Multiple Resonance Prediction through Lumped-Parameter Modeling of Transformers in High Frequency Applications

N. Davister[1], F. Henrotte[1], F. Frebel[1] and C. Geuzaine[1]

[1]University of Liège, Institut Montefiore B28, Dept. of Electrical Engineering and Computer Science, B-4000 Liège, Belgium

The accurate prediction of coupled inductive and capacitive effects in electromagnetic coils is of crucial importance in many industrial applications, due to the increase of operating frequency or to the increase of voltage levels. In this paper, we propose a light-weight coupled electric circuit model that avoids solving a large 3D Maxwell full-wave problem, and is still able to predict not only the first resonance but also the next few resonances as accurately as experimental characterizations would do. The resistive, inductive and capacitive lumped circuit parameters of the magnetic device are identified by means of 2D finite element modelling, and then implemented in an electric circuit that realises the inductive-capacitive coupling. Moreover, the lumped parameter identification can be performed at different levels of representation of the electromagnetic coils, from turn-level to winding-level, in order to resolve additional resonances of the system, still keeping the computational complexity compatible with industrial design requirements. The efficiency of the method is confirmed by means of simulations and measurements performed on a high frequency transformer and on an air inductance.

Index Terms—**Electromagnetic modeling, Transformers, Electromagnetic transients, RLC-circuit**

I. Introduction

THE coupling of inductive and capacitive effects could long be ignored when designing electromagnetic coils in electrical engineering applications. But with the advent of extremely fast switching electronic components based on silicon carbide or gallium arsenide, resonances due to parasitic capacitances become significant at the considered frequencies, especially regarding electromagnetic compatibility issues, and the inductive-capacitive coupling can thus no longer be neglected in the design process. On the other hand, the design of magnetic components (inductors and transformers) has traditionally been (and for the most part still is) carried out analytically. The last decade has however increasingly seen the adoption of finite element software in engineering offices. To keep the computational cost acceptable, only 2D models are used usually. Moreover, magnetic and electric phenomena are in general considered decoupled, and can therefore be calculated separately [1], [2].

In this paper, a method to model efficiently magnetic and electric phenomena in magnetic components (including the stray capacitances) is presented. The method proceeds by extracting resistive, inductive and capacitive lumped parameters from conventional magnetodynamic and electrostatic finite element (FE) simulations, and by introducing them into an equivalent circuit that will realise the coupling. An interesting feature of the proposed approach is that the parameter extraction can be done at different levels of representation of the electromagnetic coils, from turn-level to winding-level. The winding-level (coarsest representation) is only able to resolve one resonance frequency per winding, which might be insufficient in practice for an accurate design. The turn-level (finest representation), on the other hand might be computationally prohibitive whenever the number of turns of the winding is large. The intermediary section level advocated in this paper allows resolving a certain number of the lowest resonance frequencies with satisfactory accuracy and at a reasonable computational cost. A comparison between numerical results obtained with the identified equivalent circuits and experimental measurements performed on representative inductances and transformers encountered in high-frequency and high-voltage applications highlights the relevance and the practical benefits of the proposed approach.

II. Equivalent Circuit Representations

Various lumped parameter approaches exist to model magnetic inductances and transformers [1]-[6]. Our objective is to build a circuit model that is able to resolve explicitly the coupled inductive and (parasitic) capacitive effects, and thus to predict resonances, but that does this with a tunable coarseness of the representation, so as to control computational costs and accuracy. To this end, the electromagnetic coils are represented at different granularity levels, as depicted in Fig. 1, by considering each turn individually (turn-level), by grouping sets of consecutive turns into so-called sections (section-level), or by considering the winding as a whole (winding-level).

In order to establish an electric circuit equivalent to a transformer FE model, one has to establish clearcut correspondences between FE quantities (regions and constraints applied to these regions) on the one hand, and electric circuit quantities (nodes, branches and lumped parameters) on the other hand. In order to establish these relationships, it is first noted that the helix geometry of a wound wire cannot be represented exactly in a 2D model [7]. Turns connected in series are instead necessarily modelled as adjacent torus-shaped conductors carrying the same current I. This is a mild approximation whenever the number of turns is reasonably large. Each torus conductor is thus associated with 4 quantities: a current, a voltage drop accounting for self and mutual inductance effects, an electrostatic voltage and an electrostatic charge determining its capacitive relationship with the other turns/torus of the coil, with those of other coils as well, and finally with the environment. The first 2 quantities pertain to the magnetodynamic problem, whereas the latter 2 pertain to the electric problem.

The number of resonance frequencies predicted by the equivalent circuit model grows of course with the complexity of the circuit. The winding-level approach predicts only one resonance frequency per winding. In order to predict more resonances of the system, a more detailed equivalent circuit is needed. A turn-level representation would however lead to a very time-consuming electric network model whenever coils with hundreds or thousands of turns would be considered. Therefore, the idea developed in this paper of a tunable section-level representation of the coils that allows controlling the computational burden. The idea is to virtually section the real coil into n_{s} sections connected in series. Each section contains a number of consecutive turns of the real coil, and different sections may have different numbers of turns. Sectioning introduces $n_{\mathrm{s}}-1$ virtual terminals in-between the physical terminals of the real coil. A sectioned coil is thus represented by two physical terminals and $n_{\mathrm{s}}-1$ virtual section terminals, i.e., $n_{\mathrm{s}}+1$ section terminals in total, which are the voltage degrees of freedom of the equivalent electric circuit. The systematic identification of the turn/torus quantities (voltages, currents) in the FE model with their counterpart in the equivalent circuit representation is the key issue of the proposed coupling method.

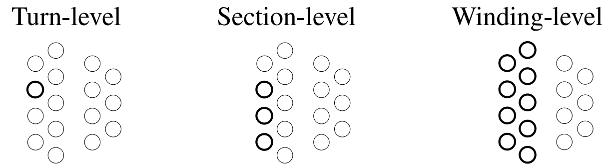

Fig. 1: Illustration of the three levels of representation in a two-coils winding.

The three representation levels of the coil in the coupling procedure are depicted in Fig. 2 with their corresponding equivalent circuits in case of a simple coil with 3 turns. Note that, for the sake of clarity, lossy elements (*i.e.*, resistances) and mutual inductances ($L_{ij}|_{i\neq j}$) are not explicitly depicted, but they are considered in all calculations. The identification method is presented in the case of the *section-level* representation only, as it covers both the *turn-level* and the *winding-level* as particular cases.

III. Lumped parameter identification

The lumped parameter matrices R, L and C are now identified. We shall use latin indices for turn/torus related quantities, and greek indices for section related quantities.

A. Magnetic Parameters

Magnetic parameters are identified by means of 2D finite element resolutions of a magneto-quasistatic formulation (eddy current problem). Individual turns are discretized in the FE model, which also contains internal circuit equations to express the series connections of the turns into windings and their connection with the voltage or current sources. In the magnetic case, section currents I_β and section voltage drops U_γ are readily expressed in terms of the internal circuit variables

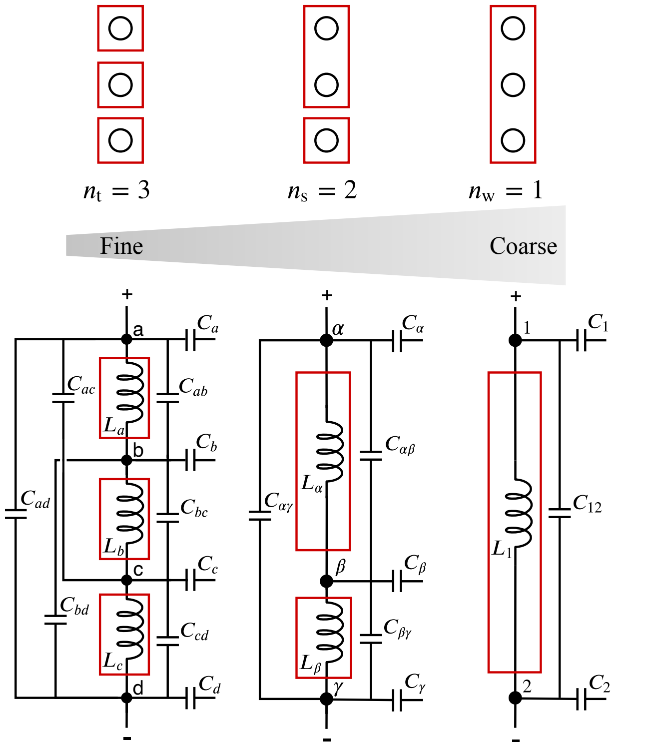

Fig. 2: Circuit representation of three different levels of representation, respectively with 3 turns, 2 sections and 1 winding.

available in the FE model by simply grouping them according to the considered sectioning of the windings. One has thus

$$U_\gamma = (R_{\gamma\beta} + \jmath\,\omega\, L_{\gamma\beta})\ I_\beta\,. \tag{1}$$

where, for each frequency of interest, the matrices $R_{\gamma\beta}$ and $L_{\gamma\beta}$ are to be identified by inspection. They have dimension $N\times N$, where N is the total number of sections, considering all coils in the system. Successively, a current $I_\beta = 1$ is imposed in each section while a current $I_{\gamma\neq\beta}=0$ is imposed in all other sections. The real part of the computed voltages U_γ constitutes then the γ^{th} column of $R_{\beta\gamma}$ whereas the imaginary part divided by the pulsation is the γ^{th} column of $L_{\beta\gamma}$ [4].

B. Electric Parameters

Likewise, the capacitance matrix C is identified by means of 2D FE electrostatic simulations. The identification procedure is however slightly more involved in this case due to the absence of an internal circuit connection in the electrostatic FE model. It is inspired by [3], [4], and developed in this paper to offer a systematic approach. The 2-turn section with terminal voltages V_β and V_γ depicted in Fig. 3 is considered as an example to explain the procedure. Under the assumption of a perfect magnetic coupling, i.e., negligible leakage flux between the turns, and assuming that all turns are of identical radius and cross-section, the electric scalar potential decreases linearly from V_β down to V_γ along the 2 turns (labelled d and e) of the section. Over each single turn, the real voltage varies thus linearly along each turn, say $\mathrm{v}^{\mathrm{t}}(\zeta), \zeta\in[0,1[$, whereas one has to attribute a unique voltage value to the equipotential torus that represents the turn in the 2D FE model. A natural choice is to choose $\zeta = 1/2$, so as to attribute to the torus the average of the scalar potential over the corresponding turn of the real coil. Generalizing now for a section with n turns, one has

$$\mathrm{v}_i^{\mathrm{t}} = \omega_{\beta i} V_\beta + \omega_{\gamma i} V_\gamma \quad , \quad i = 1, \dots n \tag{2}$$

with

$$\omega_{\beta i} = \frac{n - (i-1) - \zeta}{n} \quad , \quad \omega_{\gamma i} = 1 - \omega_{\beta i} \tag{3}$$

to express the linear relationship between turn voltages $\mathrm{v}_i^{\mathrm{t}}$ in the FE electrostatic model and the terminal voltages V_β and V_γ of the section.

One can now revert to the full system and introduce the vector $\mathbf{v}^{\mathrm{s}}$ of the voltages at the M section terminals in the system. The inspection then proceeds as usual. Successively, a voltage $V_\beta = 1$ is imposed at one section terminal while a zero voltage $V_{\gamma \neq \beta} = 0$ is imposed at all other section terminals. Using (3), the vector $\mathbf{v}^{\mathrm{t}}$ of the torus voltages is calculated as

$$\mathbf{v}^{\mathrm{t}} = \boldsymbol{\omega}\, \mathbf{v}^{\mathrm{s}}. \tag{4}$$

The torus voltages $\mathbf{v}^{\mathrm{t}}$ are imposed as electrode constraints in the electrostatic 2D problem, and the the corresponding charges carried by the individual toruses, $\mathbf{q}^{\mathrm{t}}$, are computed by post-processing the obtained FE solution. It remains to convert the torus charges $\mathbf{q}^{\mathrm{t}}$ into section charges $\mathbf{q}^{\mathrm{s}}$. For this, one uses duality. One notes that the electrostatic energy E in the system is given by

$$2E = (\mathbf{v}^{\mathrm{t}})^{\mathrm{T}} \mathbf{q}^{\mathrm{t}} = (\mathbf{v}^{\mathrm{s}})^{\mathrm{T}} \boldsymbol{\omega}^{\mathrm{T}} \mathbf{q}^{\mathrm{t}} = (\mathbf{v}^{\mathrm{s}})^{\mathrm{T}} \mathbf{q}^{\mathrm{s}}$$

so that one has by identification

$$\mathbf{q}^{\mathrm{s}} = \boldsymbol{\omega}^{\mathrm{T}}\, \mathbf{q}^{\mathrm{t}}. \tag{5}$$

The vector $\boldsymbol{\omega}^{\mathrm{T}}\, \mathbf{q}^{\mathrm{t}}$ then constitutes the β^{th} column of the matrix of influence coefficients defined by [5] as

$$\mathbf{q}^{\mathrm{s}} = \boldsymbol{\Gamma}^{\mathrm{s}}\, \mathbf{v}^{\mathrm{s}}. \tag{6}$$

It has dimension $M \times M$, where M is the total number of section terminals, considering all coils in the system. Finally, the sought capacitance C matrix is obtained from $\boldsymbol{\Gamma}^{\mathrm{s}}$ by a simple algebraic operation

$$q_i^{\mathrm{s}} = \sum_i \Gamma_{ji}^{\mathrm{s}} v_i^{\mathrm{s}} = \sum_i (-\Gamma_{ji}^{\mathrm{s}})(v_j^{\mathrm{s}} - v_i^{\mathrm{s}}) + \left(\sum_i \Gamma_{ij}^{\mathrm{s}} \right) (v_j^{\mathrm{s}} - 0),$$

which shows that $(-\Gamma_{ji}^{\mathrm{s}})$ is the capacitance of the branch connecting the section terminal j to the section terminal i in the equivalent electric circuit, and that $\left(\sum_i \Gamma_{ij}^{\mathrm{s}}\right)$ is the capacitance of the branch connecting the section terminal j to the zero ground potential.

IV. Applications

A. Multi-Layer Two-Winding Transformer

In order to analyze the performance of the proposed modeling approaches and compare them to experimental measurements, a representative transformer is first considered. It consists of a 20-turn primary made of Litz wire and a 60-turn secondary made of round wire, both wound around an ETD 34/17/11 ferromagnetic core, Fig. 4. Note that in order to use a 2D axisymmetric finite element model, the geometry of the ferromagnetic core has been adapted to keep the same equivalent cross-section and mean length.

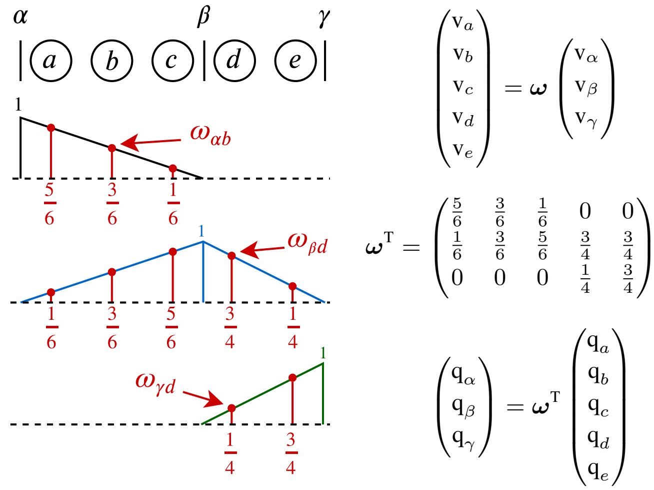

Fig. 3: Example of a 5-turn winding split into 2 sections of 3 and 2 turns, respectively, with its vectors , $\mathbf{v}^{\mathrm{s}}$, $\mathbf{v}^{\mathrm{t}}$, $\mathbf{q}^{\mathrm{s}}$ and $\mathbf{q}^{\mathrm{t}}$, and the turn-to-section matrix $\boldsymbol{\omega}$ (5).

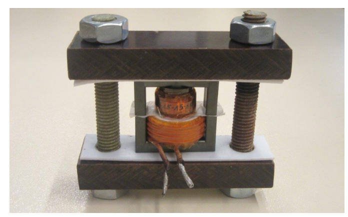

Fig. 4: High voltage two-windings transformer with primary (bottom winding) and secondary (top winding).

Fig. 5: Single layer 155-turns air inductance.

Fig. 6 highlights the good match of all models with the experimental results. The equivalent electric network identifies accurately the first resonance and anti-resonance, with all three representation levels of the windings, and it also predicts the presence of the second resonance. The maximum measurement frequency of our experimental setup being 50 MHz, we can however not conclude on the respective accuracy of the different representation levels for higher order resonances, hence the second application discussed below.

B. Single-Layer Air Inductance

In order to better assess the ability of the turn-, section- and winding-level models to capture higher resonances, the one-layer, 155-turn air inductance depicted in Figure 5 has been considered as well. The AWG #25 copper wire is wound over a PVC pipe (⌀ 50.21 mm), over a total height of 72 mm. The dielectric insulator around the copper wire is considered to be 5.176 μm thick, with a relative dielectric permittivity of $\epsilon_r = 4$.

As for the high-frequency transformer, the impedance comparison between the simulated inductance (winding-level,

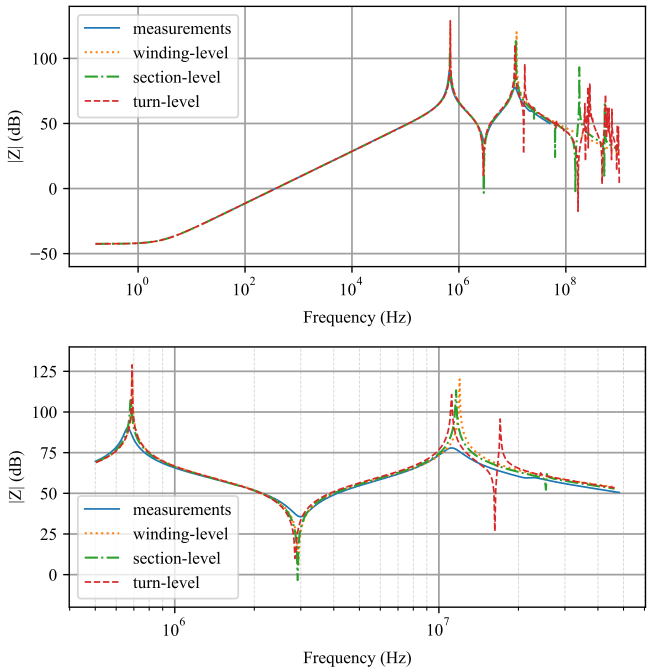

Fig. 6: Equivalent electric circuit simulations (winding-, section- and turn-levels) and measurements of the impedance $|Z(f)|$ of the transformer of Fig. 4. The impedance is seen from the primary side with the secondary left open. Below, a zoom-in on the frequency range of the first two resonances.

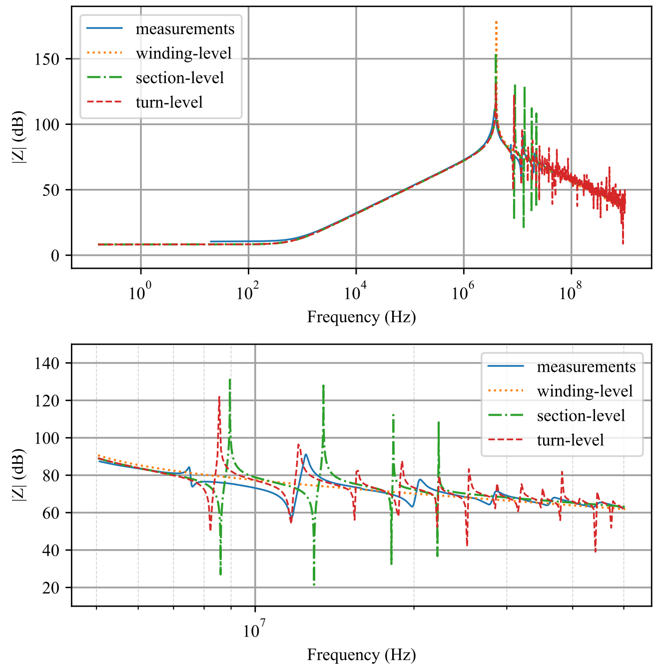

Fig. 7: Equivalent electric circuit simulations (winding-, section- and turn-levels) and measurements of the impedance $|Z(f)|$ of the single-layer air inductance. Below, a zoom-in on the frequency range above the first resonance.

TABLE I: Dimensions of the inductance and capacitance matrices,as well as the computation time needed for the identification of the equivalent electric network and the simulation time to generate the curves in Fig. 7, for the three representation levels in the case of the air inductance.

Level	**L**	**C**	$t_{\text{ident}}(s)$	$t_{\text{simul}}(s)$
155	(155×155)	(156×156)	20983	105
5	(5 × 5)	(6 × 6)	545	2
1	(1 × 1)	(2 × 2)	190	1.6

section-level and turn-level) and the measurements (Fig. 7) reveals an excellent agreement for the first resonance. Moreover, the simulations performed with the section-level approach are close enough to those performed with the finer-grained turn-level approach. They both capture the second an third resonances with a satisfactory accuracy, albeit at a drastically lower computational cost in the former case, Table I.

V. Conclusion

The proposed method combines conventional simulation tools (2D FE and circuit solver) to identify an RLC electric network equivalent to magnetic components. Different levels of complexity can be achieved thanks to the tunable section-level approach presented in this paper, so that the dielectric phenomena in the windings can be represented in the model in a more detailed fashion with controllable computational cost. In particular, the equivalent network is able to resolve the first few resonance frequencies in the windings in the system, which the simple winding-level approach cannot do. Moreover, the equivalent electric network is also a convenient representation of the studied system to carry on transient analyses. The proposed identification method offers thus an efficient lightweight modelling tool to study parasitic effects in magnetic components with an accuracy comparable to that of experimental measurements and at a limited computational cost.

Acknowledgement

This work was supported in part by the Walloon Region of Belgium under grant PIT 8139 M&SSCoT.

References

[1] J. Biela, “Using transformer parasitics for resonant converters - a review of the calculation of the stray capacitances of the transformers,” *Fourtieth IAS Annual Meeting. Conference Record of the 2005 Industry Applications Conference* vol. 44, no. 1, pp. 223–33, 2005.

[2] R. Asensi, R. Prieto, J. Cobos, and J. Uceda, “Modeling high-frequency multiwinding magnetic components using finite- element analysis,” *IEEE Transactions on Magnetics* vol. 43, no. 10, pp. 3840–3850, Oct. 2007.

[3] Z. De Grève, P. Dular, R. V. Sabariego, and C. Geuzaine, “Finite element models for studying the capacitive behaviour of wound components,” *20th International Conference on the Computation of Electromagnetic Fields*, Jun. 2015.

[4] Z. D. Greve, O. Deblecker, and J. Lobry, “Numerical modeling of capacitive effects in HF multiwinding transformers - Part I: A rigorous formalism based on the electrostatic equations,” *IEEE Transactions on Magnetics* vol. 49, no. 5, pp. 2017–2020, May 2013.

[5] É. Durand, ”Électrostatique. Tome l, Les distributions”, 1964

[6] J. Keradec, B. Cogitore, E. Laveuve and M. Bensoam. Des schémas équivalents pour les circuits couplés multi-enroulements. Journal de Physique III, EDP Sciences, 1994, 4 (4), pp.751-773.

[7] N. Davister, F. Frébel, C. Geuzaine and G. Parent. Comparison of Finite Element Formulations for the Modelling of High-Frequency Effects in Electromagnetic Coils. Proceedings of COMPUMAG, 2019, July.