Simple formula for evaluating the fire resistance of axially loaded steel square hollow section columns filled with concrete

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Abstract: Concrete filled hollow steel section (HSS) columns in the fire situation can be designed according to Eurocode 4 - Part 1.2 but this document only gives general principles for the temperature analysis and the structural calculation part is tedious to use. The given tabulated data and simplified calculation models are valid within restricted limits: for example, the percentage of reinforcing steel must be less than 5%. The formula presented here has been established on the basis of simulations performed by the computer code SAFIR, a non-linear finite element software developed at the University of Liège for the simulation of the structural behaviour under fire conditions. Three steps have been used: SAFIR results have been compared with experimental results and some calibrations have been performed; a formula for short columns with square section has been established based on SAFIR simulations taking into account the main parameters (quality of materials, dimensions, steel bars, concrete cover); the formula has then been extended to slender columns. The dimensions of the cross section can vary from 150 to 300 mm and the percentage of reinforcing steel can reach 10%.

Keywords: structural analysis; fire engineering; columns; composite steel-concrete elements; numerical simulations

1 Introduction

The use of concrete filled hollow steel section (HSS) columns for the construction of high-rise buildings, bridges, etc. has become increasingly popular in recent years. Concrete filling of these columns not only enhances the load-bearing capacity but also increases the fire resistance. The use of this type of column can lead, in an economic way, to the realization of an architectural and structural design in which steel remains visible while maintaining the fire resistance. During the last three decades experimental and theoretical investigations on fire performance of unprotected steel hollow sections filled with concrete have been carried out in the world (Kordina and Klingsch [2], Lie et al. [3,4,5], Han et al. [8,9], Wang [10], Renaud et al. [11,12]). Research on the fire resistance of HSS columns filled with plain concrete and bar-reinforced concrete was previously completed by Lie et al. [3] with the restricted limits: percentage of main reinforcing bars from 1.5 % to 5%, effective length of column from 2m to 4.5 m. Besides, Eurocode 4 part 1.2 [1] provides a simplified calculation model in Annex H applicable to axially loaded circular or rectangular HSS columns filled with concrete. But the field of application for this method is restricted too: percentage on reinforcing steel less than 5%, buckling length less than 4.5 m.

In order to give to consulting engineers more practical tools, a formula for calculating the fire resistance of HSS columns filled with concrete has been established and is presented in this paper. Furthermore, the field of applicability has been extended: effective length of column from 2m to 7m, percentage of reinforcing steel from 3.5% to 10%.

The reason of this extension comes from the fact that the developments presented in this paper are part of a large research project devoted to HSS columns filled with self-compacting concrete (SCC), or self-consolidating concrete.

In recent years, the possibility of using self-compacting concrete has been received very favourably by structural engineers. SCC may be considered as a revolution in the field of concrete technology. The self-compactability of concrete refers to the capability of the material to flow under its own weight and fill in the formwork in cast processing. Due to its rheological properties, many advantages can be mentioned and new structural applications can be realized. Among them concrete filled HSS columns with small cross-section dimensions and dense reinforcement can be envisaged.

In this paper, the numerical model SAFIR for calculating the fire resistance of HSS columns filled with concrete is presented. Using this computer program, the influence of various factors on the fire resistance of axially loaded square hollow steel section columns filled with densely bar-reinforced concrete was investigated. The parameters

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taken into account in this study are: load, dimensions, quality of materials, steel bars and concrete cover. The formulas presented here have been developed on the basis of experimental results performed using ordinary concrete. Therefore they are, at the present time, applicable only to HSS columns made with this type of concrete, and not with other particular concrete, though the thermal and mechanical properties of SCC are very close to those of normal concrete.

2 Numerical simulations performed by the computer code SAFIR

In this research, the computer program SAFIR is used to analyse the behaviour of composite columns exposed to standard fire situation. SAFIR is a non-linear finite element software developed at the University of Liege. It is specially devoted to the analysis of structures under ambient and elevated temperature conditions. The program, which is based on the Finite Element Method (FEM), can be used to study the behaviour of one, two and three-dimensional structures. SAFIR accommodates various elements for different idealizations, calculation procedures and material models incorporating stress-strain behaviour. The elements include 2-D SOLID, 3-D SOLID, BEAM, SHELL and TRUSS elements. The stress-strain material laws are generally linear-elliptic for steel and non-linear for concrete. In this research, the mechanical and thermal properties of structural steel, concrete and reinforcing steel at elevated temperatures are taken according to Eurocode 4 Part 1.2 [1].

Using the program, the analysis of a column exposed to fire consists of two steps. The first step involves predicting the temperature distribution inside the structural members, referred to as “thermal analysis”. The second step, named “structural analysis”, is carried out for the main purpose of determining the mechanical response of the structure due to static and thermal loading.

2.1 Thermal analysis

This analysis is usually performed while the structure is exposed to fire. In concrete filled HSS columns, a uniform temperature has been assumed over the height of the column. Thus, thermal analysis can be reduced to a two-dimensional problem of transient heating. The non-steady-state 2D temperature distribution within any cross-section is determined by the Fourier thermal conductivity equation:

\[
k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + Q = \rho c \frac{\partial T}{\partial t}
\]  

(1)

where \( k \) is the thermal conductivity of the material, \( T \) is the temperature, \( Q \) is the amount of heat generated in the material per unit volume, \( \rho \) is the density, \( c \) is the heat capacity, \( t \) is the time and \( x, y \) are the position coordinates.

The temperature fields within a given network are established by a finite element method in conjunction with an integration method for time steps. It is assumed that conduction is the main heat transfer mechanism in the hollow steel section and concrete core. Convection and radiation act essentially as heat transfer from the fire environment to the external hollow steel section. The influence of moisture (assumed uniformly distributed in the concrete) is treated in a simplified way: the transient temperatures in the concrete are calculated assuming that all moisture evaporates, without any transfer, at temperatures situated within a narrow range, with the heat of evaporation giving a corresponding change in the enthalpy-temperature curve. Therefore during the period of evaporation, all the heat supplied to an element is used for the moisture evaporation until the element is dry. The discretisation for plane sections of different shapes is possible by using triangular and/or quadrilateral elements. For each element the material can be defined separately. Any material can be analysed provided its physical properties at elevated temperatures are known. The variation of material properties with temperature can be considered.

In square sections, there are two axes of symmetry, therefore only one quarter of the section has to be discretized. Fig.1 shows an example of such a discretization.

Fig. 1 Discretization of one-quarter of a square section
The very thin layer "USER1" is used to take into account the thermal resistance at the steel-concrete interface. Here, the thermal resistance is taken into account with the value $R = 0.013 \text{ mK/W}$. This value has been obtained by numerical experimentation in order to get a satisfactory correlation between numerical results and twenty-five fire tests carried out in Europe (University of Braunschweig–Germany [2] and CTICM–France [12]) and in North America (National Research Council of Canada [4, 5]).

2.2 Structural analysis

The basis of the mechanical analysis of structures undergoing large displacements is the incremental form of the principle of virtual work. The formulation used in the model and the assembly of finite elements are based on the principle of virtual work expressed in a corotational description. In the model, a whole composite column is built up by means of several 2-D beam elements which are based on the following formulations and hypotheses:

- Displacement type element in a total corotational description;
- The displacement of the node line is described by the displacements of three nodes, two nodes at each end of the element supporting two translations and one rotation plus one node at mid-length supporting the non-linear part of the longitudinal displacement. The longitudinal displacement of the node line is a second-order power function of the longitudinal co-ordinate. The transversal displacement of the node line is third-order power function of the longitudinal co-ordinate;
- The Bernoulli hypothesis is considered, i.e., the cross section remains plane under bending moment;
- The hypothesis of Von Karman is used: the strains are small;
- The rotations are assumed to be small (note that they are evaluated in the co-rotated configuration);
- The longitudinal integrations are numerically calculated using Gauss’ method;
- The integration of the longitudinal stresses and stiffness on the section is based on the fibre model; the section is supposed to be made of a certain number of parallel fibres. In fact, the same discretisation as the one used for the thermal analysis is used. Each finite element of the thermal analysis, with its known material type and temperature, is considered as a fibre;
- More information is given by Franssen [6, 7].

2.3 Experimental verification

Validity of the thermal model

Twenty-five fire tests considered here were carried out in Europe (University of Braunschweig–Germany [2] and CTICM–France [12]) and in North America (National Research Council of Canada [4, 5]). The main structural characteristics of the columns tested are reported in Table 1.

<table>
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<tr>
<th>Section</th>
<th>Size (mm)</th>
<th>Thickness (mm)</th>
<th>Reinforcements</th>
<th>Concrete cover (mm)</th>
<th>Laboratory</th>
<th>N°</th>
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</table>
Fig. 2 presents a comparison between calculated and measured temperatures in the column cross section of one particular test. Fig. 3 to 5 present in another way the same comparison for all the reference tests at the points indicated in Fig. 2.

![Figure 2: Comparison between calculated and measured temperatures in the cross-section of one particular test](image)

![Figure 3: Comparison between calculated and measured temperatures at steel surface of the reference tests](image)

The following comments can be drawn:

- Calculated temperatures on the hollow section are in good agreement with the measured temperatures provided thermal resistance defined previously is used in the calculation.
- Temperatures calculated in the longitudinal reinforcements are simulated satisfactorily but the differences between calculations and tests are more important than those found on the hollow steel section. These differences are explained on one hand by a certain uncertainty of the real moisture content (fixed at 4% in calculations) and on the other hand by a water accumulation close to reinforcements subsequent to water migration (not taken into account in the model). In fact, part of this water moves to the external steel section, but can only escape through two holes generally drilled at the top and the bottom of the steel tube. This explains why the amount of vapour is more important than in classical reinforced concrete columns. Furthermore, another part of the vapour migrates toward the coldest zones where it condenses again, which results in a slowing down of the vaporisation phase.
- With regard to the point inside the concrete, the agreement is not so good especially at low temperatures. Once more the difference between theoretical and experimental curves is primarily the result of the length of the vaporisation stage. Differences between theoretical and experimental curves do not affect much the variation of material properties: for low temperatures, the concrete mechanical properties are not affected and for higher temperatures, the agreement is good enough. On the contrary these differences influence the structural behaviour, as second order effects are produced by thermal gradients.

![Figure 4: Comparison between calculated and measured temperatures in the steel reinforcement of the reference tests](image)

![Figure 5: Comparison between calculated and measured temperatures at the central point of the concrete core of the reference tests](image)

Validity of the structural model

The following assumptions have been made for the numerical simulations of the columns:
- A uniform temperature has been assumed over the height of the column;
- An out-of-straightness of L/500 has been used in numerical simulations (tolerance given by EN 10210-2 for hot rolled structural hollow sections). It has always been considered that the effect of this eccentricity and the one of loading eccentricity are cumulative;
- In each column, there is no slip between the concrete core and the hollow steel section;
- The effects of residual stresses have been assumed negligible.

To examine the validity of the structural model, a comparison between the fire resistance of seventeen fire tests and corresponding calculation results has been performed. The seventeen fire tests considered here were carried out at the University of Braunschweig—Germany [2] and by the National Research Council of Canada [5]. Some other tests have been chosen here because in some of the preceding ones, not enough information was given in order to perform structural calculations. The results of Table 2 show that there is a good agreement between tests and numerical results.

<table>
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<th>Reference</th>
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<th>Rebars</th>
<th>Length (mm)</th>
<th>Test loading</th>
<th>Measured failure time $R_{\text{test}}$ (min)</th>
<th>Calculated failure time $R_{\text{cal}}$ (min)</th>
<th>$R_{\text{cal}}/R_{\text{test}}$</th>
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$R_{\text{test}}$ = 0.95

3 Factors influencing the fire resistance of short columns

Using SAFIR programme, the influence of various factors on the fire resistance of very short hollow steel section columns filled with bar-reinforced concrete was investigated through computer-simulated fire tests. These factors include cross-sectional dimensions, quality of materials, steel bar area and concrete cover to the steel bars. The variables that were considered in this research study are given below ($f_c$ and $f_y$ represent in this section the characteristic strength of concrete and the characteristic yielding stress of steel):

- The outside dimension of square HSS columns (D) varies from 150mm to 300mm.
- The wall thickness $t$ varies from 5mm to 10mm and $D/t$ is less than $52\sqrt{235/f_y}$ to neglect the effect of local buckling in the steel wall according to Eurocode 4-Part 1.1 [1].
- Only the quality of steel $f_y=335$ Mpa has been considered because it is now the most commonly used material for hollow steel sections.
- Three concrete strengths, namely, $f_c=30$MPa, $f_c=40$MPa, $f_c=50$MPa have been used.
- The amount of steel reinforcement varies from 3.5% to 10%.
- Only the quality of reinforcing steel S500 has been considered in this study because it is now the most commonly used material for reinforcing steel.
- Concrete cover which represents the distance between the axis of longitudinal reinforcements and the border of the concrete core has been fixed at a minimum value of 30 mm. According to the dimensions of the cross-sections, other concrete covers, namely 35, 40, 45, 50 mm have also been used.
- Concentrated loads from 20% $N_{ax}$ to 70% $N_{ax}$ have been considered ($N_{ax}=f_cA_c+f_yA_s$).

The following symbols have been used in the figures:

N2000 stands for axial load $N_a=2000$ kN
S200-6.3 stands for square section with dimension $D=200$ mm, steel wall thickness $t=6.3$ mm
C40 stands for compression strength of concrete on cylinder $f_c=40$ MPa
Dr45 stands for concrete cover $D_r=45$ mm
12D18 stands for the case: reinforcement in the section consists of 12 bars with diameter 18mm
3.1 Load

The influence of the load on the fire resistance of HSS columns is shown in Fig. 6 where the fire resistance is plotted as a function of the axial load for various other parameters. It can be seen that the relationship between fire resistance and load can be expressed as \( R_{\text{short}} = a \cdot \sqrt{N_x} + b \) (\( a, b \) are constant for a given \( N_x \)) whatever the cross section dimension, the steel bar area and concrete strength.

![Fig.6: Fire resistance as a function of load for various cross section dimensions and concrete strengths](image)

![Fig.7: Fire resistance as a function of \( N_x \) for various cross section dimensions, loads and concrete covers](image)

3.2 Concrete area and strength

The influence of the concrete area and strength on the fire resistance of HSS columns has been studied by finding the relationship between the fire resistance and \( N_c = A_e \cdot f_c \). In Fig. 7 the fire resistance is plotted as a function of \( N_c \) for various other parameters. It can be seen that the relationship between fire resistance and load is linear whatever the cross section dimension, the steel bar area, concrete strength and concrete cover.

3.3 Steel bar area

The influence of the steel bar area on the fire resistance of HSS columns has been studied by finding the relationship between the fire resistance and \( N_s = A_e \cdot f_s \). In Fig. 8 the fire resistance is plotted as a function of \( N_s \) for various values of concrete cover. It can be seen that the relationship between fire resistance and steel bar area is a curve affected by the value of concrete cover.

![Fig.8: Fire resistance as a function of \( N_s \) for various concrete covers](image)

![Fig.9: Fire resistance as a function of \( D_r \) for various loads, concrete strengths and cross-section dimensions](image)

3.4 Concrete cover

An increase of concrete cover increases the fire resistance of short columns as can be seen in Fig. 9.

4 Formula for calculating the fire resistance of short columns

Based on the data from parametric studies, a formula has been developed for the calculation of the fire resistance of square HSS columns filled with bar-reinforced concrete. The proposed formula is:
\[ R_{\text{short}} = a + b \cdot t + c \cdot N_c + d \cdot \sqrt{N_B} + e \cdot \sqrt{N_r} \times D + f \cdot D \]  

(2)

where \( a, b, c, d, e \) and \( f \) are constants; \( t, D_r \) and \( D \) in mm; \( N_c, N_B \) and \( N_r \) in kN

With more than 4000 numerical calculations concerning almost all cases and using MATLAB program, regression coefficients \( a, b, c, d, e, f \) have been computed by performing a least squares fit. The following values have been obtained:

\[
\begin{align*}
  a &= 50 \\
  b &= 0.33 \\
  c &= 0.018 \\
  d &= -3.93 \\
  e &= 0.33 \\
  f &= 0.42
\end{align*}
\]

A comparison between the results obtained from the numerical model and the simplified method is shown in Fig.10

![Fig.10 Comparison of fire resistance of columns from Eq.(2) with predictions of the numerical model](image)

![Fig.11 Fire resistance as a function of column length](image)

5 Formula for calculating the fire resistance of slender columns

5.1 Influence of effective length

The correlation between the fire resistance of slender columns and column length is shown in Fig.11. It can be seen that the relationship between fire resistance and column length is almost linear except for some ranges corresponding to very small values of the fire resistance (never used in practice).

5.2 Relationship between fire resistances of short and slender columns

In the range studied, it can be seen that the fire resistance of a column with a determined length can be linearity interpolated between two points: point A (Fig.11) corresponding to the fire resistance of very short column (with a length of 0.5 m), point B corresponding to a slender column with a fire resistance of 10 minutes. It is now needed to find the length corresponding to a fire resistance of the column of 10 minutes, called here \( L_{10} \).

To this aim the following considerations can be made. After 10 minutes in fire, with sectional dimensions from 150 mm to 300 mm and steel wall thicknesses from 5 mm to 8 mm, the temperatures in reinforcing steel and concrete are below 100°C, the temperatures in steel wall are approximately 400°C. Therefore the mechanical properties of concrete and reinforcing steel are unchanged compare to those at room temperatures. The strength of the steel wall is unchanged but the modulus of steel wall at 400°C is about 70% of that at room temperature. The value \( L_{10} \) can therefore be obtained using the method for analysing axially loaded composite columns at normal temperature. The proposed formula is:

\[
R_{\text{long}} = R_{\text{short}} - \frac{R_{\text{short}} - 10}{L_{10} - 0.5} (L - 0.5)
\]

(3)

with \( R_{\text{long}} \) and \( R_{\text{short}} \) in minutes; \( L_{10} \) and \( L \) in meters

6 Conclusions

In this paper a formula for calculating the fire resistance of hollow steel section columns filled with concrete has been presented.

The numerical model used in this study is capable of predicting the fire resistance of composite columns under fire conditions taking into account the thermal resistance at the steel-concrete interface in thermal analysis and material and geometrical nonlinearities in structural analysis. The validation of the model has been performed by comparing computed results with test results in both thermal and structural analysis.
The formula has been first established for short columns, and then extended to slender columns. Using the formula, the fire resistance of square hollow section columns filled with concrete can be evaluated for any value of the significant parameters such as the load, column section dimensions, column length and reinforcing steel provided these parameters are in the range studied. In the near future, the numerical model will be used to perform a very wide series of simulations for concrete filled steel section columns under eccentric loading with various kinds of cross-section such as circular section, section with embedded steel profile, double steel tube section. Referring to this base of numerical results, it may be expected that a suitable simplified method with a wider range of application will be established.

References