Validation of data-driven wall models on the upper and lower walls of the two-dimensional periodic hill

ACOMEN 2022, Liège

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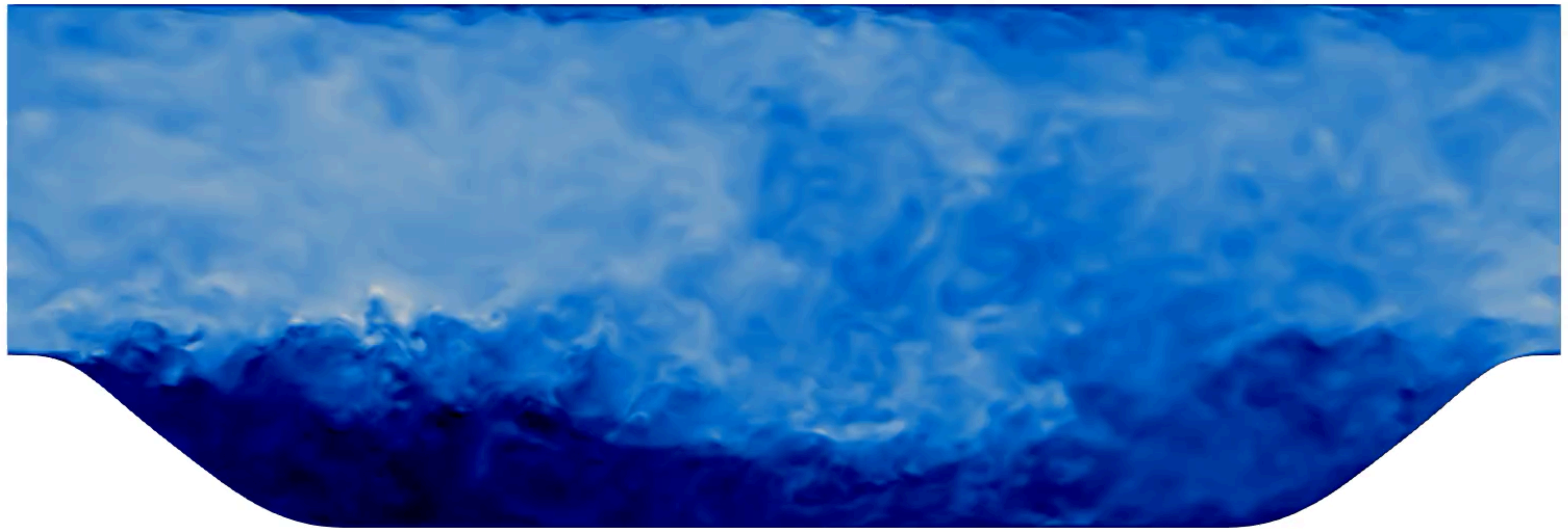
ULiege, UCLouvain, Cenaero, Safran Tech

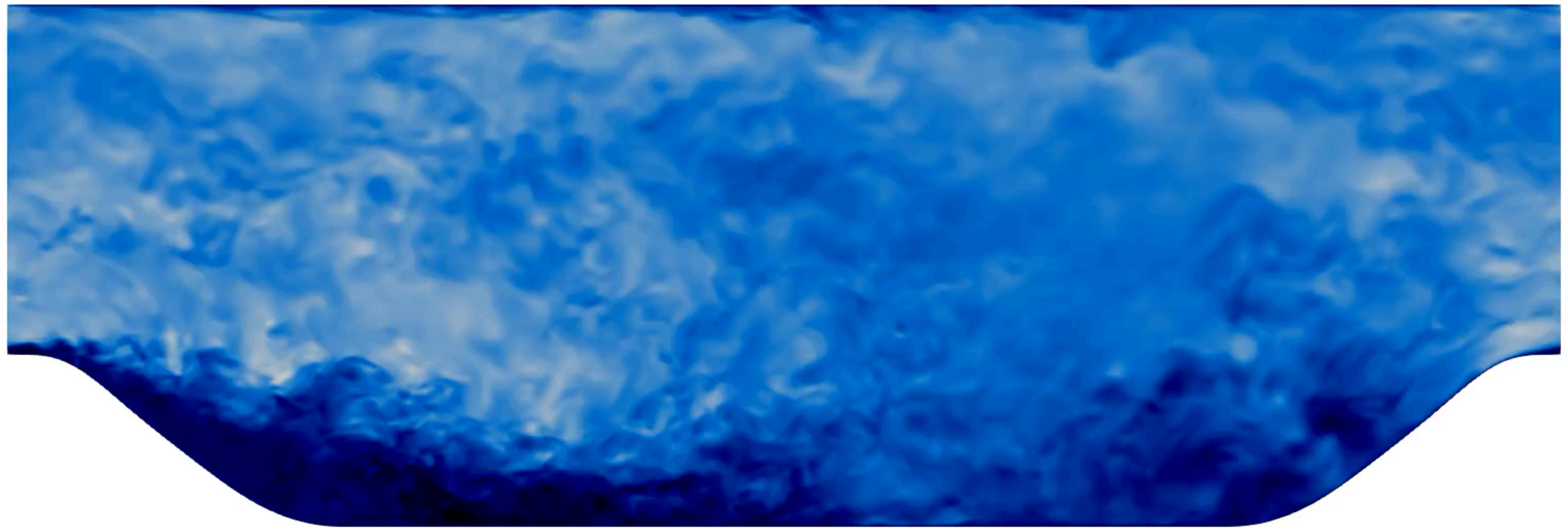
Contact: margaux.boxho@cenaero.be

Doc. ref.: 2019030-THESELDP-SAFRAN

PROD-F-015-02

What's the difference?





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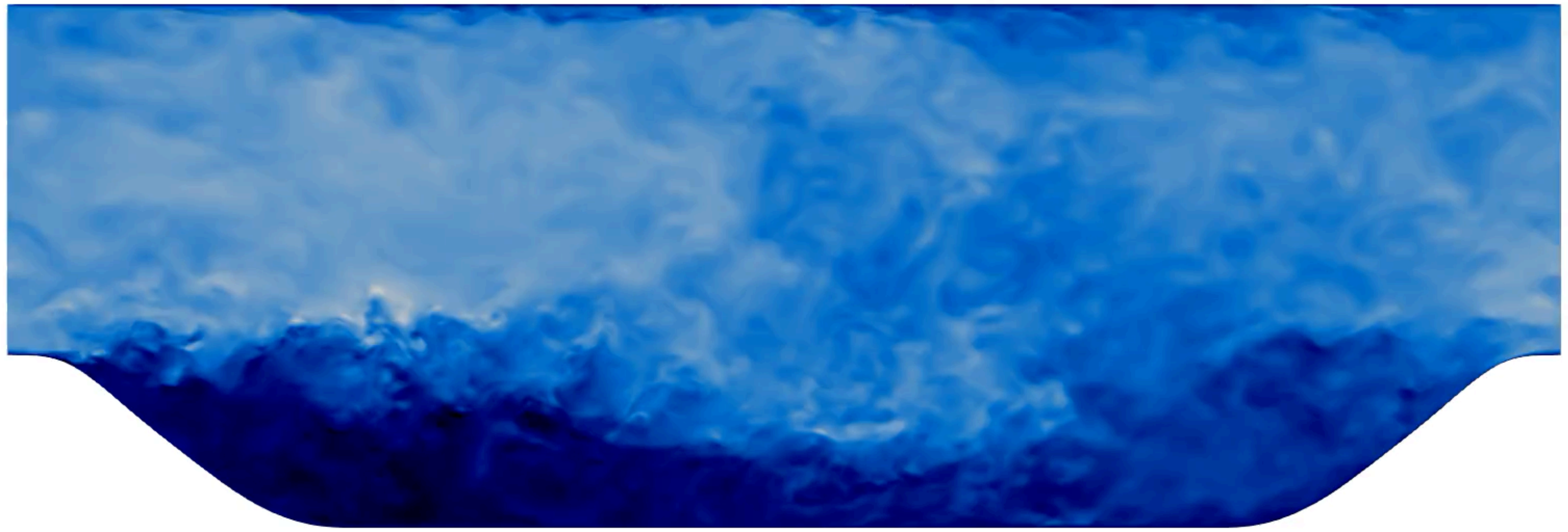

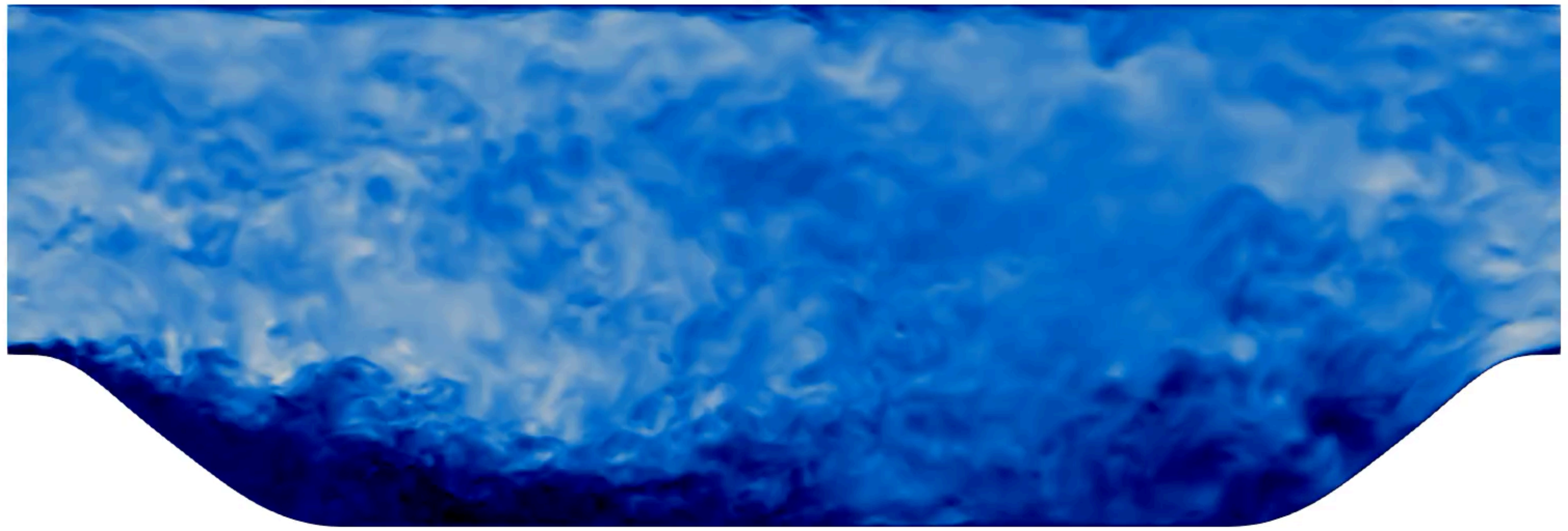

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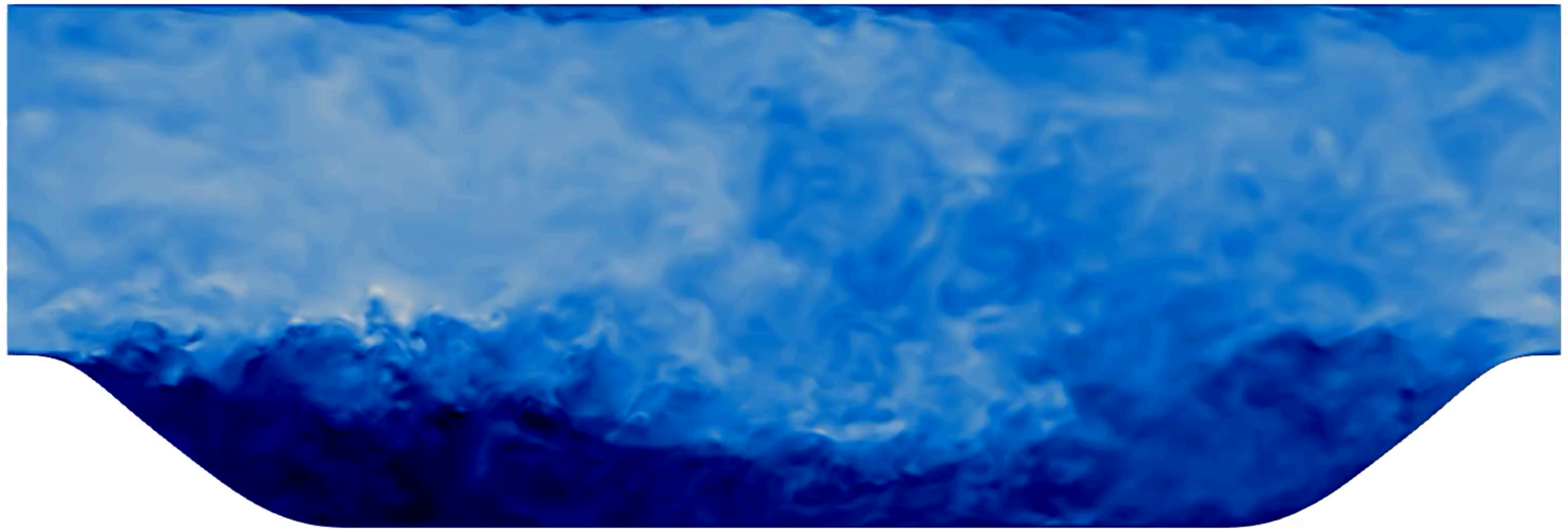

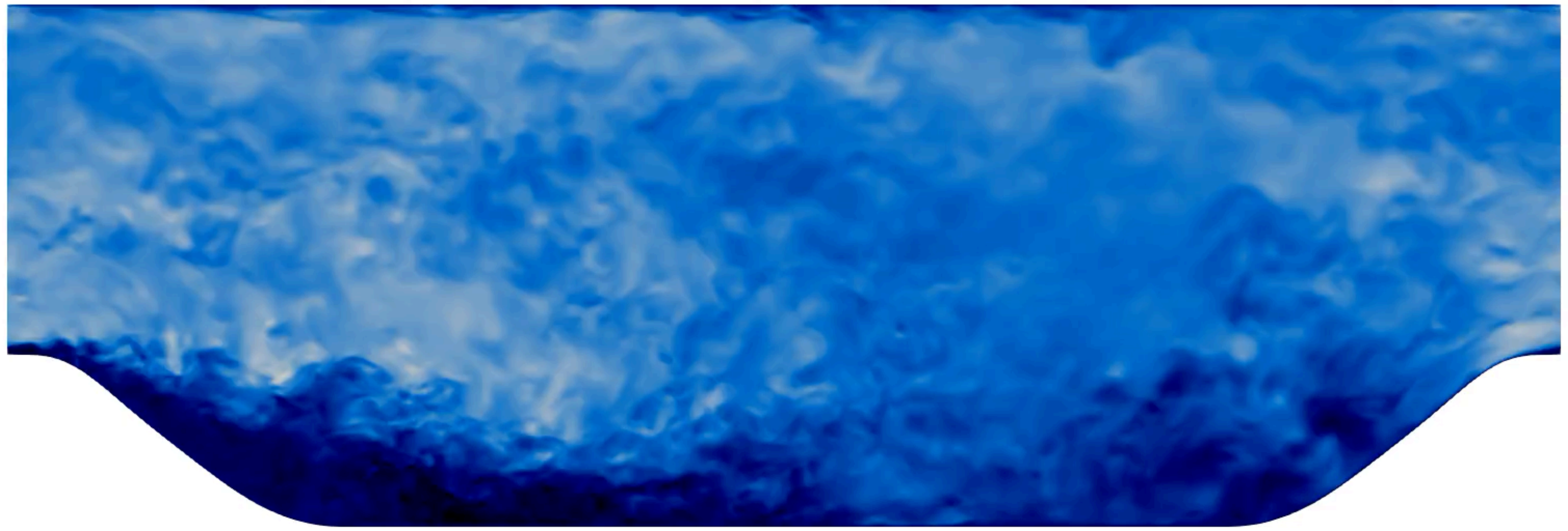

Cenaero

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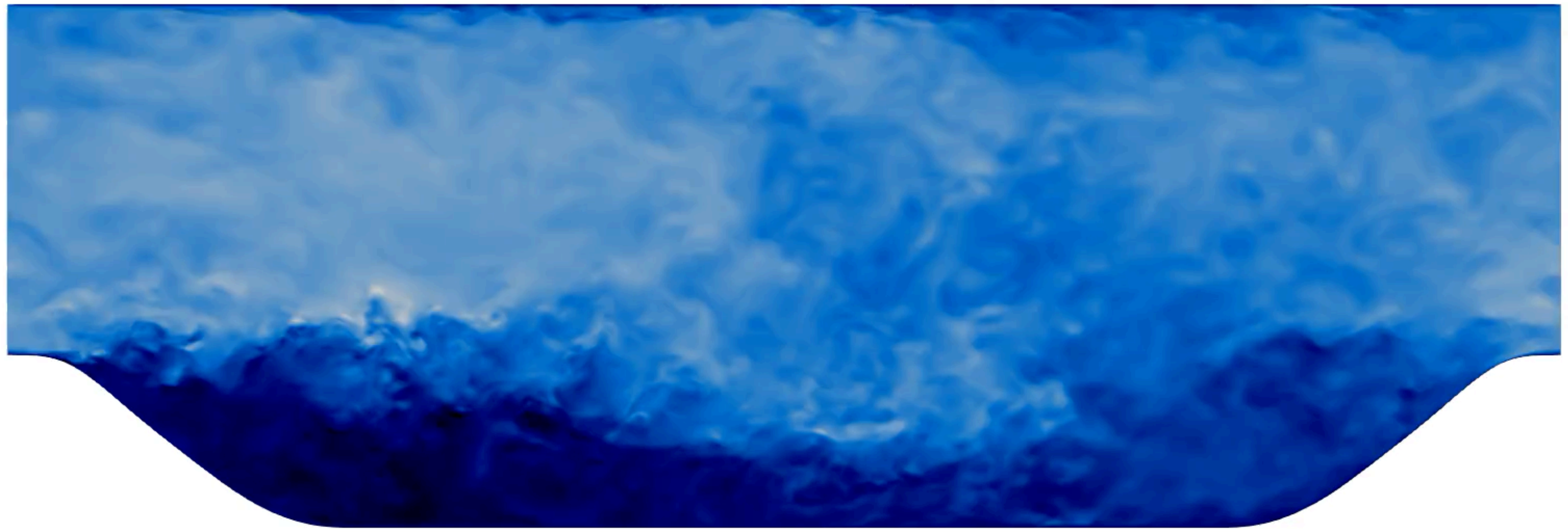	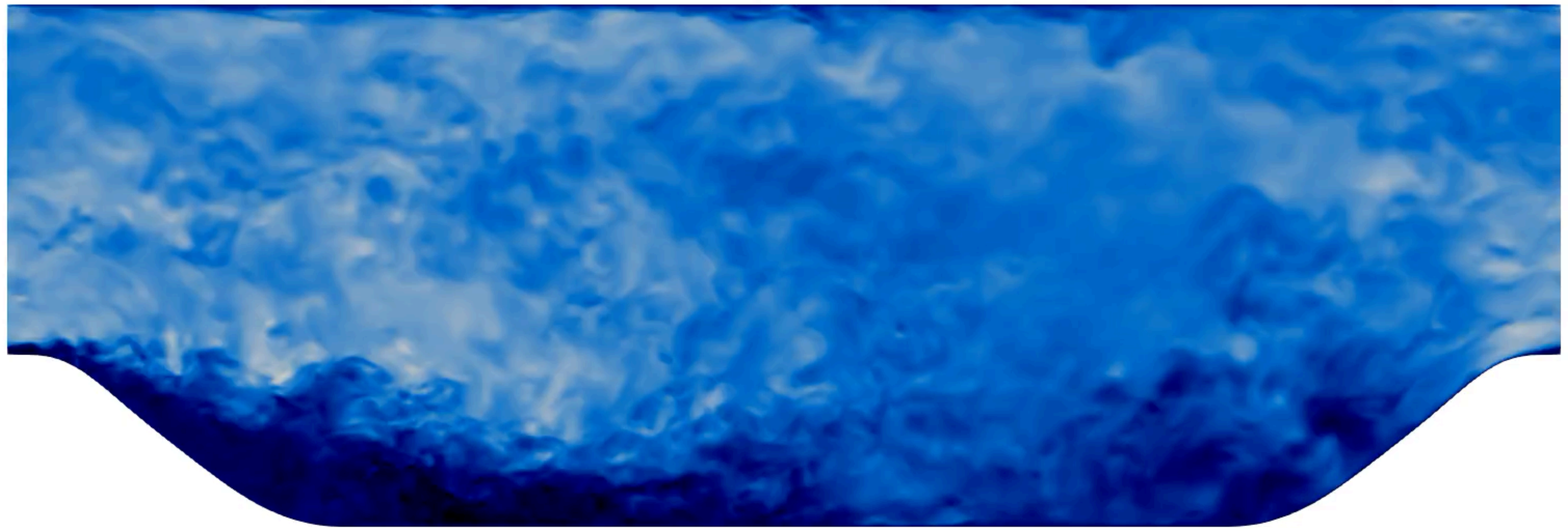

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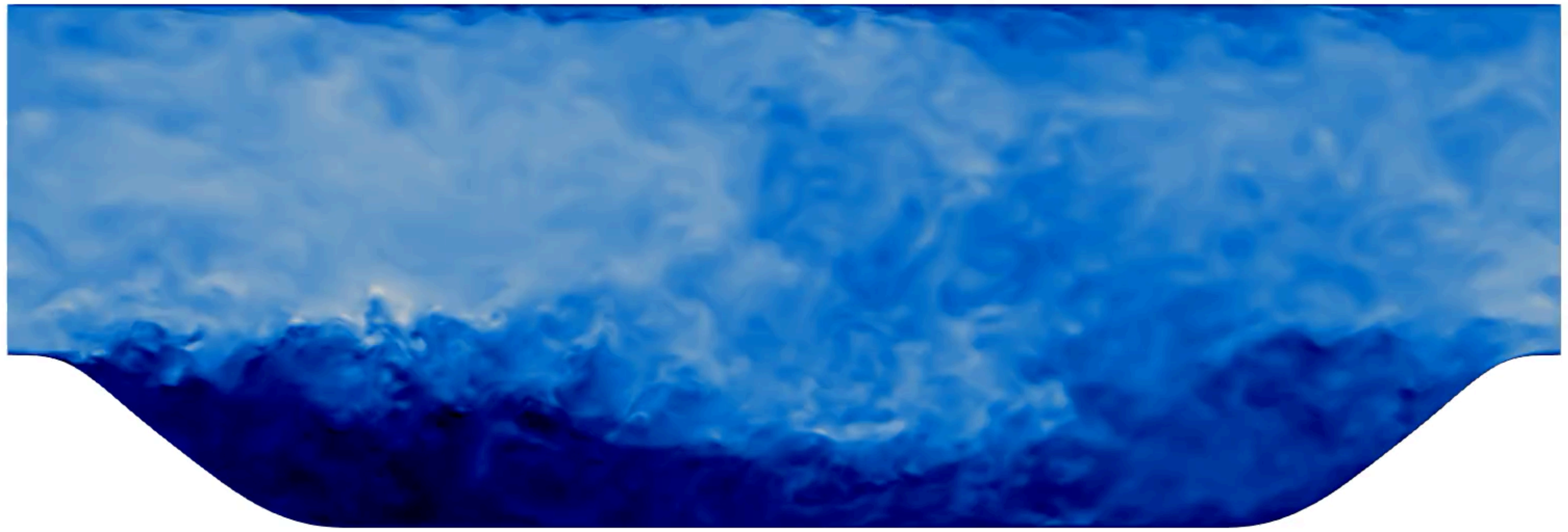

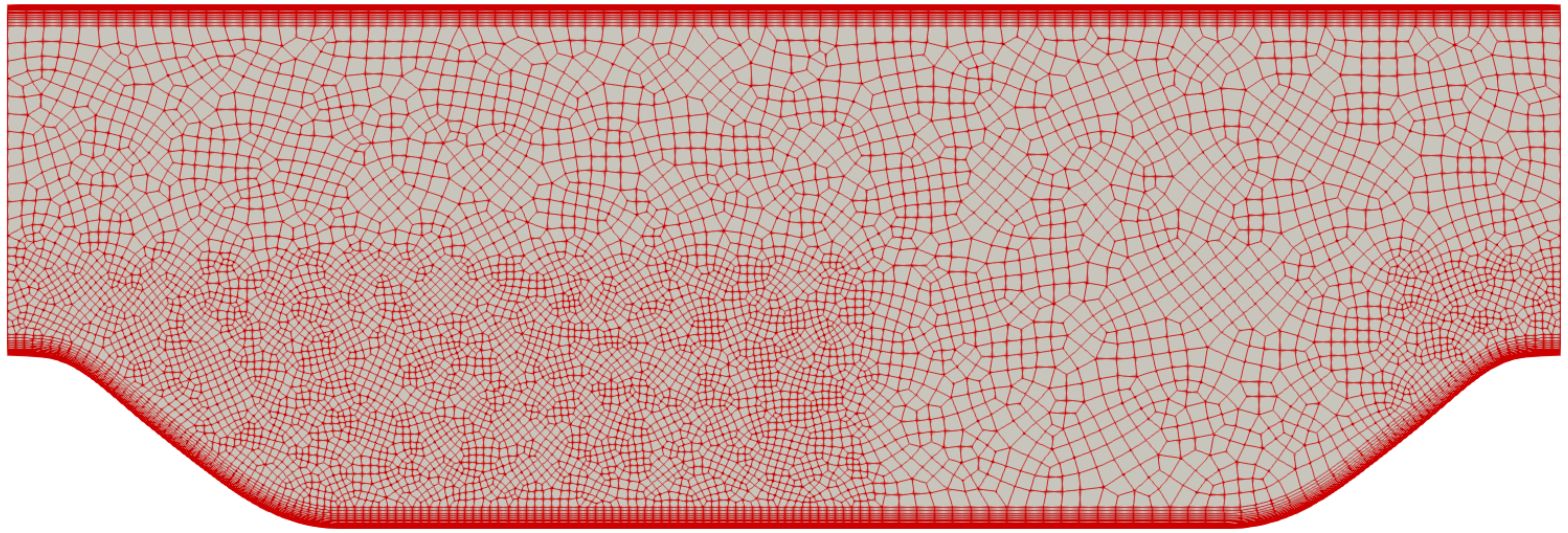

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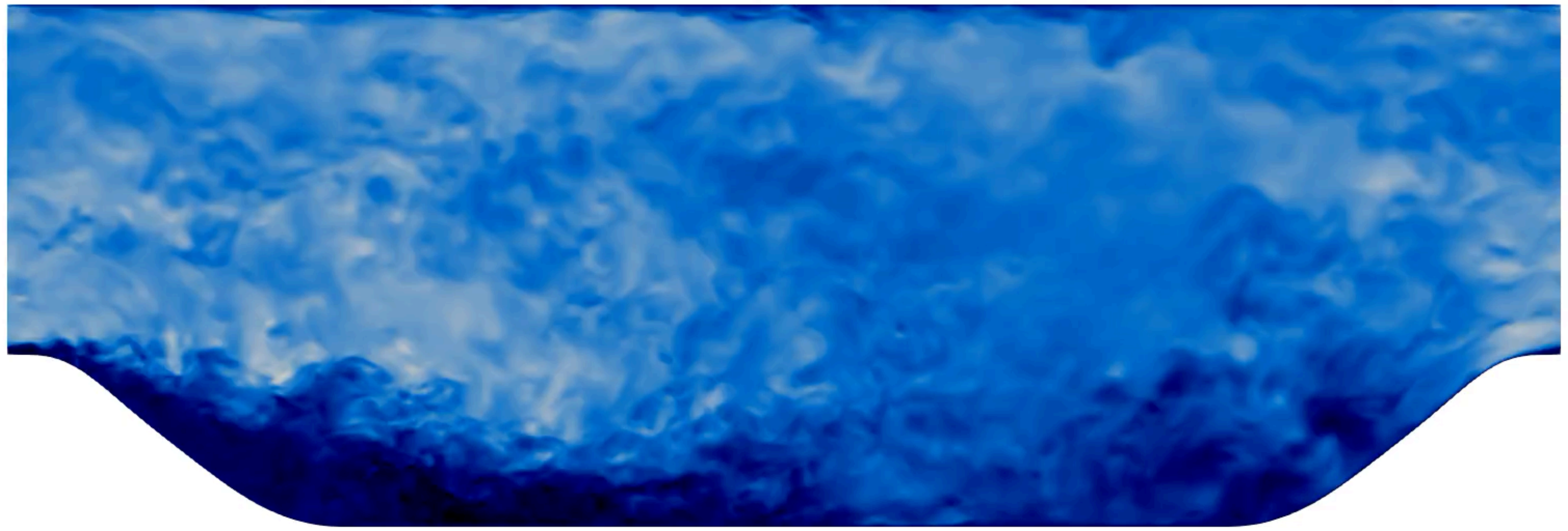

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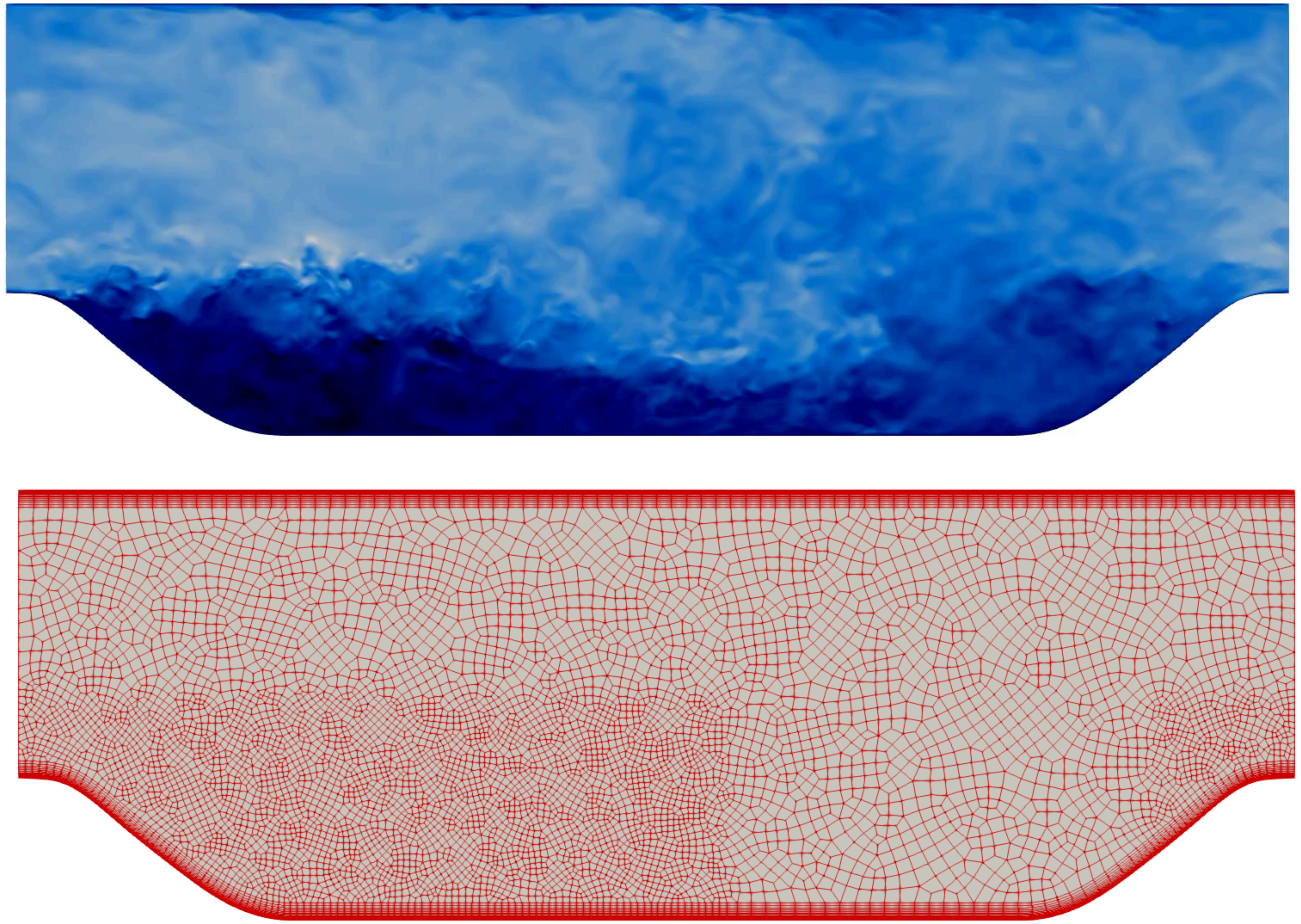	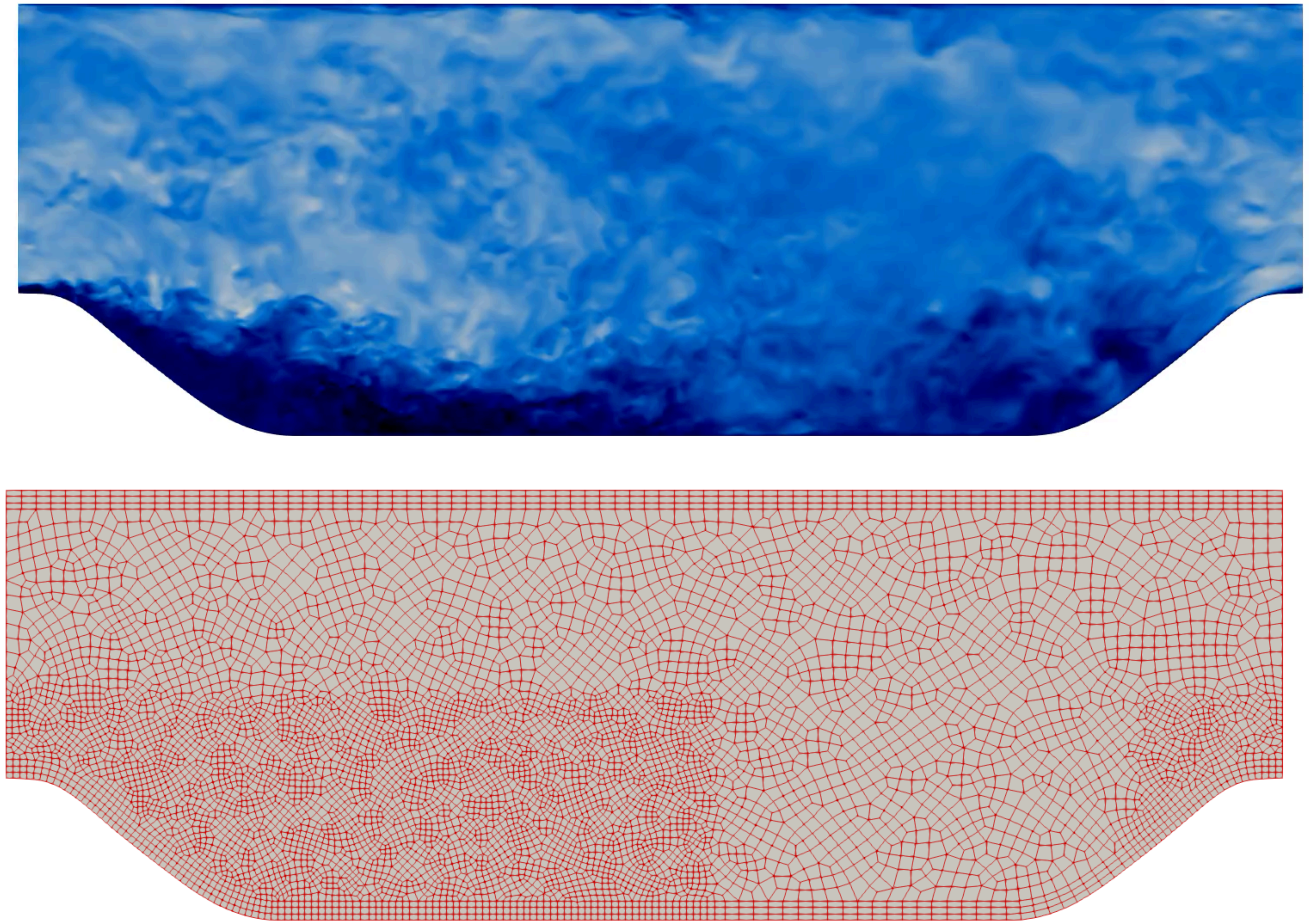

PROD-F-015 02

Cenaero

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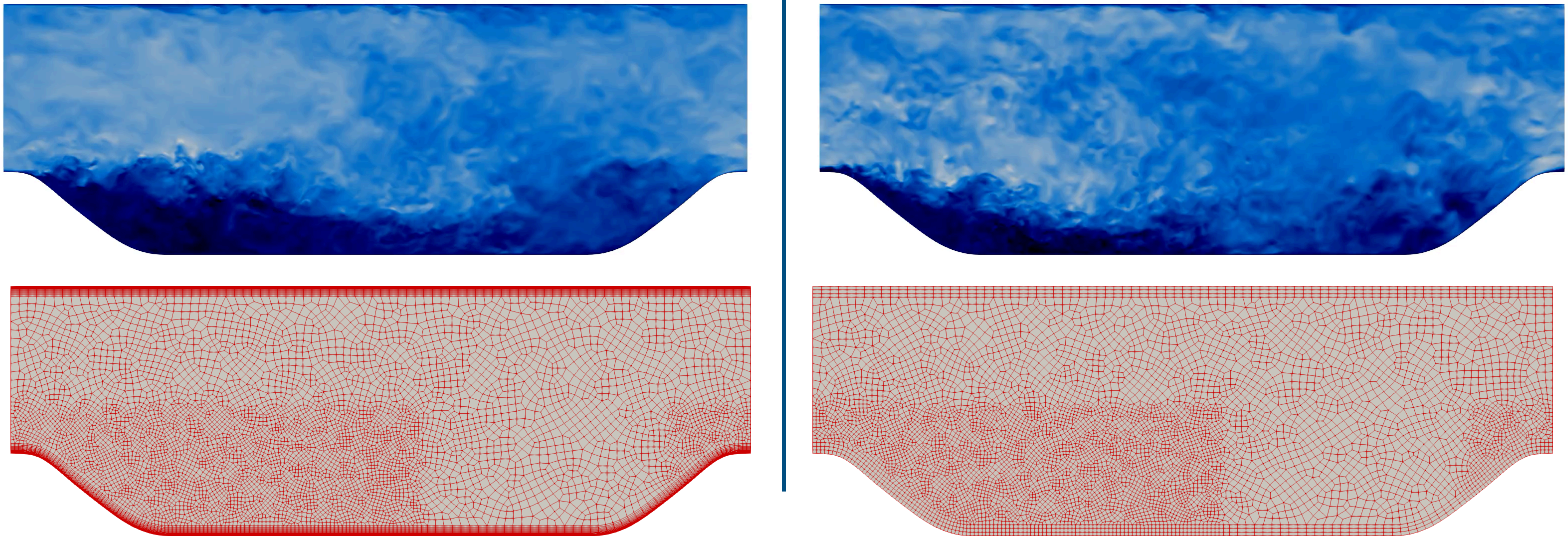

Number of degrees of freedom
28,480,320

PROD-F-015 02

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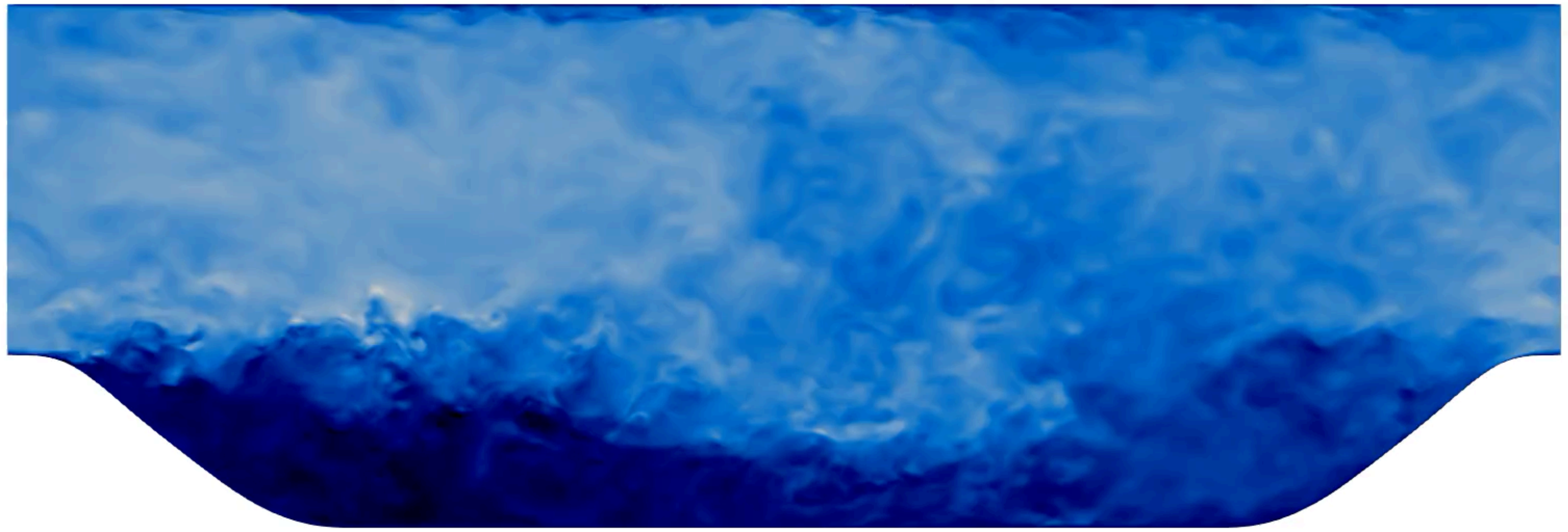

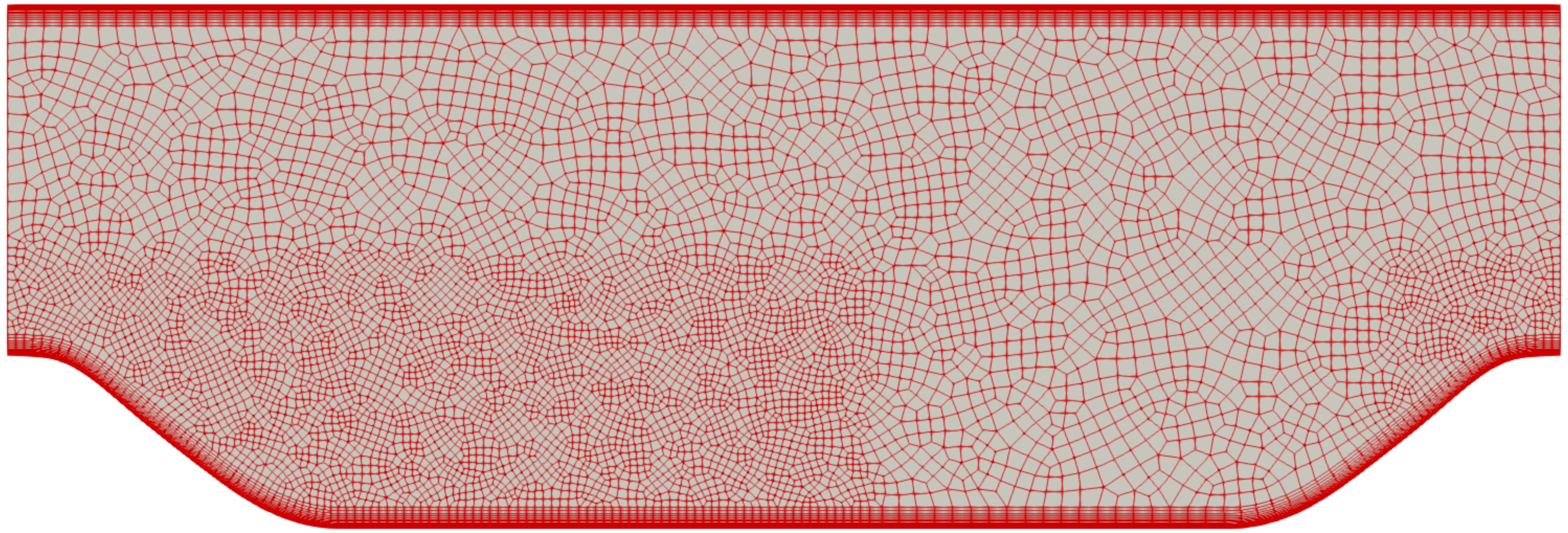

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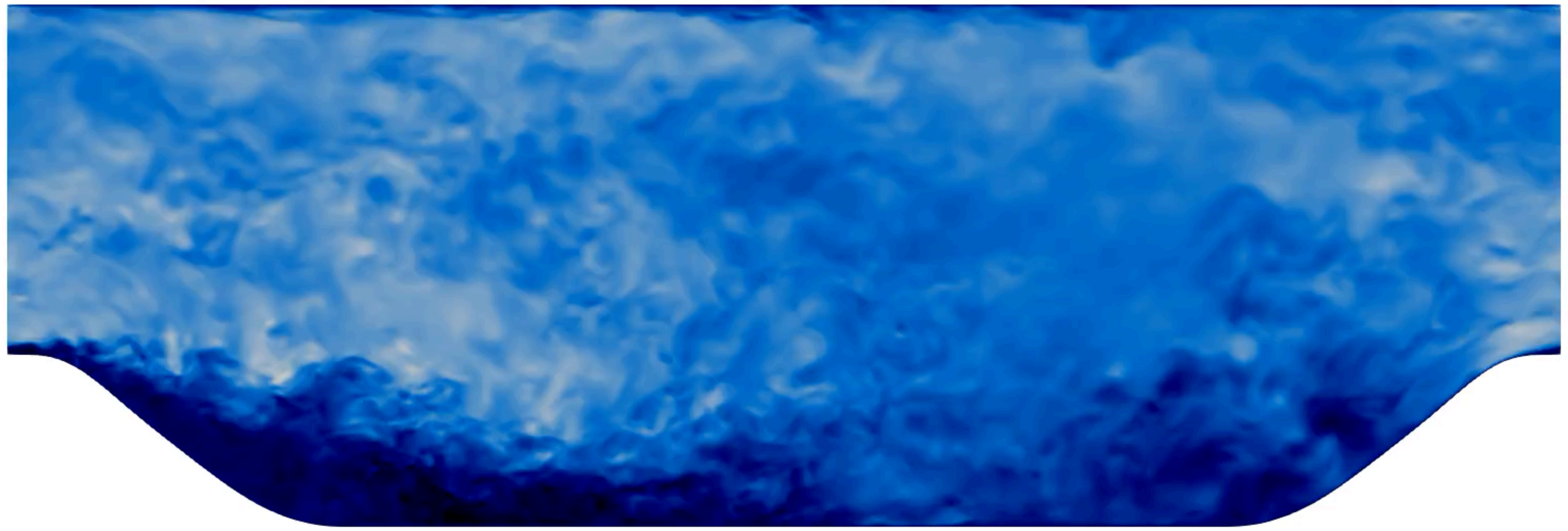

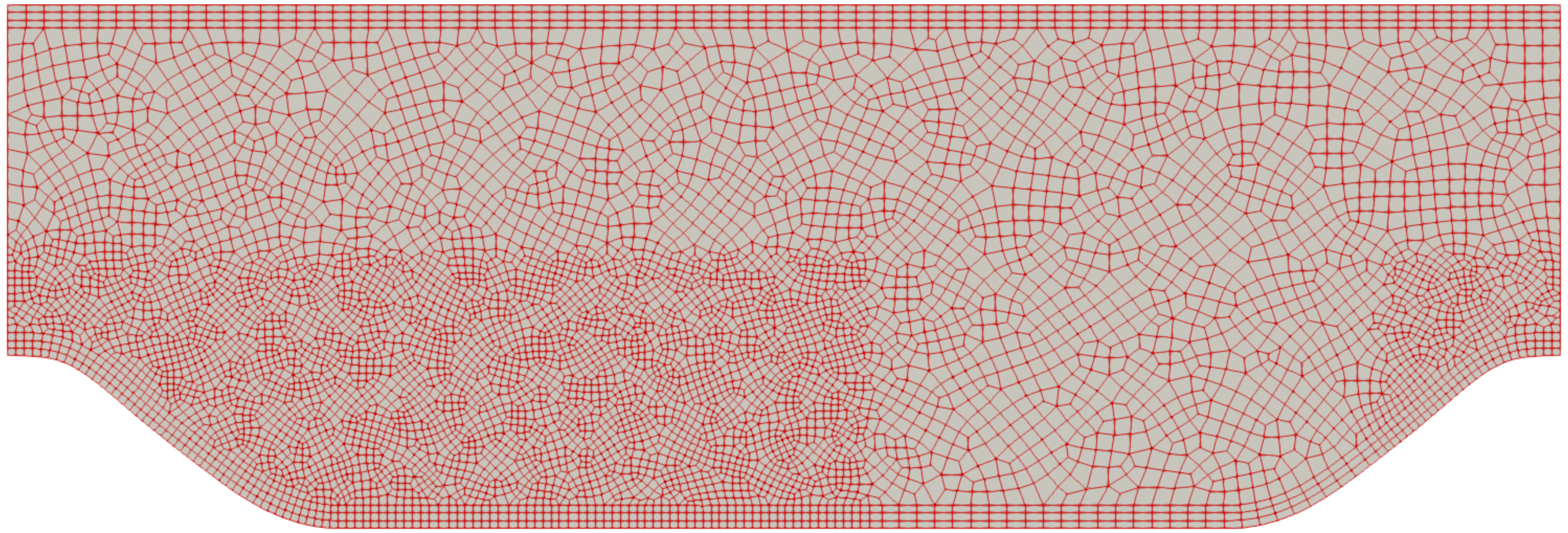

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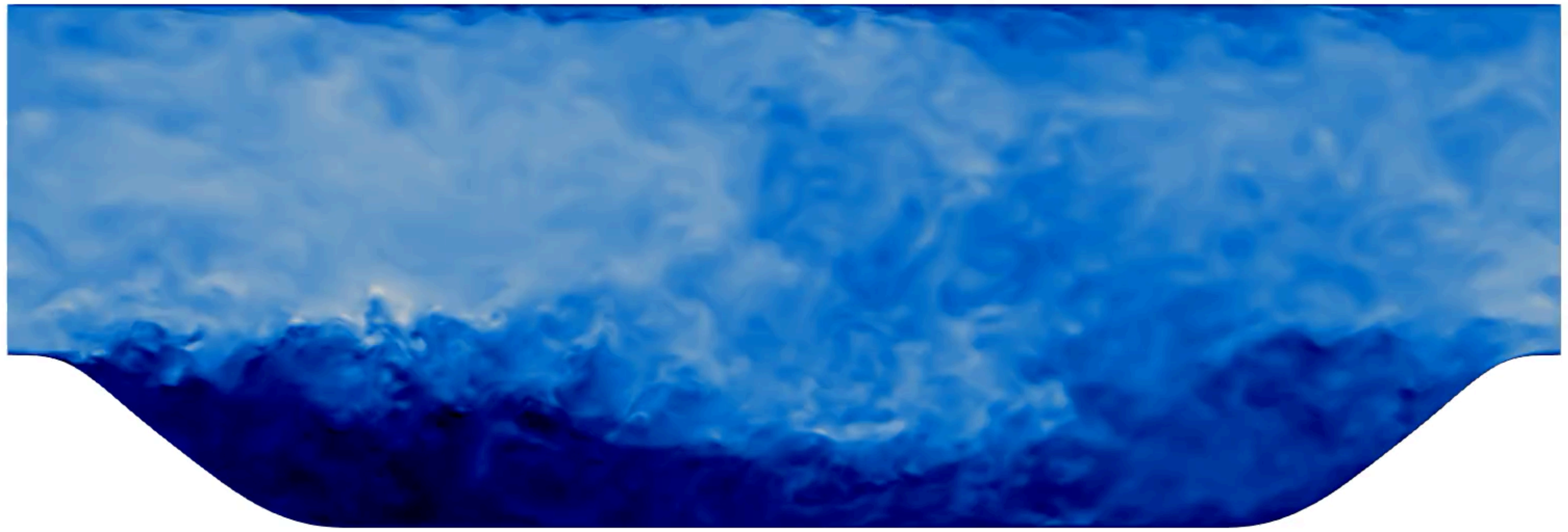

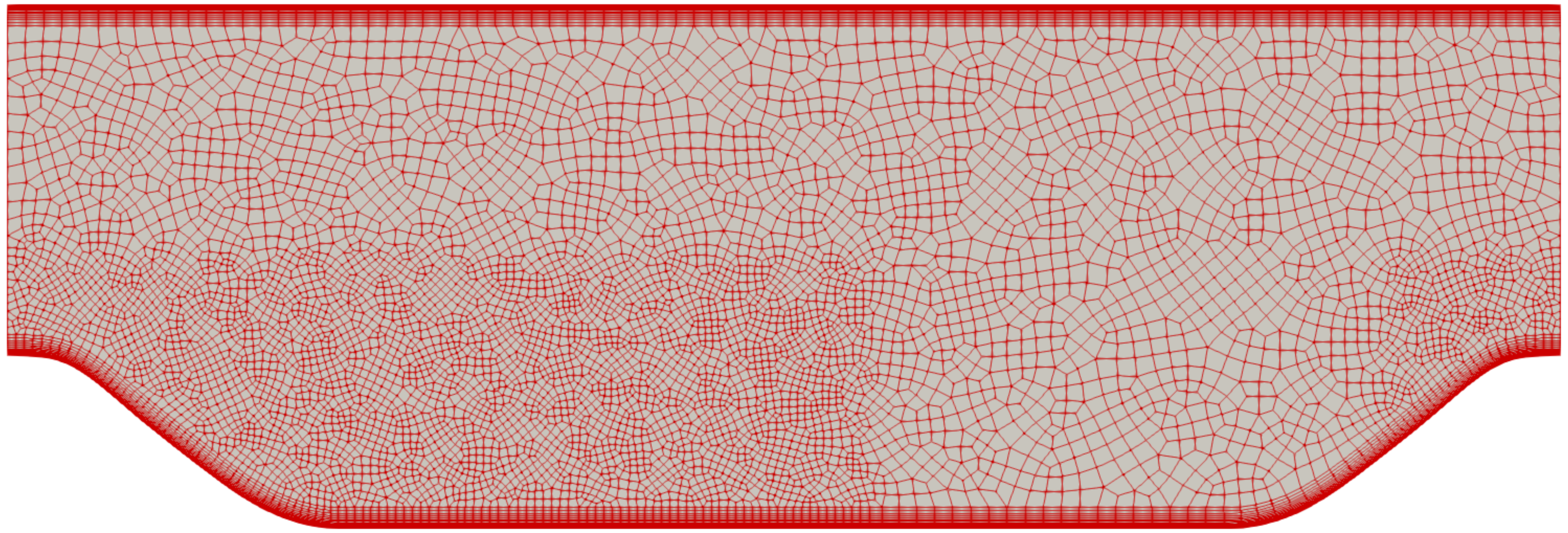

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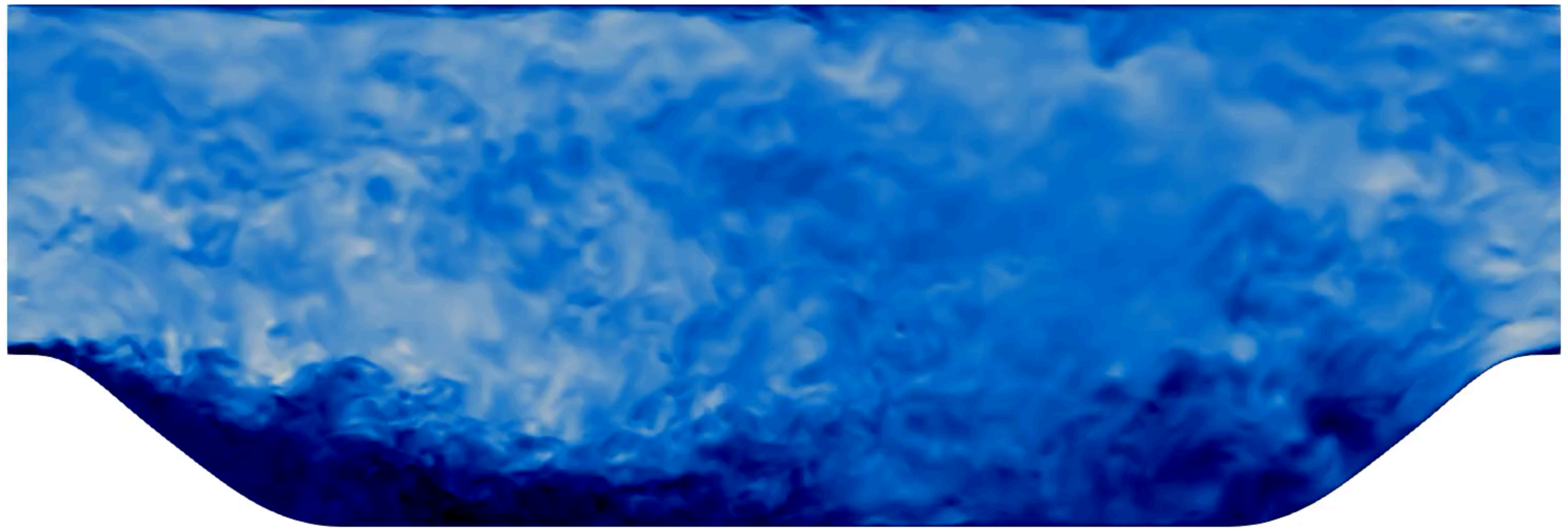

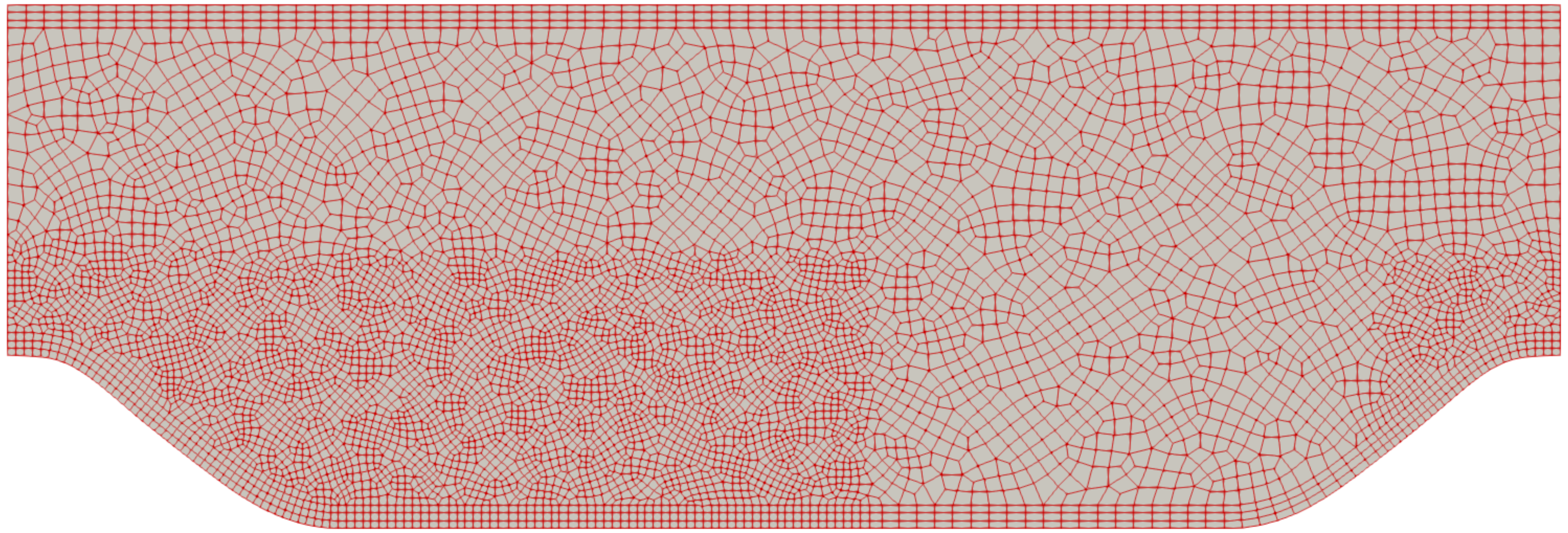

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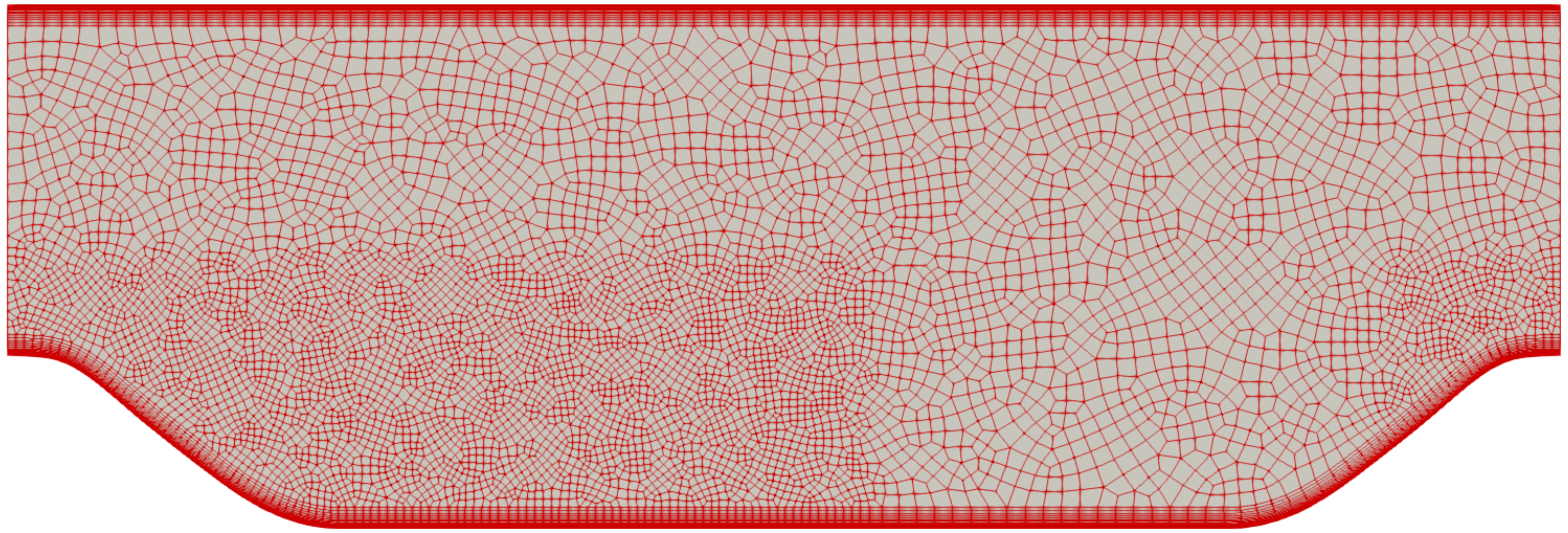

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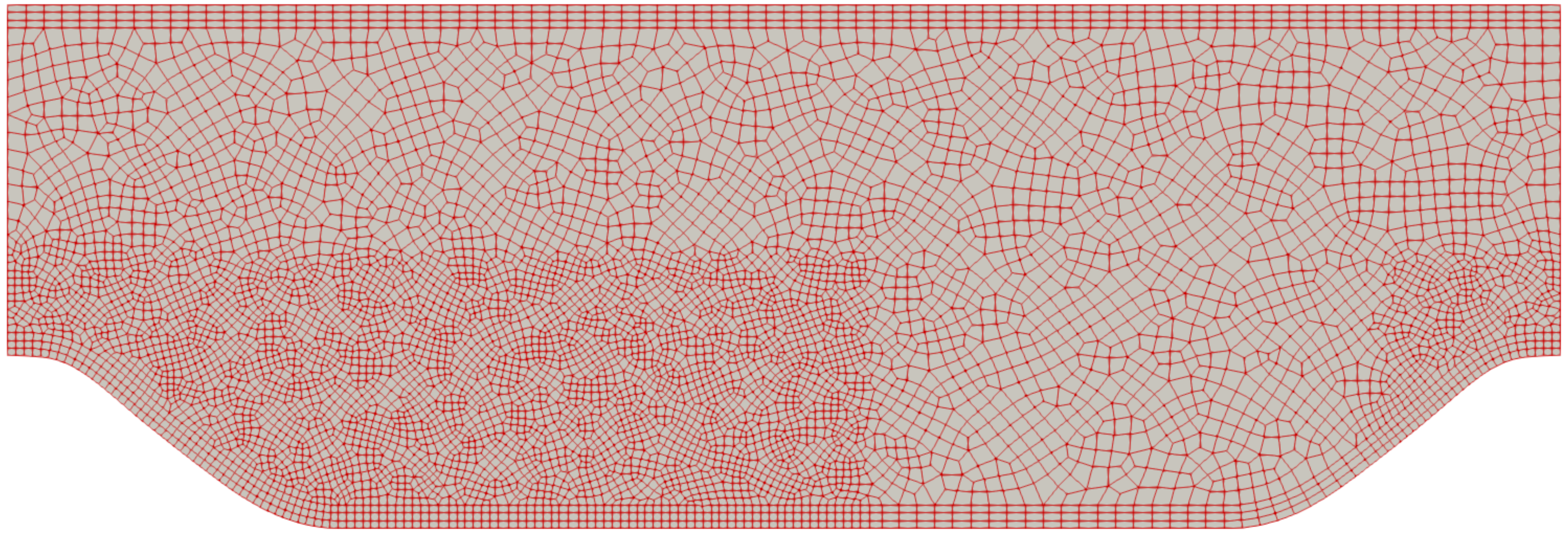

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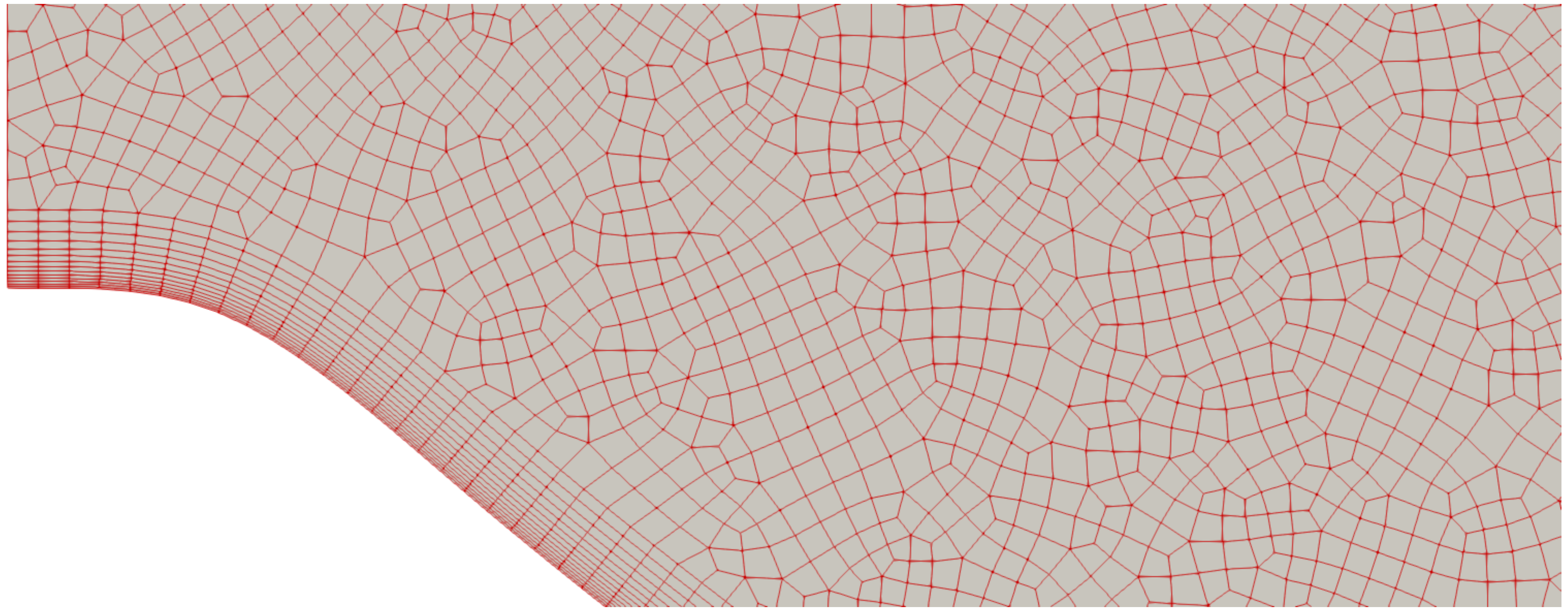

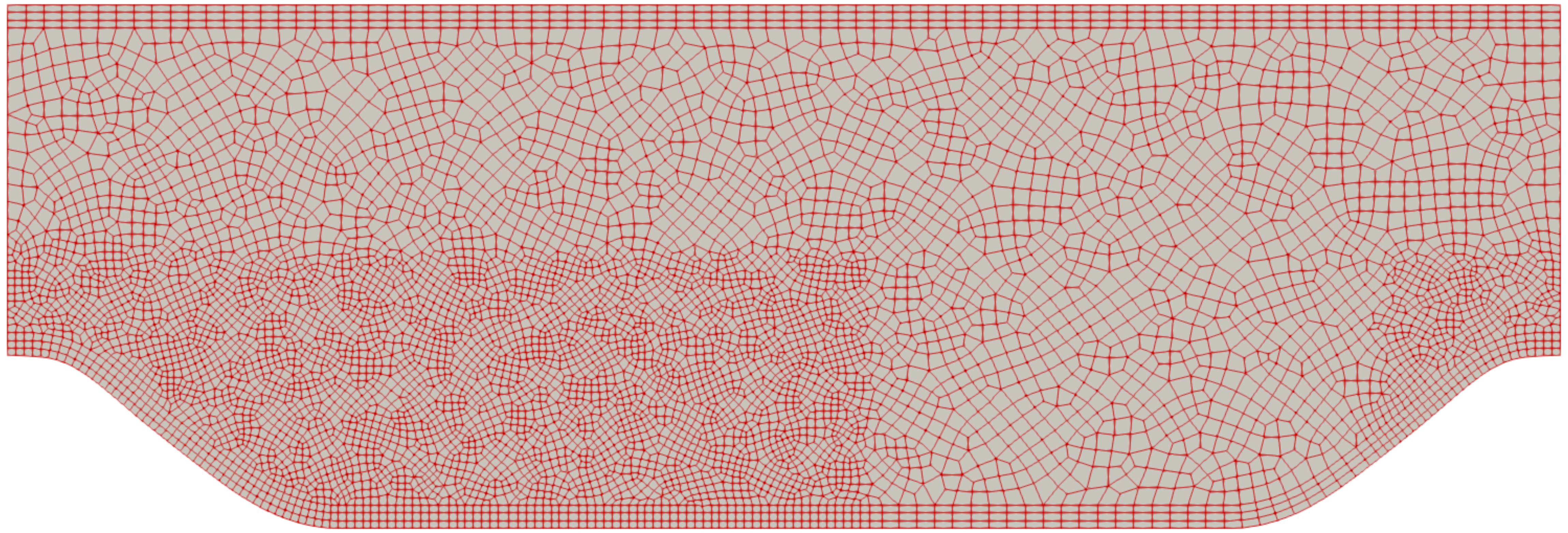

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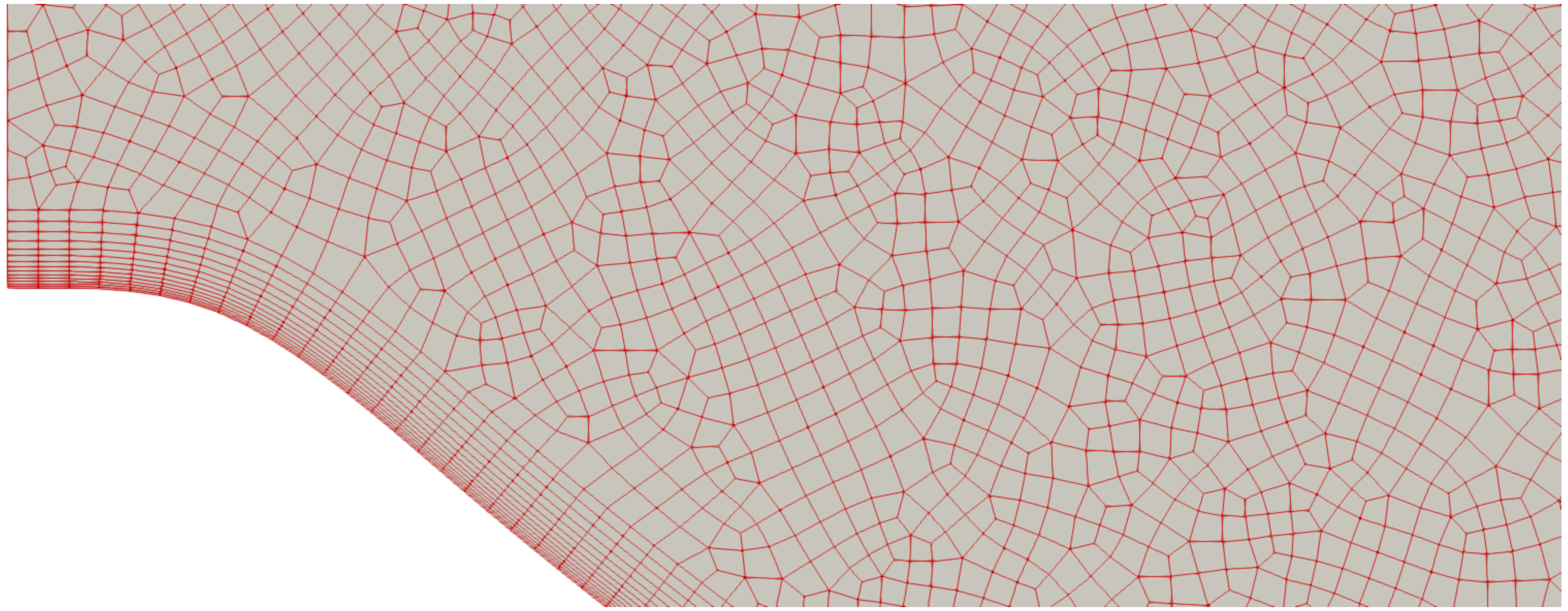

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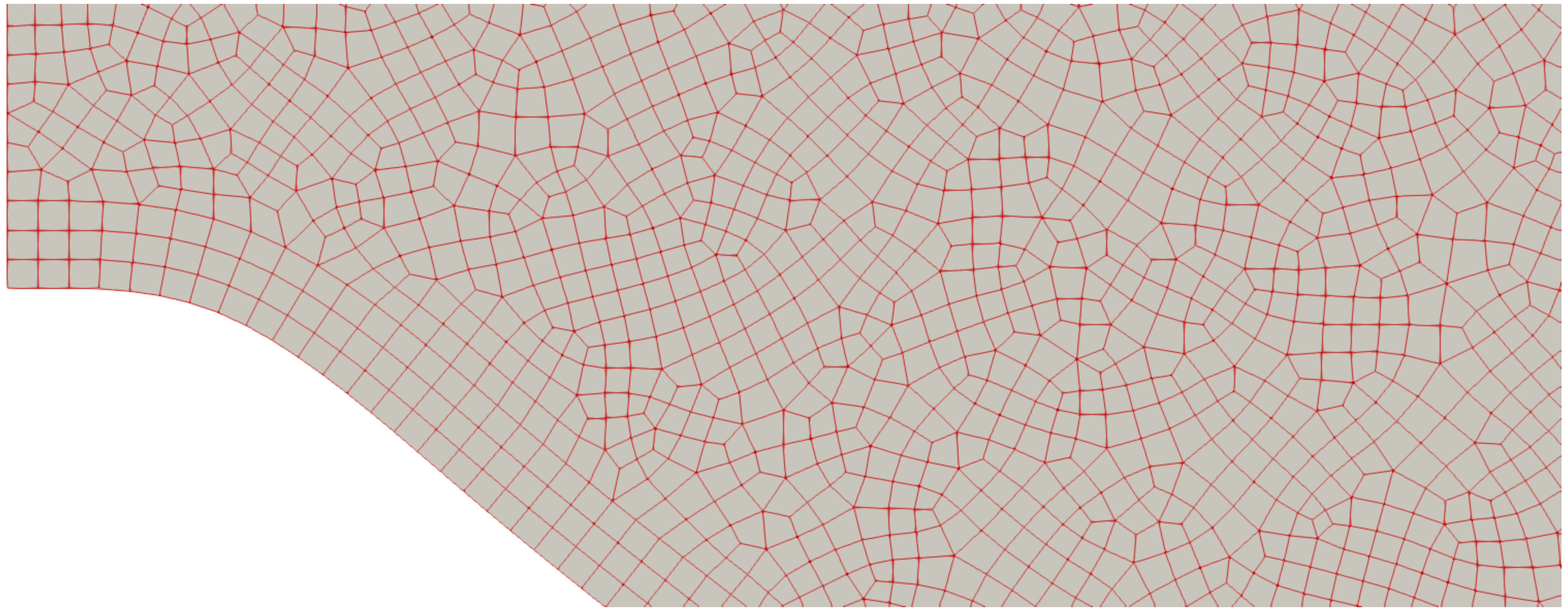

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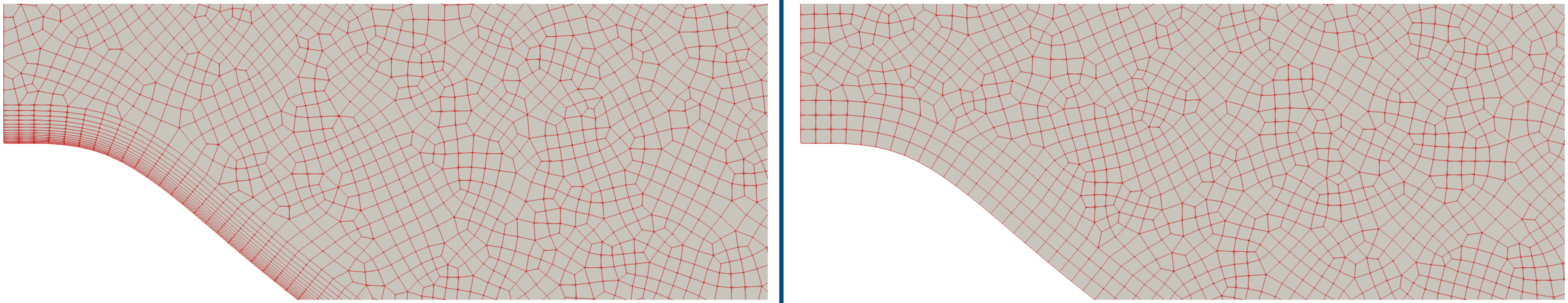

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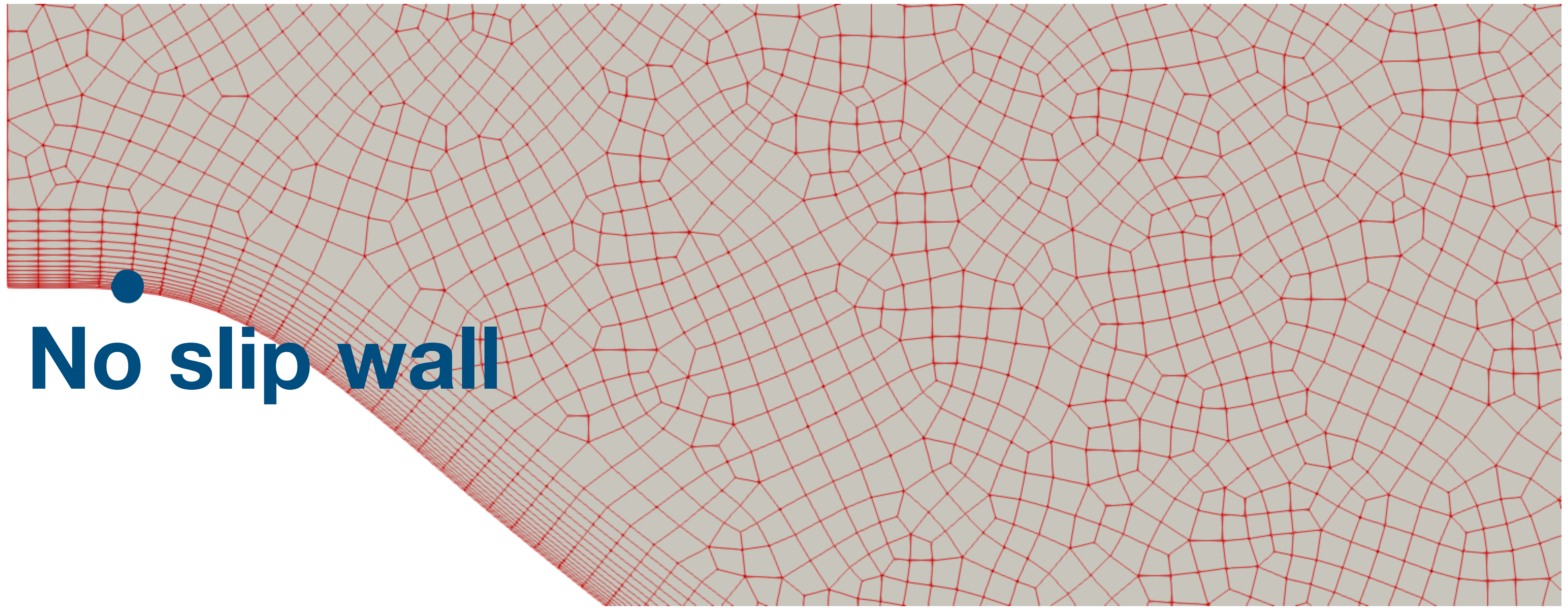

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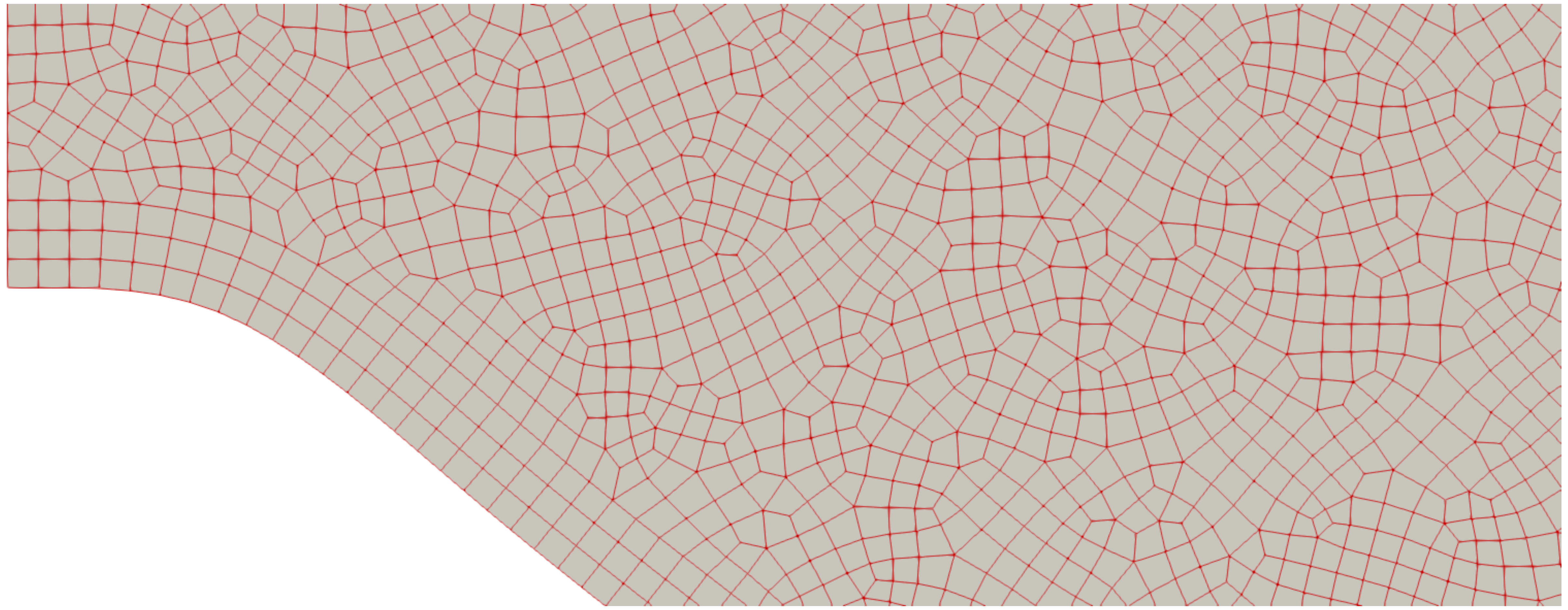

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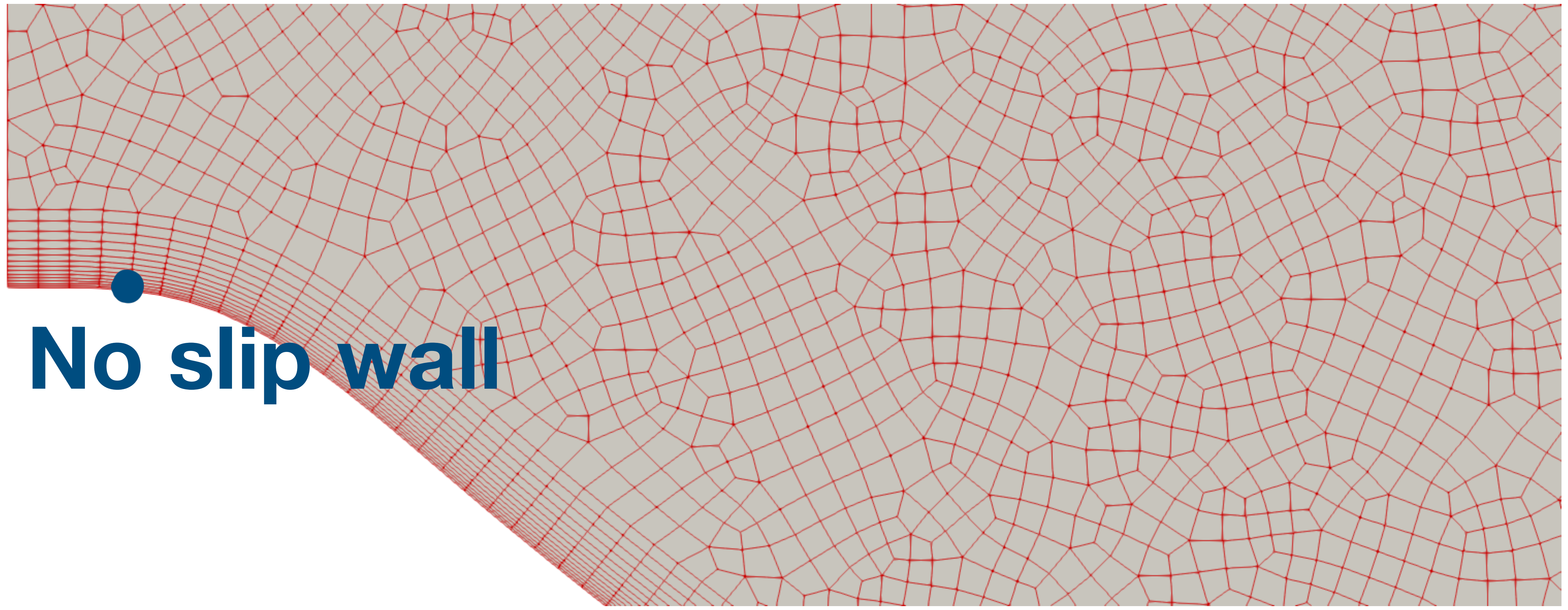

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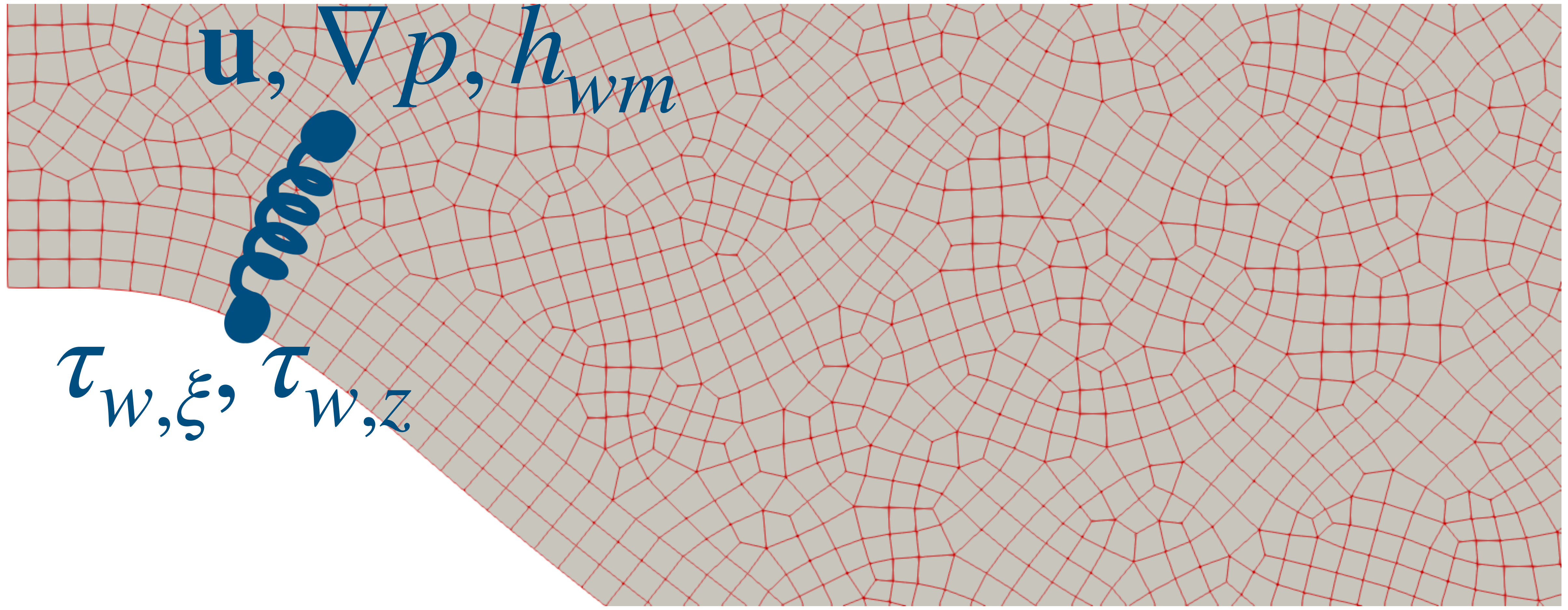

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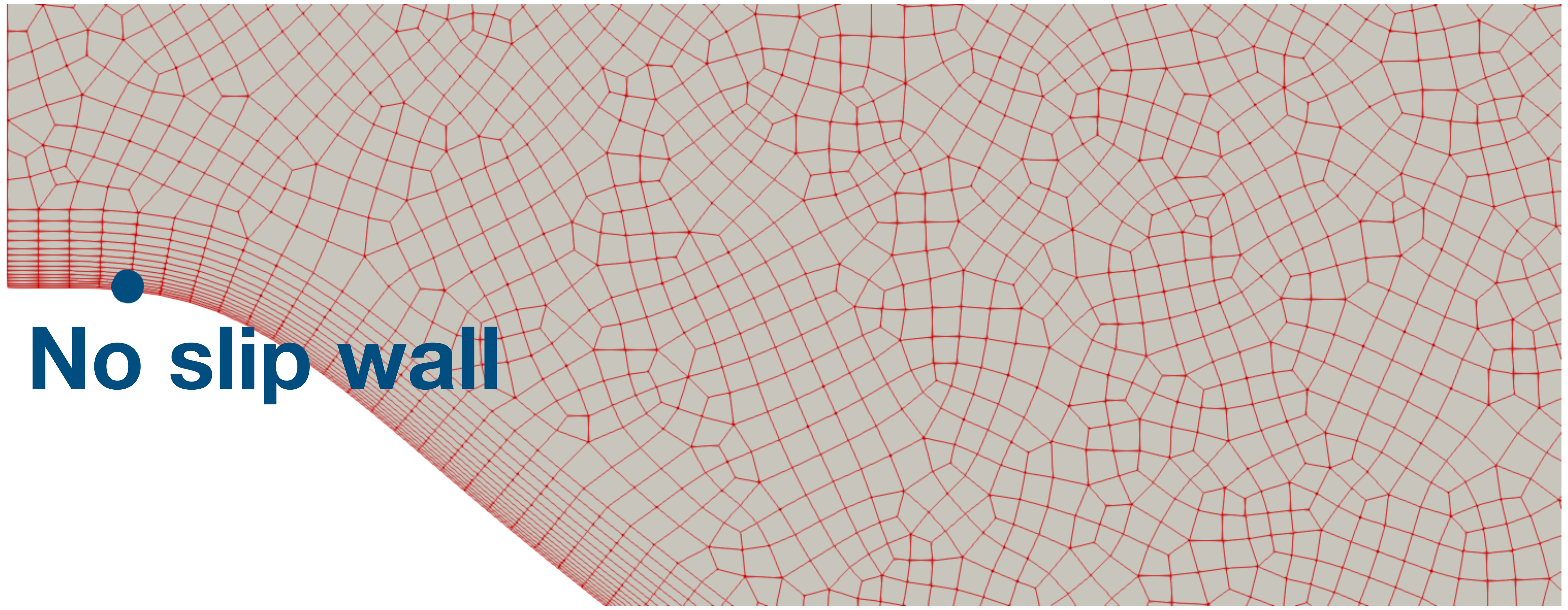

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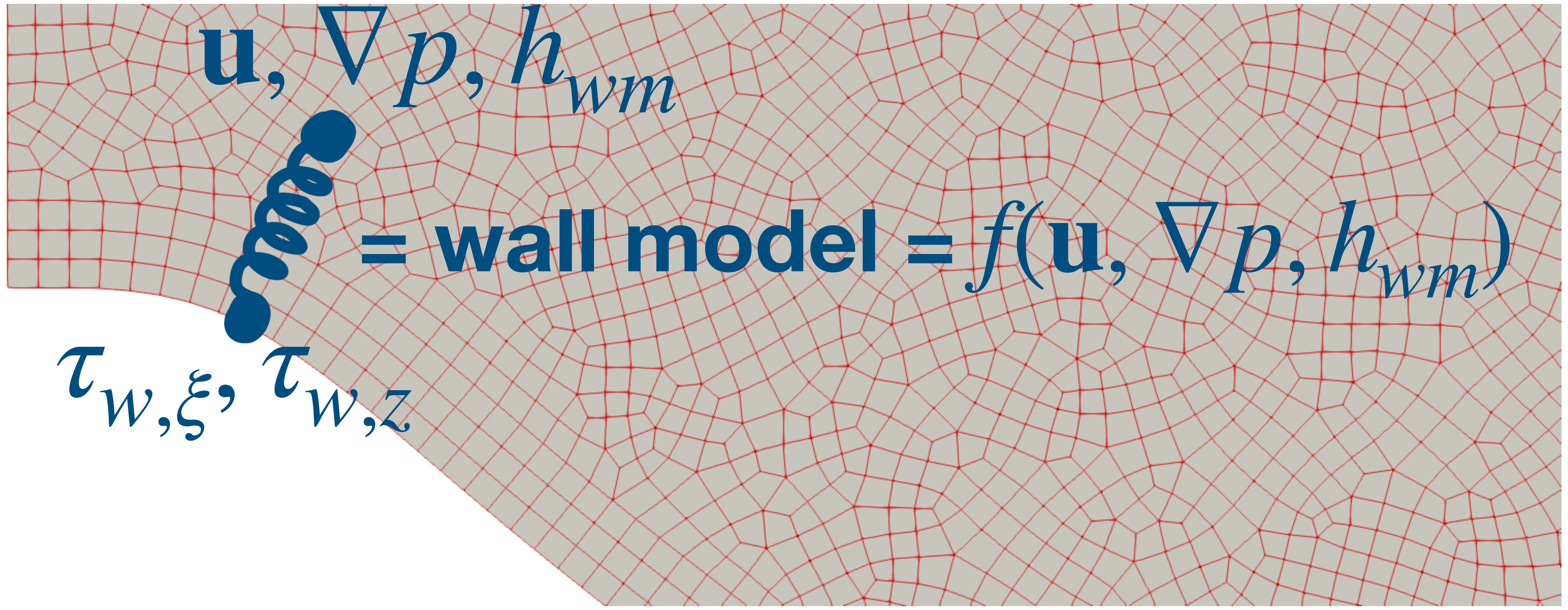

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$\mathbf{u}, \nabla p, h_{wm}$

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Cenaero

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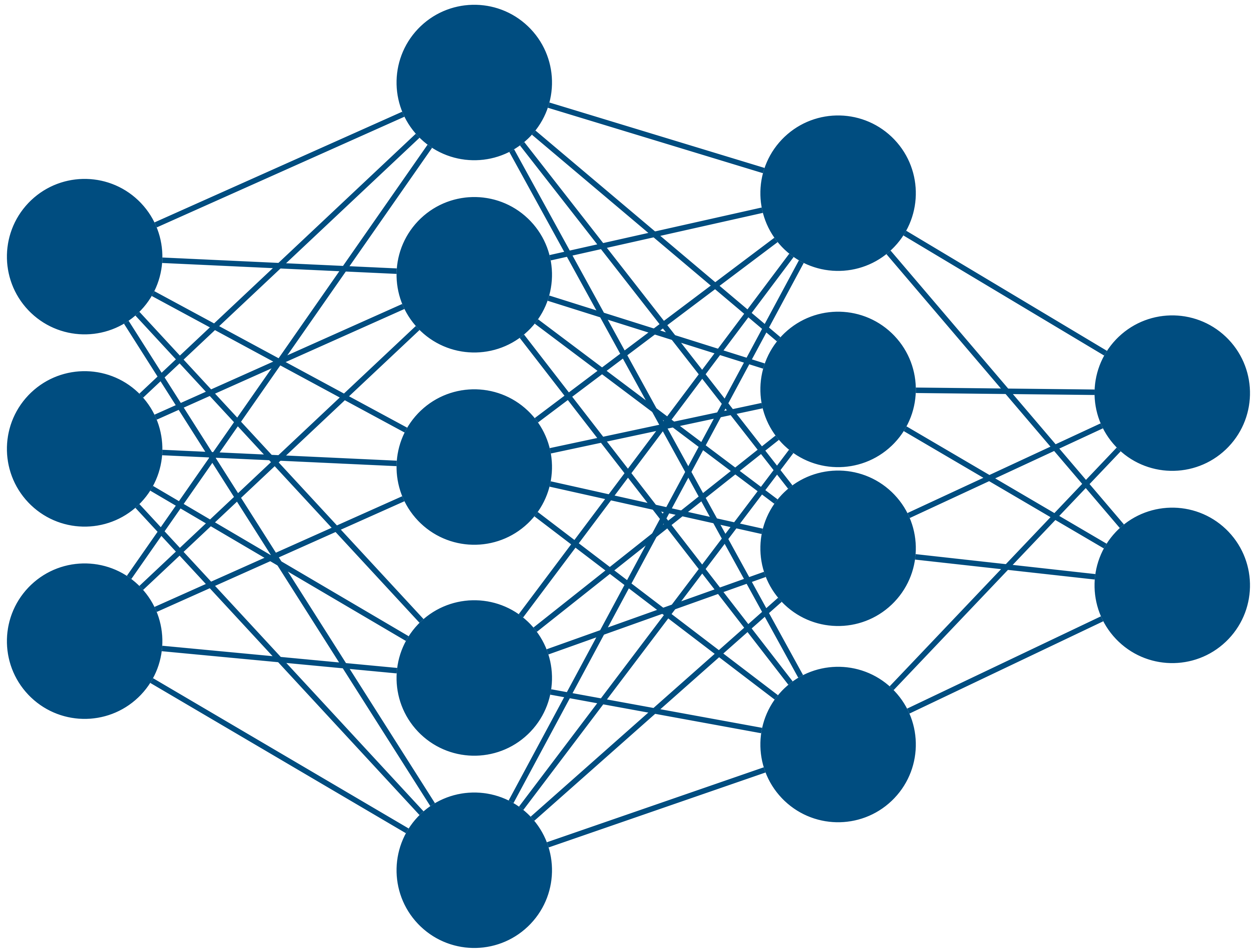

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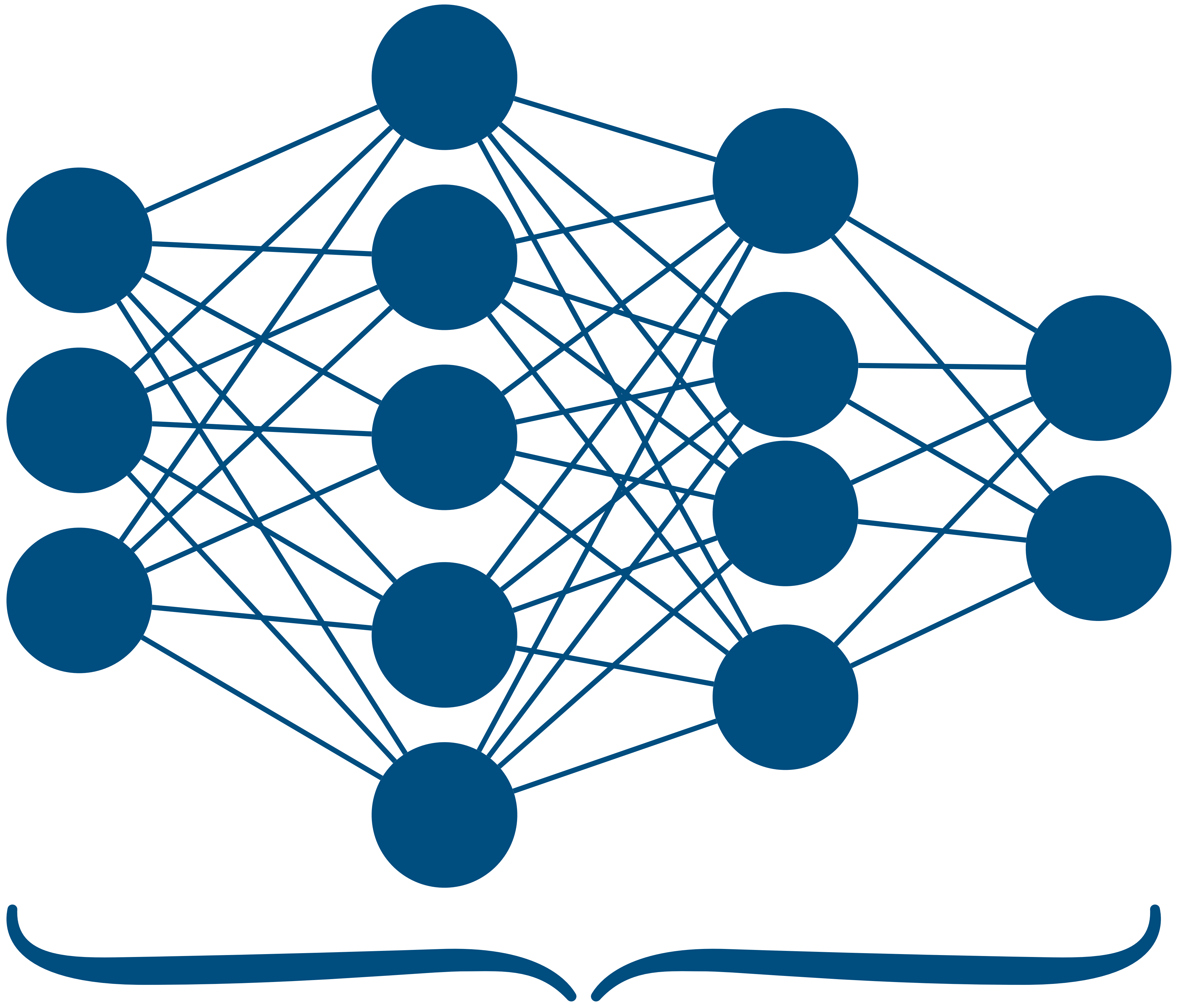

Neural network

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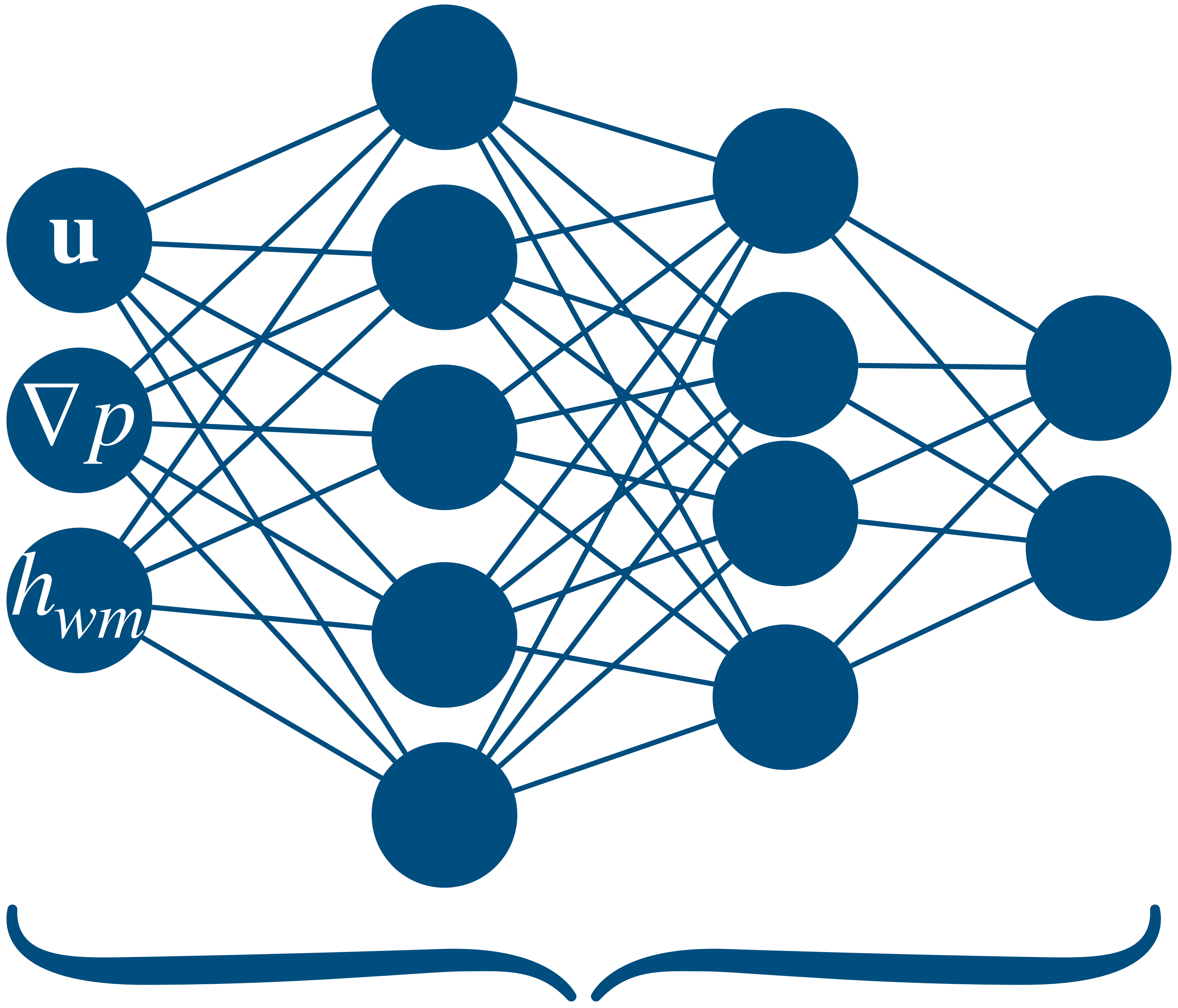

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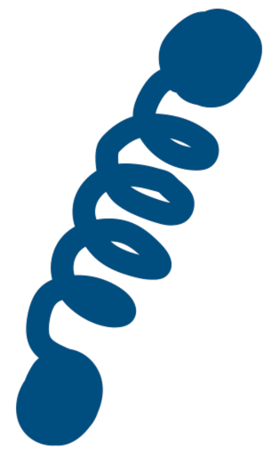

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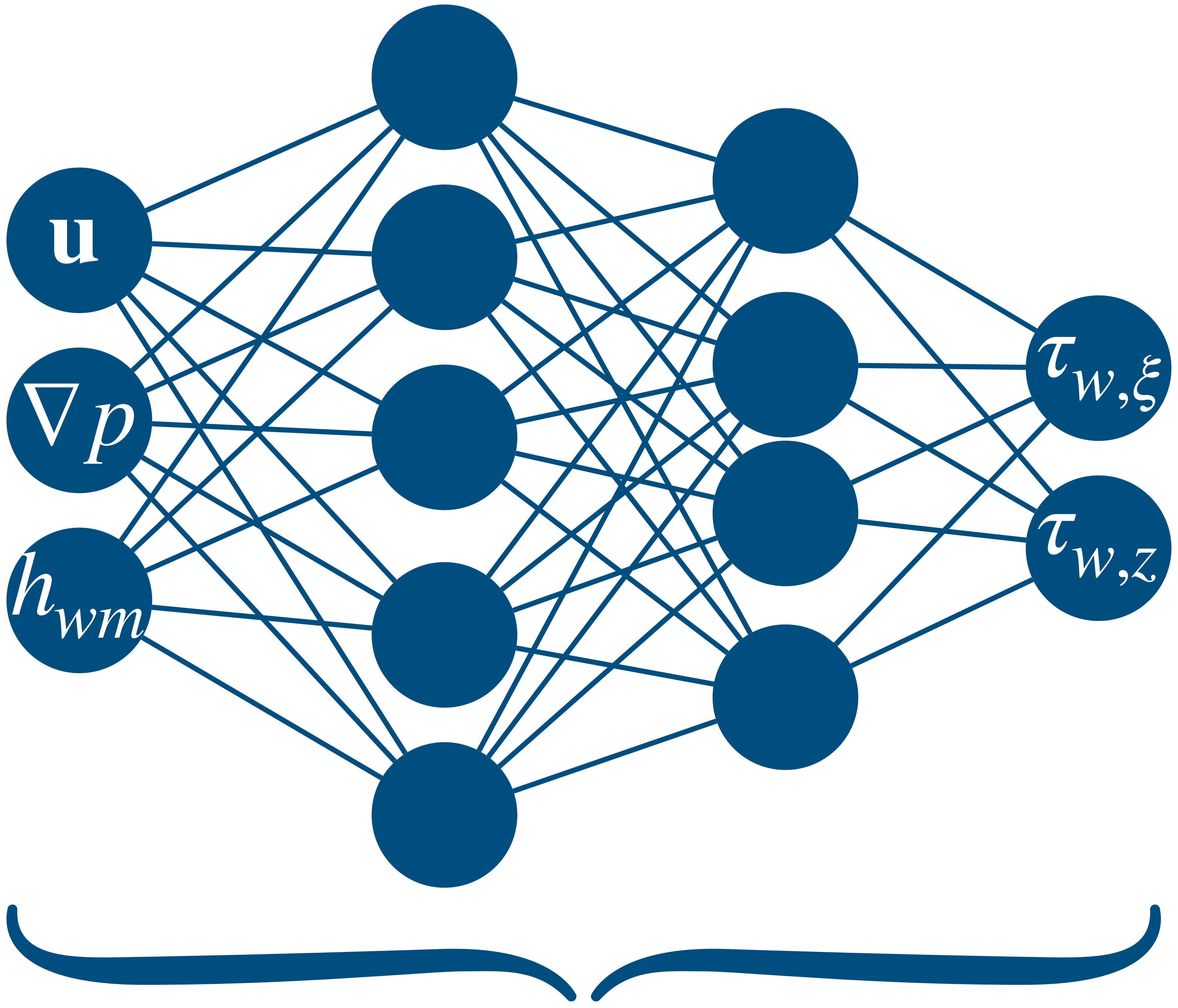

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To apply such a wall model, the following **questions** should be answered:

- **Where** the inputs are extracted (i.e., input stencil)?
- **How** to normalize the inputs and the outputs?
- **Which** neural network to use?

Brief explanation of the test case

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Two-dimensional periodic hill: bi-periodic flow evolving between two wall featuring a streamwise constriction, the flow is controlled by a pressure gradient to match the bulk Reynolds number of 10,595

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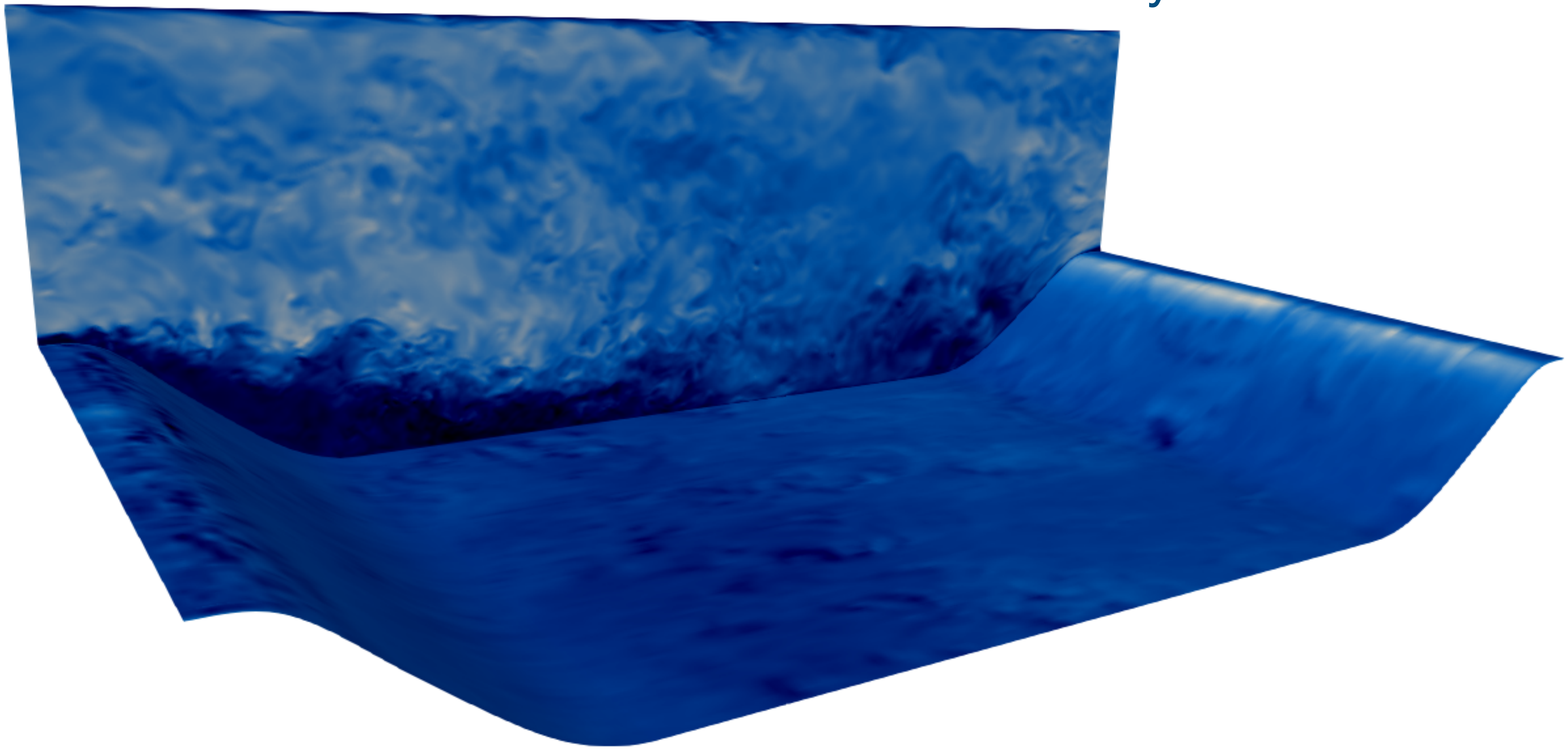

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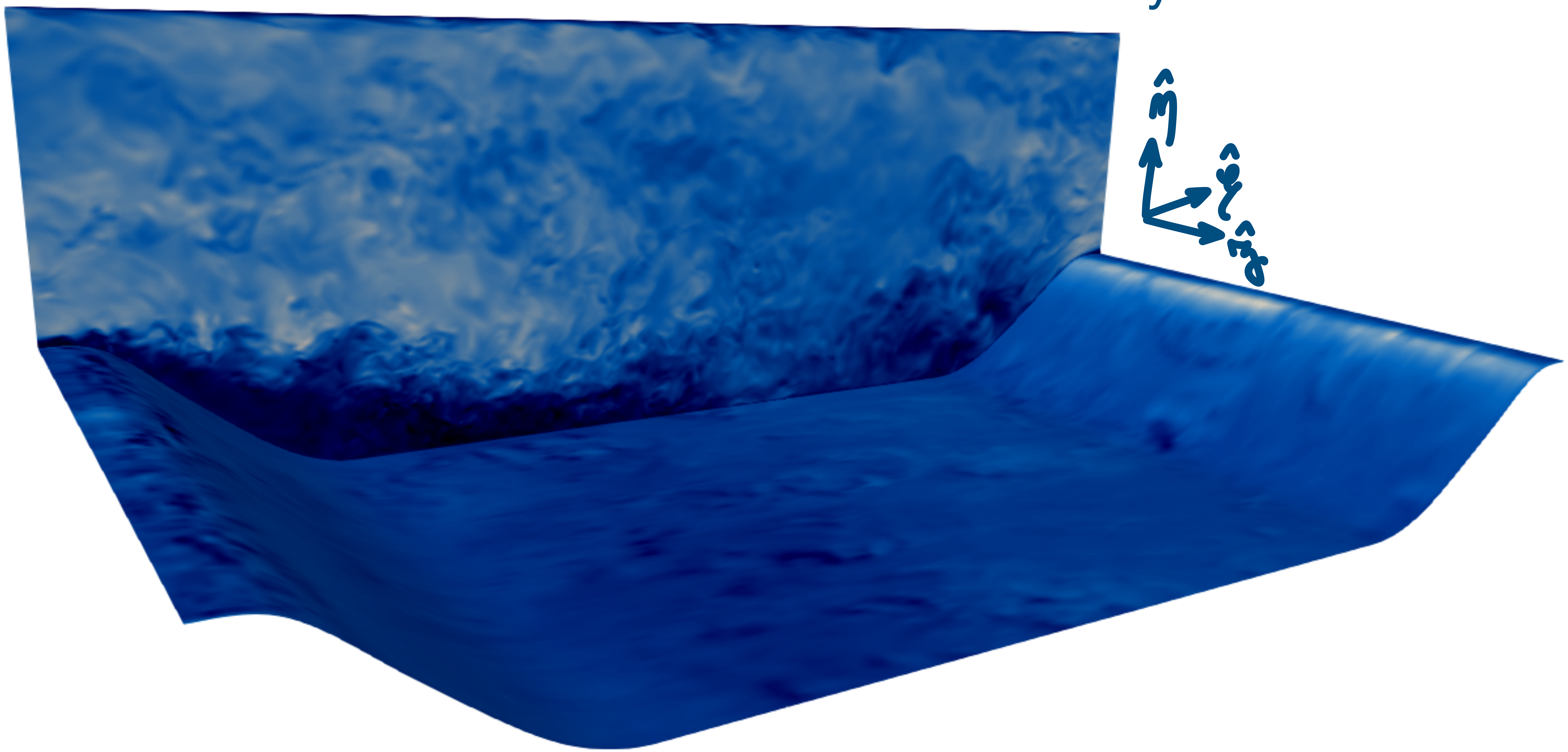

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[1] Boxho, M. *et al.*: Analysis of space-time correlations to support the development of wall-modeled LES. Flow, Turbulence and Combustion. (2022).

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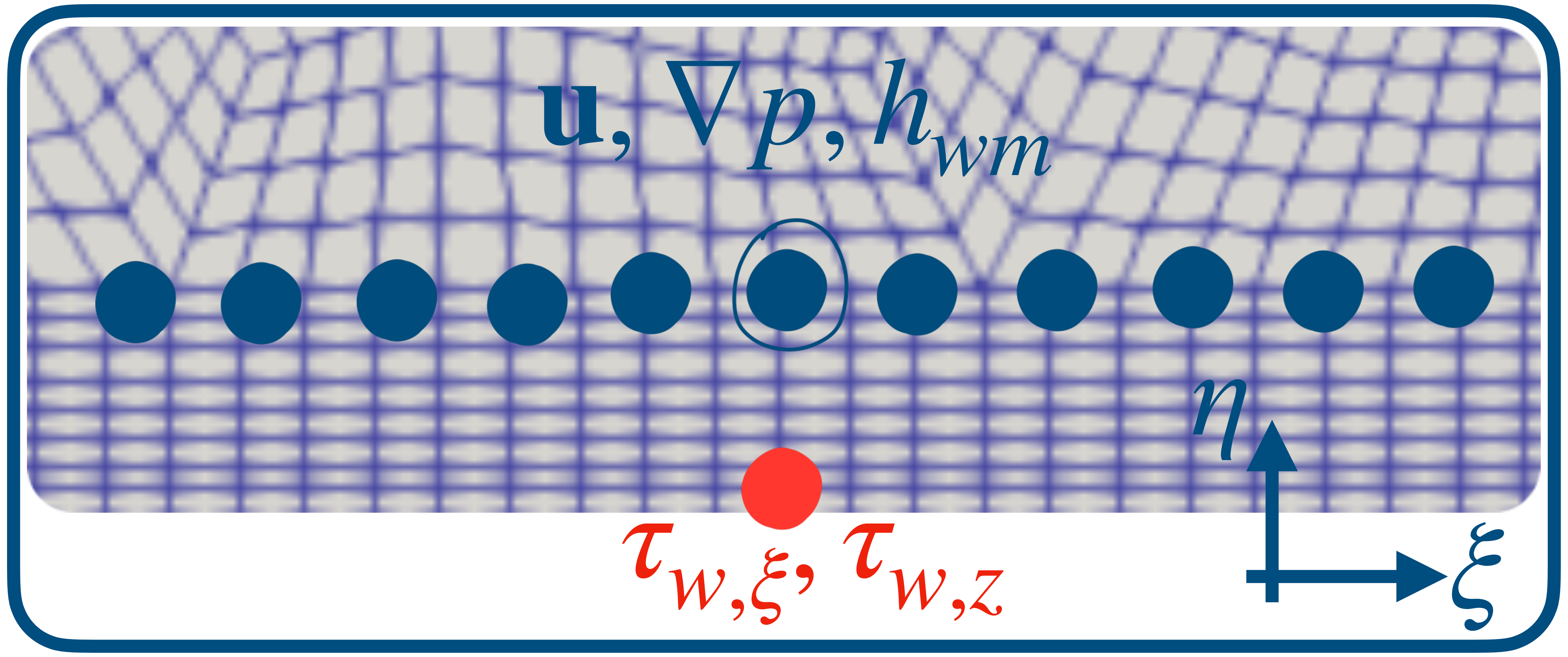
Input stencil



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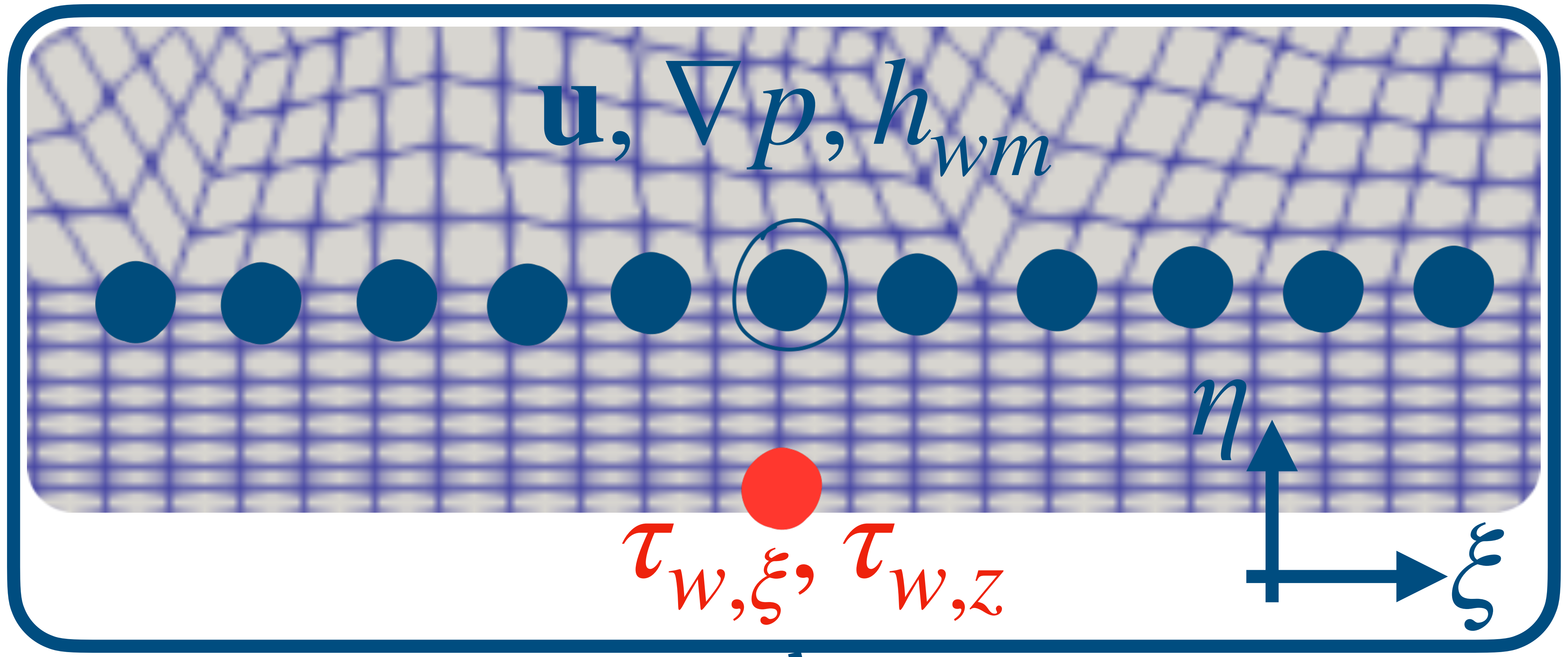

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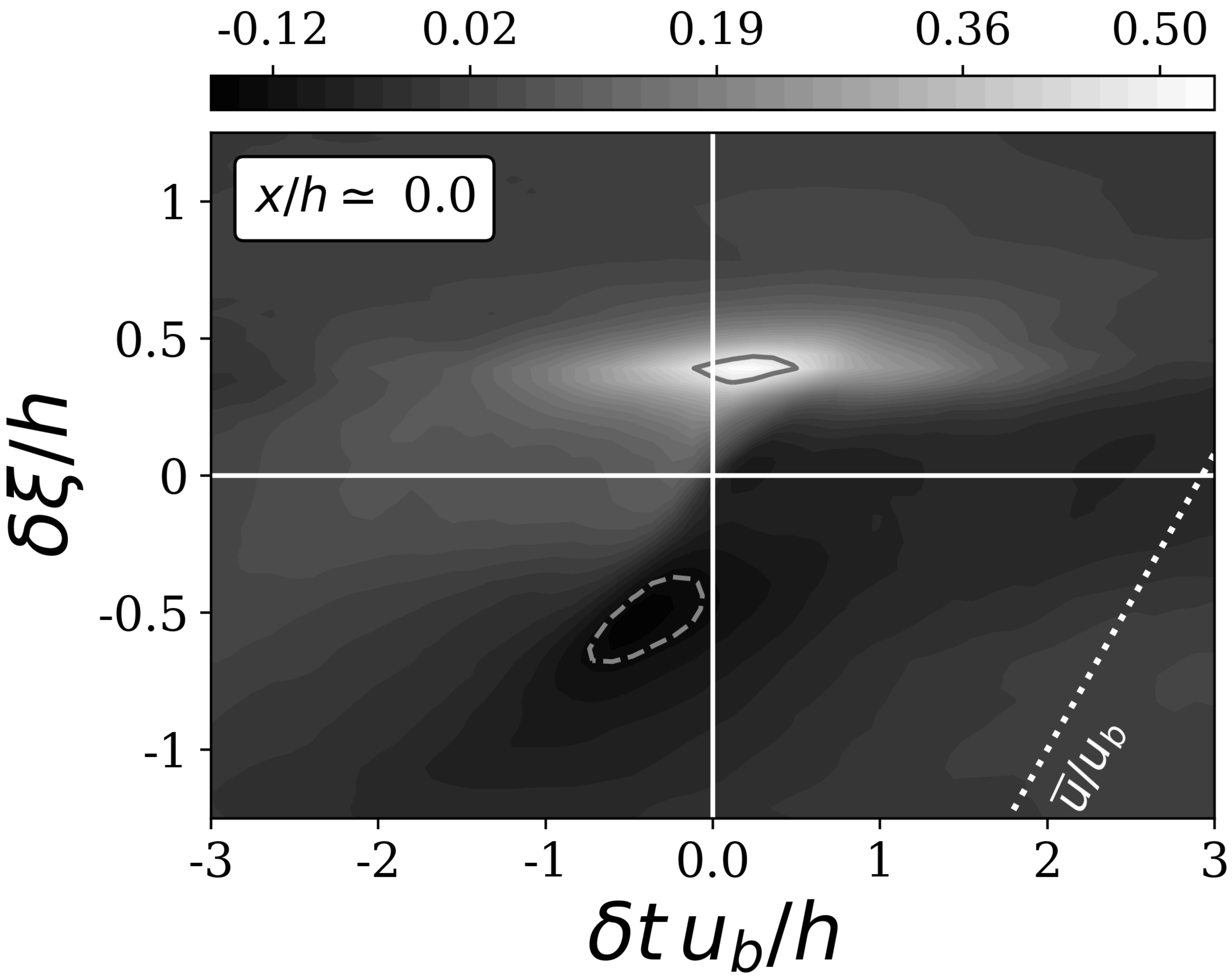

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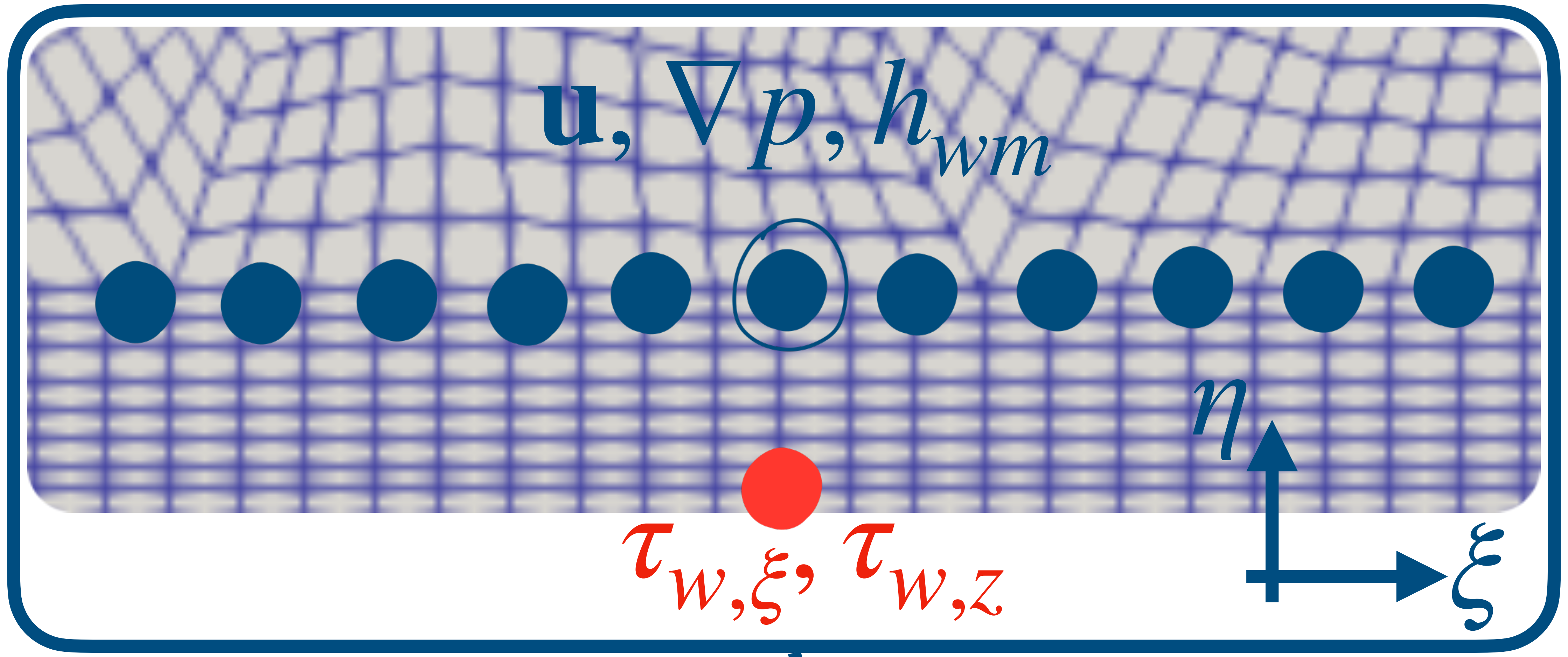

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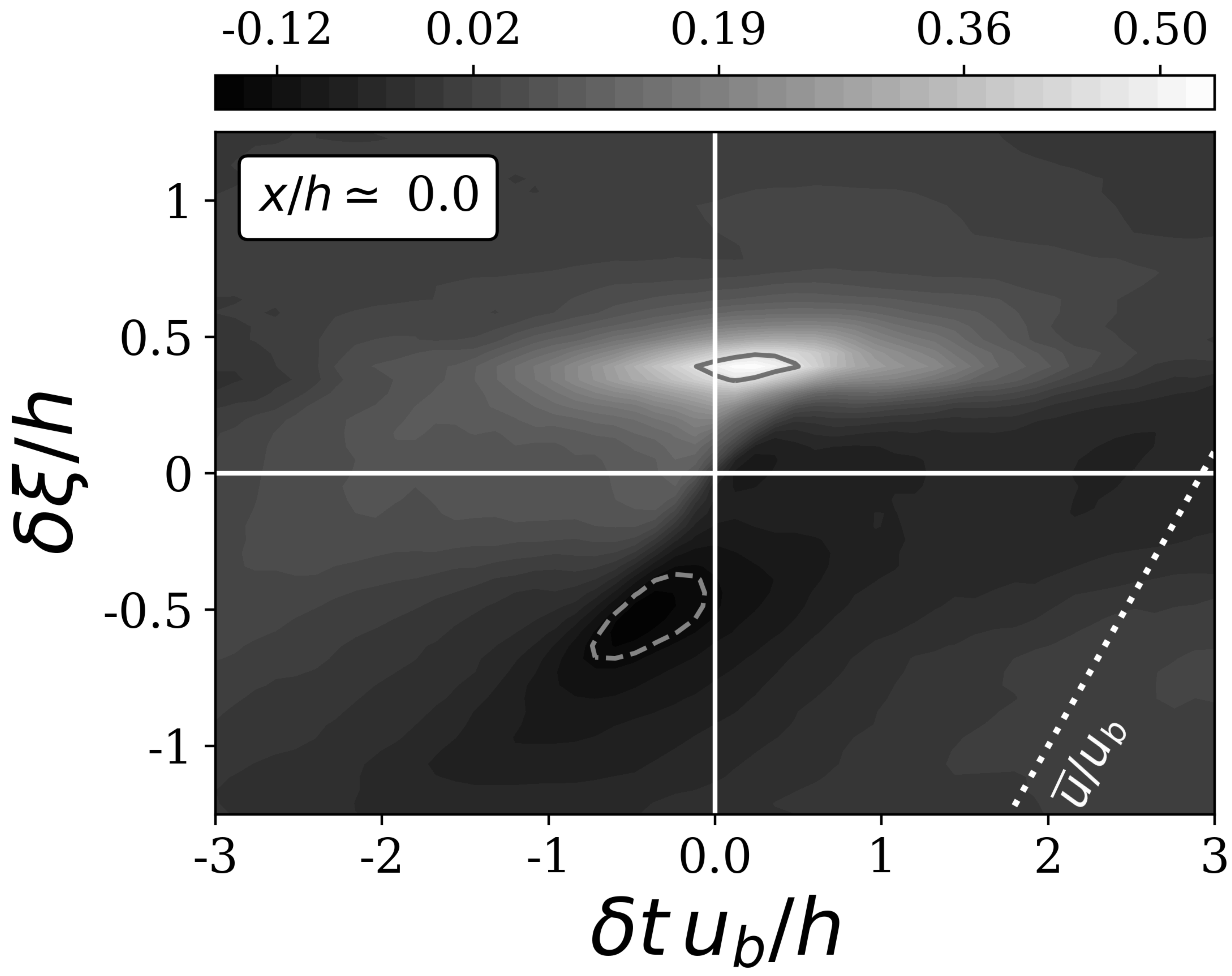

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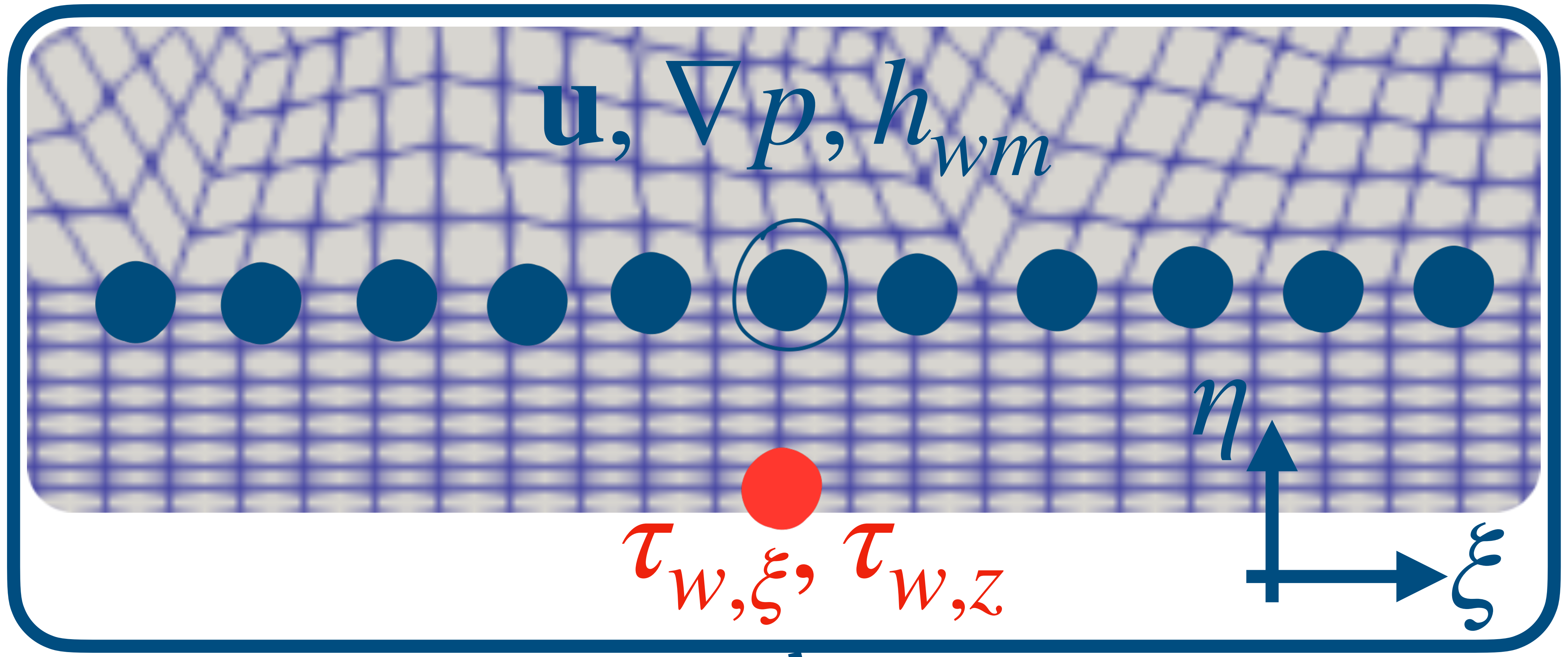

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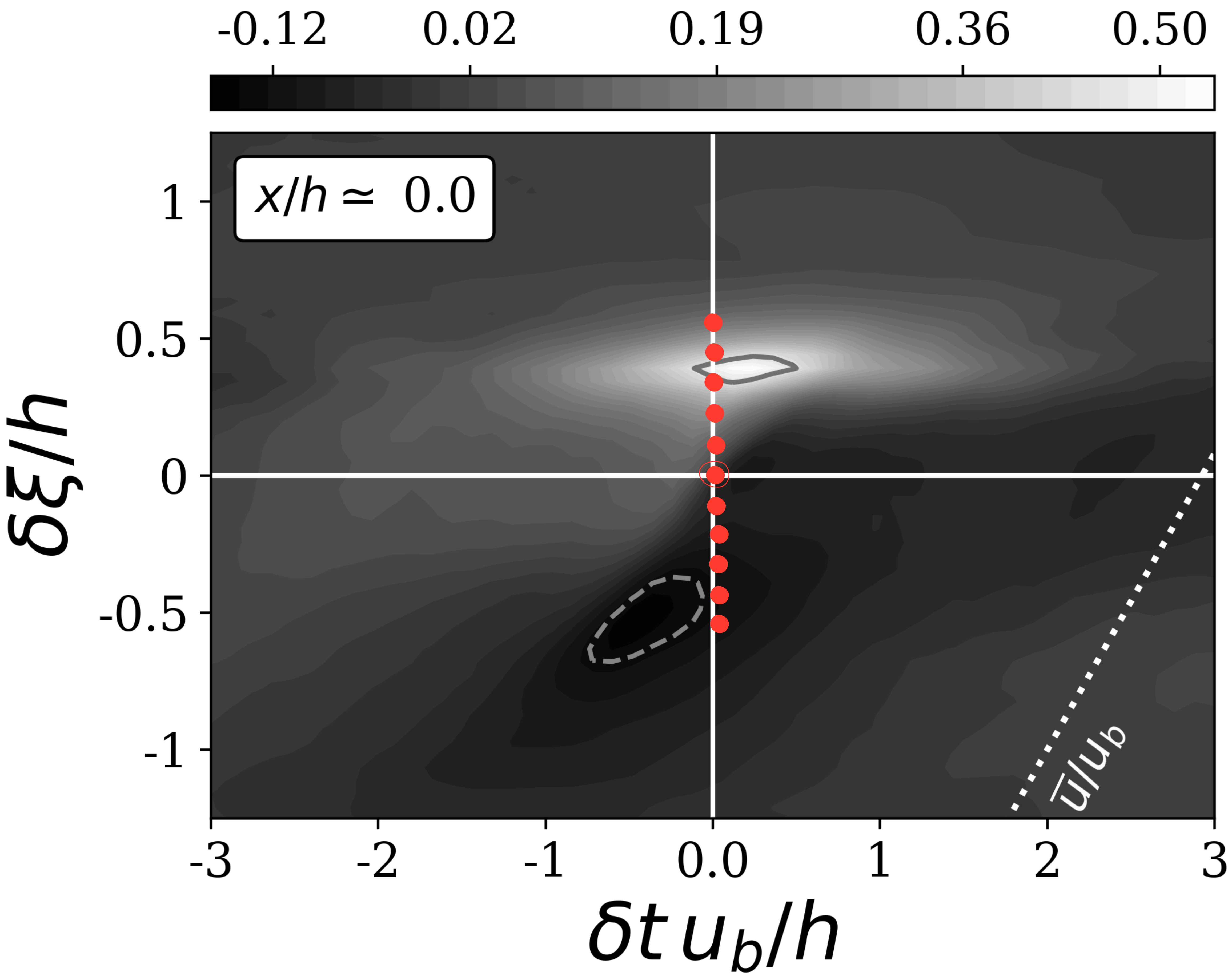

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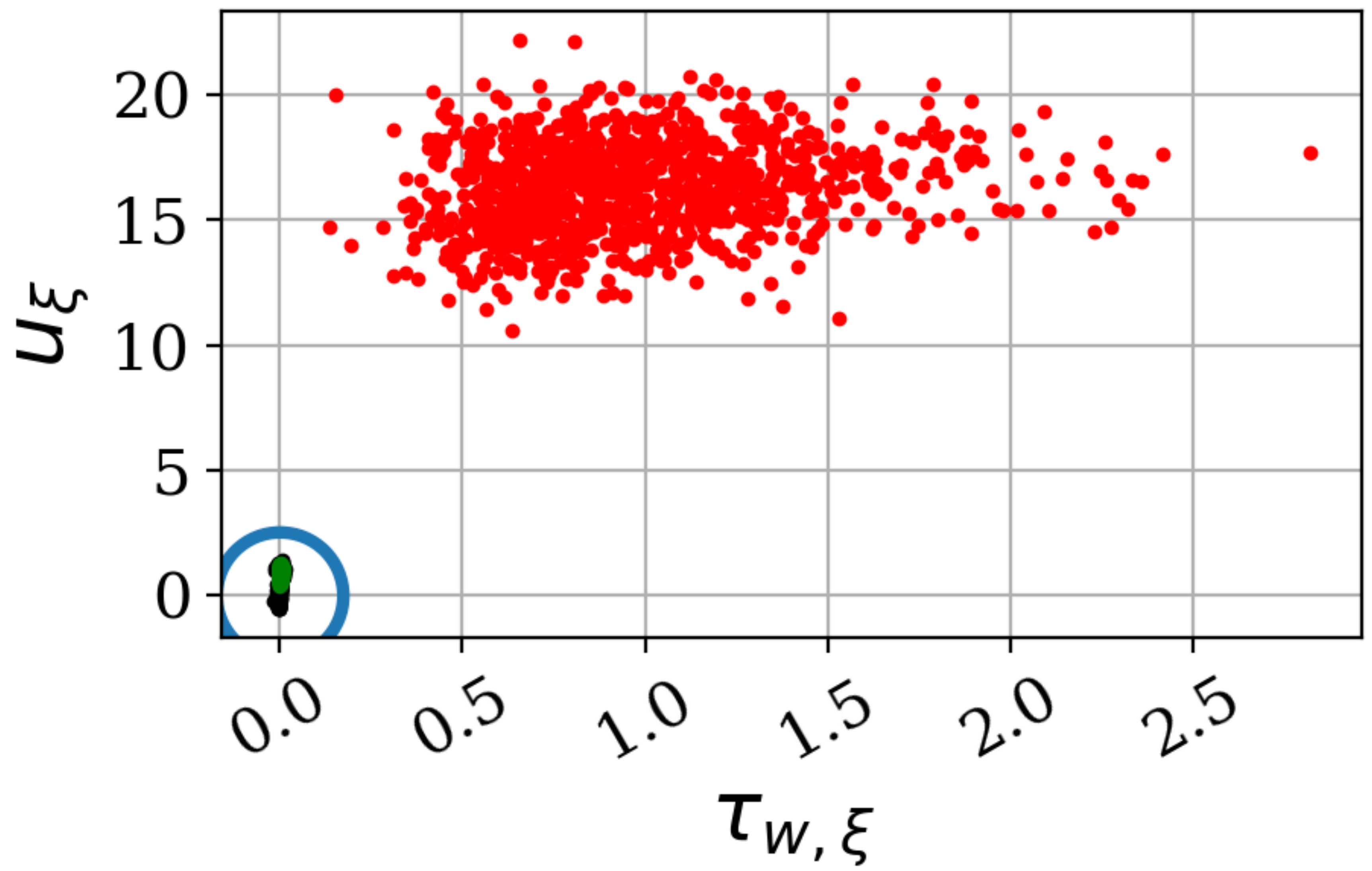

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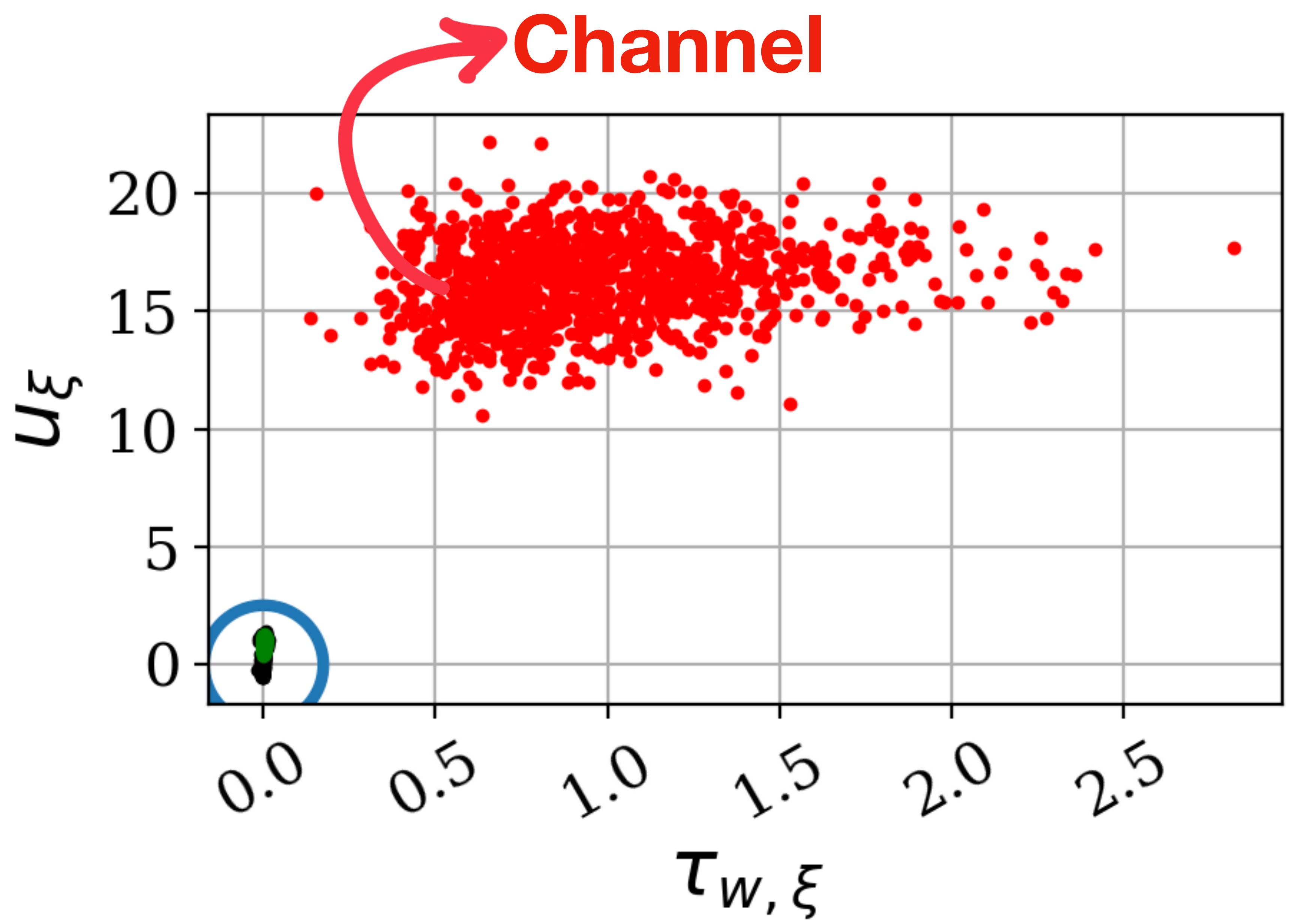

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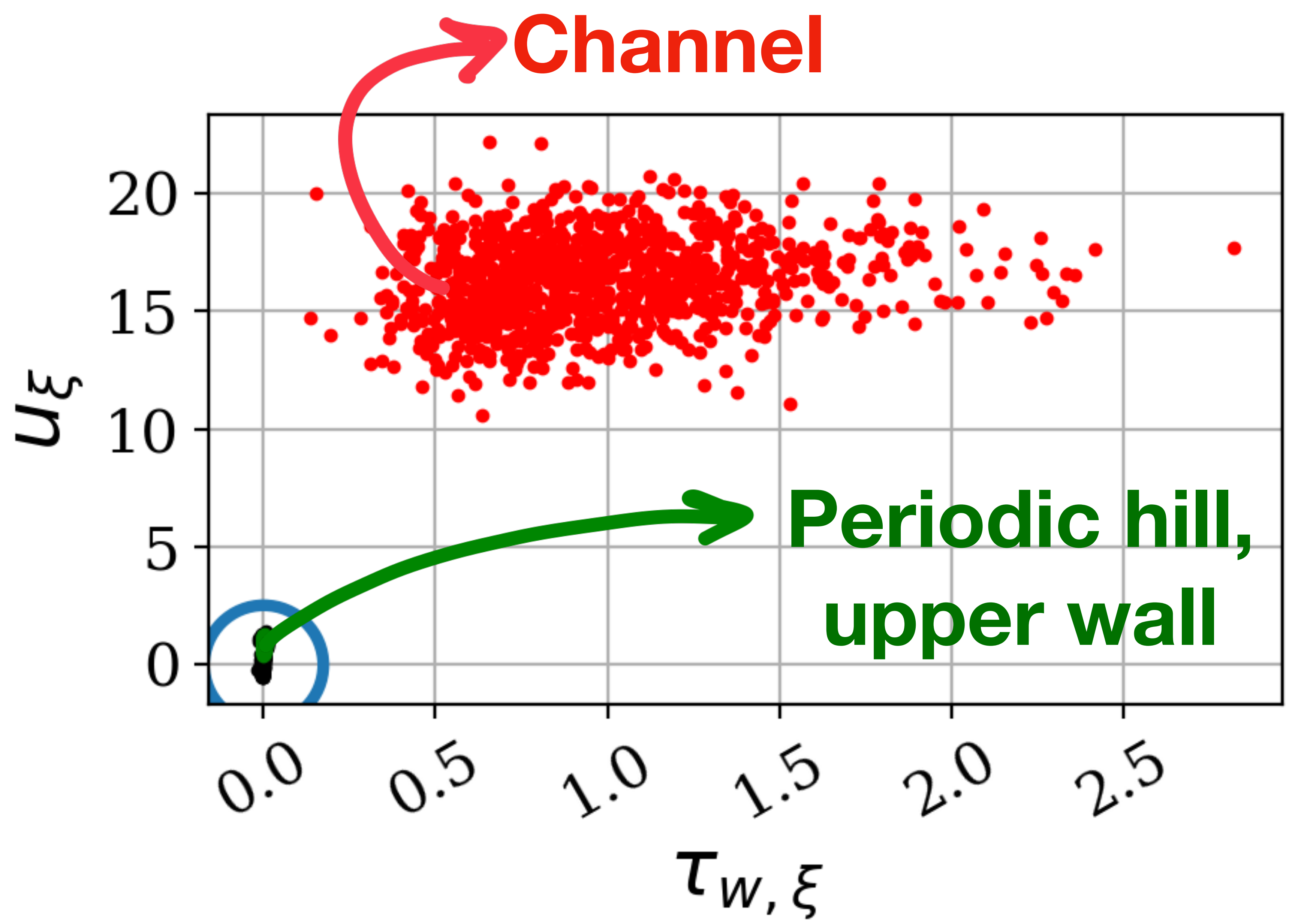

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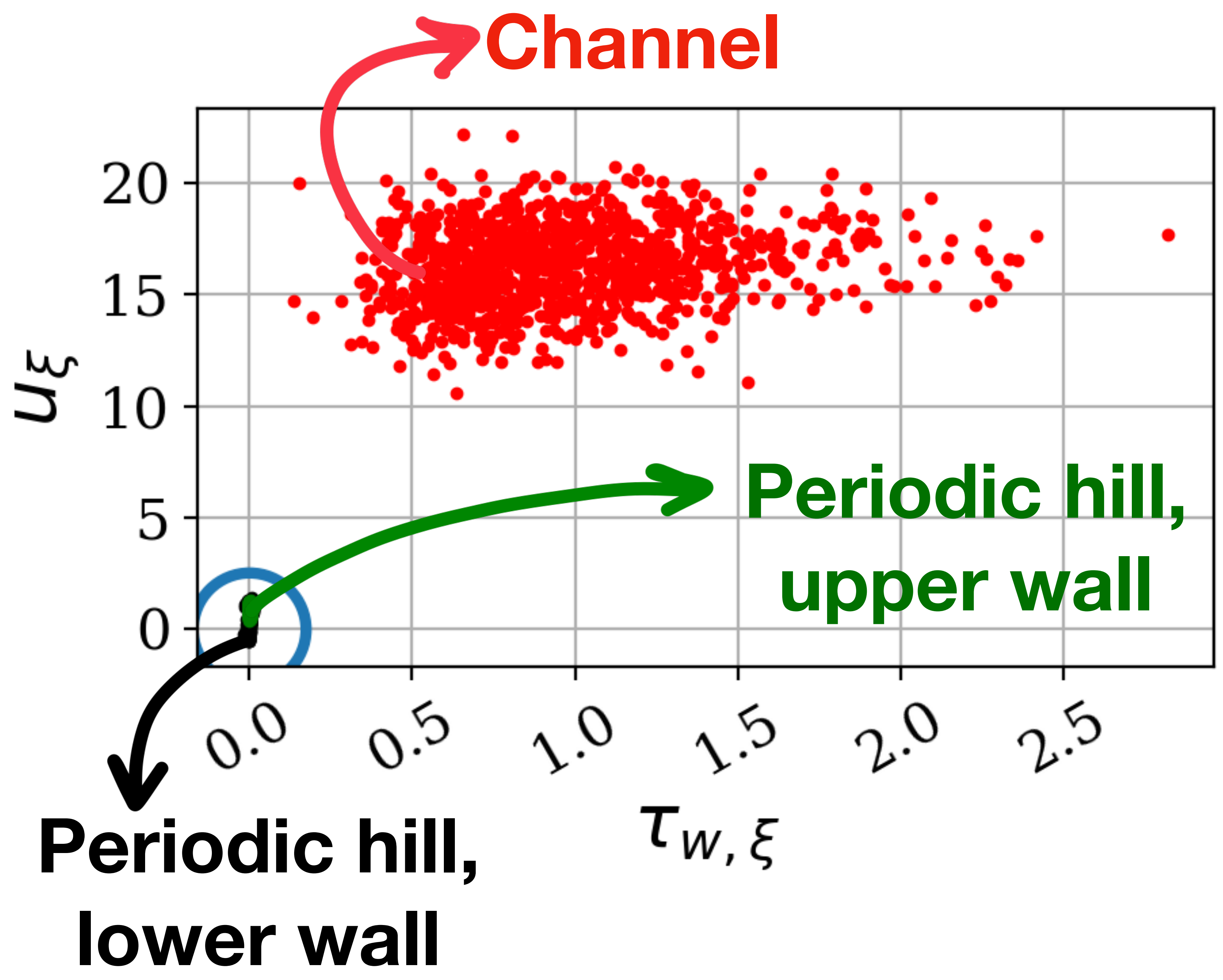

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Cenaero

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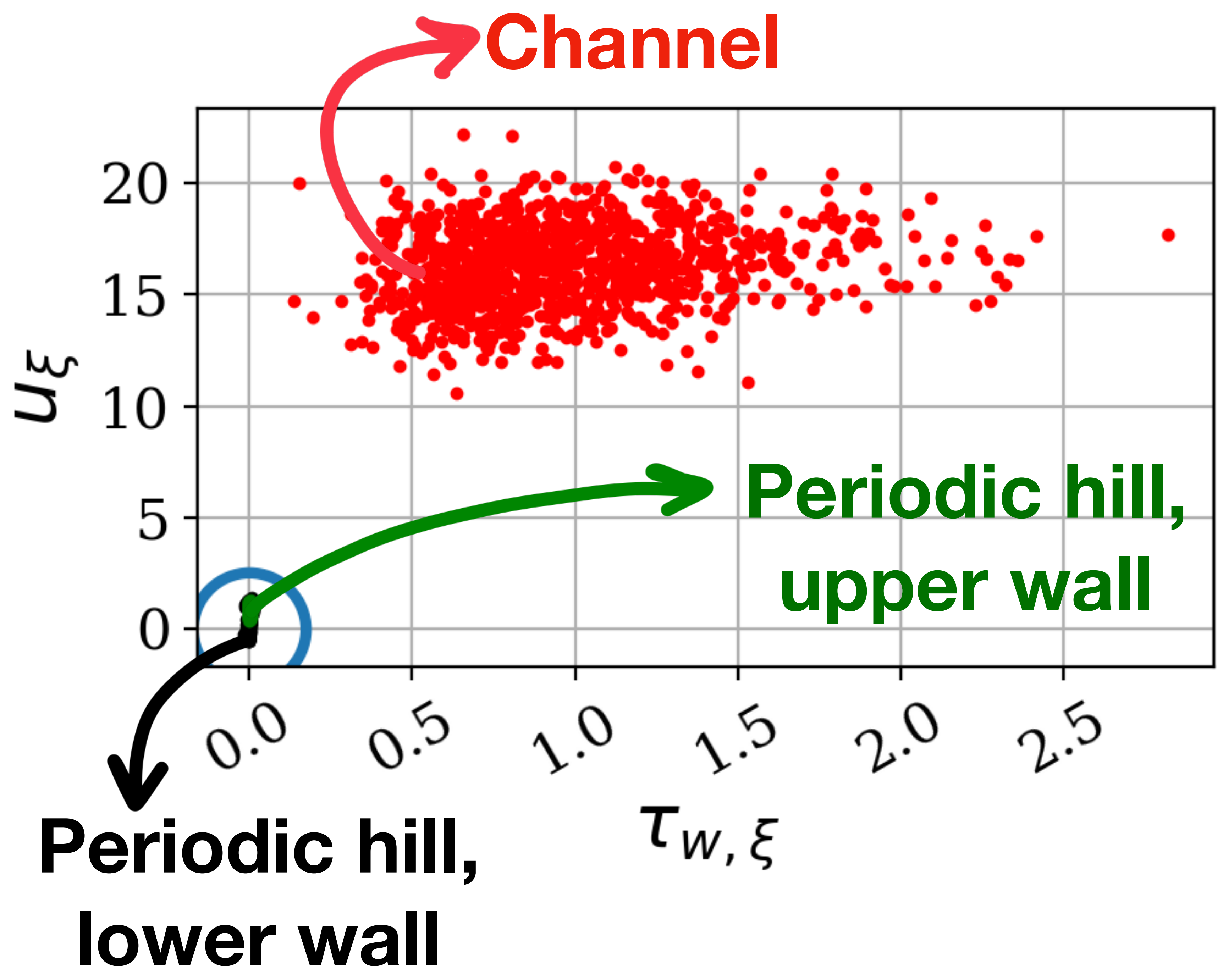

Normalization

The distribution between the input and output for different databases does not fit on each other →bad behavior for the training

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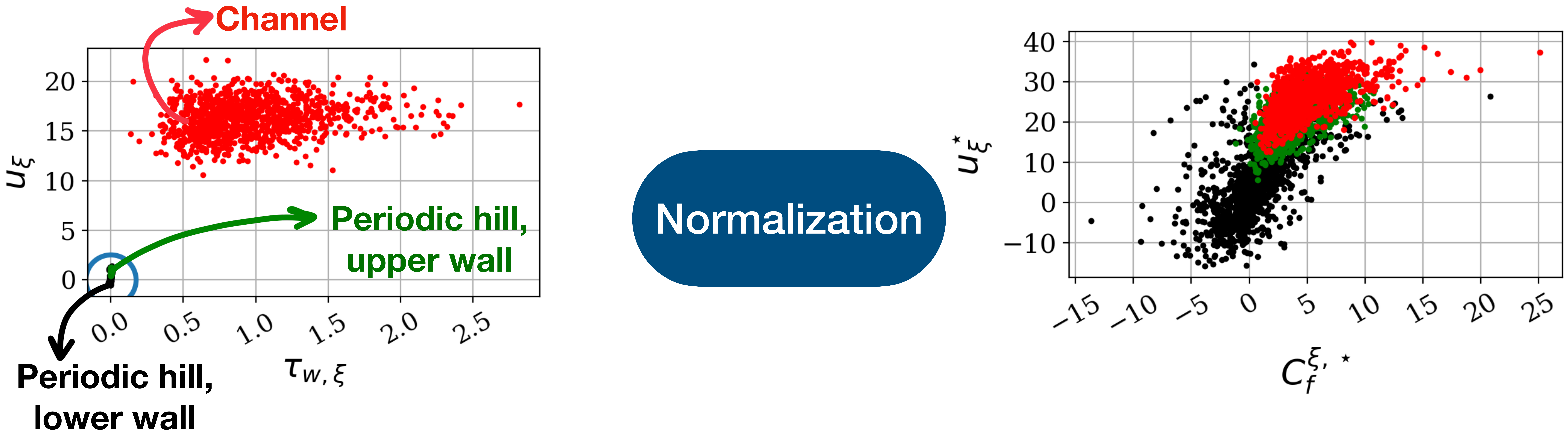

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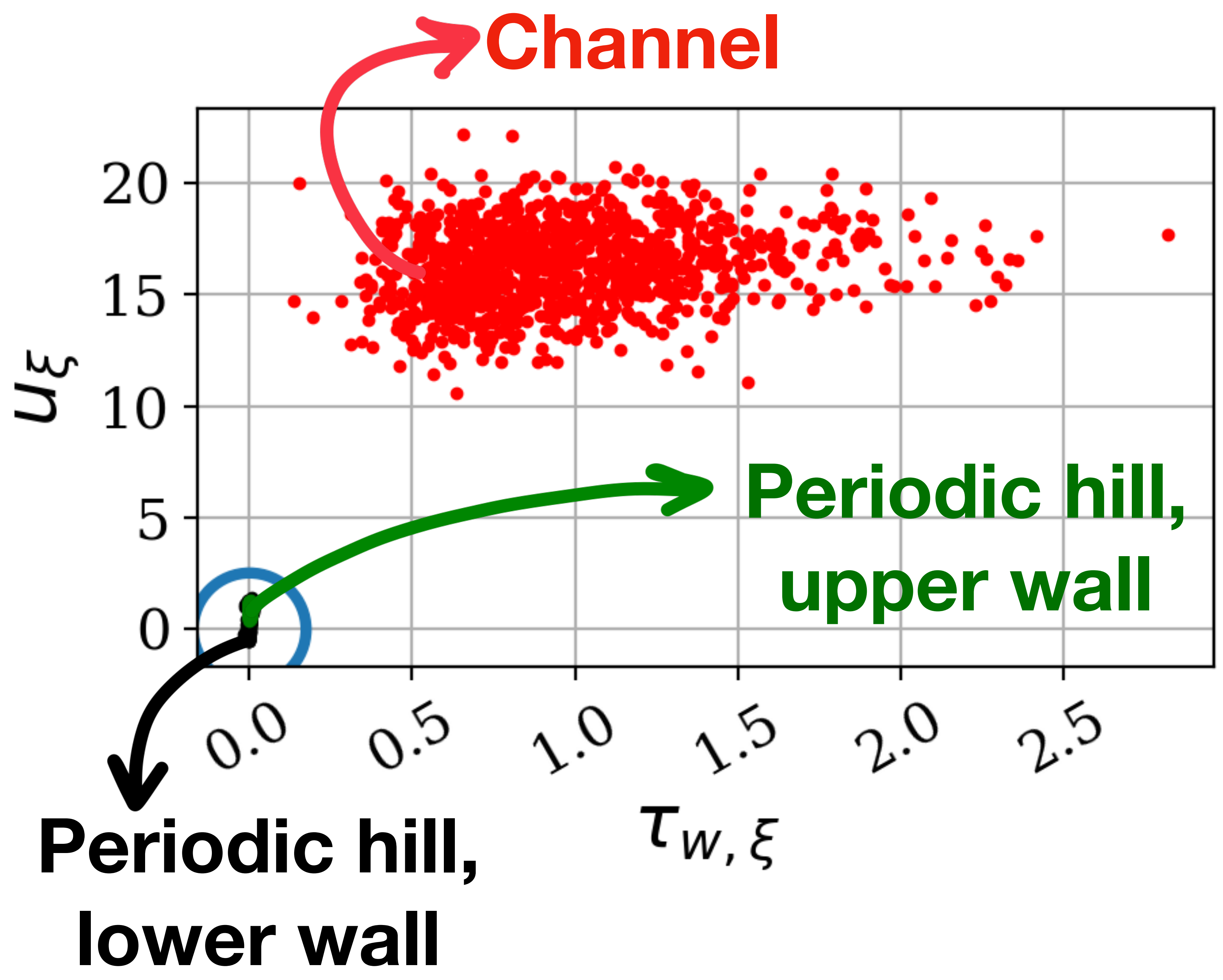
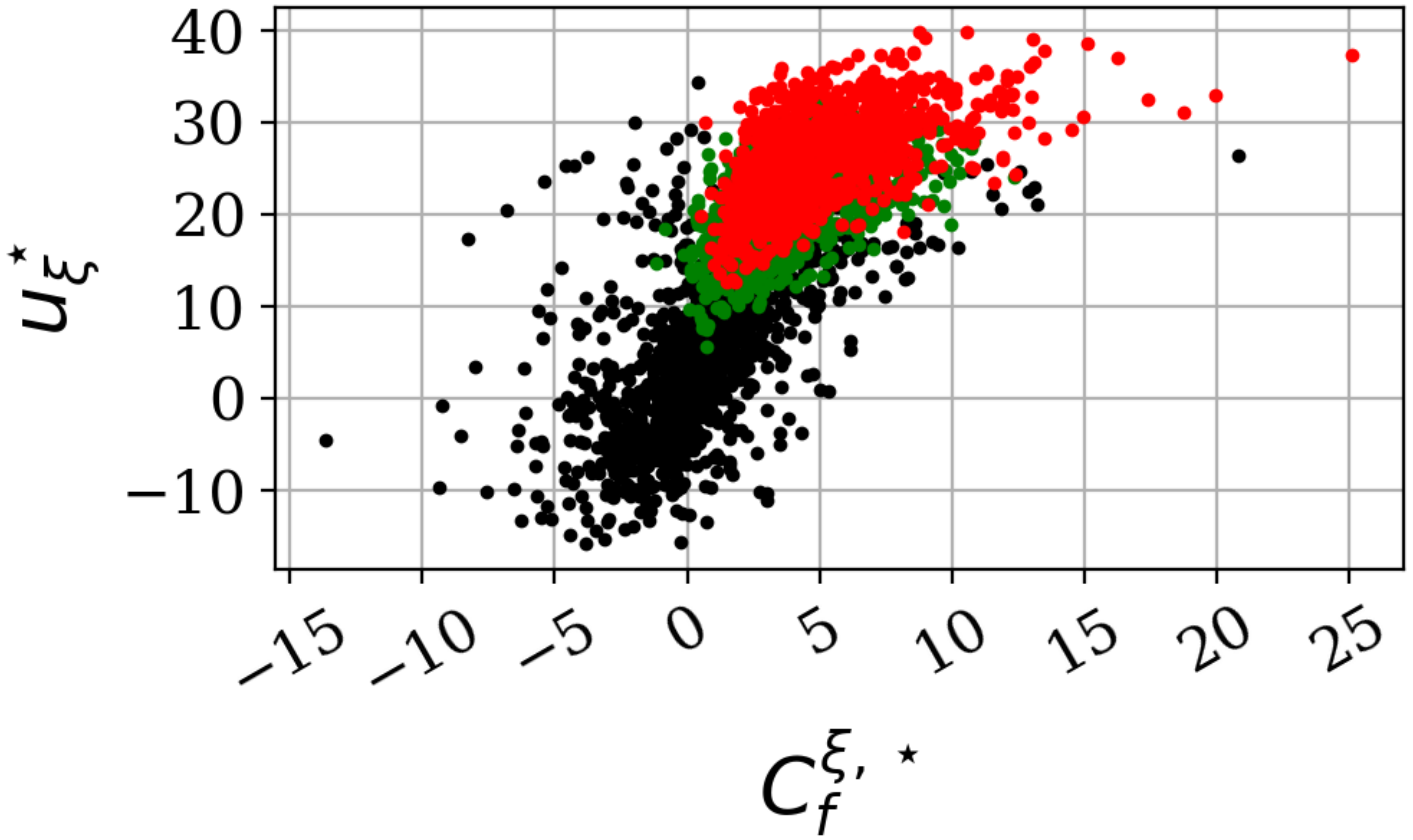
PROD-F-015-02

How to normalize the input/output pairs?



Normalization



The normalization allows a better fitting between the different databases →better behavior for the training

How to normalize the input/output pairs?

How to normalize the input/output pairs?

$$u^* = \frac{u}{u_{vp}}$$

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$$u^* = \frac{u}{u_{\tau p}}$$

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PROD-F-015-02

Cenaero

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Cenaero

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PROD-F-015-02

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$$u_v = \sqrt{\frac{\nu \|u\|}{h_{wm}}}$$

$$u_p = \left|\frac{\nu}{\rho} \frac{\partial p}{\partial x}\right|^{1/3}$$

PROD-F-015-02

Cenaero

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PROD-F-015-02

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How to normalize the input/output pairs?

$$u^* = \frac{u}{u_{vp}} \qquad \nabla p^* = \frac{h_{wm}}{\rho\, u_{vp}^2} \nabla p \qquad h_{wm}^* = \ln\left(\frac{h_{wm}}{y_{vp}}\right) \qquad \Delta\rho^* = \frac{\Delta\rho}{y_{vp}}$$

$$u_{vp} = \sqrt{u_v^2 + u_p^2}$$

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PROD-F-015-02

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Concerning the output, we get

PROD-F-015-02

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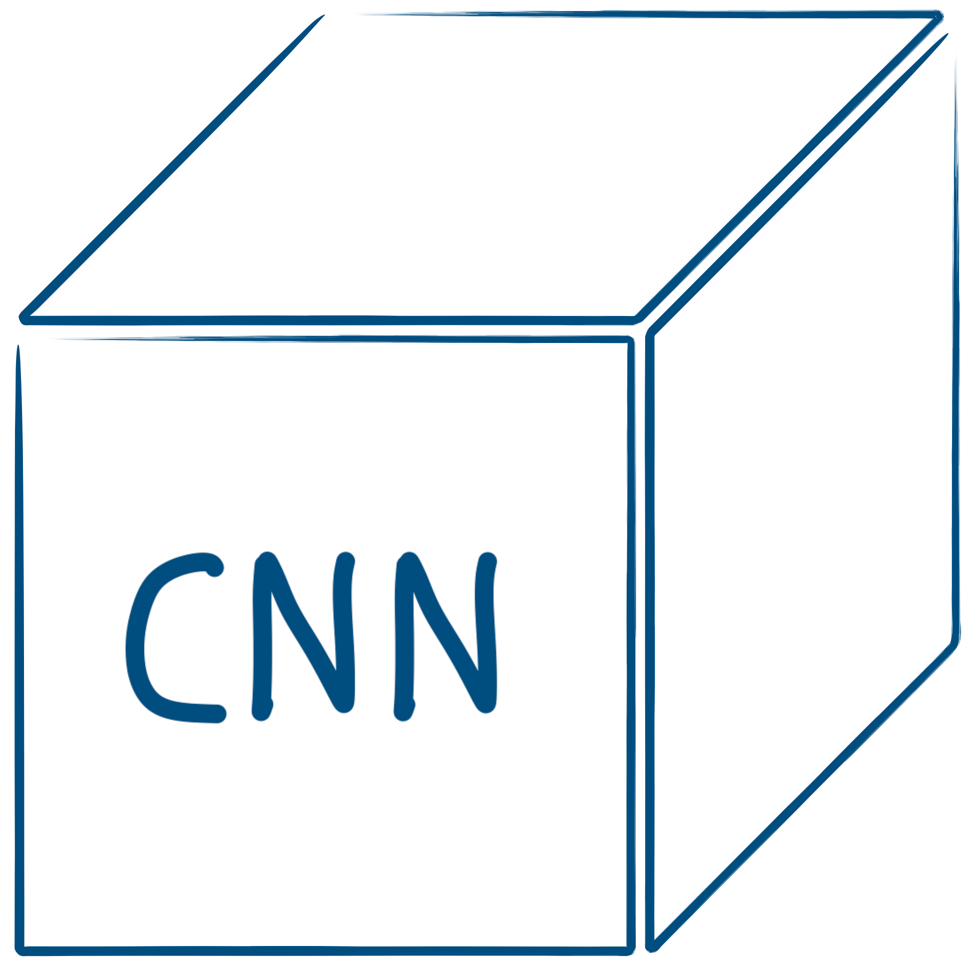
$$y_{vp} = \frac{\nu}{u_{vp}}$$

Concerning the output, we get

$$\tau_{w\xi}^* = \frac{\tau_{w\xi}}{\frac{1}{2} \rho \langle u_{vp}^2 \rangle_{\xi z}}$$

PROD-F-015-02

Which neural network to use?

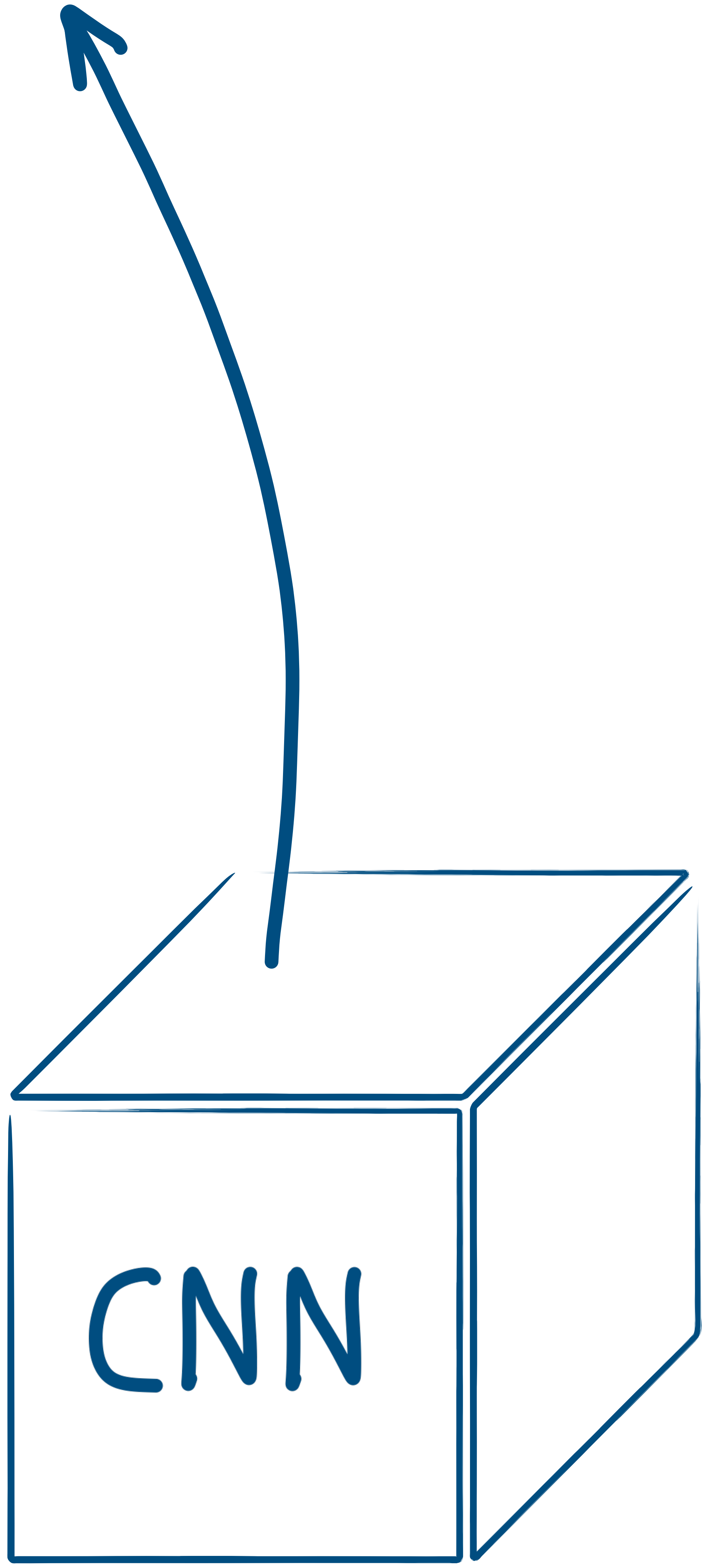


PROD-F-015-02

Cenaero

Which neural network to use?

Learnable convolutional kernel



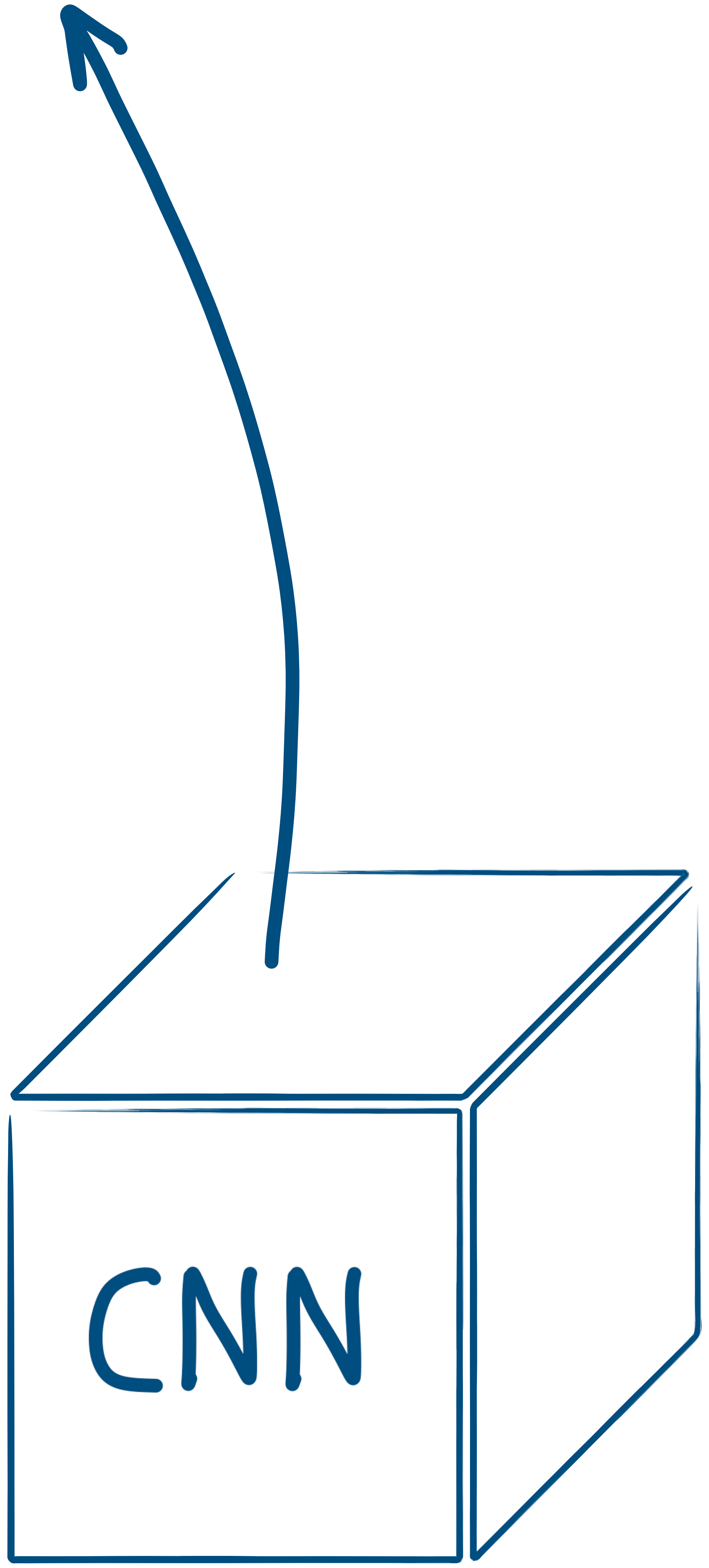
PROD-F-015-02

Cenaero

Which neural network to use?

Learnable convolutional kernel

0	1	0
1	1	1
0	1	0



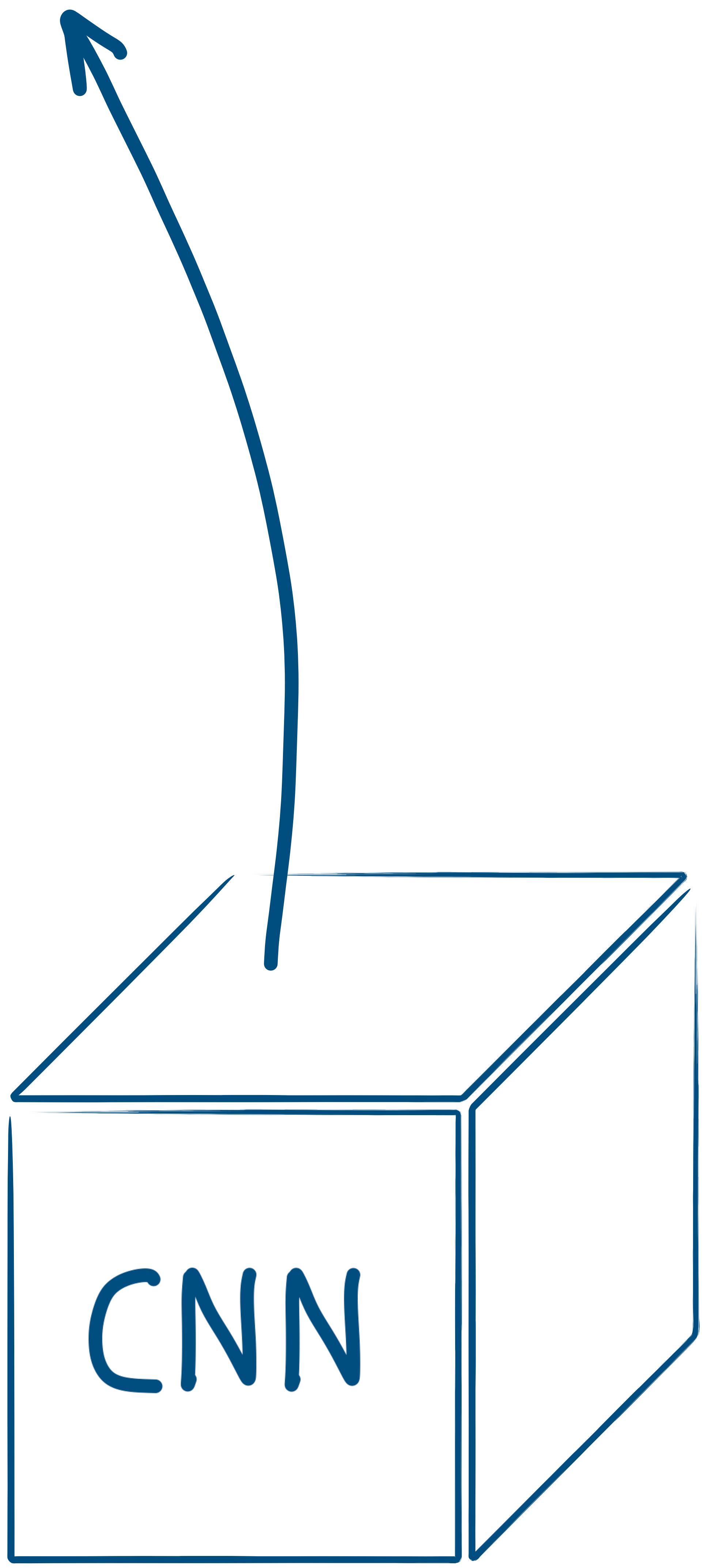
PROD-F-015-02

Cenaero

Learnable convolutional kernel

0	0.5	0.8	0.2	0.3
0.1	0	0.4	0.6	0.6
0.9	1	0	0	0.8
0.9	1	0.7	0.8	0.1

0	1	0
1	1	1
0	1	0



PROD-F-015-02

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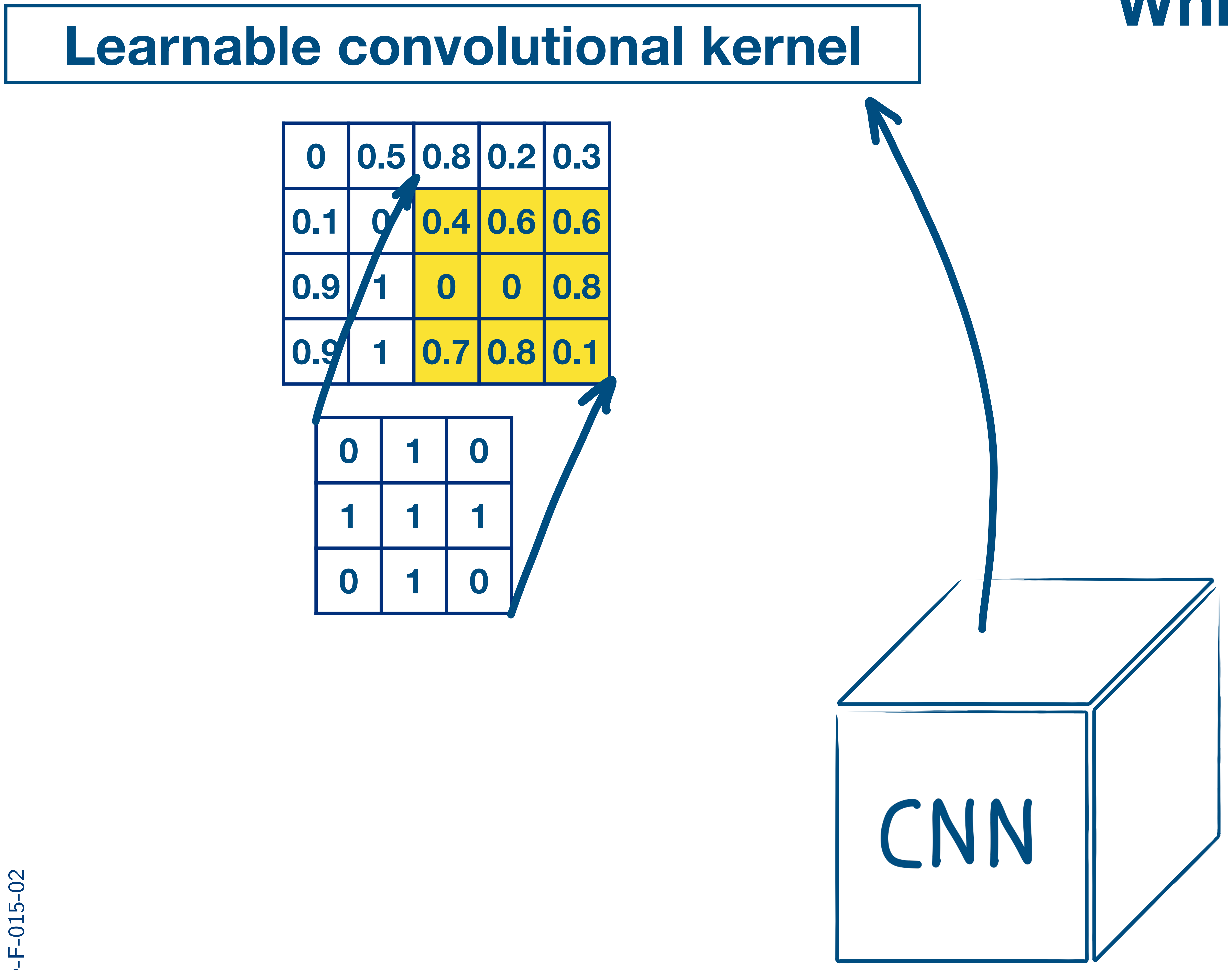

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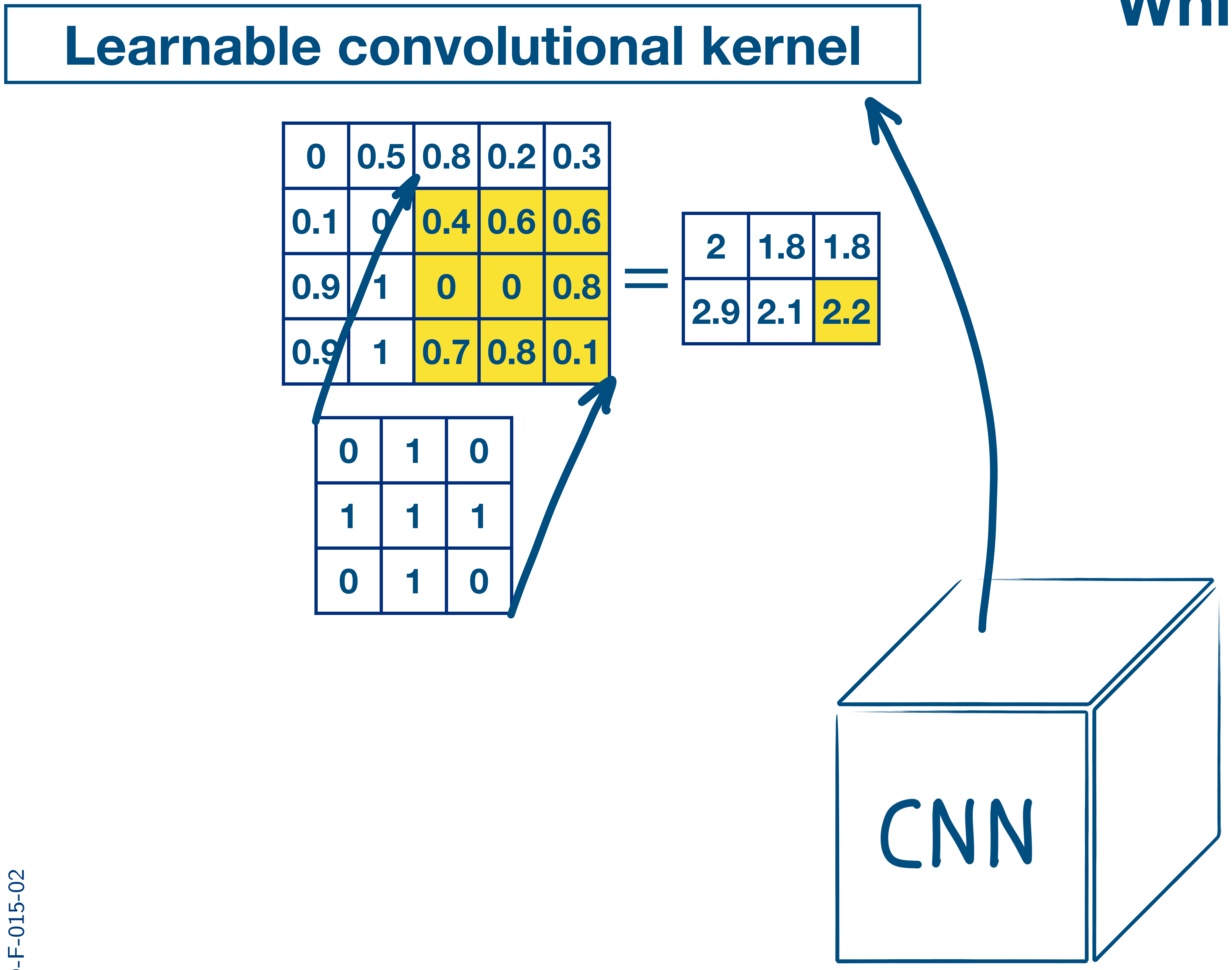

PROD-F-015-02

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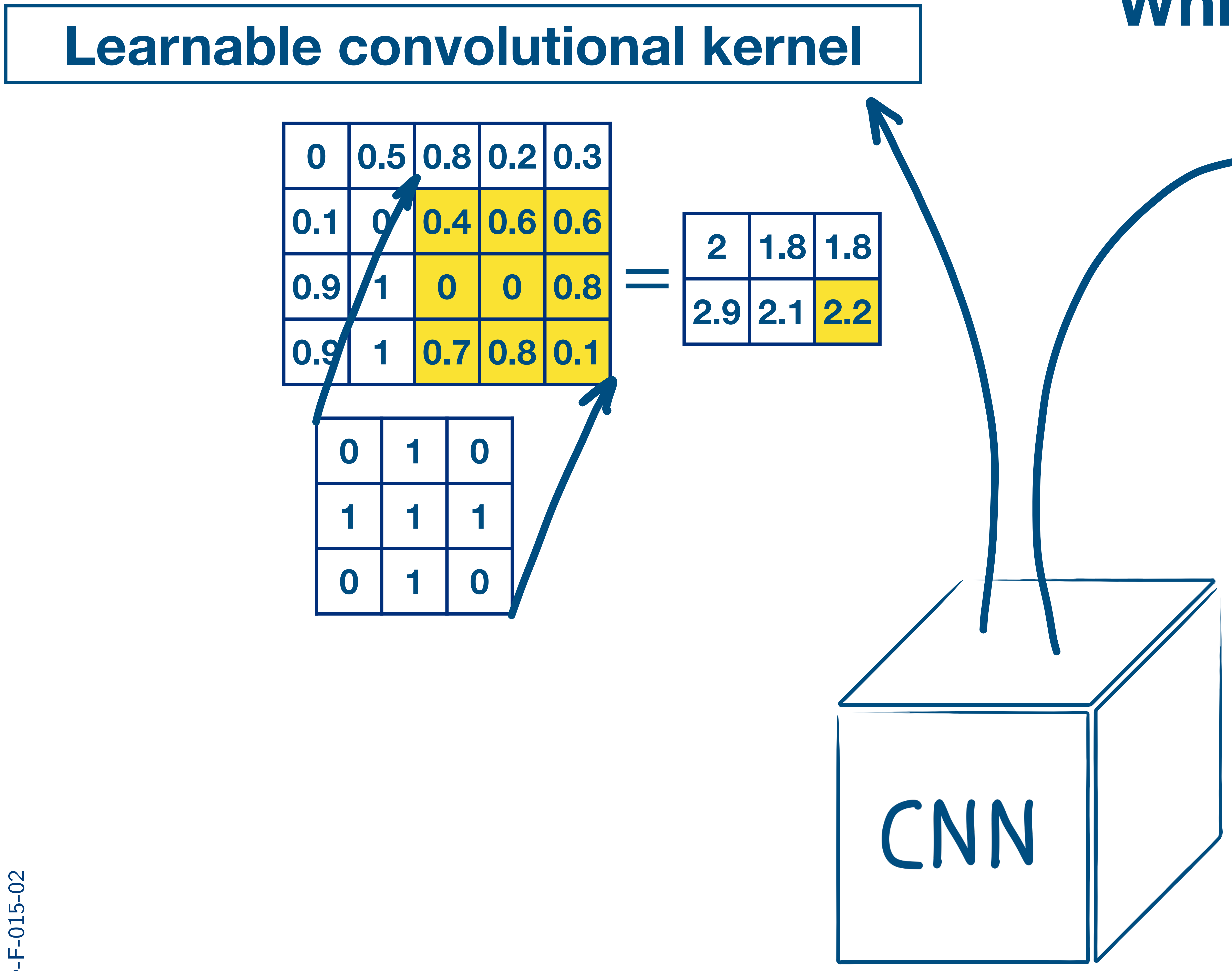

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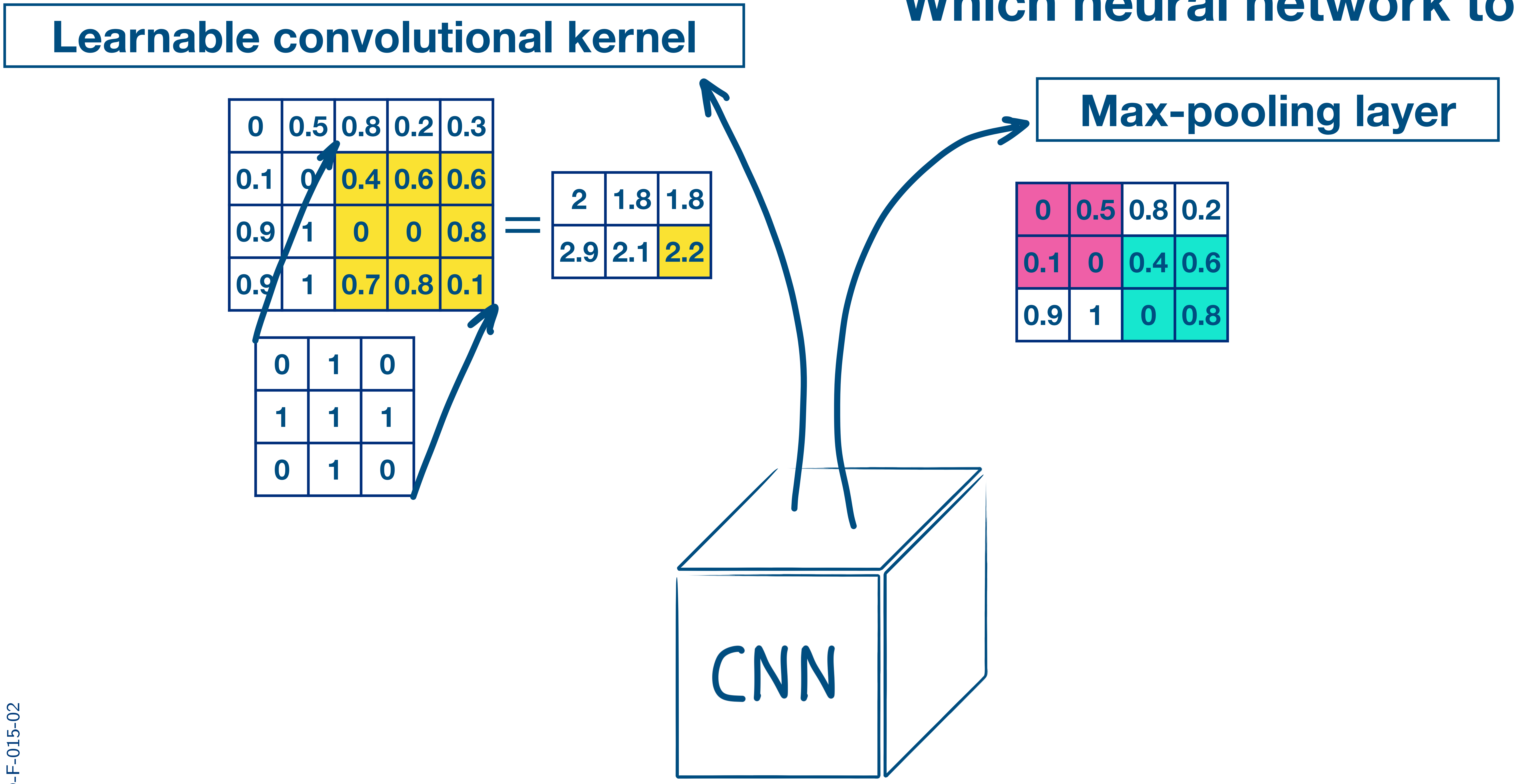

PROD-F-015-02

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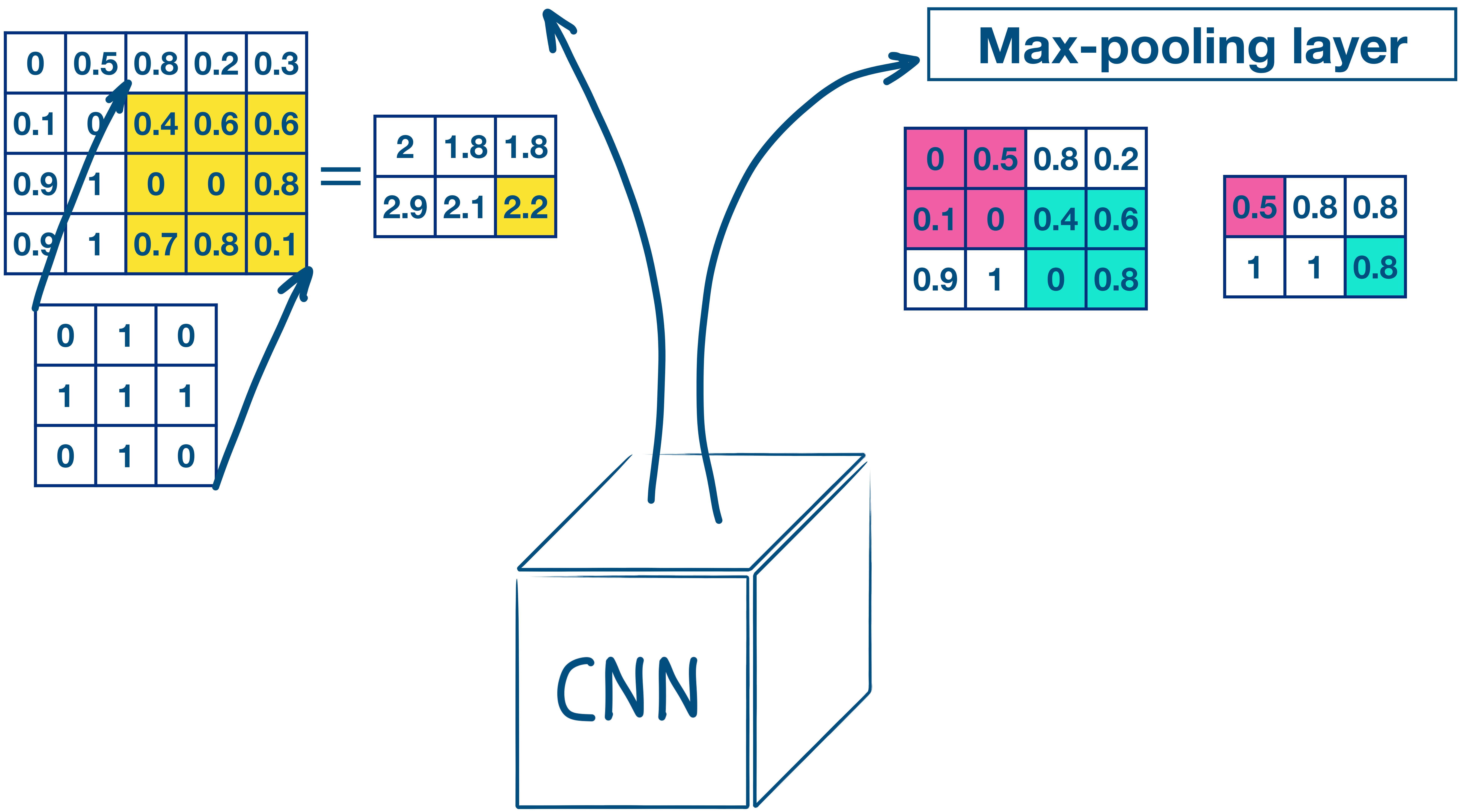
Learnable convolutional kernel



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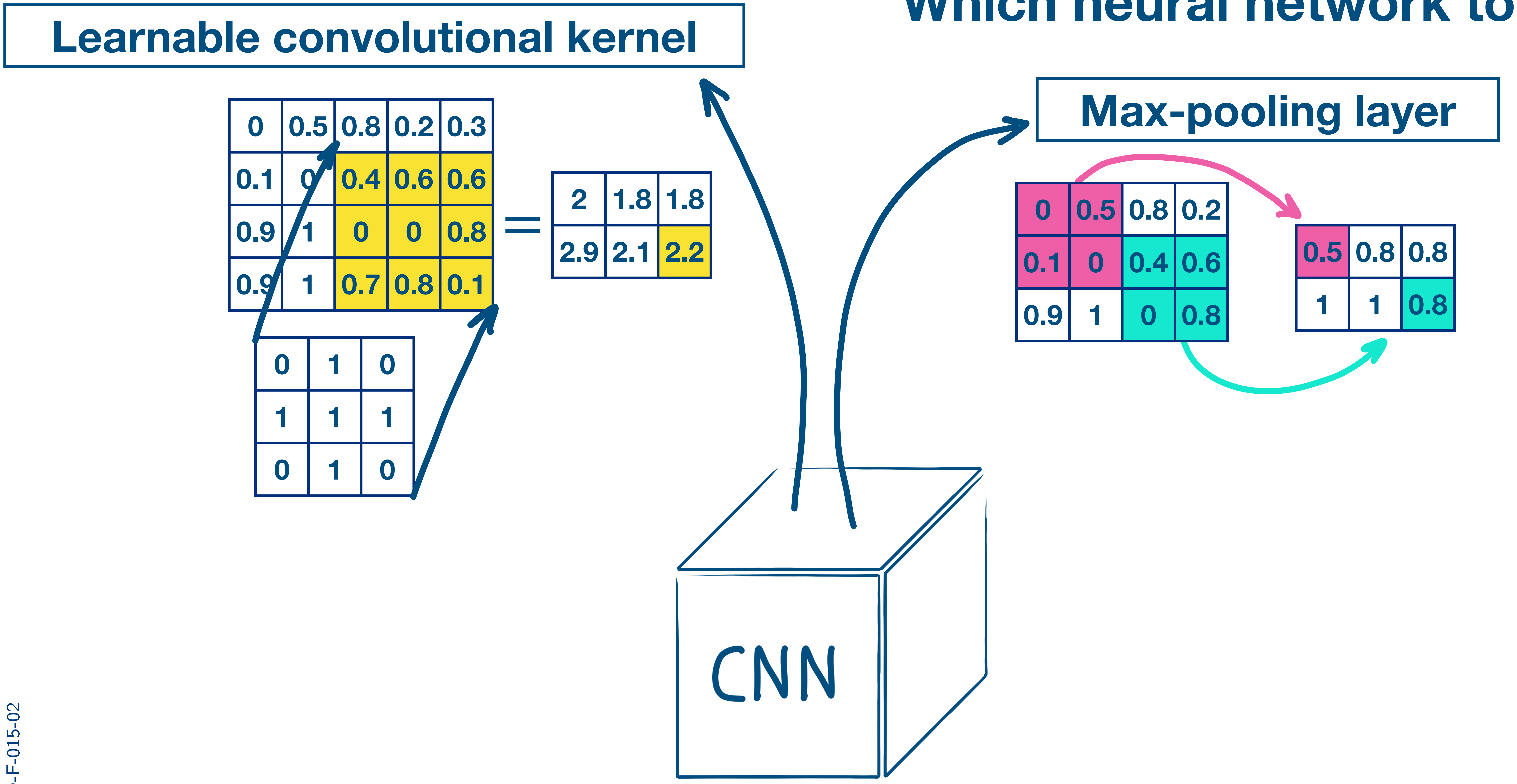

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Which neural network to use?

Which neural network to use?

ReLU$(x) = \max(0,x)$

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Cenaero

Which neural network to use?

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Cenaero

Which neural network to use?

Learnable convolutional kernel

0	0.5	0.8	0.2	0.3
0.1	0	0.4	0.6	0.6
0.9	1	0	0	0.8
0.9	1	0.7	0.8	0.1

0	1	0
1	1	1
0	1	0

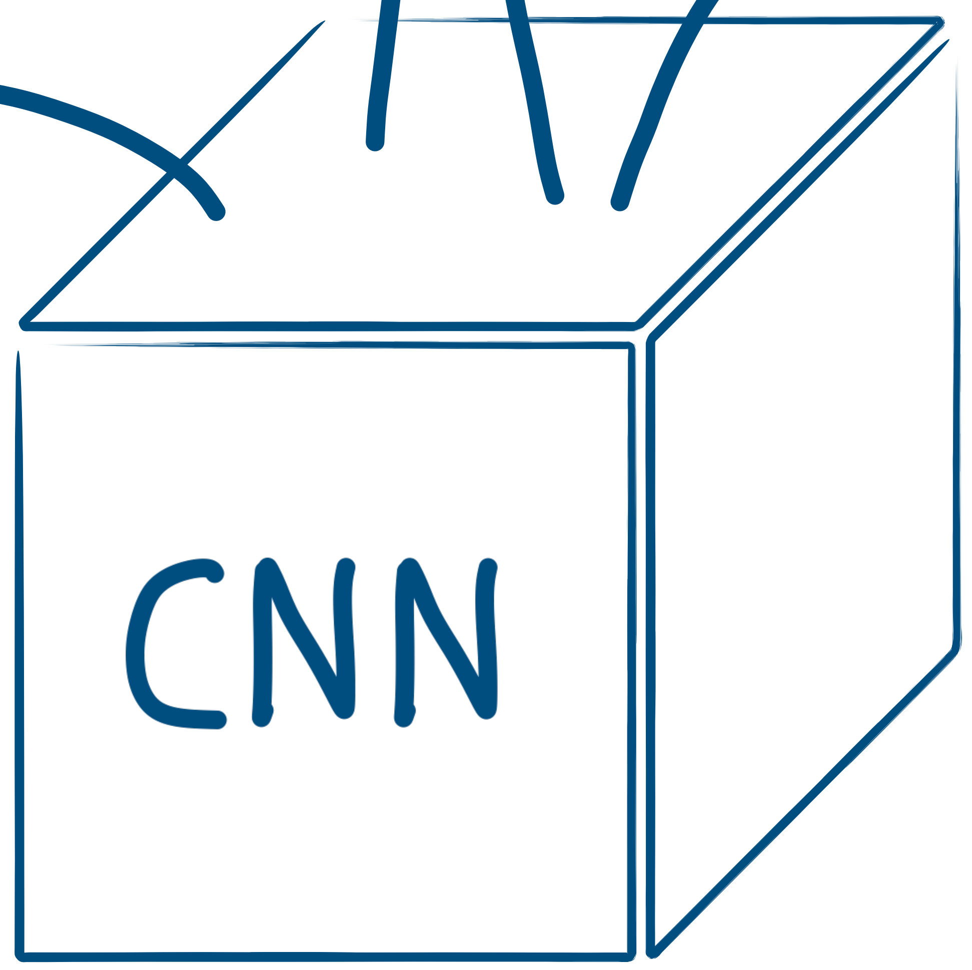
=

2	1.8	1.8
2.9	2.1	2.2

Max-pooling layer

0	0.5	0.8	0.2
0.1	0	0.4	0.6
0.9	1	0	0.8

0.5	0.8	0.8
1	1	0.8



Self-attention layer

$$O(x) = \text{softmax}\left(\frac{QK^T}{\sqrt{e}}\right)V \in \mathbb{R}^{t\times e}$$

Activation function σ

$$\text{ReLU}(x) = \max(0,x)$$

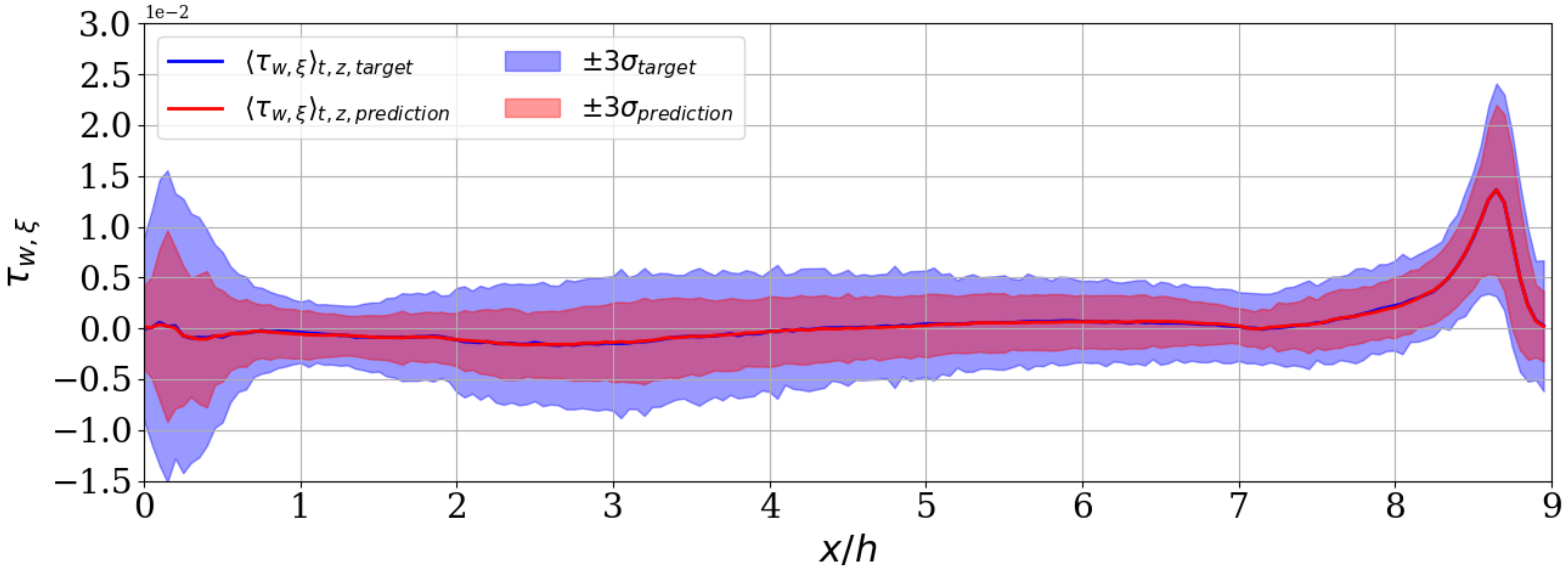
A Priori Validation on the two-dimensional periodic hill

A Priori Validation on the two-dimensional periodic hill

PROD-F-015-02

Cenaero

A Priori Validation on the two-dimensional periodic hill

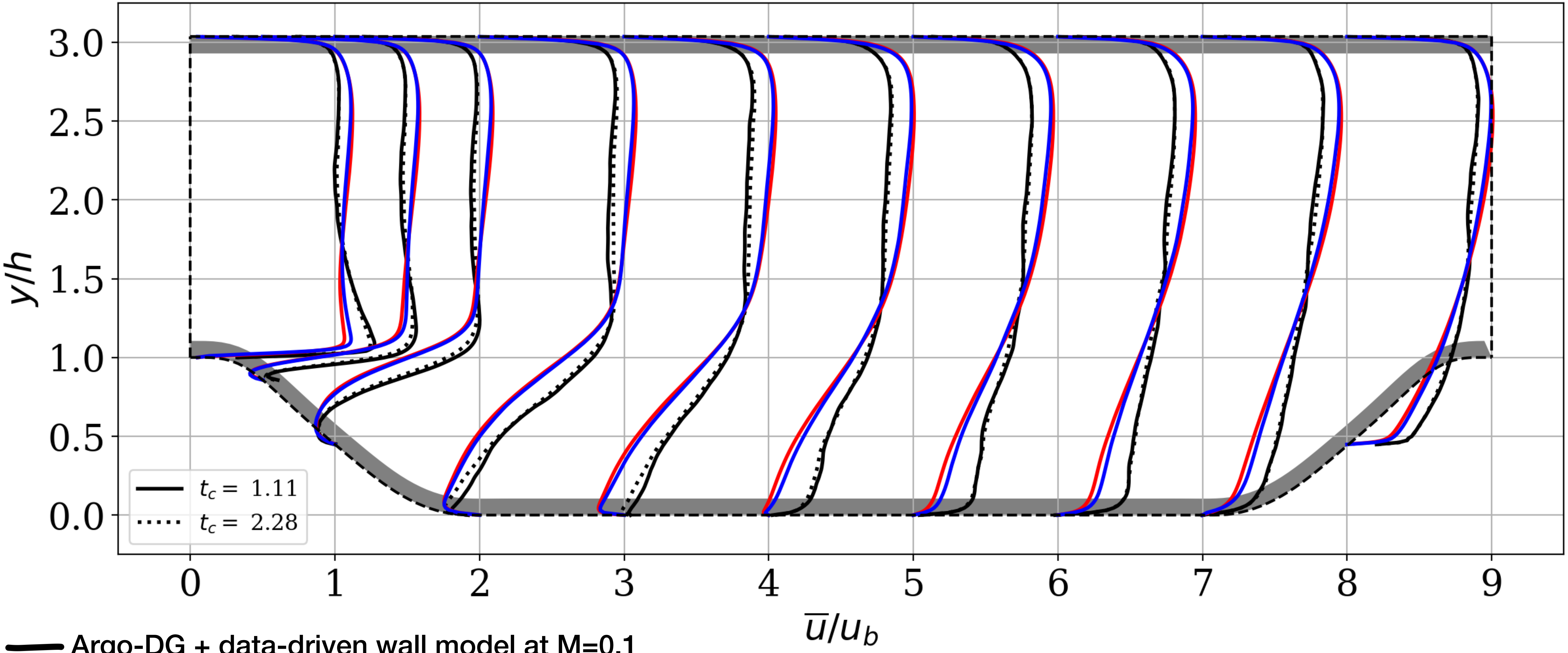


Cenaero

A Posteriori Validation on the two-dimensional periodic hill

Argo-DG + data-driven wall model at M=0.1
Breuer *et al.* (2009)
Gloerflt et Cinnella at M=0.1

A Posteriori Validation on the two-dimensional periodic hill



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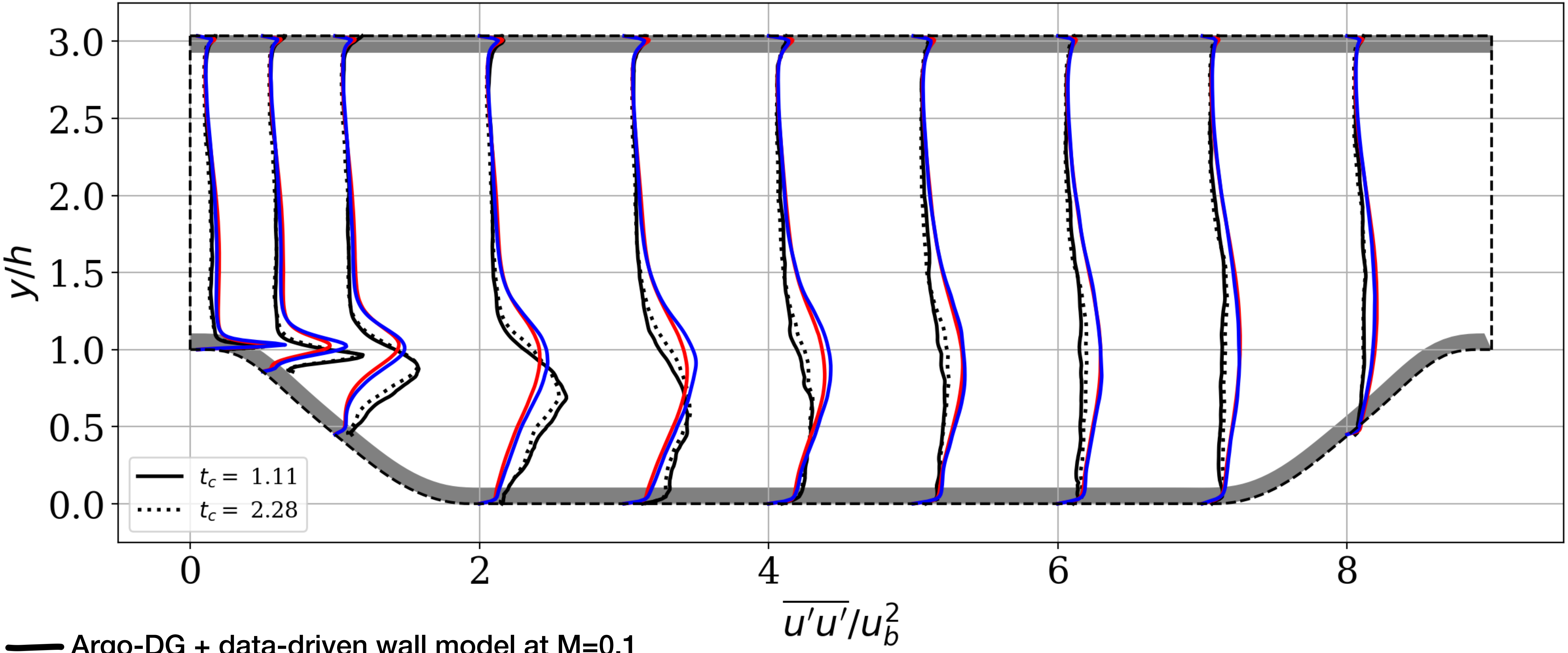

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PROD-F-015-02

Cenaero

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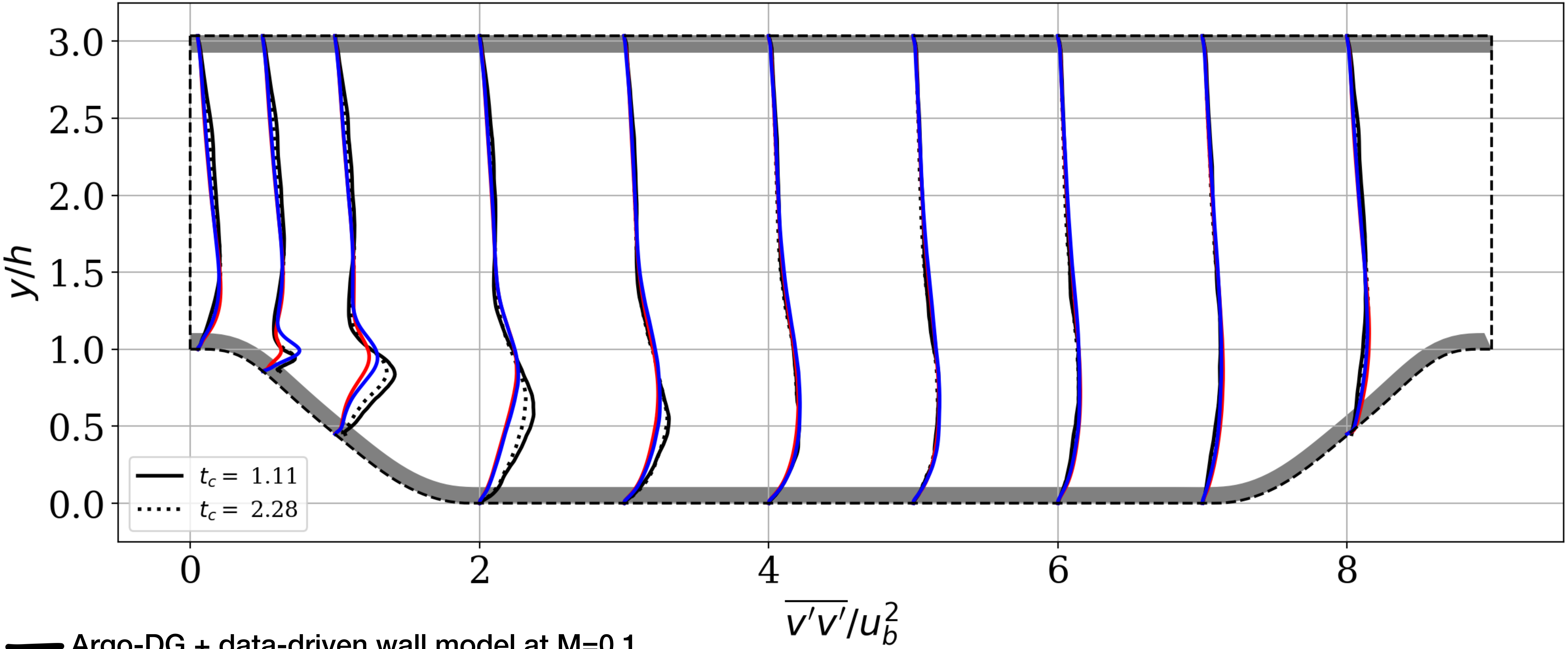

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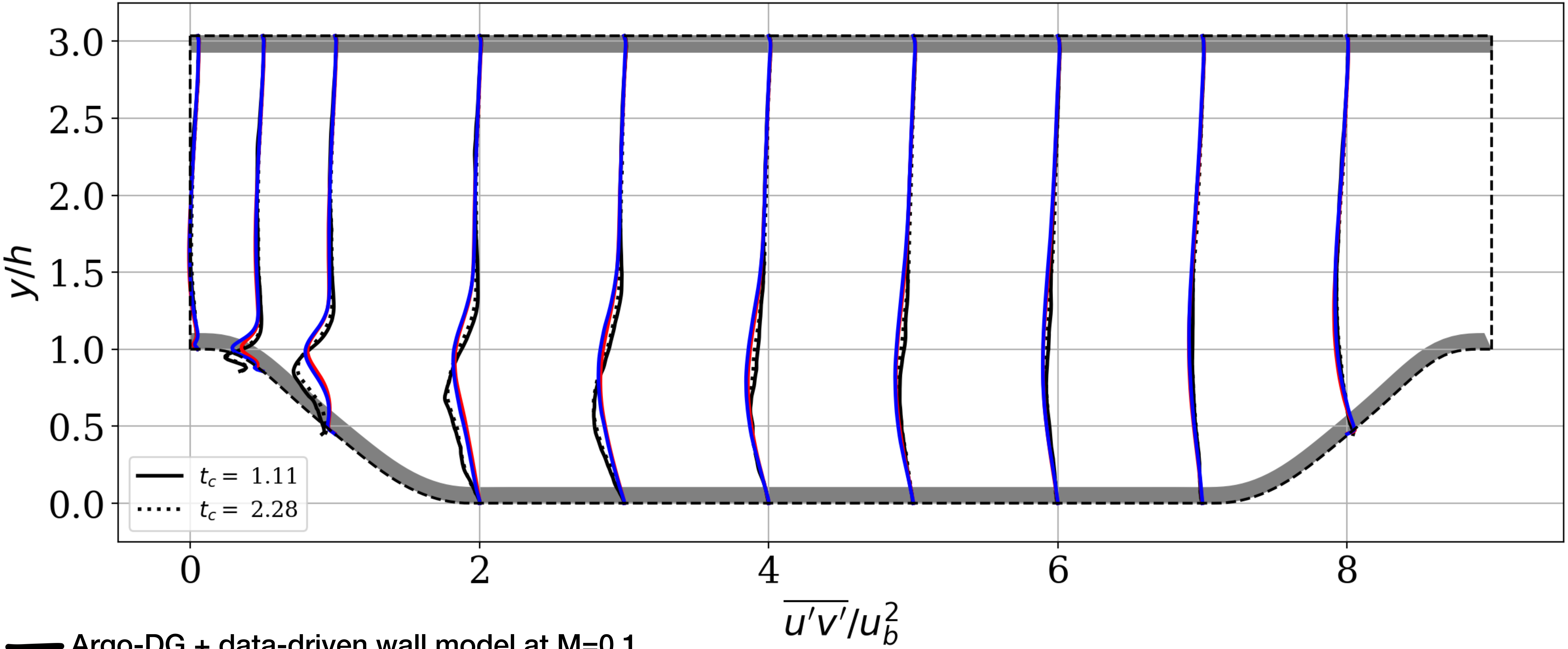

Argo-DG + data-driven wall model at M=0.1
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PROD-F-015-02

Cenaero

Conclusion and perspectives

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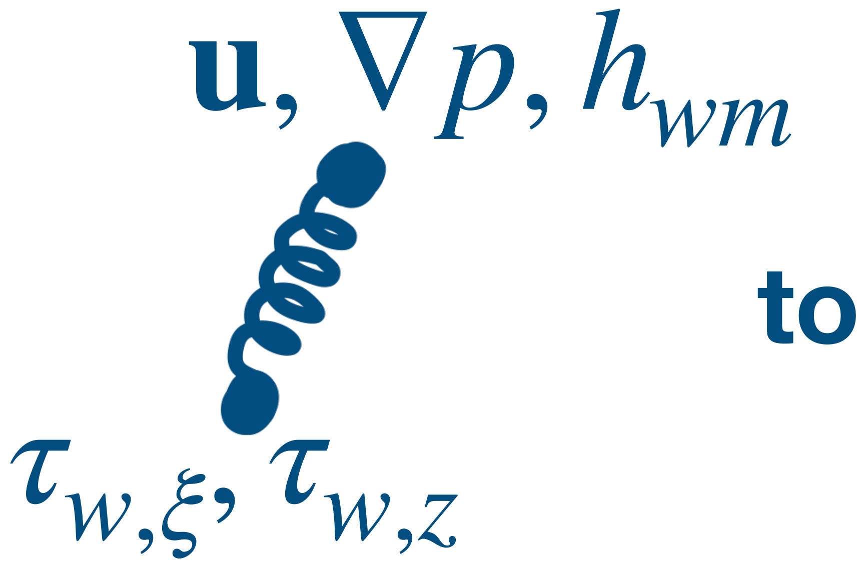
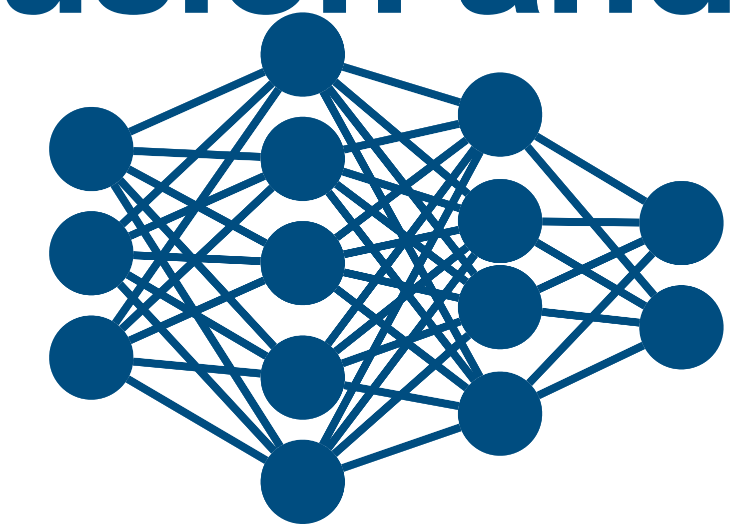


to tackle separation using deep neural network

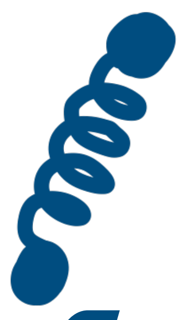


PROD-F-015-02

Cenaero

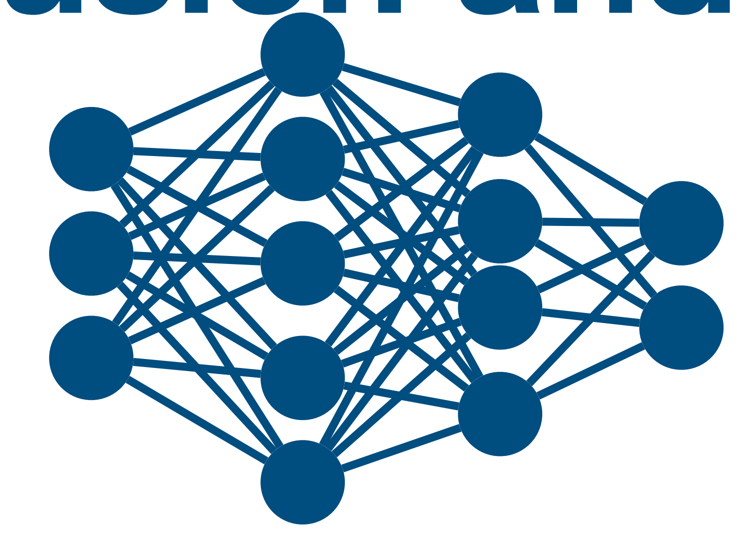
Conclusion and perspectives

$\mathbf{u}, \nabla p, h_{wm}$



$\tau_{w,\xi}, \tau_{w,z}$

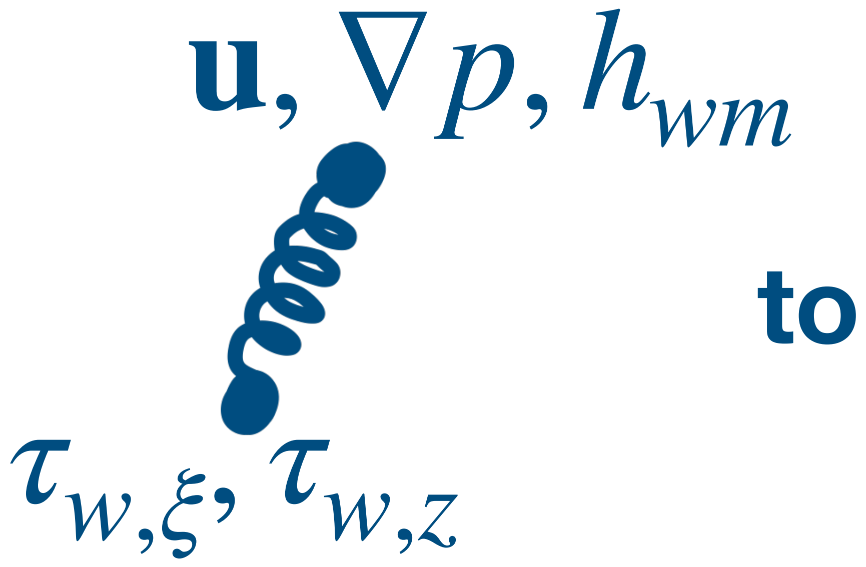
to tackle separation using deep neural network



1) Generating database from channel and periodic hill flows

PROD-F-015-02

Conclusion and perspectives



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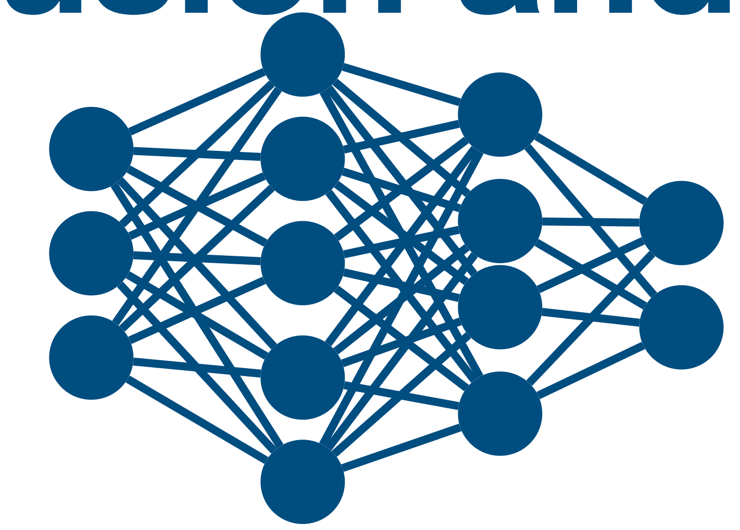
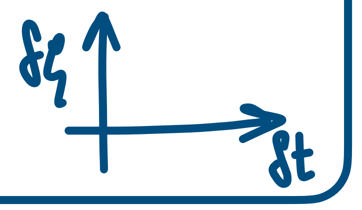


1) Generating database from channel and periodic hill flows

→

2) Analyzing space-time correlations with the wall shear stress (τ_w)



→

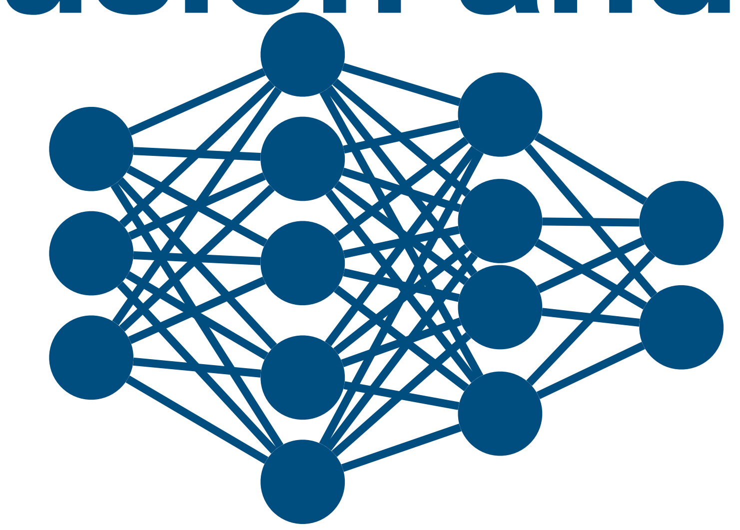
PROD-F-015-02

Conclusion and perspectives

$\mathbf{u}, \nabla p, h_{wm}$

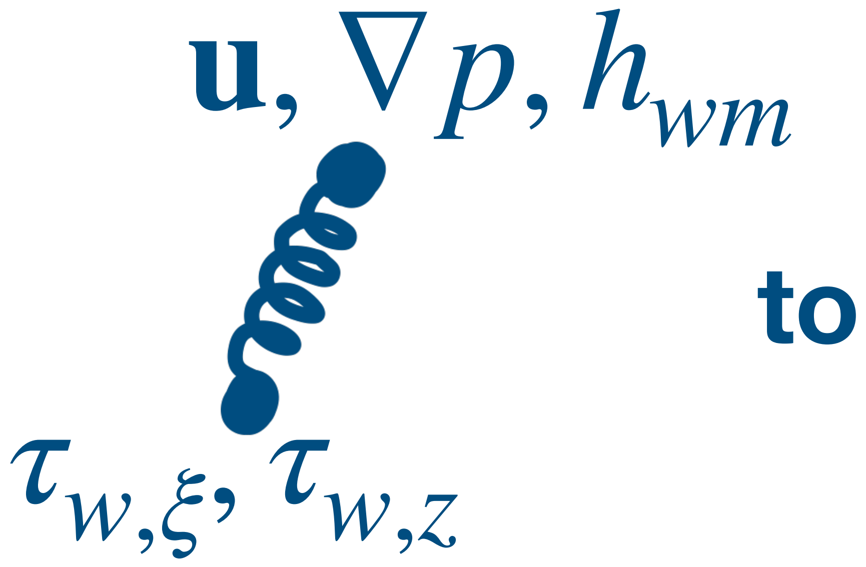
$\tau_{w,\xi}, \tau_{w,z}$

to tackle separation using deep neural network



1) Generating database from channel and periodic hill flows → **2) Analyzing space-time correlations with the wall shear stress (τ_w)** → **3) Training CNN and GMN for the prediction of τ_w**

Conclusion and perspectives



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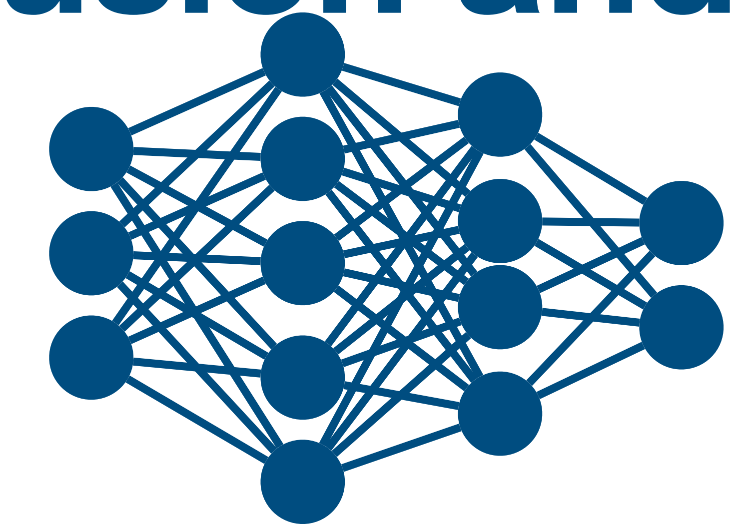

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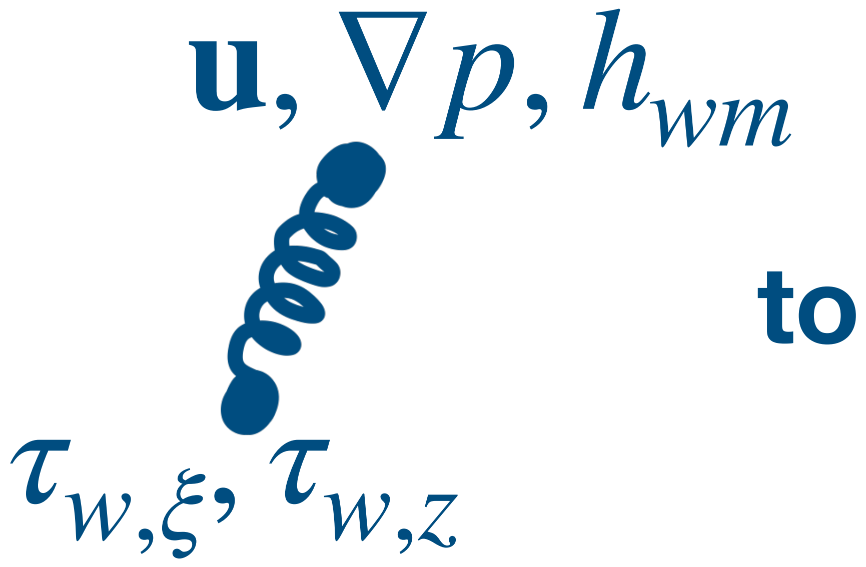
A Posteriori Validation

- **Channel at $Re_\tau = 950$**
- **Periodic hill at $Re_b = 10595$**

Conclusion and perspectives



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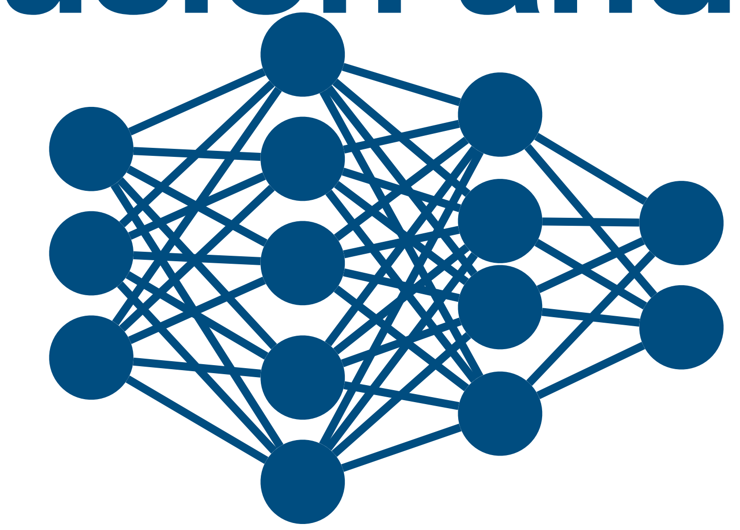

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A Posteriori Validation

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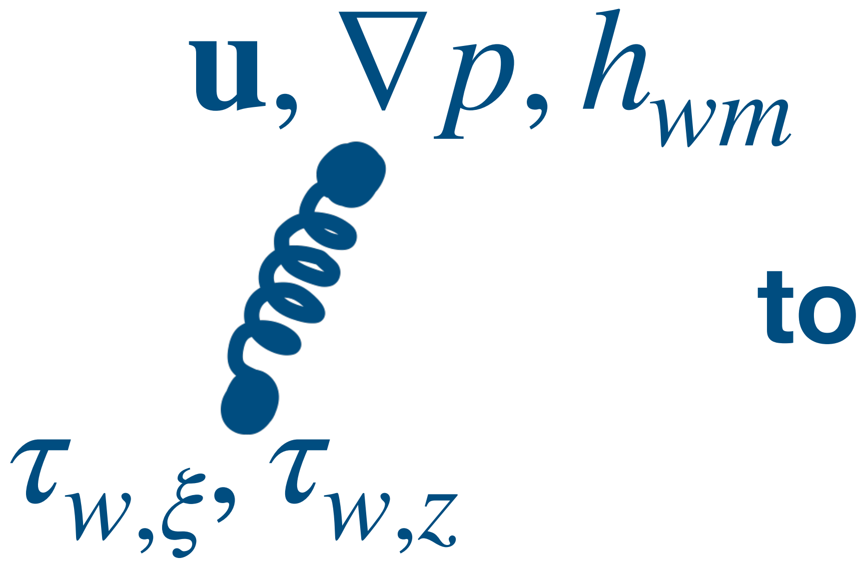
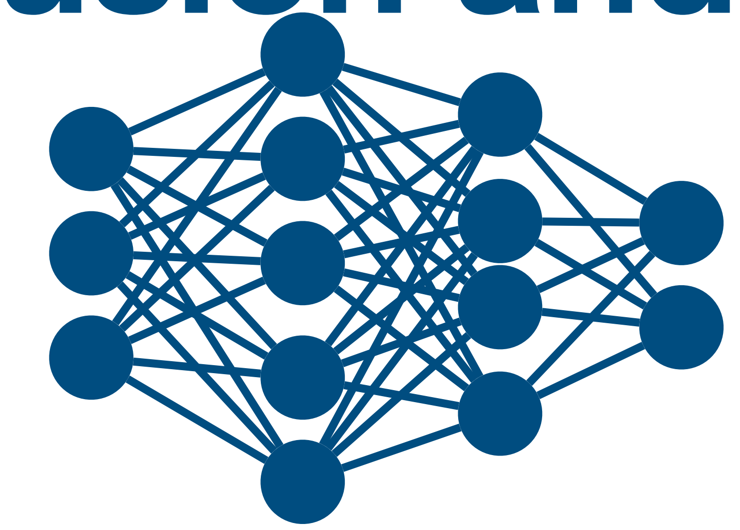
Future challenges:

PROD-F-015-02

Conclusion and perspectives



to tackle separation using deep neural network



1) Generating database from channel and periodic hill flows → 2) Analyzing space-time correlations with the wall shear stress (τ_w) → 3) Training CNN and GMN for the prediction of τ_w

A Posteriori Validation

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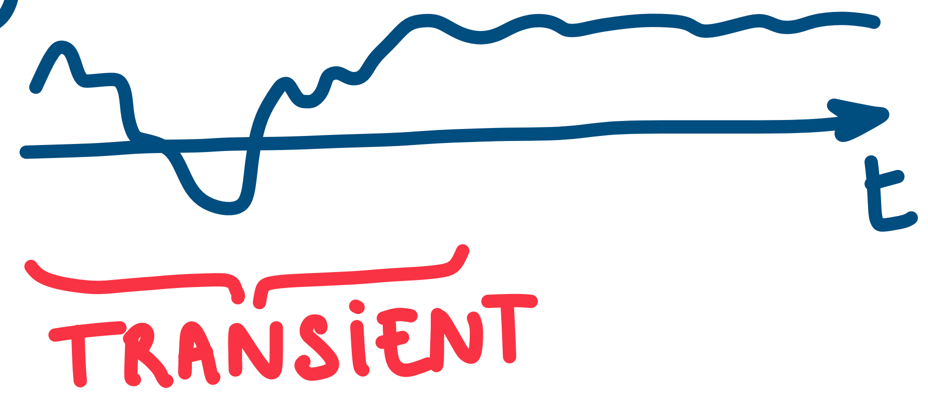
Future challenges:

①



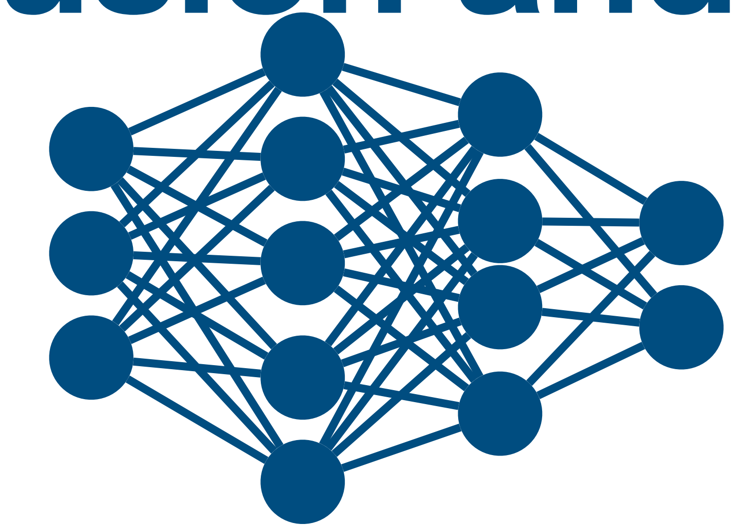
PROD-F-015-02

Conclusion and perspectives

$\mathbf{u}, \nabla p, h_{wm}$

$\tau_{w,\xi}, \tau_{w,z}$

to tackle separation using deep neural network



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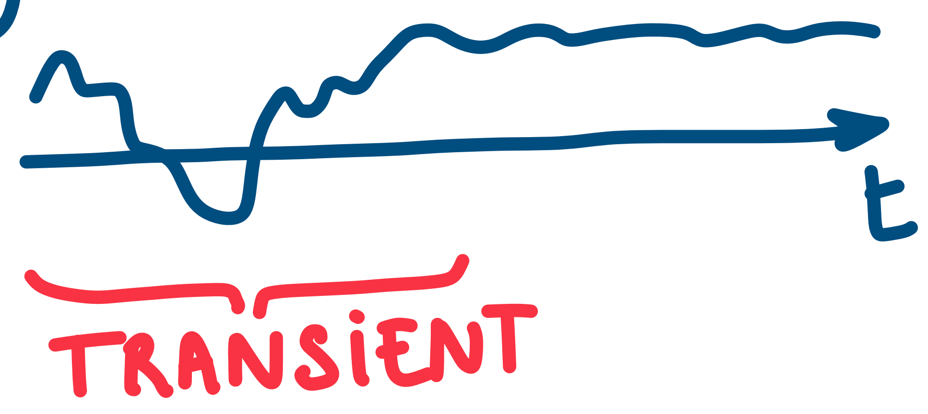
Future challenges:

①



② A posteriori validation at higher Re!

PROD-F-015-02

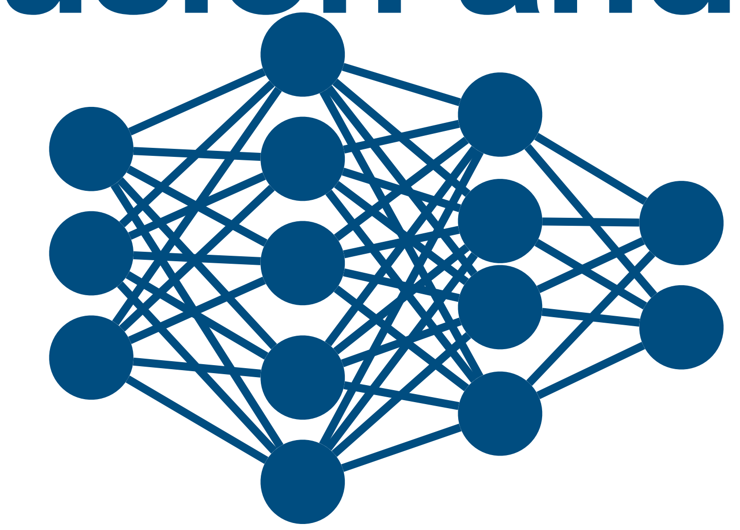
Cenaero

Conclusion and perspectives

$\mathbf{u}, \nabla p, h_{wm}$

$\tau_{w,\xi}, \tau_{w,z}$

to tackle separation using deep neural network



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A Posteriori Validation

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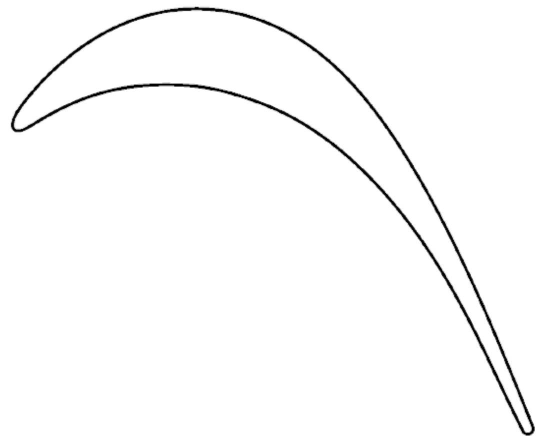
Future challenges:

①

② A posteriori validation at higher Re!

③ Industrial configuration



PROD-F-015-02

Cenaero