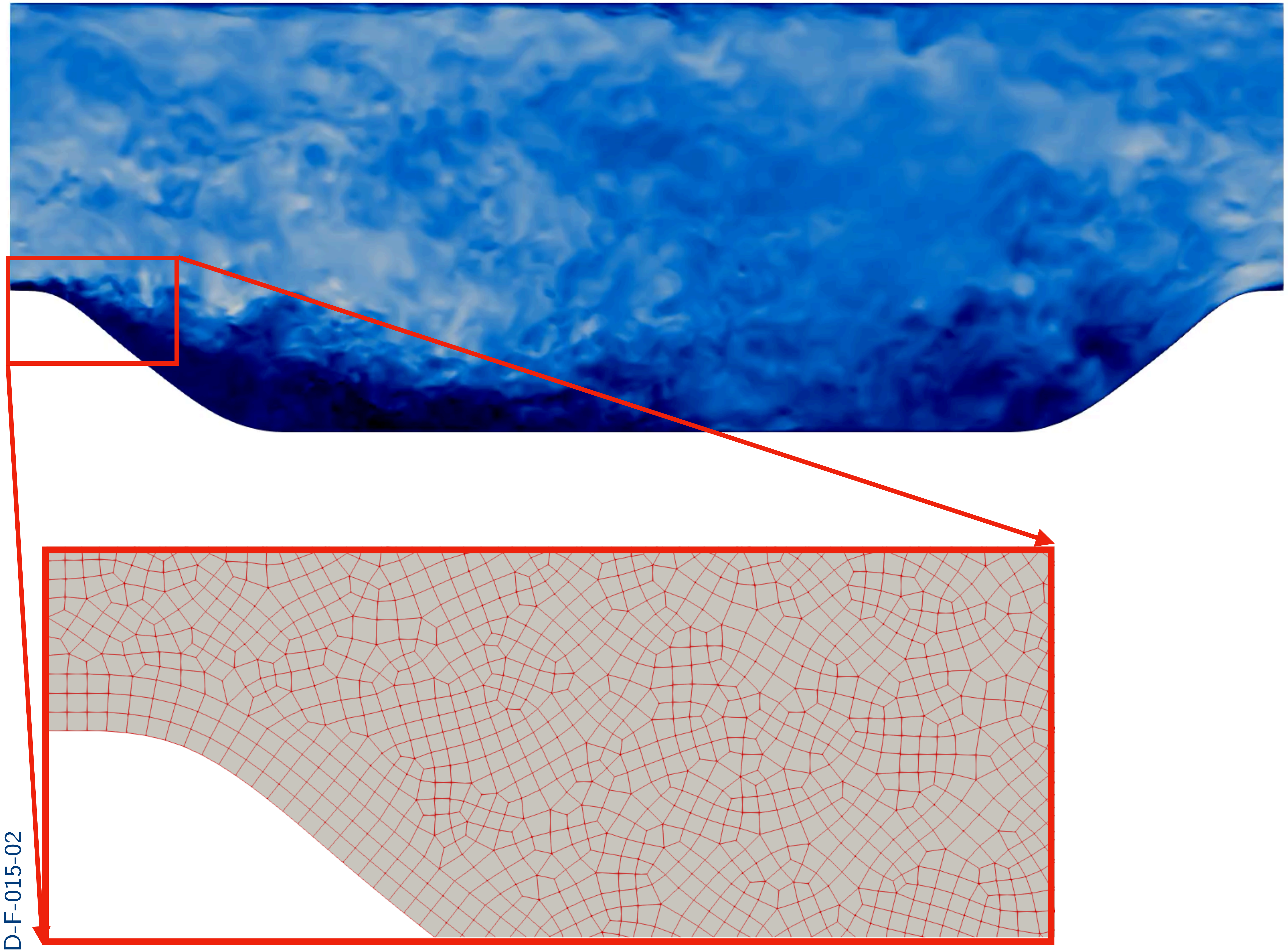
Wall modeled LES: context



Wall modeled LES: context



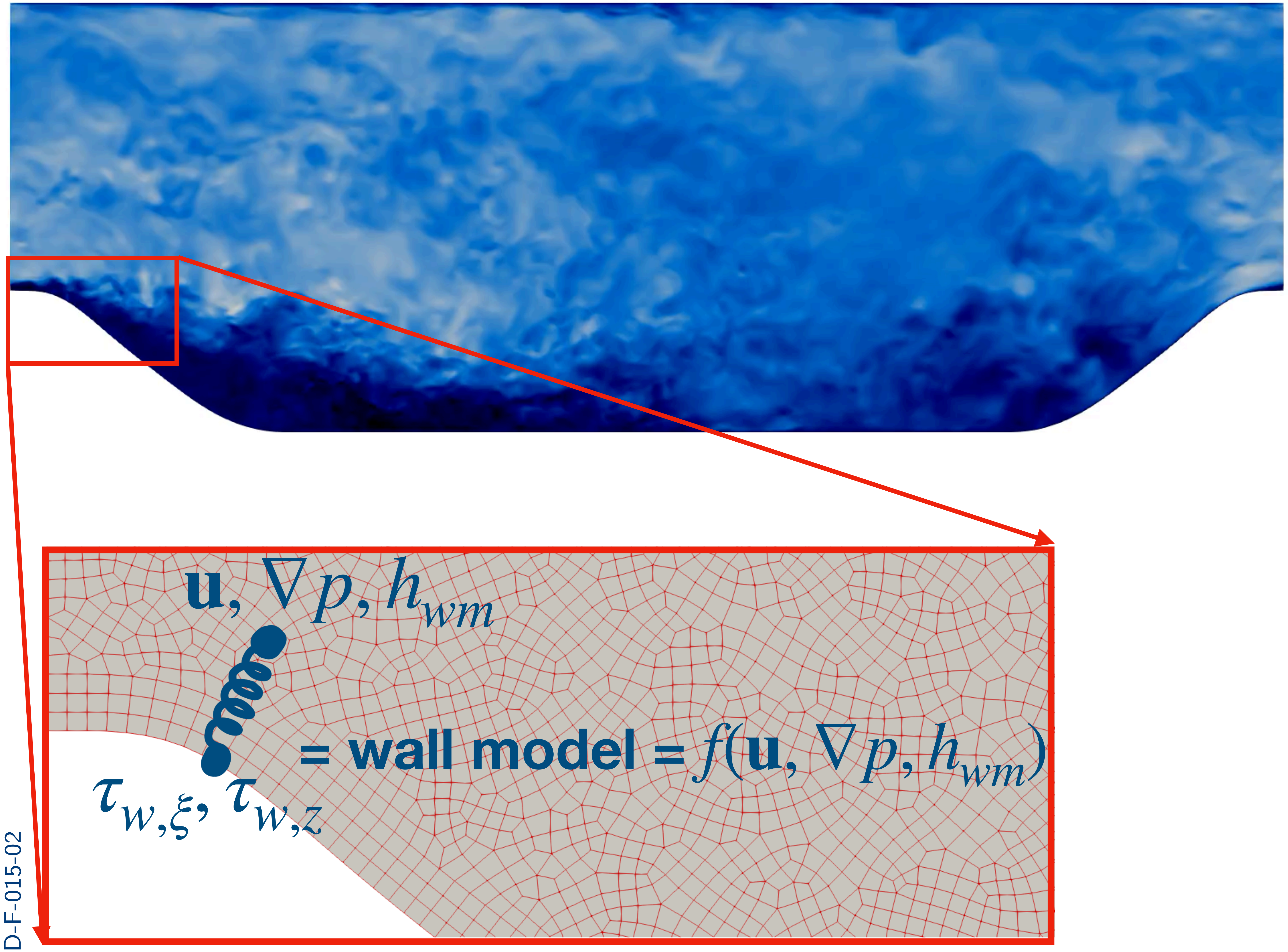
Wall modeled LES: context



PROD-F-015-02

Cenaero

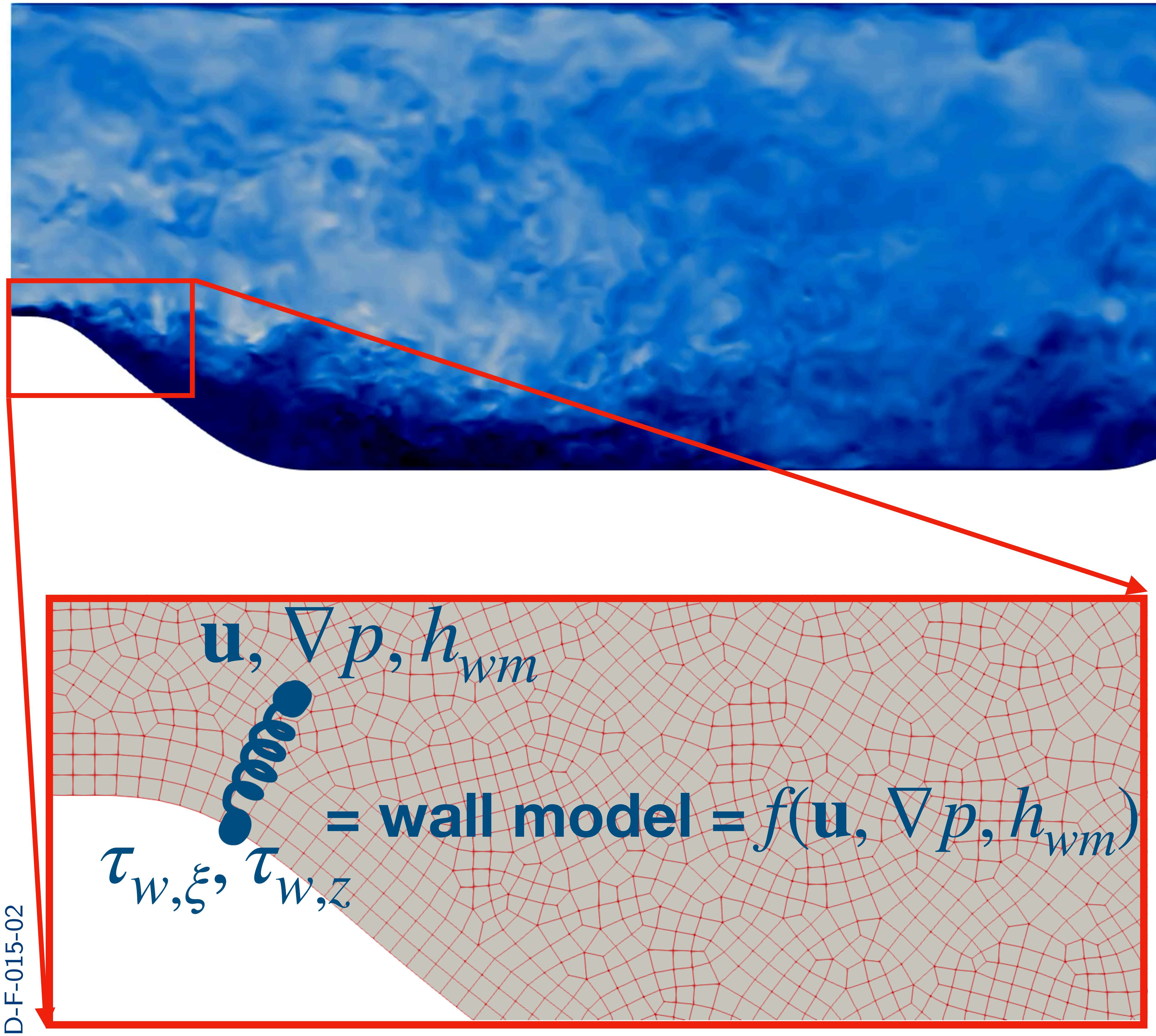
Wall modeled LES: context



Problem definition: finding a complex and dynamic relation between instantaneous fields, geometrical parameters and the wall shear stress

PROD-F-015-02

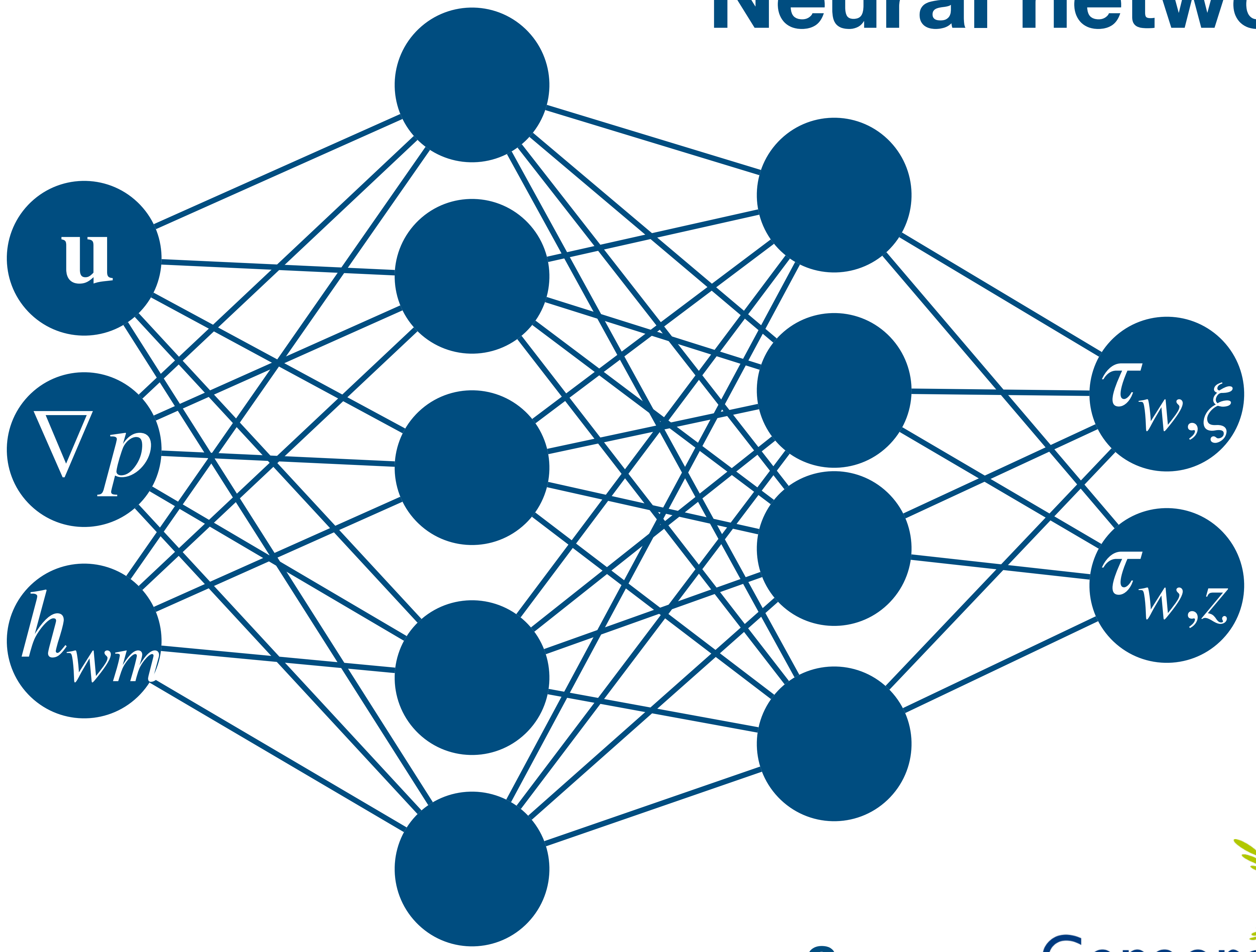
Wall modeled LES: context



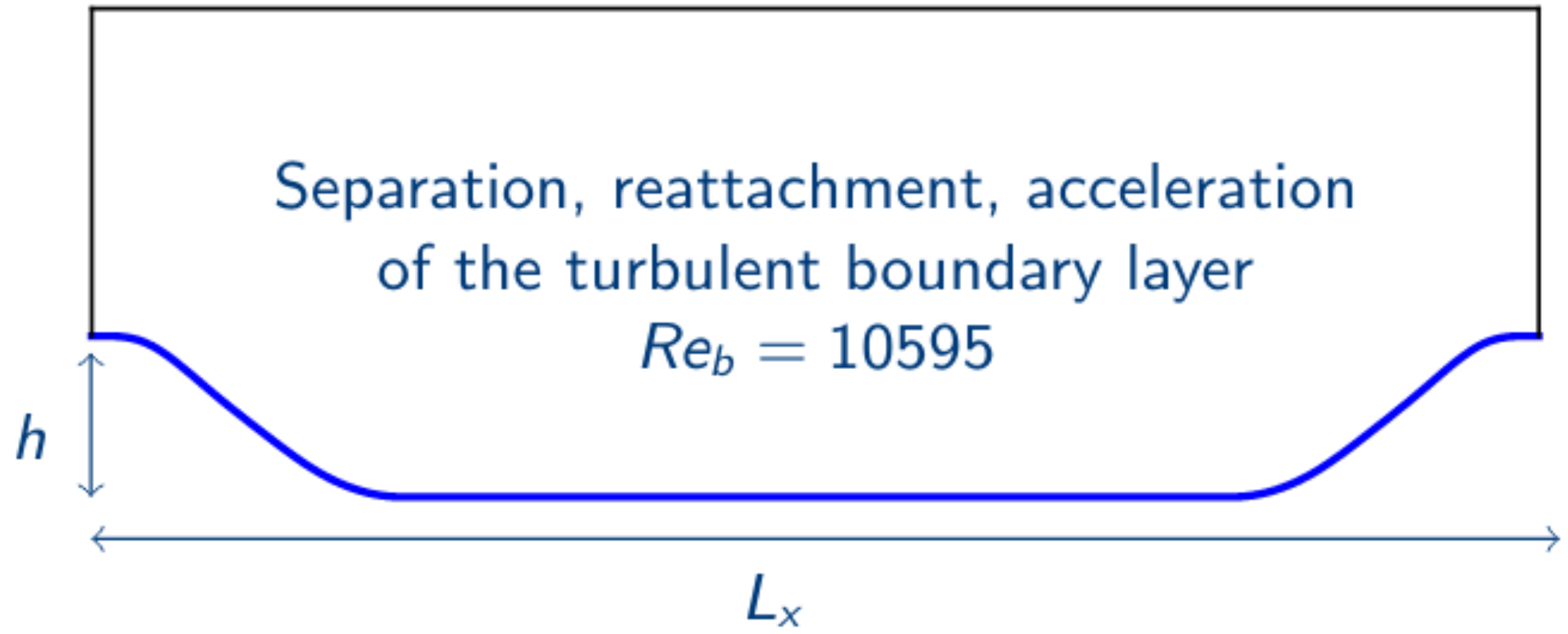
Problem definition: finding a complex and dynamic relation between instantaneous fields, geometrical parameters and the wall shear stress

=

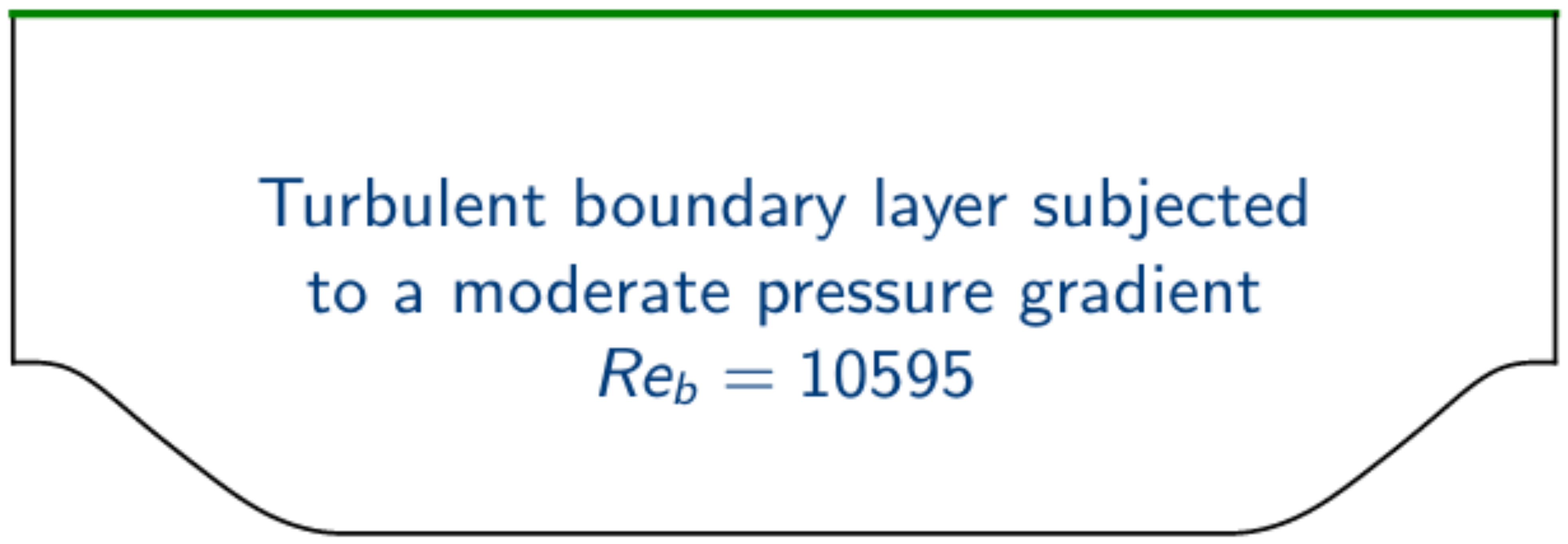
Neural network



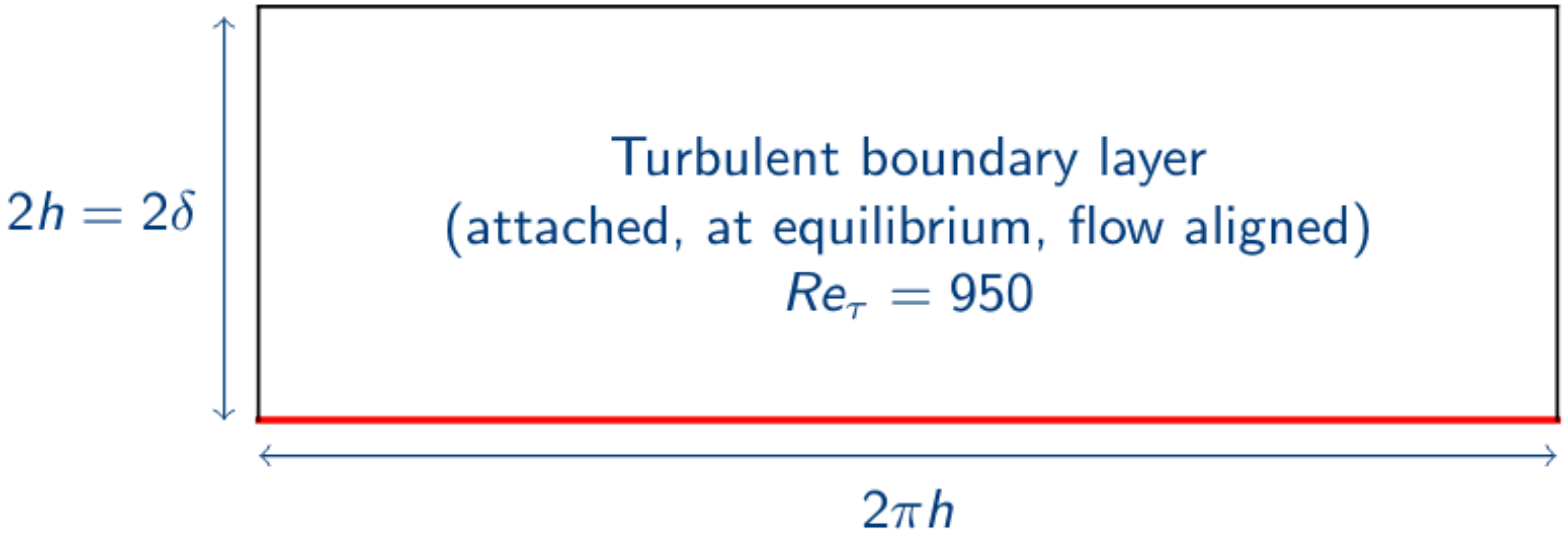
TC1 : Periodic Hill - Lower wall



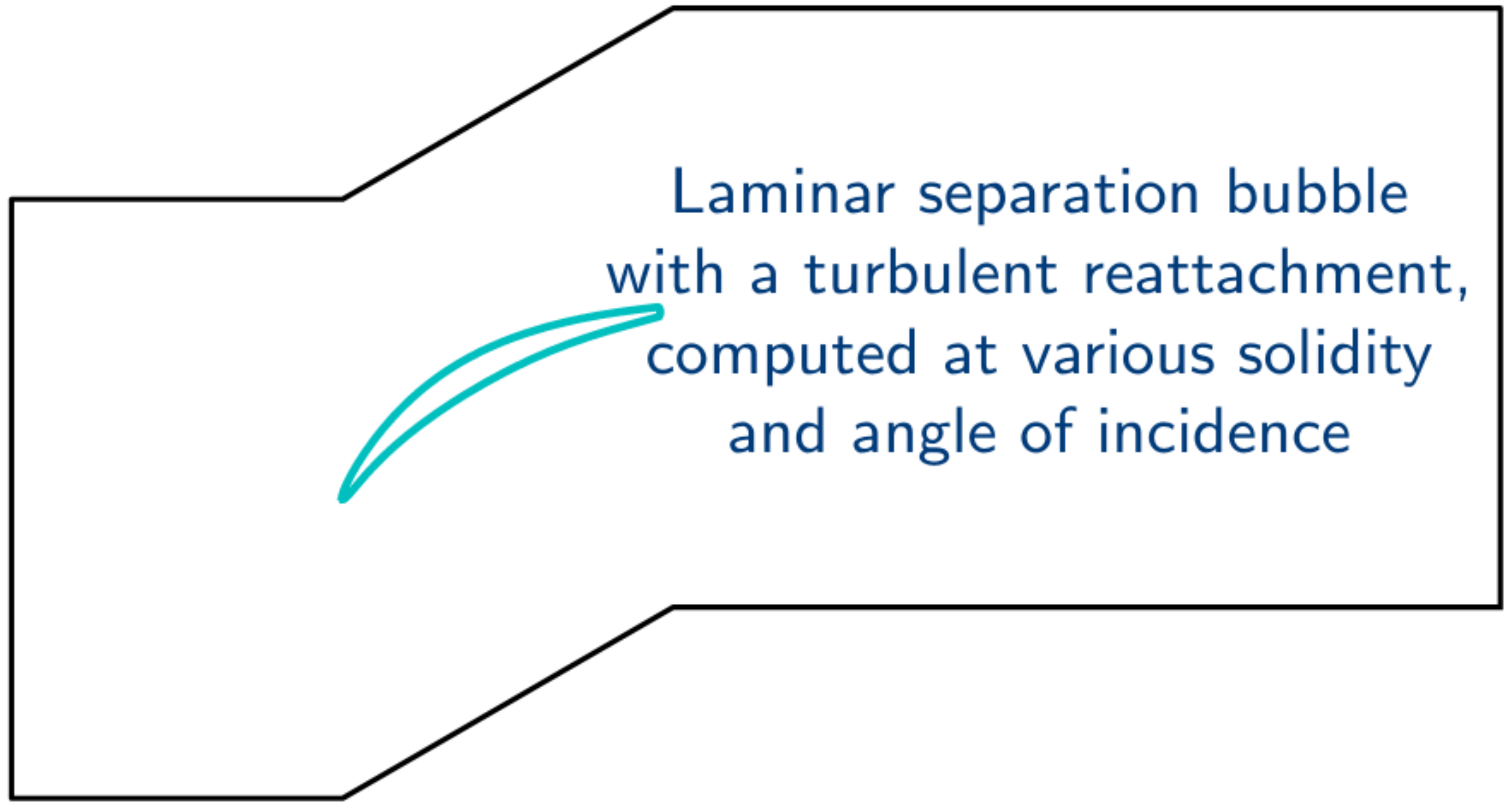
TC2 : Periodic Hill - Upper wall



TC3 : Channel wall



TC4 : SOLIDITY - Compressor Blade



Where is the data extracted ?

PROD-F-015-

[1] Boxho, M. *et al.*: Analysis of space-time correlations to support the development of wall-modeled LES. Flow, Turbulence and Combustion. (2022).

Where is the data extracted ?

[1] Boxho, M. *et al*.: Analysis of space-time correlations to support the development of wall-modeled LES. Flow, Turbulence and Combustion. (2022).

Where is the data extracted ?

PROD-F-015-

[1] Boxho, M. *et al.*: Analysis of space-time correlations to support the development of wall-modeled LES. Flow, Turbulence and Combustion. (2022).

Where is the data extracted ?

PROD-F-015-

[1] Boxho, M. *et al*.: Analysis of space-time correlations to support the development of wall-modeled LES. Flow, Turbulence and Combustion. (2022).

Cenaero

Where is the data extracted ?

PROD-F-015-

[1] Boxho, M. *et al.*: Analysis of space-time correlations to support the development of wall-modeled LES. Flow, Turbulence and Combustion. (2022).

Cenaero

Where is the data extracted ?

PROD-F-015-

[1] Boxho, M. *et al.*: Analysis of space-time correlations to support the development of wall-modeled LES. Flow, Turbulence and Combustion. (2022).

Where is the data extracted ?

PROD-F-015-

[1] Boxho, M. *et al.*: Analysis of space-time correlations to support the development of wall-modeled LES. Flow, Turbulence and Combustion. (2022).

Cenaero

Where is the data extracted ?

PROD-F-015-

[1] Boxho, M. *et al*.: Analysis of space-time correlations to support the development of wall-modeled LES. Flow, Turbulence and Combustion. (2022).

Where is the data extracted ?

PROD-F-015-

[1] Boxho, M. *et al.*: Analysis of space-time correlations to support the development of wall-modeled LES. Flow, Turbulence and Combustion. (2022).

Where is the data extracted ?

PROD-F-015-

[1] Boxho, M. *et al.*: Analysis of space-time correlations to support the development of wall-modeled LES. Flow, Turbulence and Combustion. (2022).

Cenaero

Where is the data extracted ?

[1] Boxho, M. *et al.*: Analysis of space-time correlations to support the development of wall-modeled LES. Flow, Turbulence and Combustion. (2022).

Where is the data extracted ?

PROD-F-015-

[1] Boxho, M. *et al*.: Analysis of space-time correlations to support the development of wall-modeled LES. Flow, Turbulence and Combustion. (2022).

Cenaero

Where is the data extracted ?

PROD-F-015-

[1] Boxho, M. *et al*.: Analysis of space-time correlations to support the development of wall-modeled LES. Flow, Turbulence and Combustion. (2022).

Where is the data extracted ?

PROD-F-015-

[1] Boxho, M. *et al*.: Analysis of space-time correlations to support the development of wall-modeled LES. Flow, Turbulence and Combustion. (2022).

Cenaero

Where is the data extracted ?

PROD-F-015-

[1] Boxho, M. *et al.*: Analysis of space-time correlations to support the development of wall-modeled LES. Flow, Turbulence and Combustion. (2022).

Cenaero

Where is the data extracted ?

PROD-F-015-

[1] Boxho, M. *et al.*: Analysis of space-time correlations to support the development of wall-modeled LES. Flow, Turbulence and Combustion. (2022).

Where is the data extracted ?

PROD-F-015-

[1] Boxho, M. *et al.*: Analysis of space-time correlations to support the development of wall-modeled LES. Flow, Turbulence and Combustion. (2022).

Cenaero

Where is the data extracted ?

PROD-F-015-

[1] Boxho, M. *et al.*: Analysis of space-time correlations to support the development of wall-modeled LES. Flow, Turbulence and Combustion. (2022).

Cenaero

Where is the data extracted ?

[1] Boxho, M. *et al*.: Analysis of space-time correlations to support the development of wall-modeled LES. Flow, Turbulence and Combustion. (2022).

PROD-F-015-

Cenaero

Where is the data extracted ?

Using space-time correlations (Pearson and Distance correlations [1])

PROD-F-015-

[1] Boxho, M. *et al.*: Analysis of space-time correlations to support the development of wall-modeled LES. Flow, Turbulence and Combustion. (2022).

Cenaero

Where is the data extracted ?

Using space-time correlations (Pearson and Distance correlations [1])

PROD-F-015-

[1] Boxho, M. *et al.*: Analysis of space-time correlations to support the development of wall-modeled LES. Flow, Turbulence and Combustion. (2022).

Cenaero

How to normalize the input/output pairs ?

How to normalize the input/output pairs ?

$$u^* = \frac{u}{u_{vp}}$$

$$\nabla p^* = \frac{h_{wm}}{\rho\, u_{vp}^2} \nabla p$$

$$h_{wm}^* = \ln\left(\frac{h_{wm}}{y_{vp}}\right)$$

$$\Delta \varphi^* = \frac{\Delta \varphi}{y_{vp}}$$

PROD-F-015-02

Cenaero

How to normalize the input/output pairs ?

$$u^* = \frac{u}{u_{vp}}$$

$$\nabla p^* = \frac{h_{wm}}{\rho\, u_{vp}^2} \nabla p$$

$$h_{wm}^* = \ln\left(\frac{h_{wm}}{y_{vp}}\right)$$

$$\Delta \xi^* = \frac{\Delta \xi}{y_{vp}}$$

PROD-F-015-02

Cenaero

How to normalize the input/output pairs ?

$$u^* = \frac{u}{u_{vp}}$$

$$\nabla p^* = \frac{h_{wm}}{\rho\, u_{vp}^2} \nabla p$$

$$h_{wm}^* = \ln\left(\frac{h_{wm}}{y_{vp}}\right)$$

$$\Delta \xi^* = \frac{\Delta \xi}{y_{vp}}$$

$$u_{vp} = \sqrt{u_v^2 + u_p^2}$$

Cenaero

How to normalize the input/output pairs ?

$$u^* = \frac{u}{u_{vp}}$$

$$\nabla p^* = \frac{h_{wm}}{\rho\, u_{vp}^2} \nabla p$$

$$h_{wm}^* = \ln\left(\frac{h_{wm}}{y_{vp}}\right)$$

$$\Delta \xi^* = \frac{\Delta \xi}{y_{vp}}$$

$$u_{vp} = \sqrt{u_v^2 + u_p^2}$$

$$u_v = \sqrt{\frac{\nu \|u\|}{h_{wm}}}$$

PROD-F-015-02

Cenaero

How to normalize the input/output pairs ?

$$u^* = \frac{u}{u_{vp}} \qquad \nabla p^* = \frac{h_{wm}}{\rho\, u_{vp}^2} \nabla p \qquad h_{wm}^* = \ln\left(\frac{h_{wm}}{y_{vp}}\right) \qquad \Delta \varrho^* = \frac{\Delta \varrho}{y_{vp}}$$

$$u_{vp} = \sqrt{u_v^2 + u_p^2}$$

$$u_v = \sqrt{\frac{\nu \|u\|}{h_{wm}}} \qquad u_p = \left|\frac{\nu}{\rho}\frac{\partial p}{\partial x}\right|^{1/3}$$

PROD-F-015-02

Cenaero

How to normalize the input/output pairs ?

$$u^* = \frac{u}{u_{vp}}$$

$$\nabla p^* = \frac{h_{wm}}{\rho\, u_{vp}^2} \nabla p$$

$$h_{wm}^* = \ln\left(\frac{h_{wm}}{y_{vp}}\right)$$

$$\Delta \xi^* = \frac{\Delta \xi}{y_{vp}}$$

$$u_{vp} = \sqrt{u_\nu^2 + u_p^2}$$

$$u_\nu = \sqrt{\frac{\nu \|u\|}{h_{wm}}} \qquad u_p = \left| \frac{\nu}{\rho} \frac{\partial p}{\partial x} \right|^{1/3}$$

PROD-F-015-02

How to normalize the input/output pairs ?

$$u^* = \frac{u}{u_{\nu p}}$$

$$\nabla p^* = \frac{h_{wm}}{\rho\, u_{\nu p}^2} \nabla p$$

$$h_{wm}^* = \ln\left(\frac{h_{wm}}{y_{\nu p}}\right)$$

$$\Delta \xi^* = \frac{\Delta \xi}{y_{\nu p}}$$

$$u_{\nu p} = \sqrt{u_\nu^2 + u_p^2}$$

$$u_\nu = \sqrt{\frac{\nu \|u\|}{h_{wm}}} \qquad u_p = \left| \frac{\nu}{\rho} \frac{\partial p}{\partial x} \right|^{1/3}$$

$$y_{\nu p} = \frac{\nu}{u_{\nu p}}$$

How to normalize the input/output pairs ?

$$u^* = \frac{u}{u_{vp}} \qquad \nabla p^* = \frac{h_{wm}}{\rho\, u_{vp}^2} \nabla p \qquad h_{wm}^* = \ln\left(\frac{h_{wm}}{y_{vp}}\right) \qquad \Delta\xi^* = \frac{\Delta\xi}{y_{vp}}$$

$$u_{vp} = \sqrt{u_v^2 + u_p^2}$$

$$y_{vp} = \frac{\nu}{u_{vp}}$$

$$u_v = \sqrt{\frac{\nu \|u\|}{h_{wm}}} \qquad u_p = \left|\frac{\nu}{\rho}\frac{\partial p}{\partial x}\right|^{1/3}$$

Concerning the output, we get

$$\tau_{w\xi}^* = \frac{\tau_{w\xi}}{\frac{1}{2}\rho\langle u_{vp}^2\rangle_{\xi,z}}$$

PROD-F-015-02

Neural Networks : Properties and Architecture

Convolutional Neural Network (CNN)

Mixture Density Network (MDN)

Neural Networks : Properties and Architecture

	Convolutional Neural Network (CNN)	Mixture Density Network (MDN)
Properties		
Architecture		
Objective function		

PR

Cenaero

Neural Networks : Properties and Architecture

	Convolutional Neural Network (CNN)	Mixture Density Network (MDN)
Properties	Invariant to translation	
Architecture		
Objective function		

PR

Cenaero

Neural Networks : Properties and Architecture

	Convolutional Neural Network (CNN)	Mixture Density Network (MDN)
Properties	Invariant to translation	Produce a distribution as a linear combination of Gaussian distributions: $p(y\,|\,\mathbf{x}) = \sum_{k=1}^{K} \pi_k \mathcal{N}(y; \mu_k, \sigma_k^2)$
Architecture		
Objective function		

PR

Neural Networks : Properties and Architecture

Convolutional Neural Network (CNN)

Mixture Density Network (MDN)

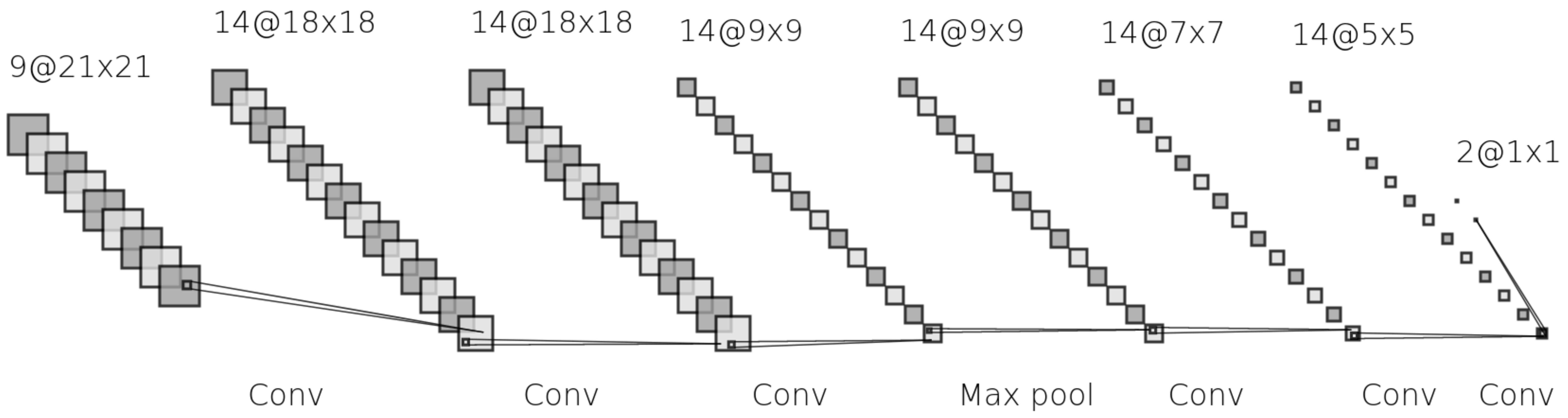
Properties

Invariant to translation

Produce a distribution as a linear combination of Gaussian distributions:

$$p(y\,|\,\mathbf{x}) = \sum_{k=1}^{K} \pi_k \mathcal{N}(y; \mu_k, \sigma_k^2)$$

Architecture



Objective function

PR

Cenaero

Neural Networks : Properties and Architecture

Convolutional Neural Network (CNN)

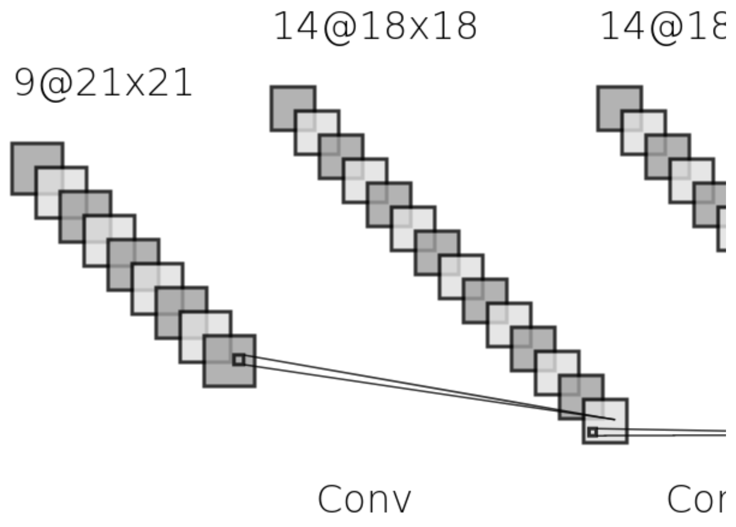
Mixture Density Network (MDN)

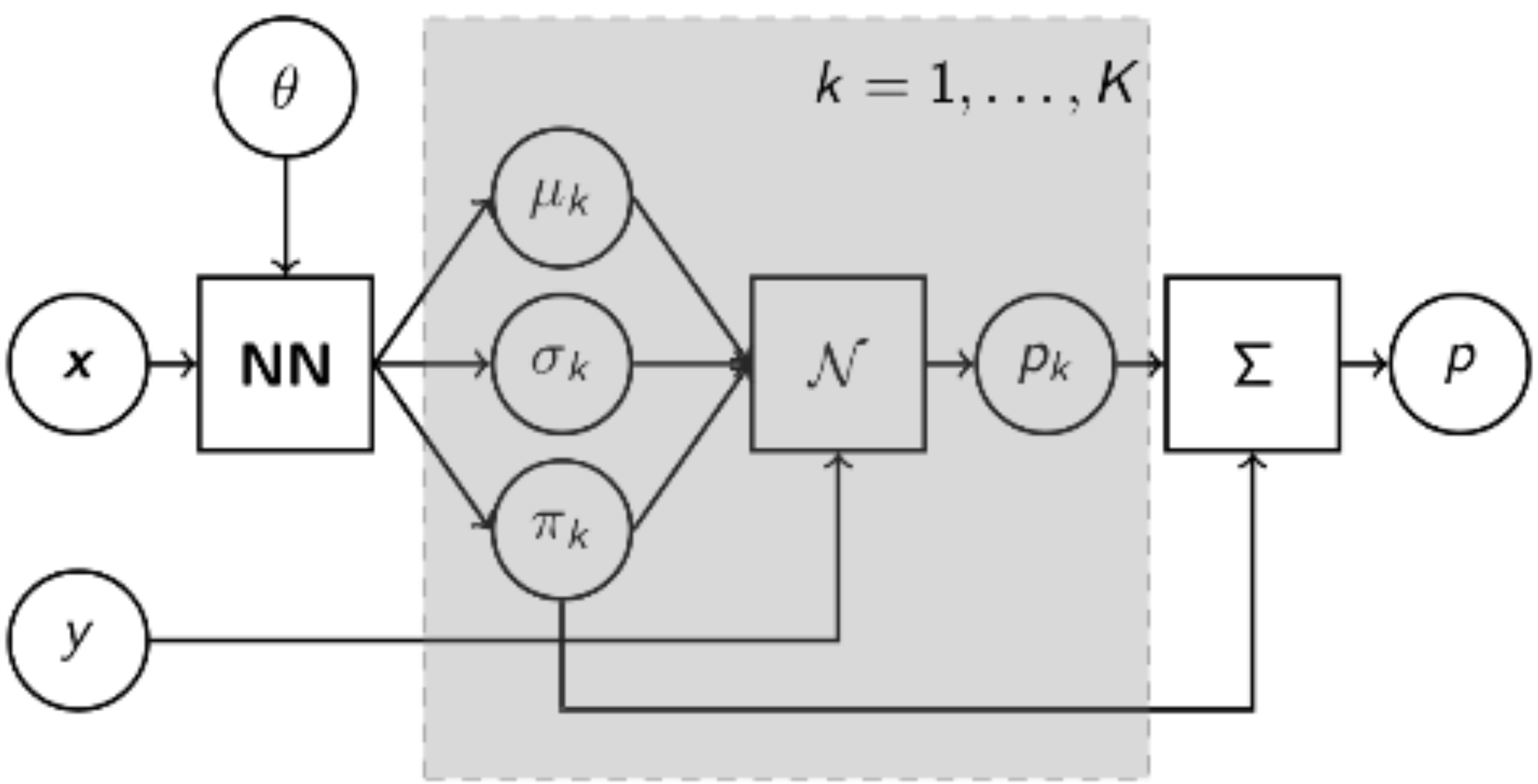
Properties

Invariant to translation

Produce a distribution as a linear combination of Gaussian distributions:

Architecture



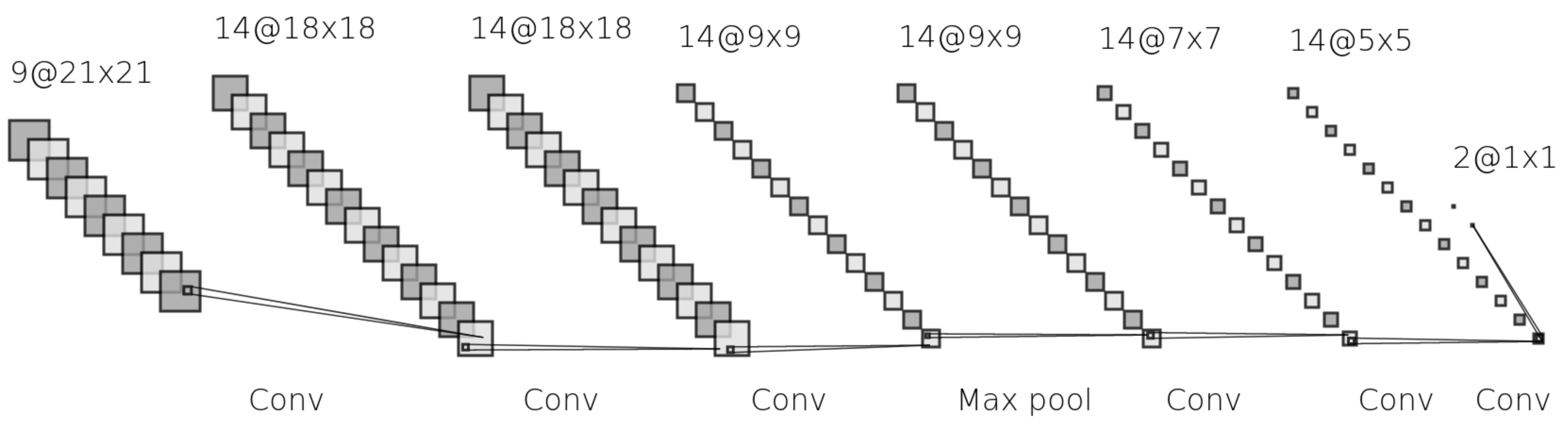
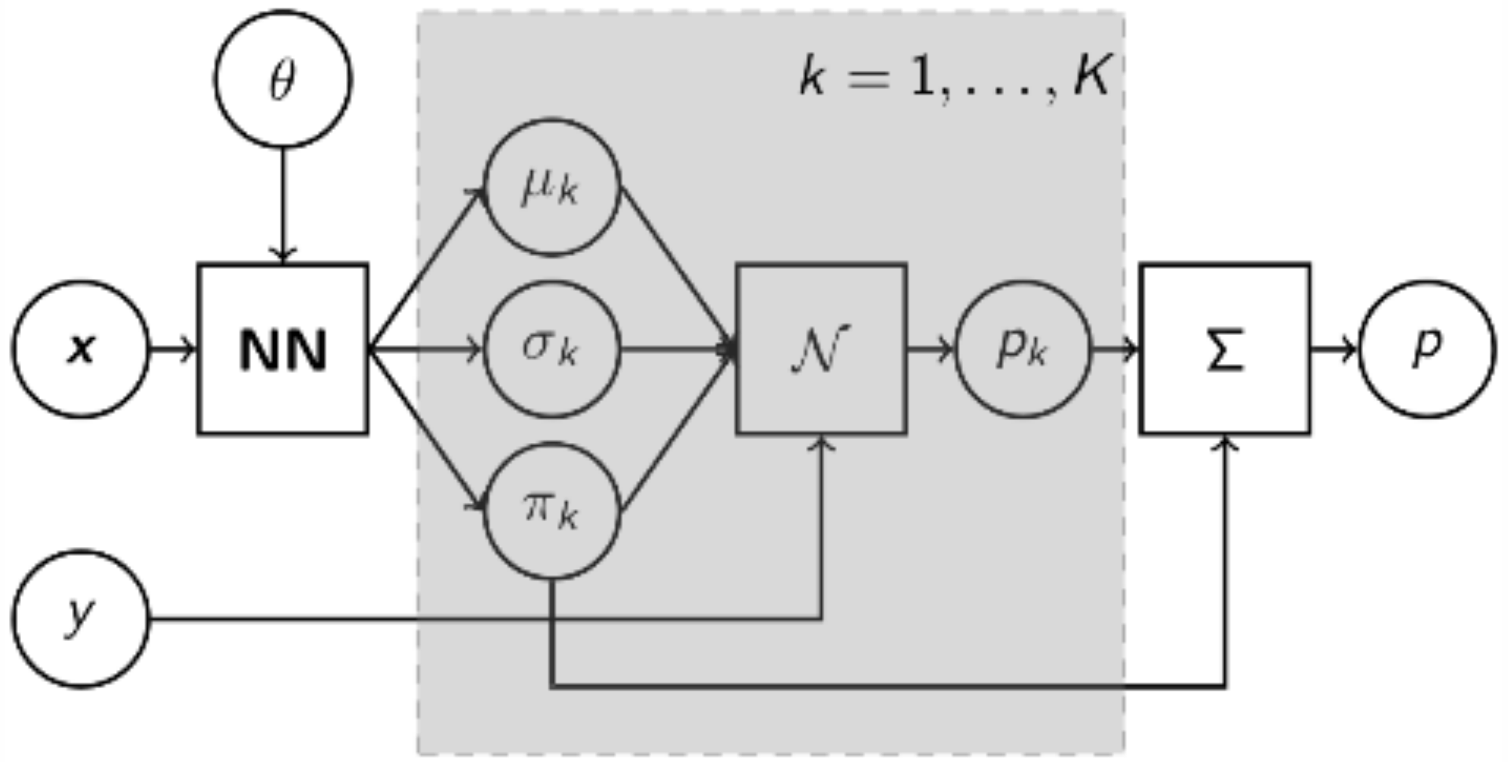


Objective function

PR

Cenaero

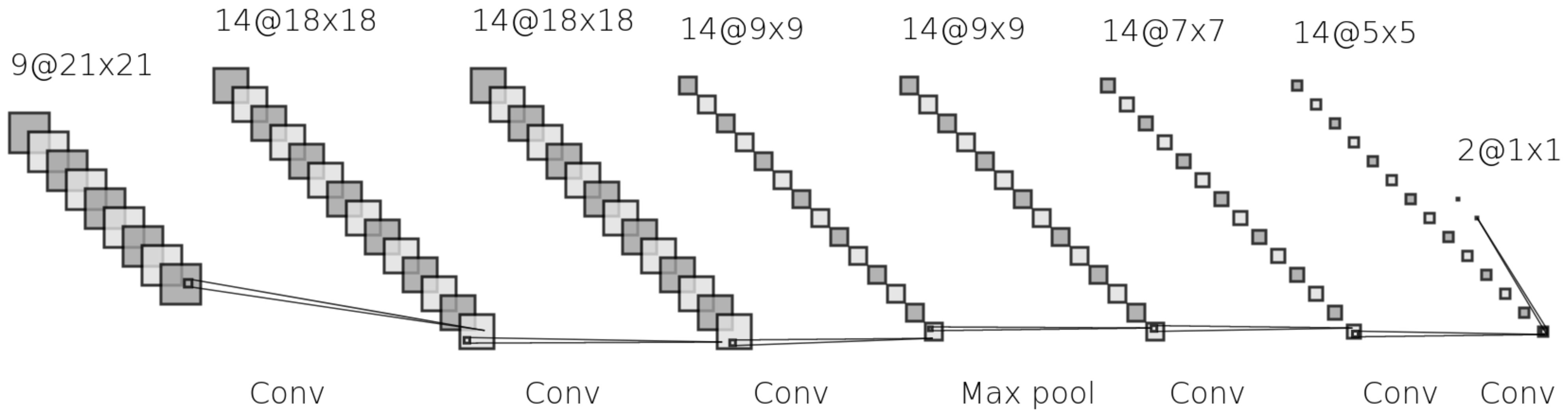
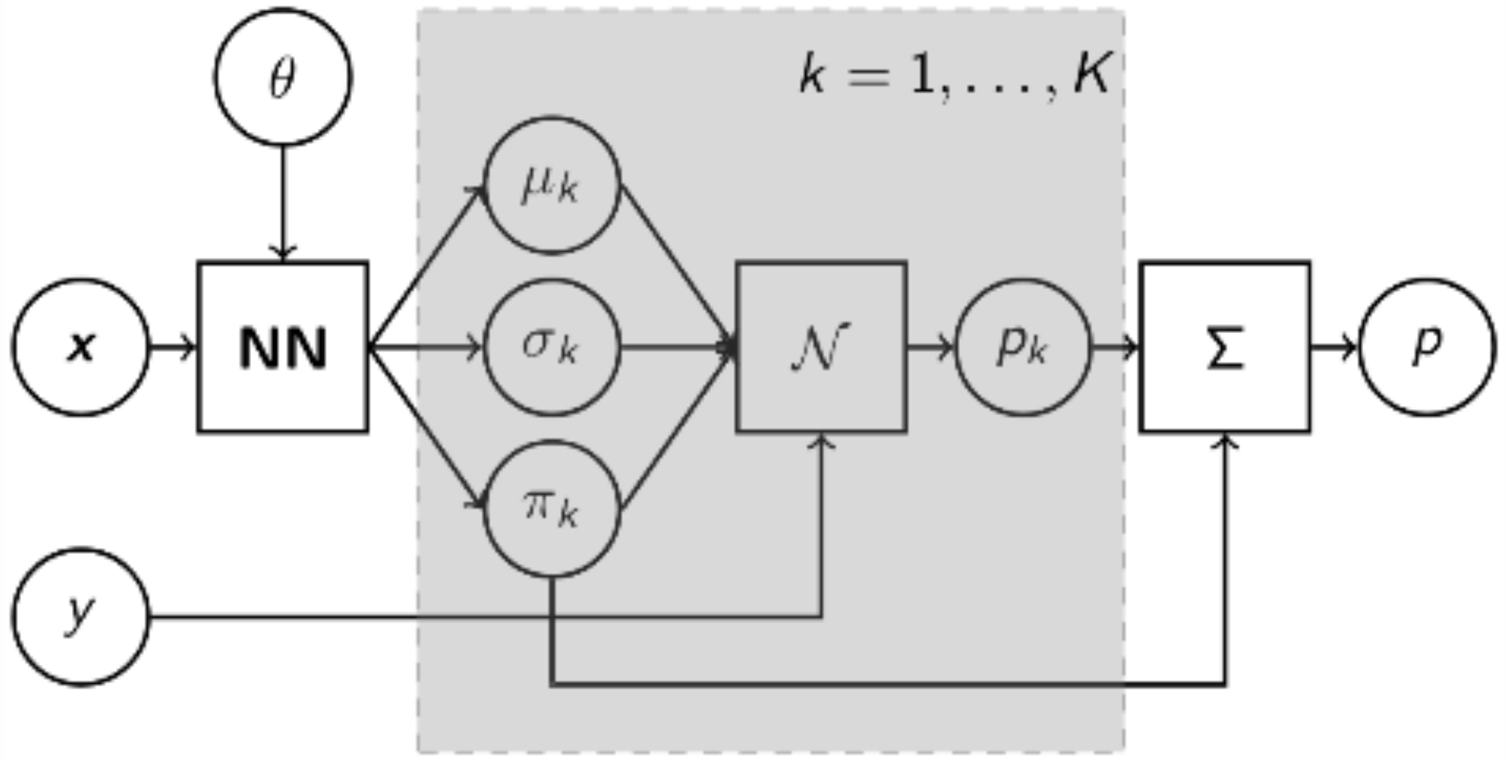
Neural Networks : Properties and Architecture

	Convolutional Neural Network (CNN)	Mixture Density Network (MDN)
Properties	Invariant to translation	Produce a distribution as a linear combination of Gaussian distributions: $$p(y\,|\,\mathbf{x}) = \sum_{k=1}^{K} \pi_k \mathcal{N}(y; \mu_k, \sigma_k^2)$$
Architecture		
Objective function	$$arg\min_{\theta} \sum_{\mathbf{x}_i, y_i \in \mathbf{d}} (y_i - \hat{y}_i)^2$$	

PR

Cenaero

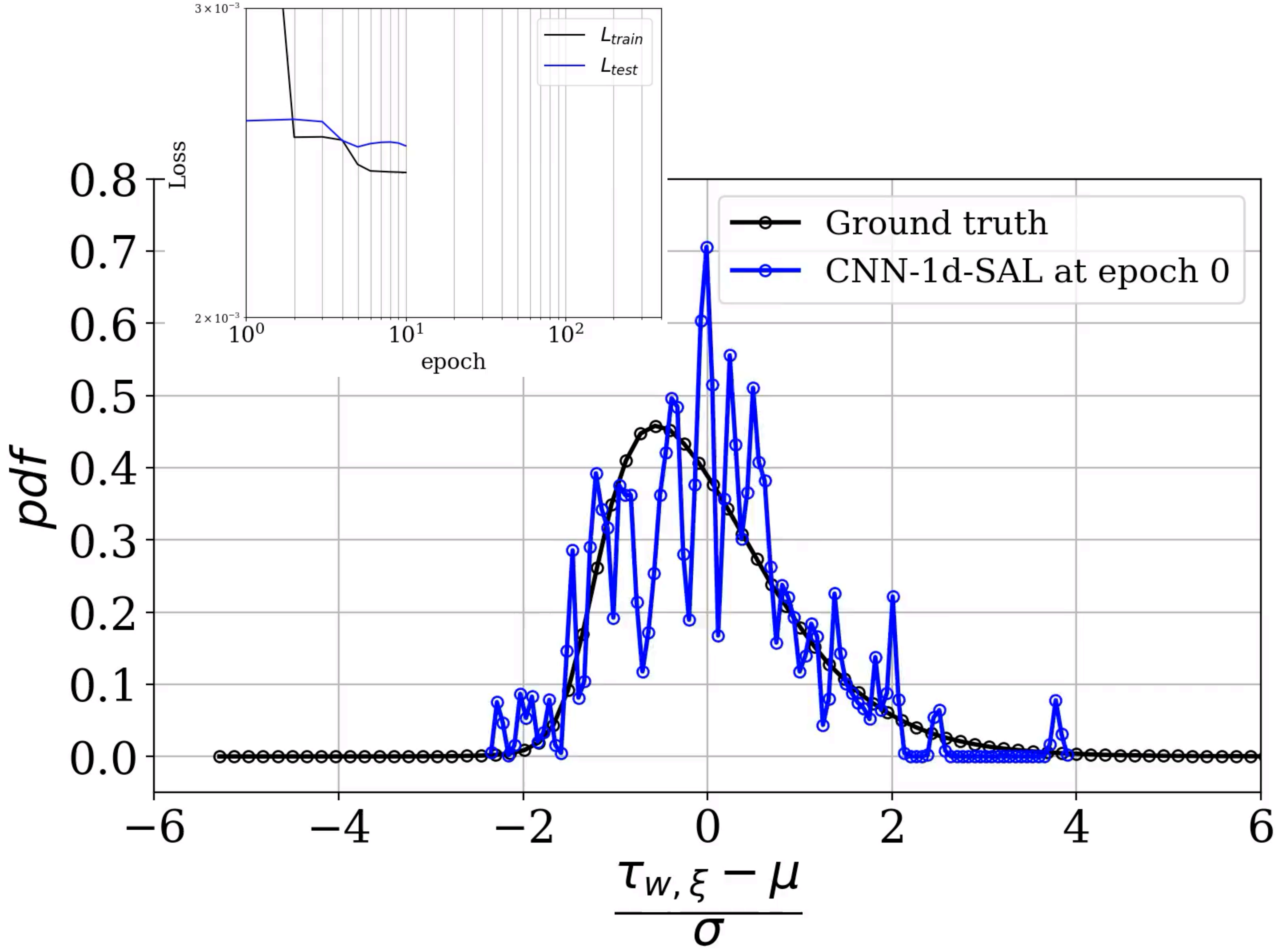
Neural Networks : Properties and Architecture

	Convolutional Neural Network (CNN)	Mixture Density Network (MDN)
Properties	Invariant to translation	Produce a distribution as a linear combination of Gaussian distributions: $p(y\,\vert\,\mathbf{x}) = \sum_{k=1}^{K} \pi_k \mathcal{N}(y; \mu_k, \sigma_k^2)$
Architecture		
Objective function	$arg\min_{\theta} \sum_{\mathbf{x}_i, y_i \in \mathbf{d}} (y_i - \hat{y}_i)^2$	$arg\min_{\theta} \sum_{\mathbf{x}_i, y_i \in \mathbf{d}} \frac{\left(y_i - \mu(\mathbf{x}_i)\right)^2}{2\sigma(\mathbf{x}_i)} + \log(\sigma(\mathbf{x}_i)) + C$

PR

Cenaero

Training a CNN-1d-SAL on the channel $Re_\tau = 950$

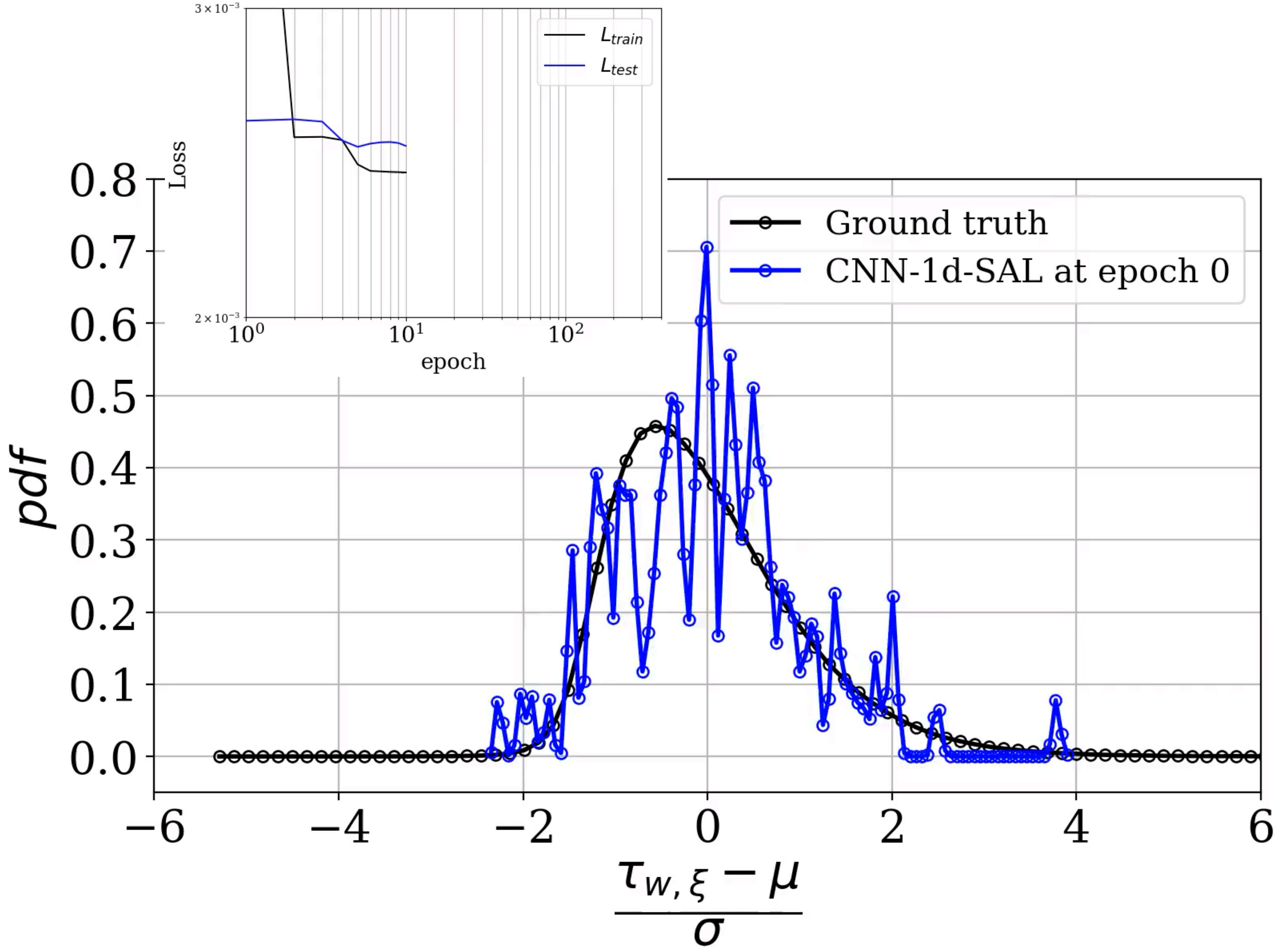


#Parameters: 6,440
Learning rate: 0.005
Batch size: 512
Database size: 181,370
Training time: 3h33min
#Epoch: 400

PROD-F-015-02

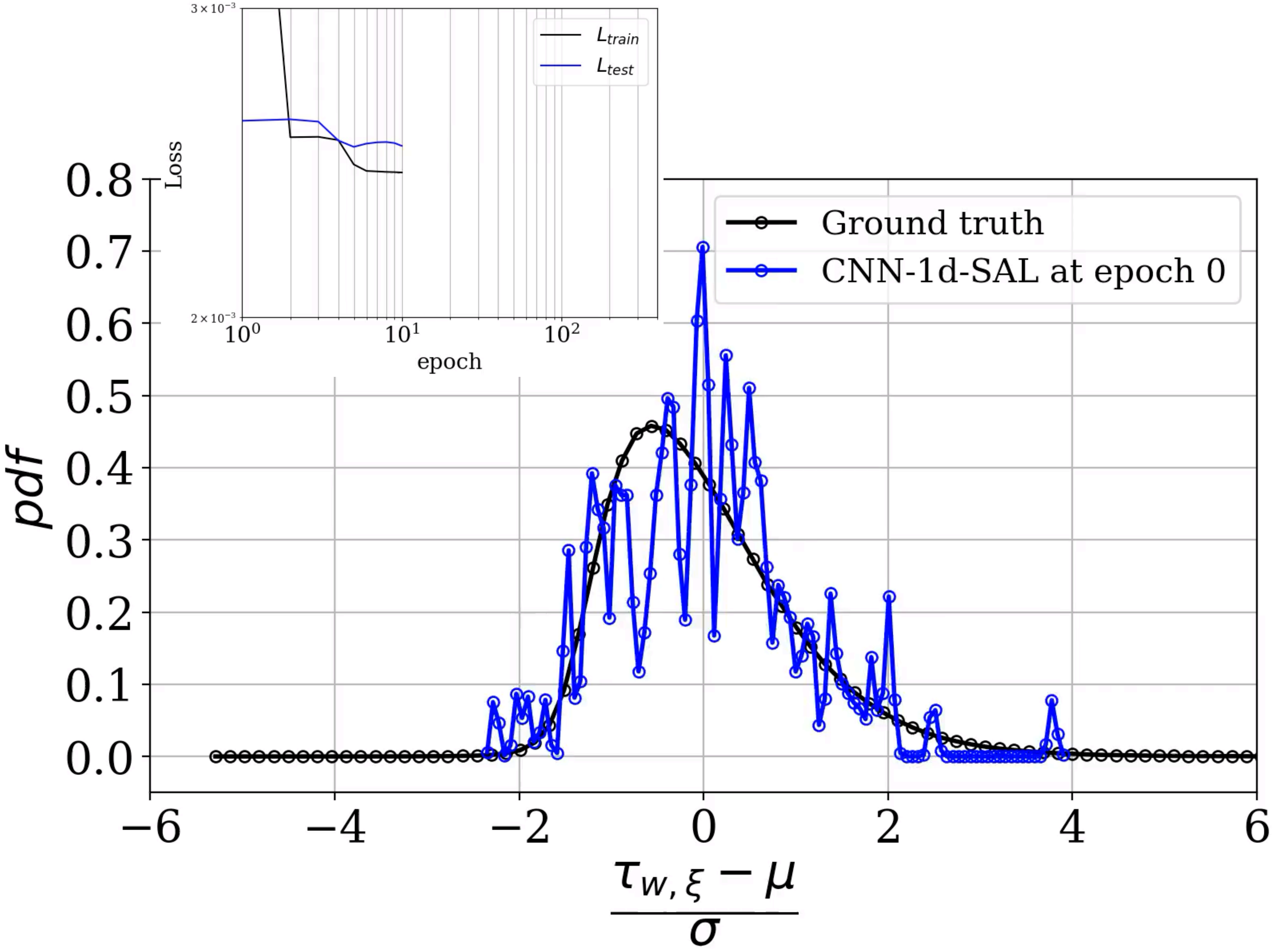
Cenaero

Training a CNN-1d-SAL on the channel $Re_\tau = 950$



#Parameters: 6,440
Learning rate: 0.005
Batch size: 512
Database size: 181,370
Training time: 3h33min
#Epoch: 400

Training a CNN-1d-SAL on the channel $Re_\tau = 950$



#Parameters: 6,440
Learning rate: 0.005
Batch size: 512
Database size: 181,370
Training time: 3h33min
#Epoch: 400

- Difficulties to capture the **negative values** of the normalized distribution during the first epochs
- Match the **mean** of the distribution
- Not able to capture the **variance** and the **skewness**

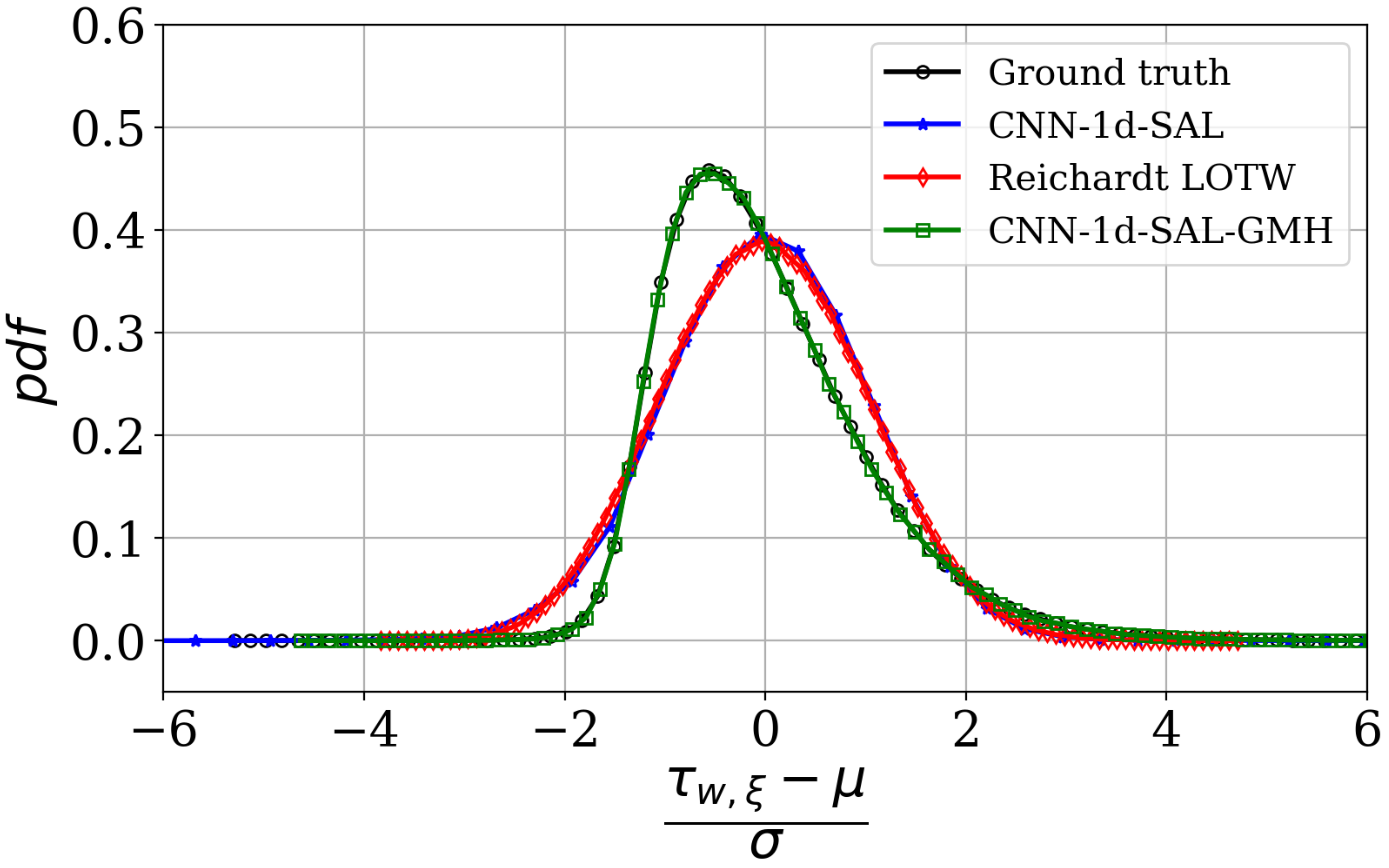
PROD-F-015-02

Cenaero

Training a CNN-1d-SAL on the channel $Re_\tau = 950$

PROD-F-015-02

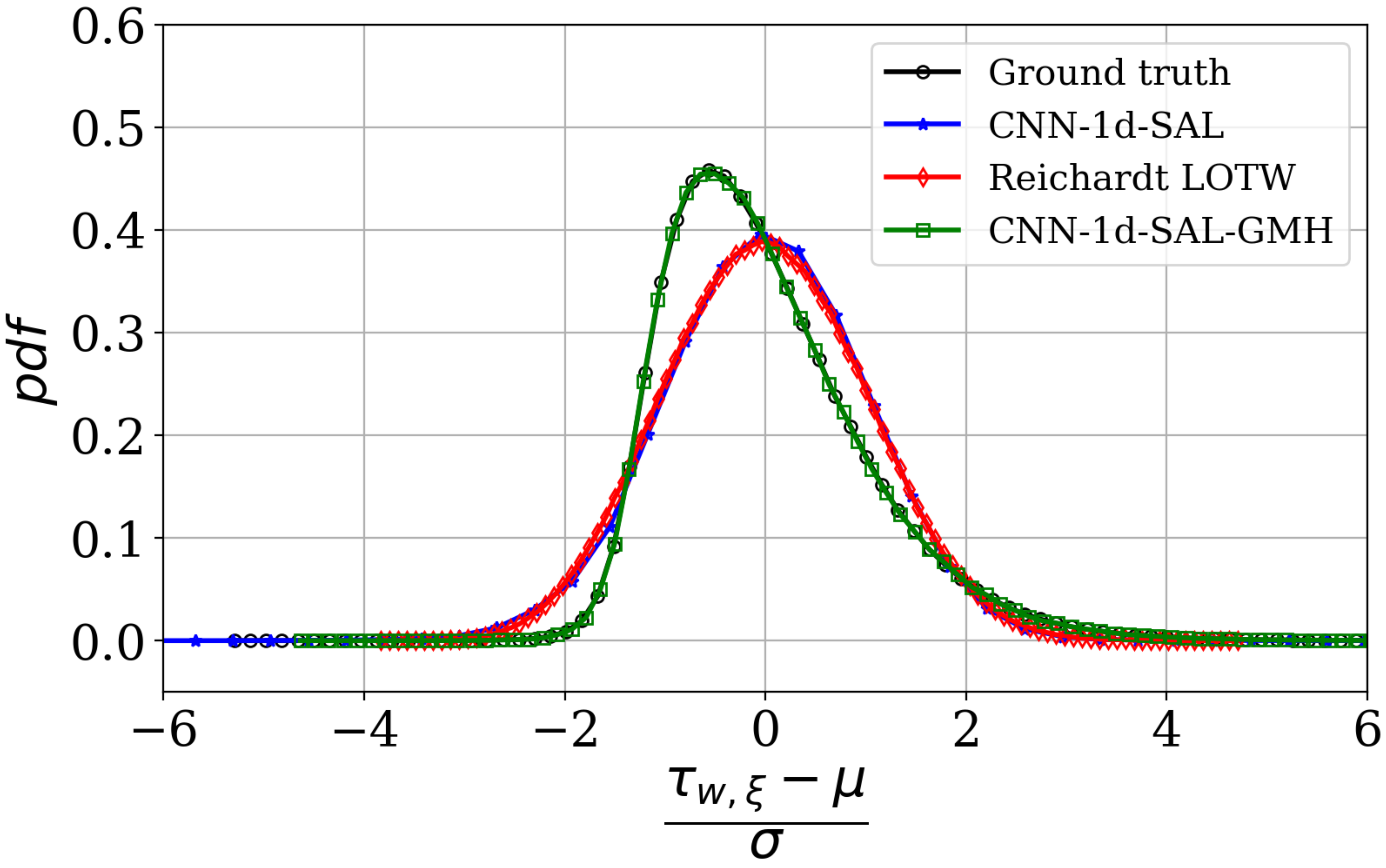
Training a CNN-1d-SAL on the channel $Re_\tau = 950$



PROD-F-015-02

Cenaero

Training a CNN-1d-SAL on the channel $Re_\tau = 950$

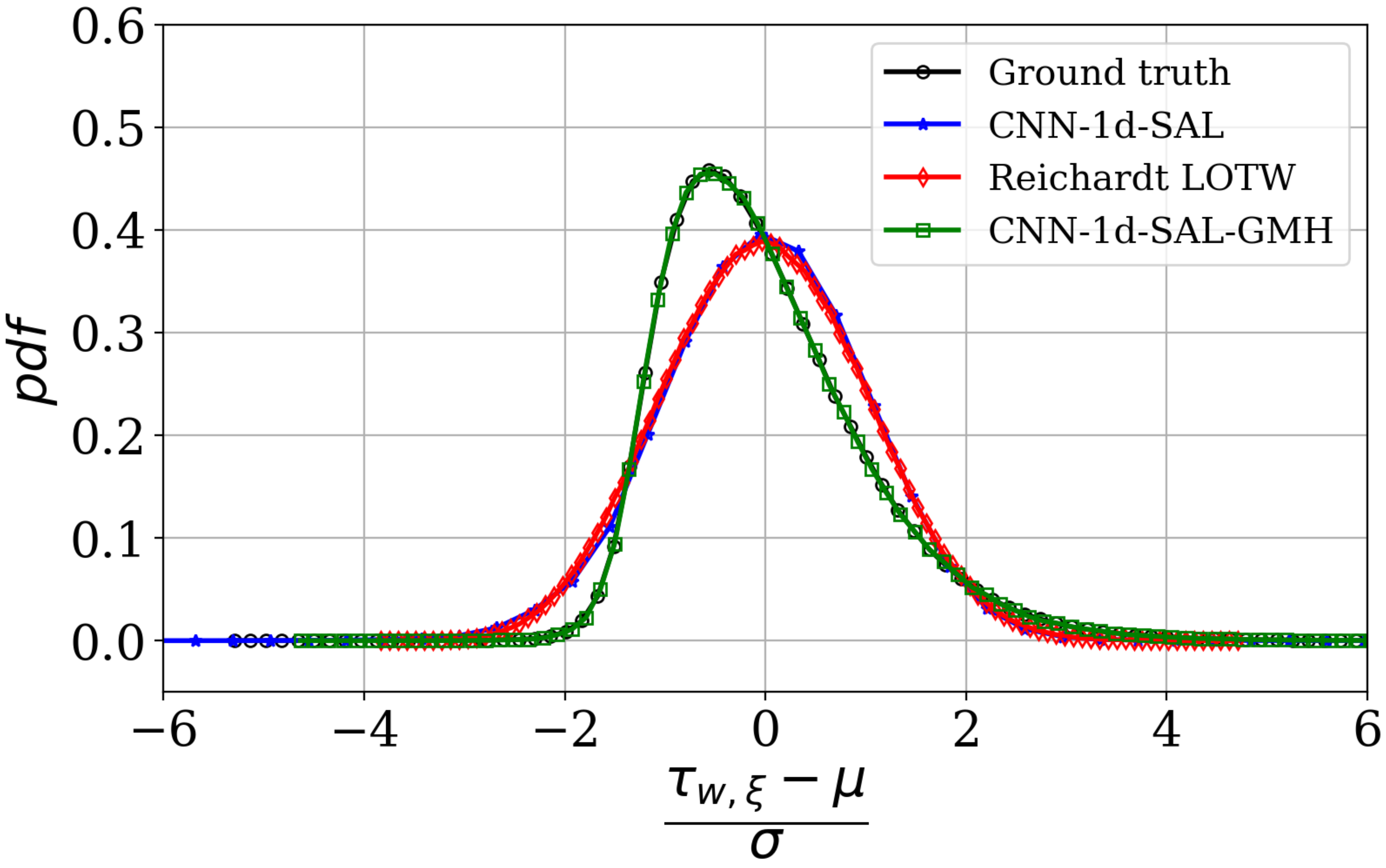


	μ	σ	κ
CNN-1d-SAL	1.00798	0.1265	-0.0508
CNN-1d-SAL-GMH	1.0087	0.4242	1.0242
Reichardt LOTW	1.0206	0.1939	0.0689
wrLES	1.0085	0.4234	1.0156

PROD-F-015-02

Cenaero

Training a CNN-1d-SAL on the channel $Re_\tau = 950$



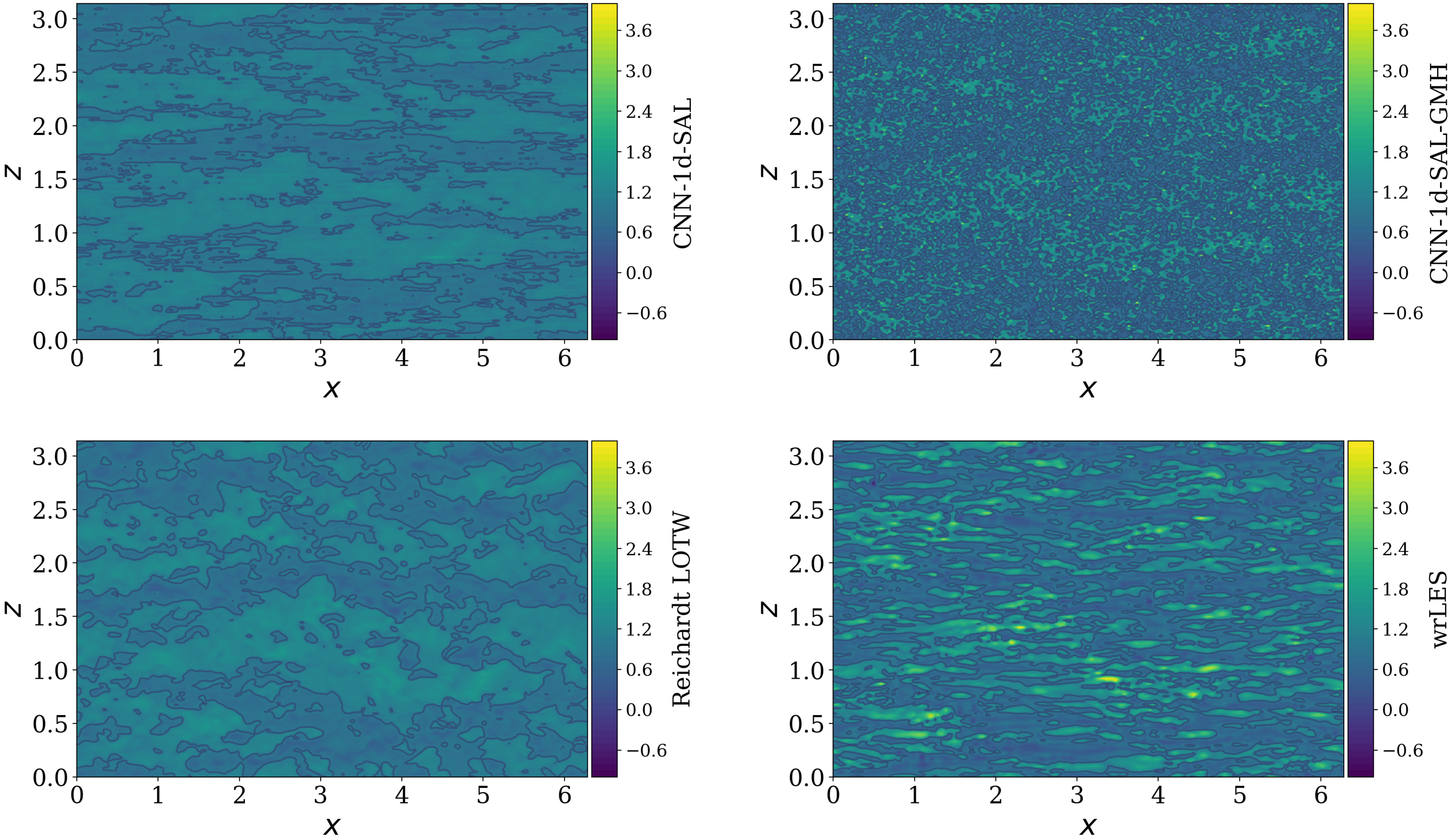
	μ	σ	κ
CNN-1d-SAL	1.00798	0.1265	-0.0508
CNN-1d-SAL-GMH	1.0087	0.4242	1.0242
Reichardt LOTW	1.0206	0.1939	0.0689
wrLES	1.0085	0.4234	1.0156

The Mixture Density Network is the only one able to capture the **first three moments of the true distribution.**

PROD-F-015-02

Cenaero

Instantaneous contours of $\tau_{w,\xi}$ on the channel $Re_\tau = 950$



CNN-1d-SAL and Reichardt are the only one to reproduce the long streamwise streaks of the wrLES field.

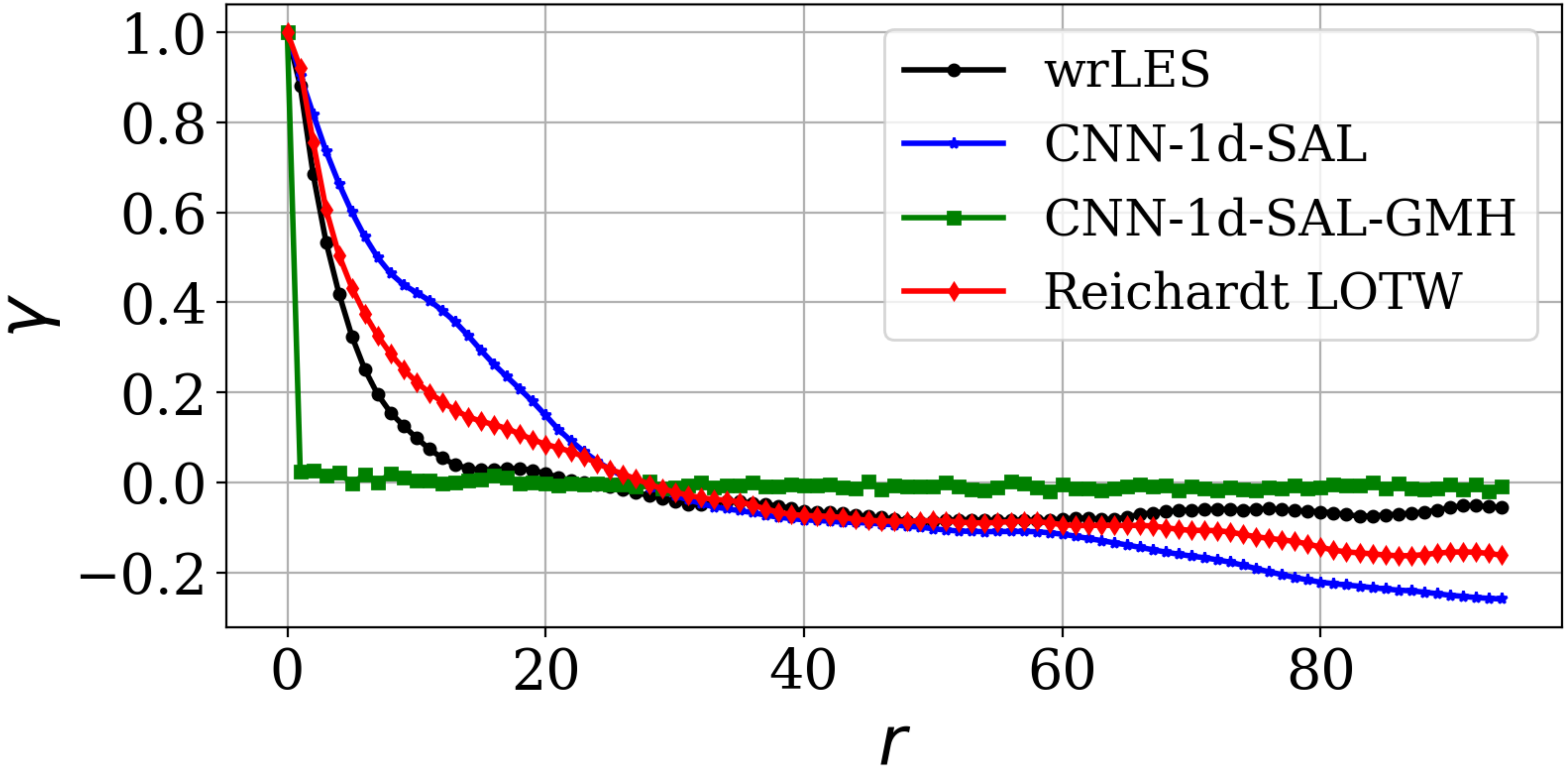
PROD-F-015-02

Cenaero

Correlation and PSD of $\tau_{w,\xi}$ on the channel $Re_\tau = 950$

PROD-F-015-02

Correlation and PSD of $\tau_{w,\xi}$ on the channel $Re_\tau = 950$

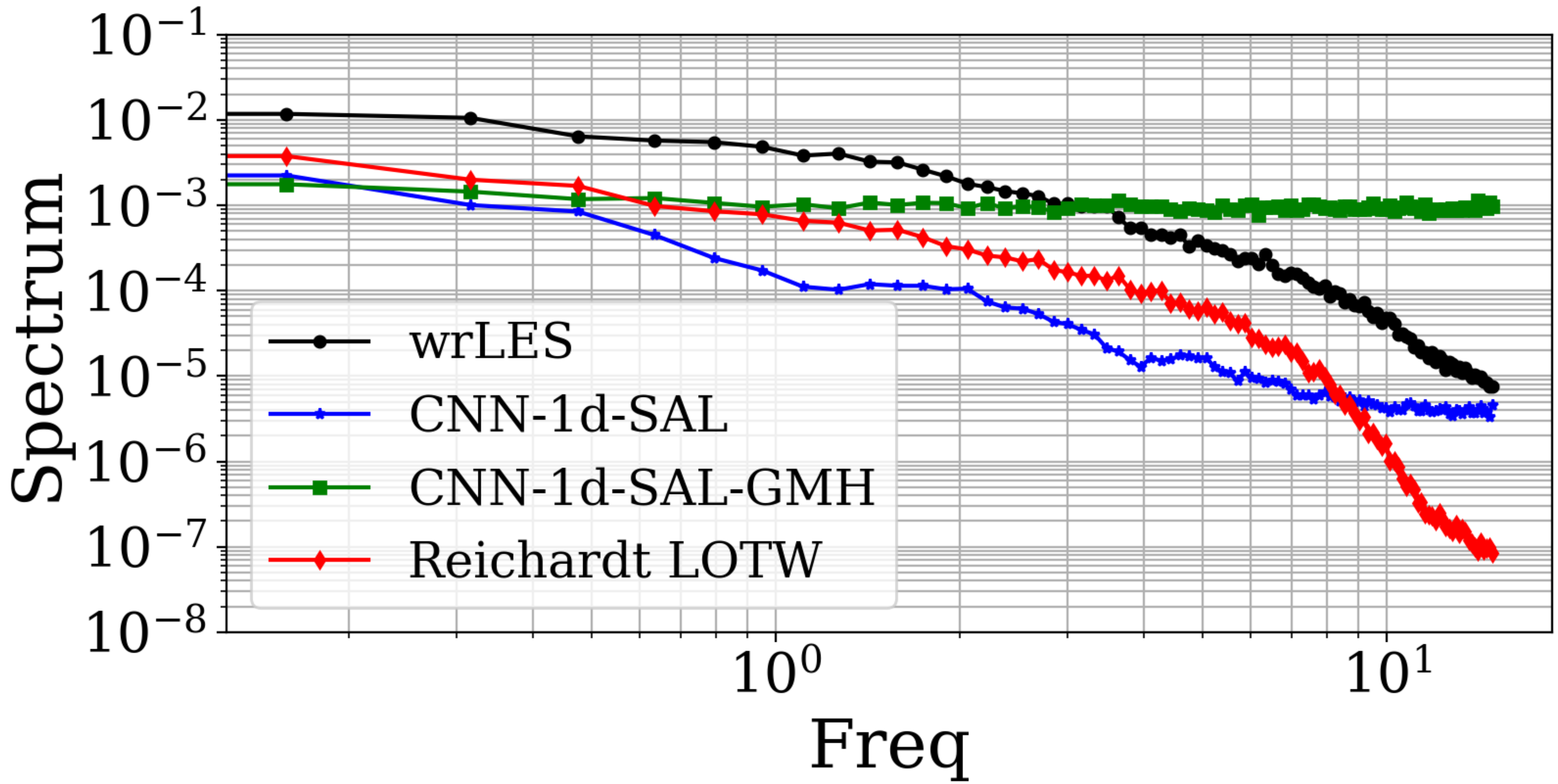


- The correlation measured on the Reichardt predictions is the one that closely match the true correlation function.
- The Mixture Density Network completely misses the correlation and predicts independent samples.

PROD-F-015-02

Cenaero

Correlation and PSD of $\tau_{w,\xi}$ on the channel $Re_\tau = 950$

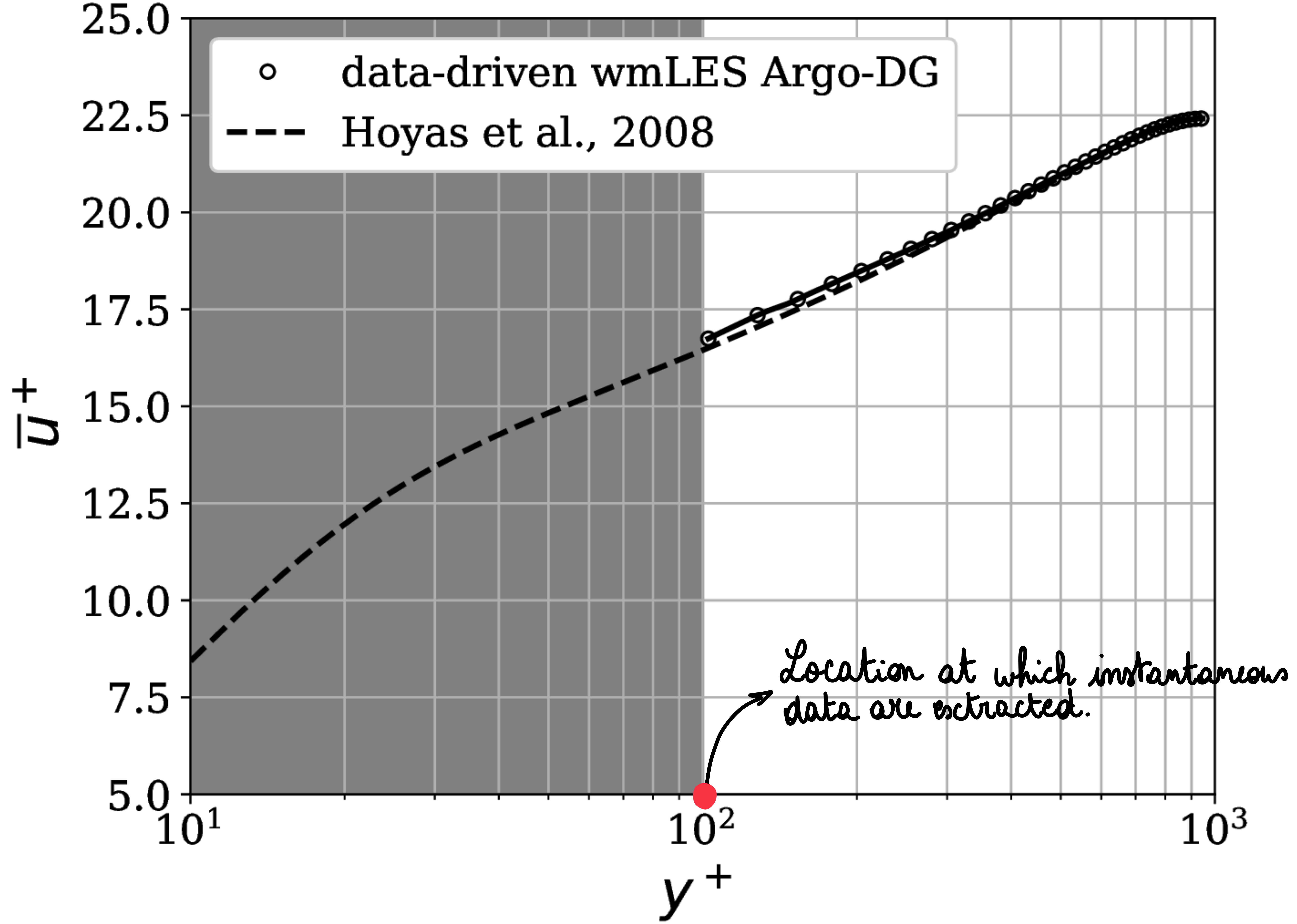


- The predictions under-estimates the wrLES PSD (Power Spectral Density) of $\tau_{w,\xi}$
- The Mixture Density Network puts the same amount of energy at every scales.

PROD-F-015-02

Cenaero

A posteriori validation on the channel $Re_\tau = 950$



$$\frac{dof_{wmLES}}{dof_{wrLES}} = 30$$

Mesh: $n_x \times n_y \times n_z = 26 \times 13 \times 13$

to get

$(\Delta x^+, \Delta y^+, \Delta z^+) = (76{,}25{,}76)$

No stretching is applied at the wall

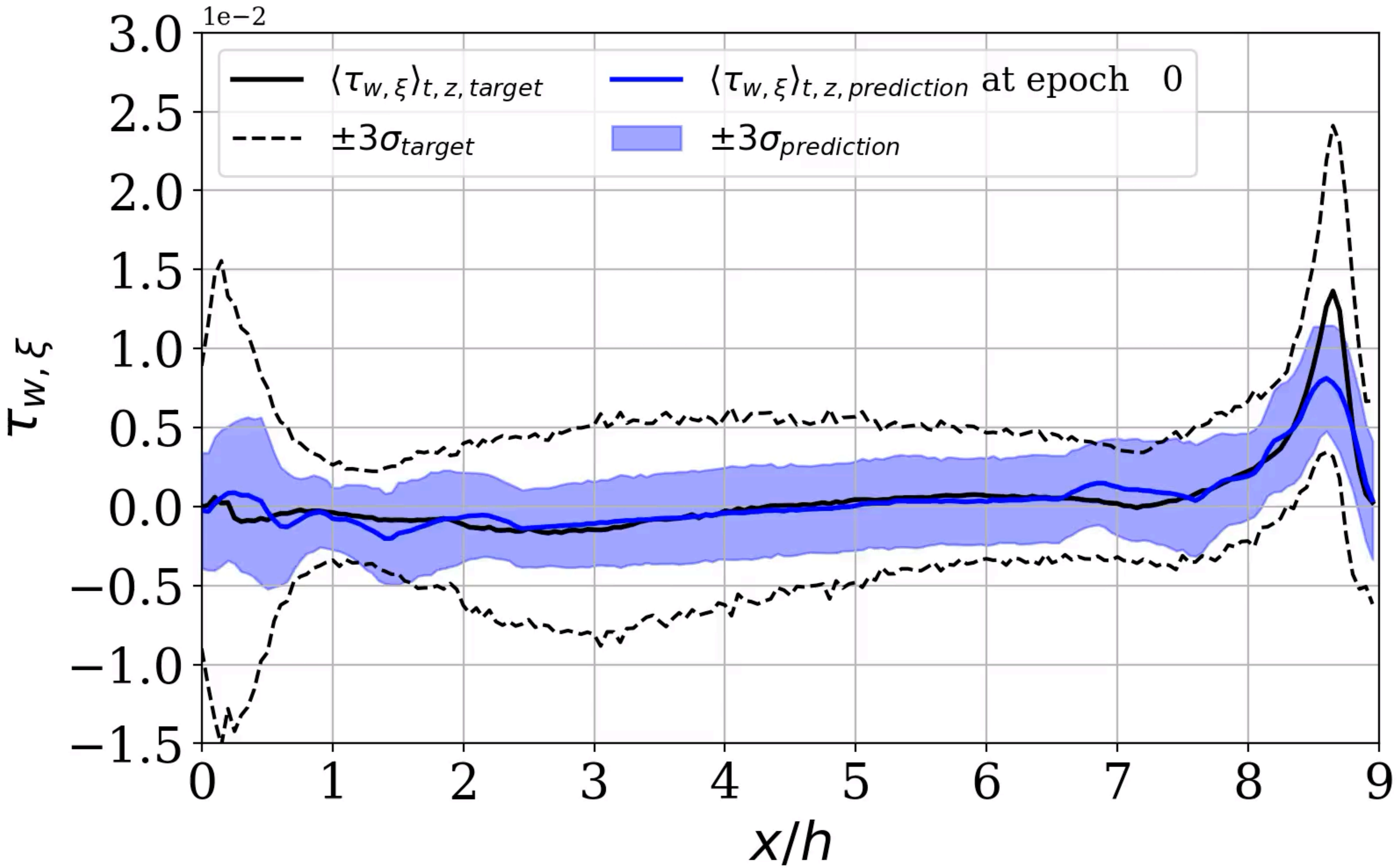
Model: CNN-1d-SAL

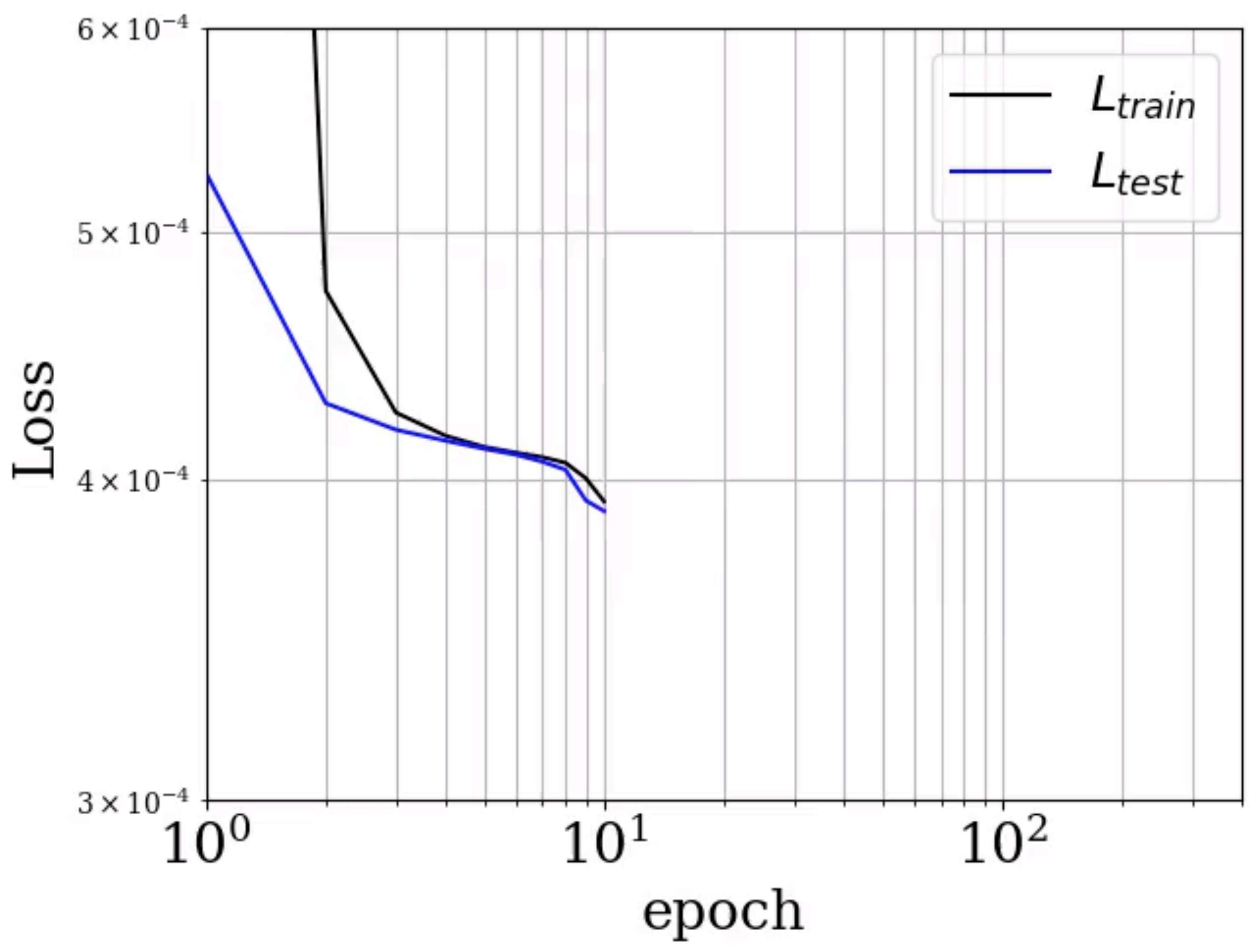
Small **overpredicts** of less than 5% near the gray area.

PROD-F-015-02

Cenaero

Training a CNN-1d-SAL on the periodic hill $Re_b = 10595$

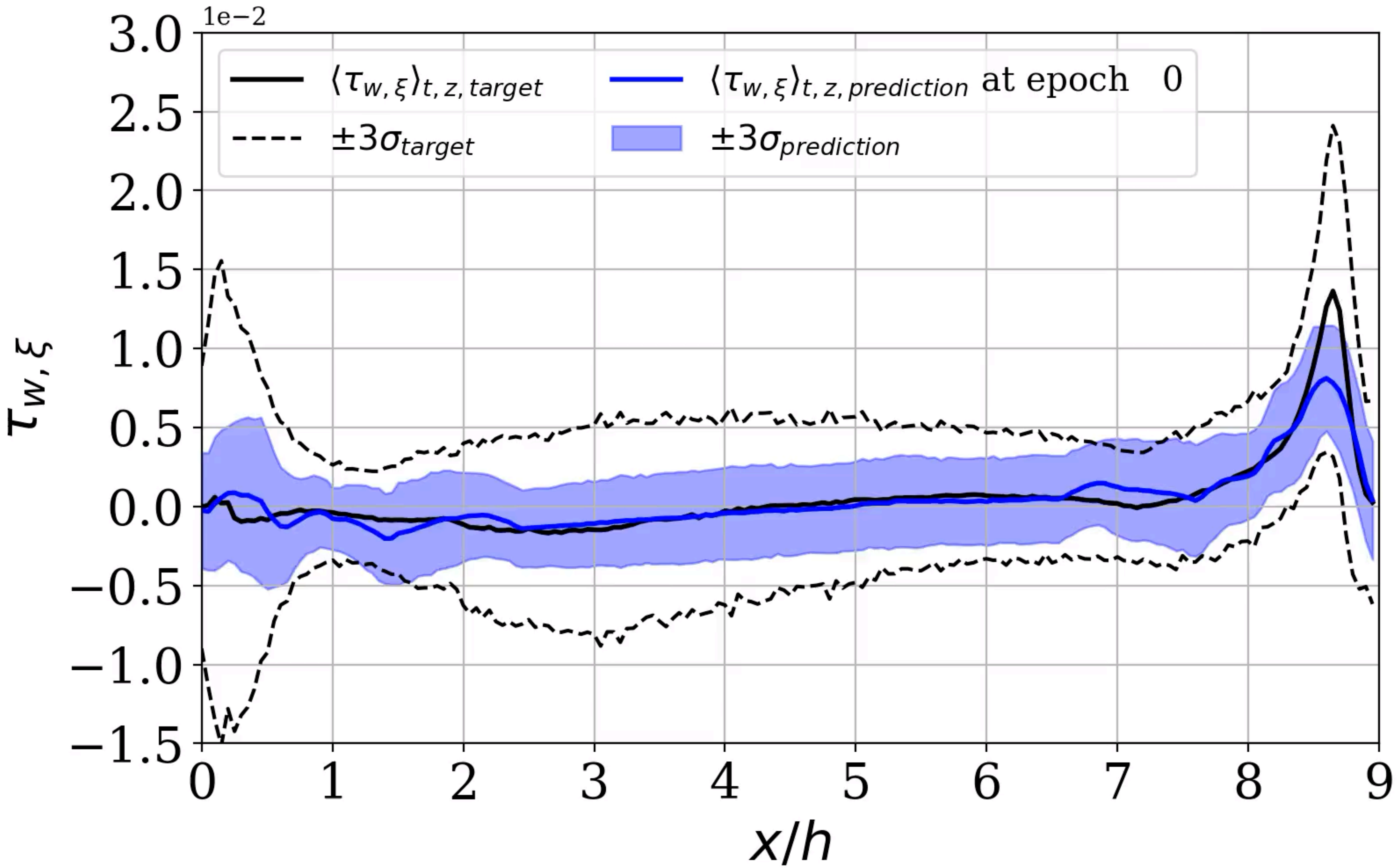


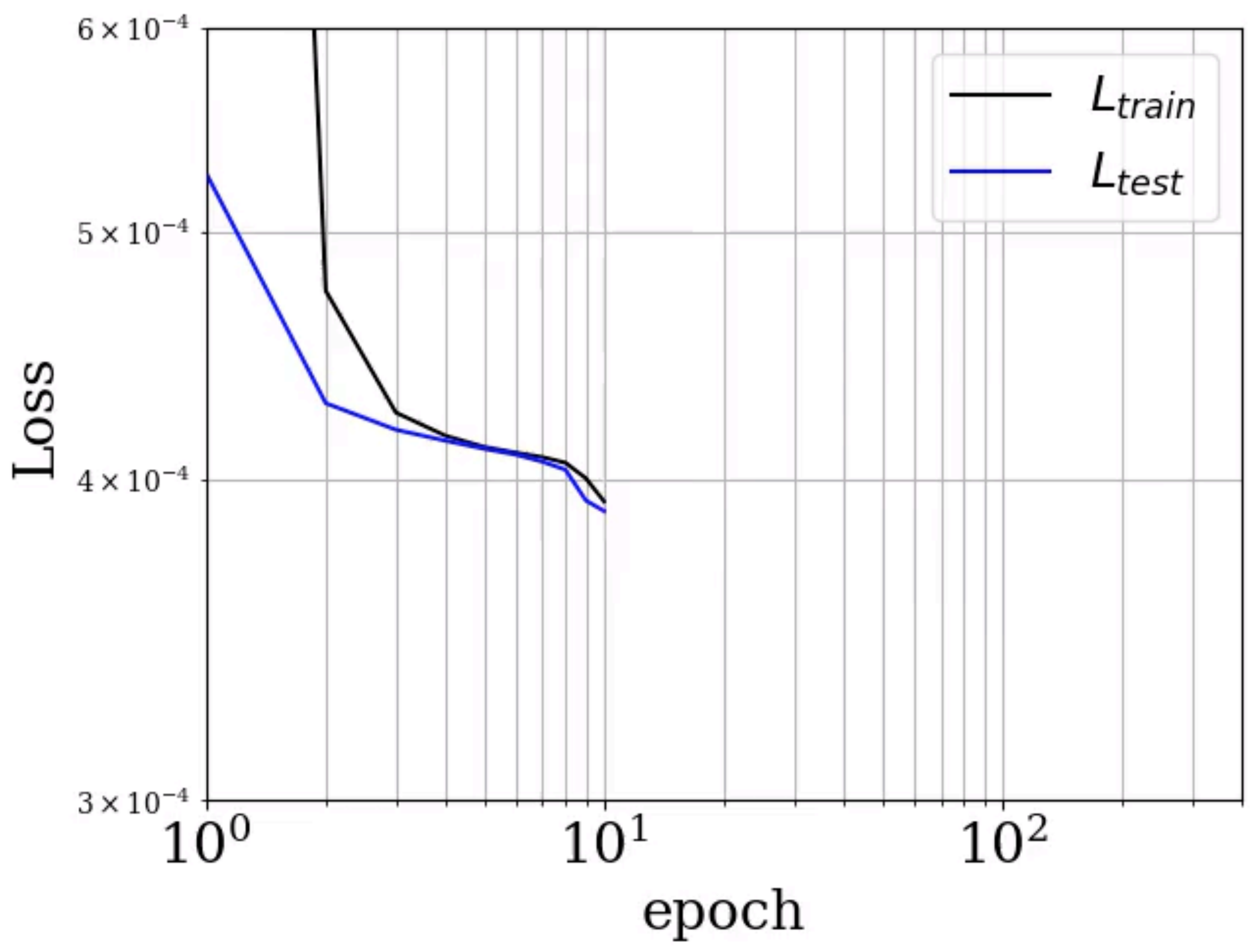


#Parameters: 6,440
Learning rate: 0.001
Batch size: 1024
Database size: 2,736,00
Training time: 5h13min
#Epoch: 400

Cenaero

Training a CNN-1d-SAL on the periodic hill $Re_b = 10595$

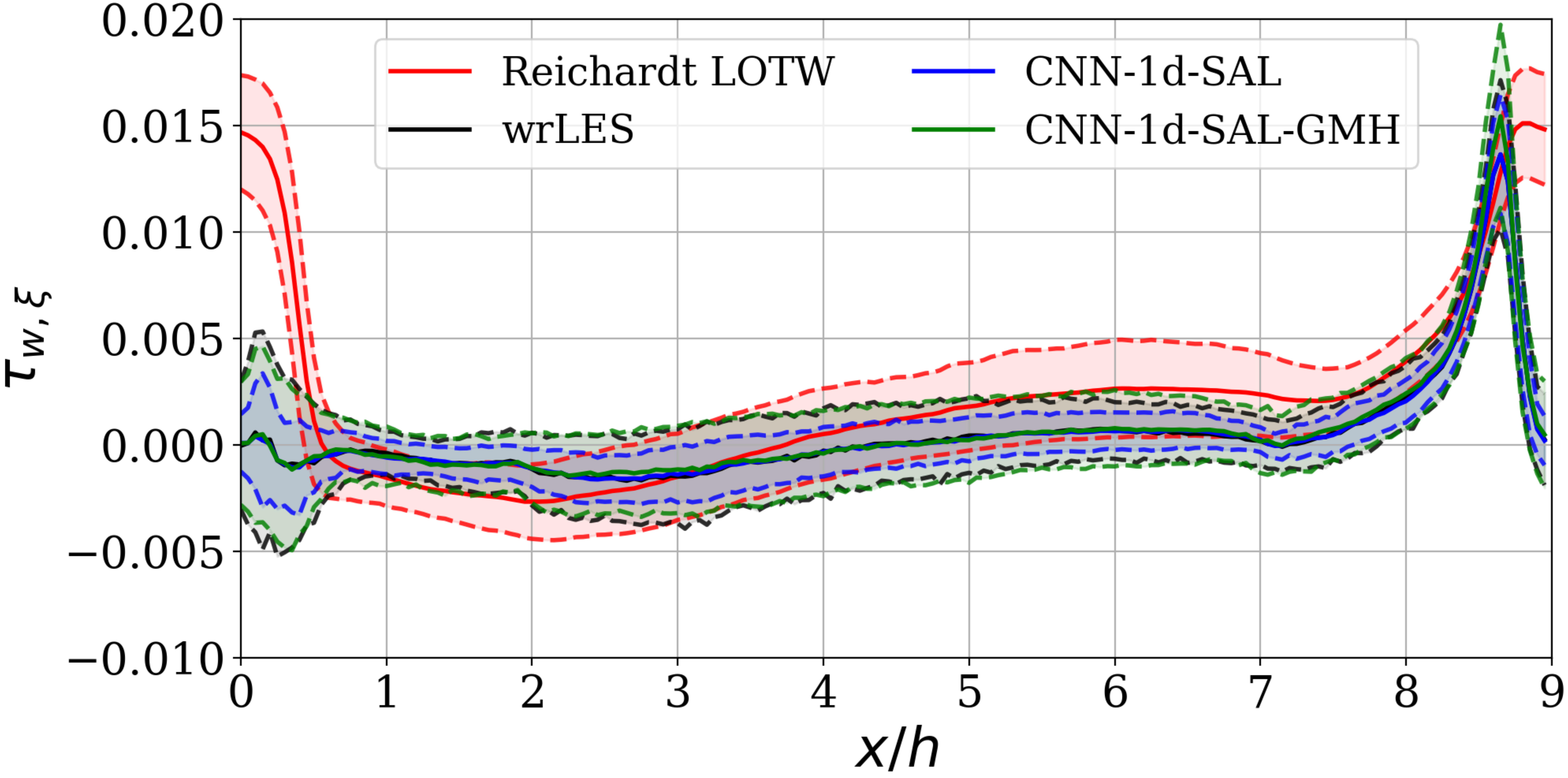




#Parameters: 6,440
Learning rate: 0.001
Batch size: 1024
Database size: 2,736,00
Training time: 5h13min
#Epoch: 400

Cenaero

Training a CNN-1d-SAL on the periodic hill $Re_b = 10595$

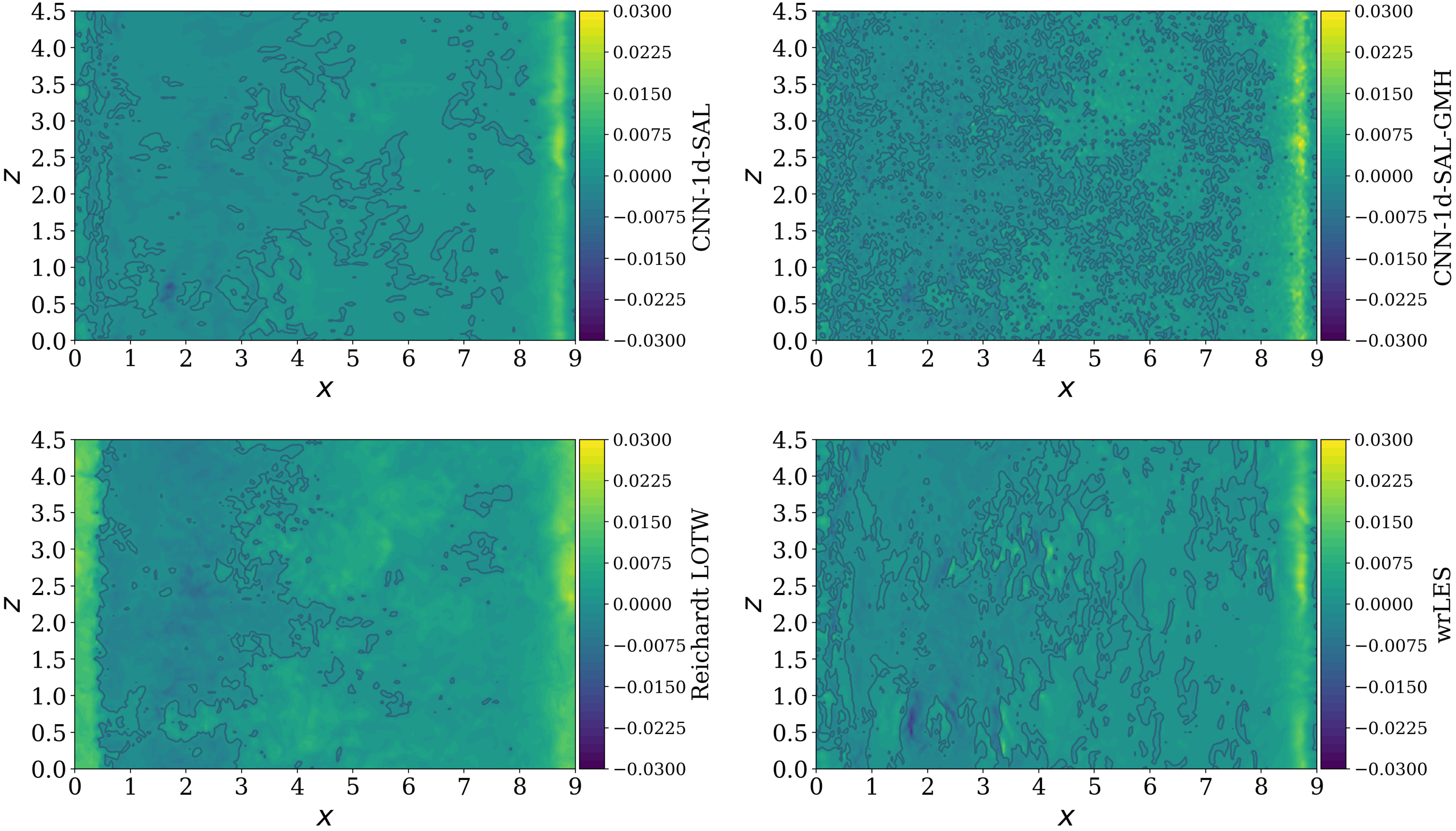


- **CNN-1d-SAL** predicts well the mean but mispredicts the variance especially in the **separation vicinity.**
- **CNN-1d-SAL-GMH** predicts well both the mean and the variance.
- **Reichardt** totally mispredicts the separation and never matches the mean behavior on the lower wall.

PROD-F.

Cenaero

Instantaneous contours of $\tau_{w,\xi}$ on the periodic hill $Re_b = 10595$



CNN-1d-SAL gives good structures sizes while the Reichardt LOTW misses the separation.

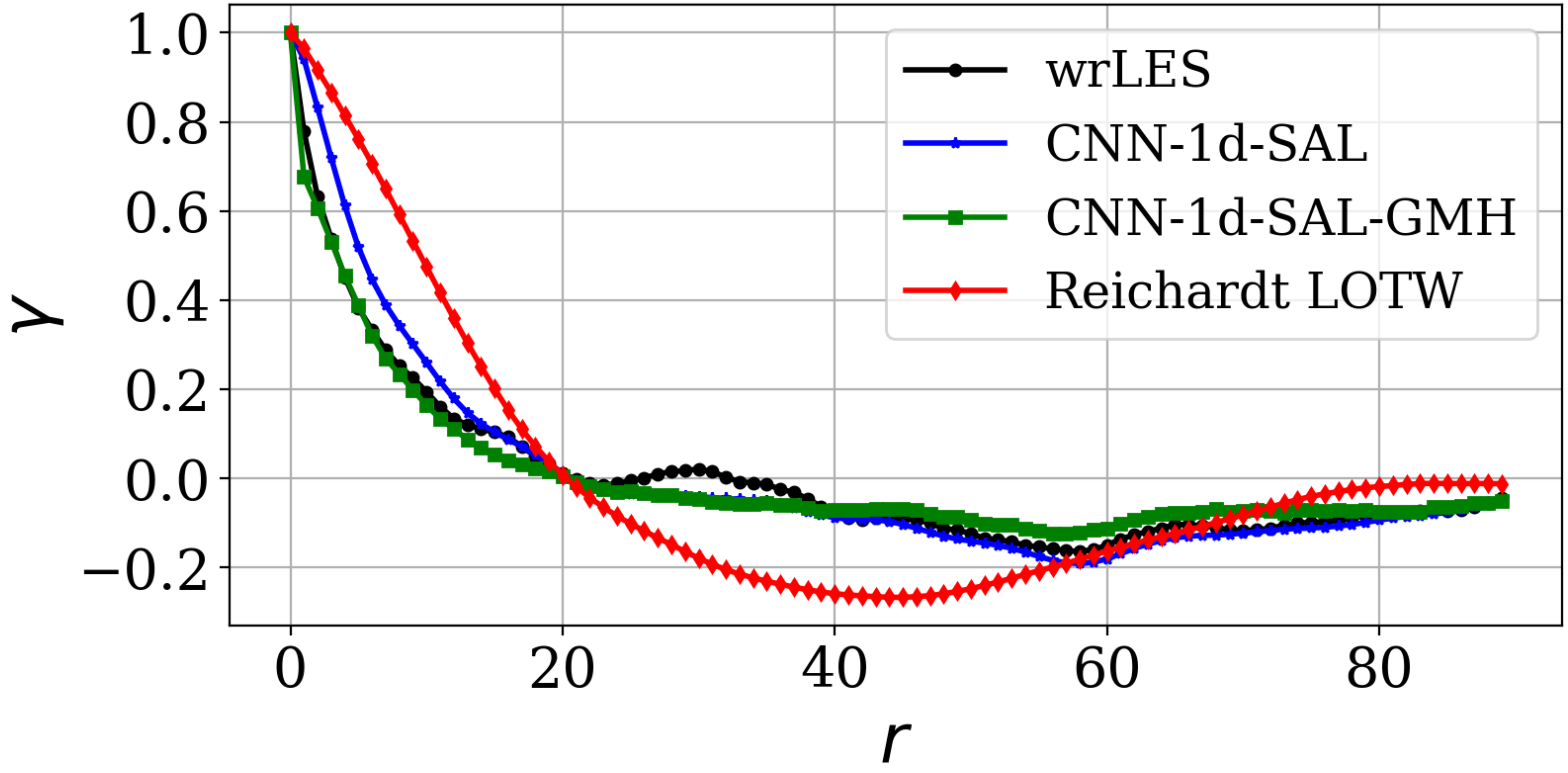
Note that the **CNN-1d-SAL-GMH** produces smaller stuctures.

PROD-F-015-02

Cenaero

Correlation and PSD of $\tau_{w,\xi}$ on the periodic hill $Re_b = 10595$

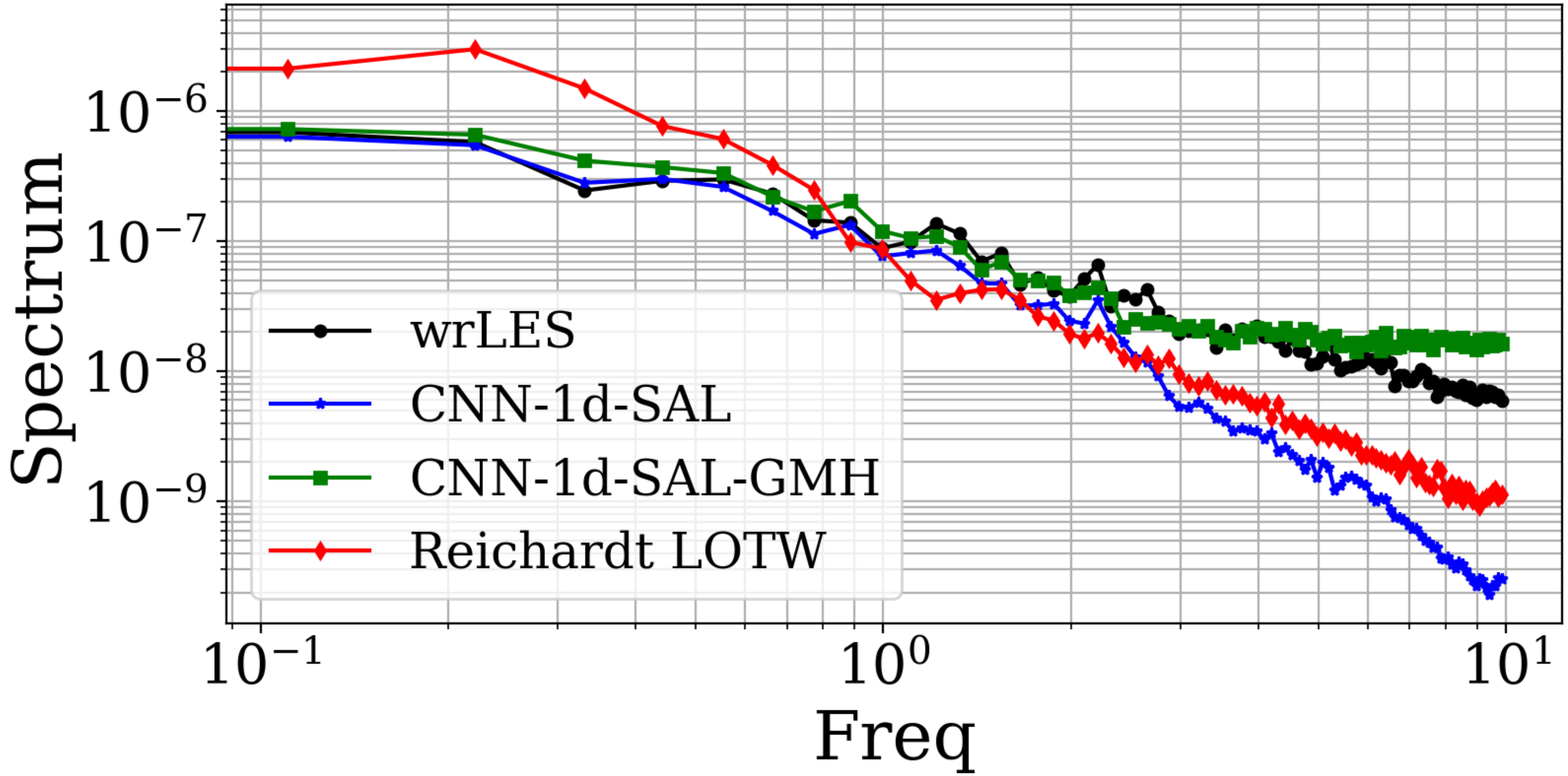
Correlation and PSD of $\tau_{w,\xi}$ on the periodic hill $Re_b = 10595$



- The correlation measured on the **Mixture Density Network** predictions is the one that matches the wrLES correlation function.

Cenaero

Correlation and PSD of $\tau_{w,\xi}$ on the periodic hill $Re_b = 10595$



- The **Mixture Density Network** predicts well the PSD.

PROD-F-015-0?

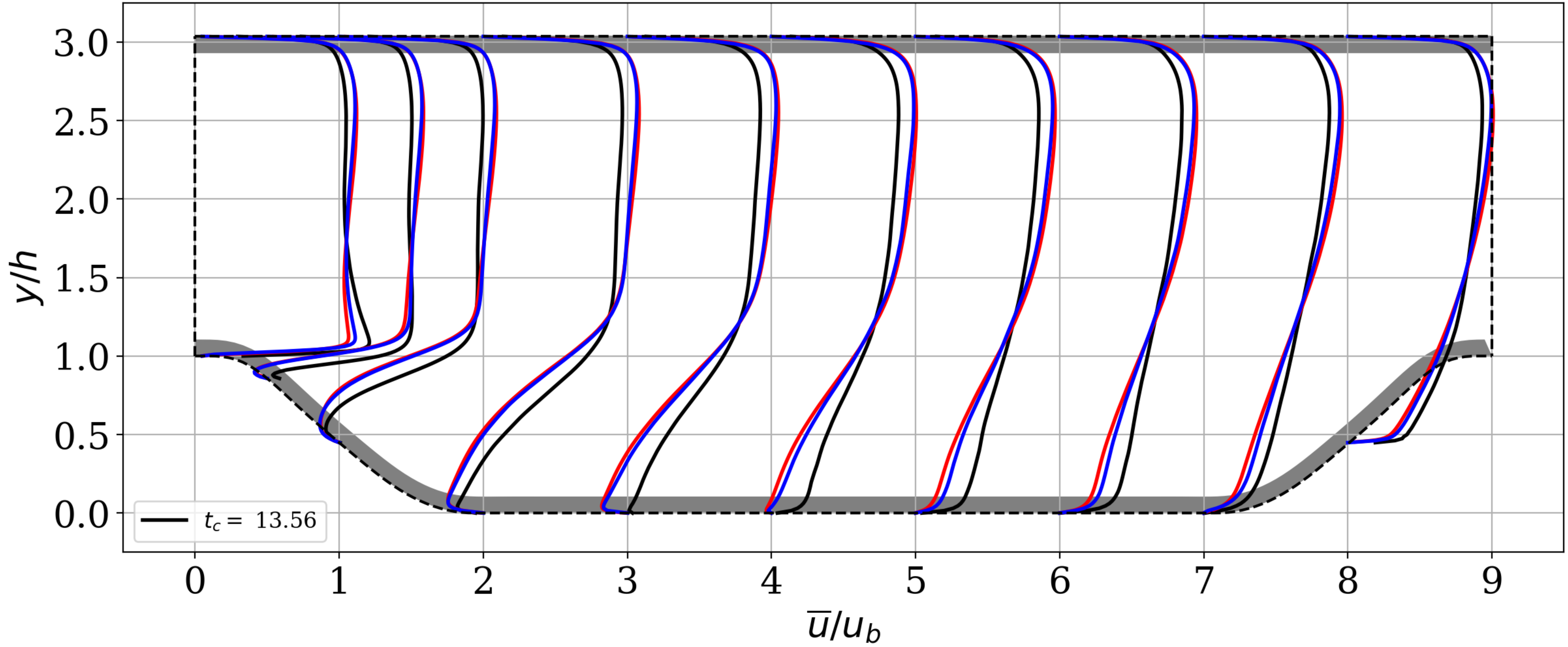
Cenaero

A posteriori validation on the periodic hill $Re_b = 10595$

PROD-F-015-02

Argo-DG + data-driven wall model (CNN-1d-SAL) at M=0.1
Breuer *et al.* (2009)
Gloerflt and Cinnella at M=0.1

A posteriori validation on the periodic hill $Re_b = 10595$

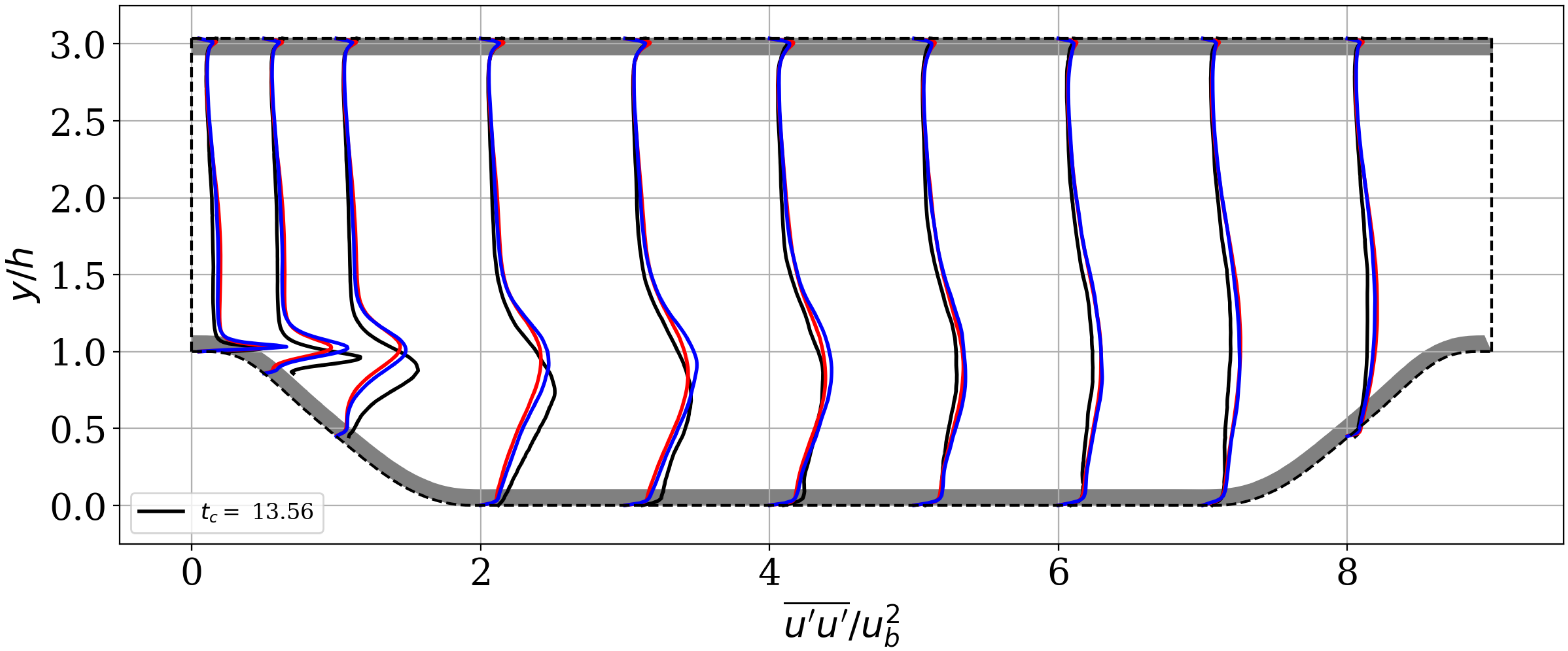


Argo-DG + data-driven wall model (CNN-1d-SAL) at M=0.1
Breuer *et al.* (2009)
Gloerflt and Cinnella at M=0.1

PROD-F-015-02

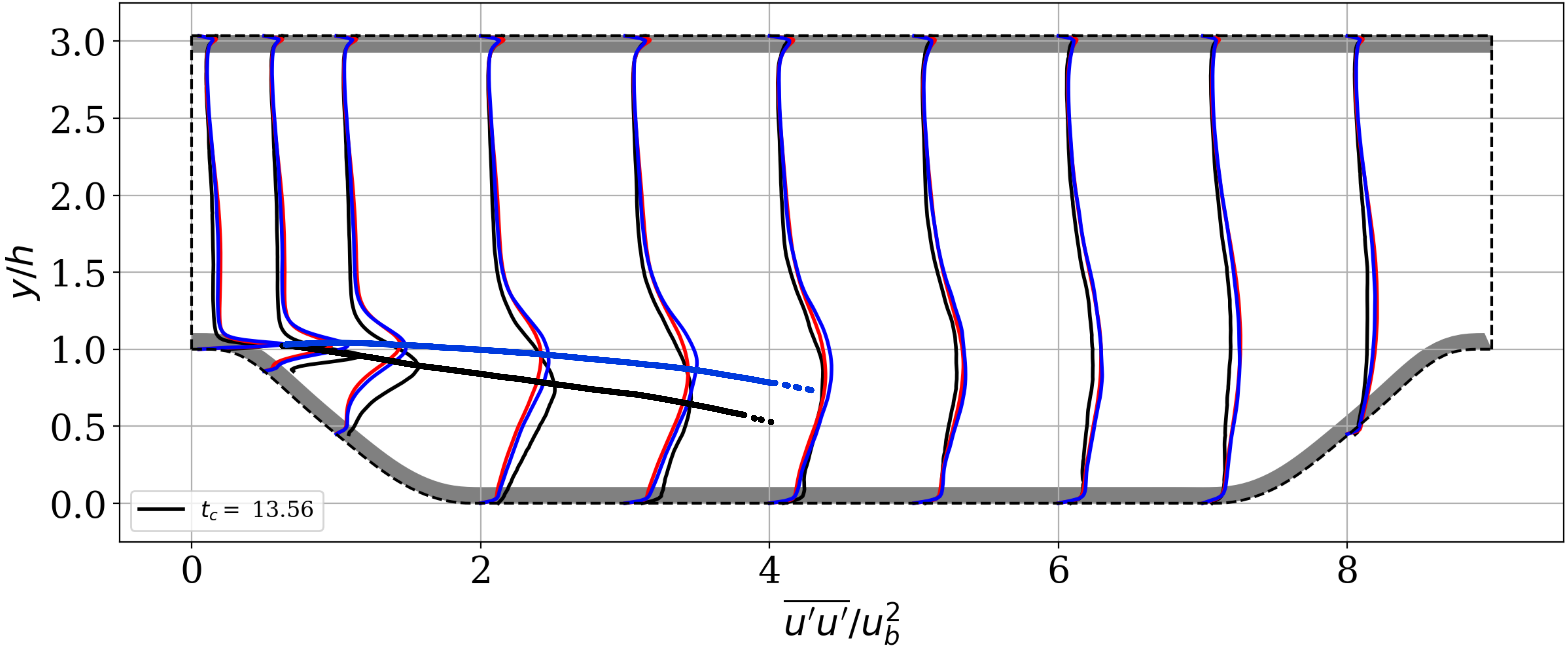
Cenaero

A posteriori validation on the periodic hill $Re_b = 10595$



Argo-DG + data-driven wall model (CNN-1d-SAL) at M=0.1
Breuer *et al.* (2009)
Gloerflt and Cinnella at M=0.1

A posteriori validation on the periodic hill $Re_b = 10595$



Argo-DG + data-driven wall model (CNN-1d-SAL) at M=0.1
Breuer *et al.* (2009)
Gloerflt and Cinnella at M=0.1

PROD-F-015-02

Cenaero

Conclusion

Conclusion

- **Goal:** Developing new wall models to be applied to the separation/reattachment phenomenon using Machine Learning techniques
- **New topic of research** that needs to
  - Generate databases by computing wrLES of various test cases
  - Analyze them using space-time correlations
  - Train various neural networks architectures

Conclusion

- **Goal:** Developing new wall models to be applied to the separation/reattachment phenomenon using Machine Learning techniques
- **New topic of research** that needs to
  - Generate databases by computing wrLES of various test cases
  - Analyze them using space-time correlations
  - Train various neural networks architectures
- **A priori validation:**
  - Gives good results on both the channel wall and the lower wall of the periodic hill
  - MDN gives the best results on the periodic hill (i.e., mean and variance) while it has some difficulties to replicat the « physical » aspects of $\tau_{w,\xi}$ on the channel.
  - MDN generates independent samples on the channel but replicats well the correlation function on the periodic hill
  - Needs to investigate why the MDN works better on the periodic hill than on the channel

PROD-F-015-02

Cenaero

Conclusion

- **Goal:** Developing new wall models to be applied to the separation/reattachment phenomenon using Machine Learning techniques
- **New topic of research** that needs to
  - Generate databases by computing wrLES of various test cases
  - Analyze them using space-time correlations
  - Train various neural networks architectures
- **A priori validation:**
  - Gives good results on both the channel wall and the lower wall of the periodic hill
  - MDN gives the best results on the periodic hill (i.e., mean and variance) while it has some difficulties to replicat the « physical » aspects of $\tau_{w,\xi}$ on the channel.
  - MDN generates independent samples on the channel but replicats well the correlation function on the periodic hill
  - Needs to investigate why the MDN works better on the periodic hill than on the channel
- A **posteriori validation:**
  - Good behavior of the CNN-1d-SAL on the channel at $Re_\tau = 950$ with results comparable to a Reichardt LOTW
  - On the periodic hill, the recirculation bubble size is shorter and impacts the whole physics
    - The exploitation of the prediction still raises some questions
    - It may not be an issue from the model but from the mesh, the sampling, …
    - Deeper analysis is underway