Conserved quantities in cosmology

Phillip Helbig

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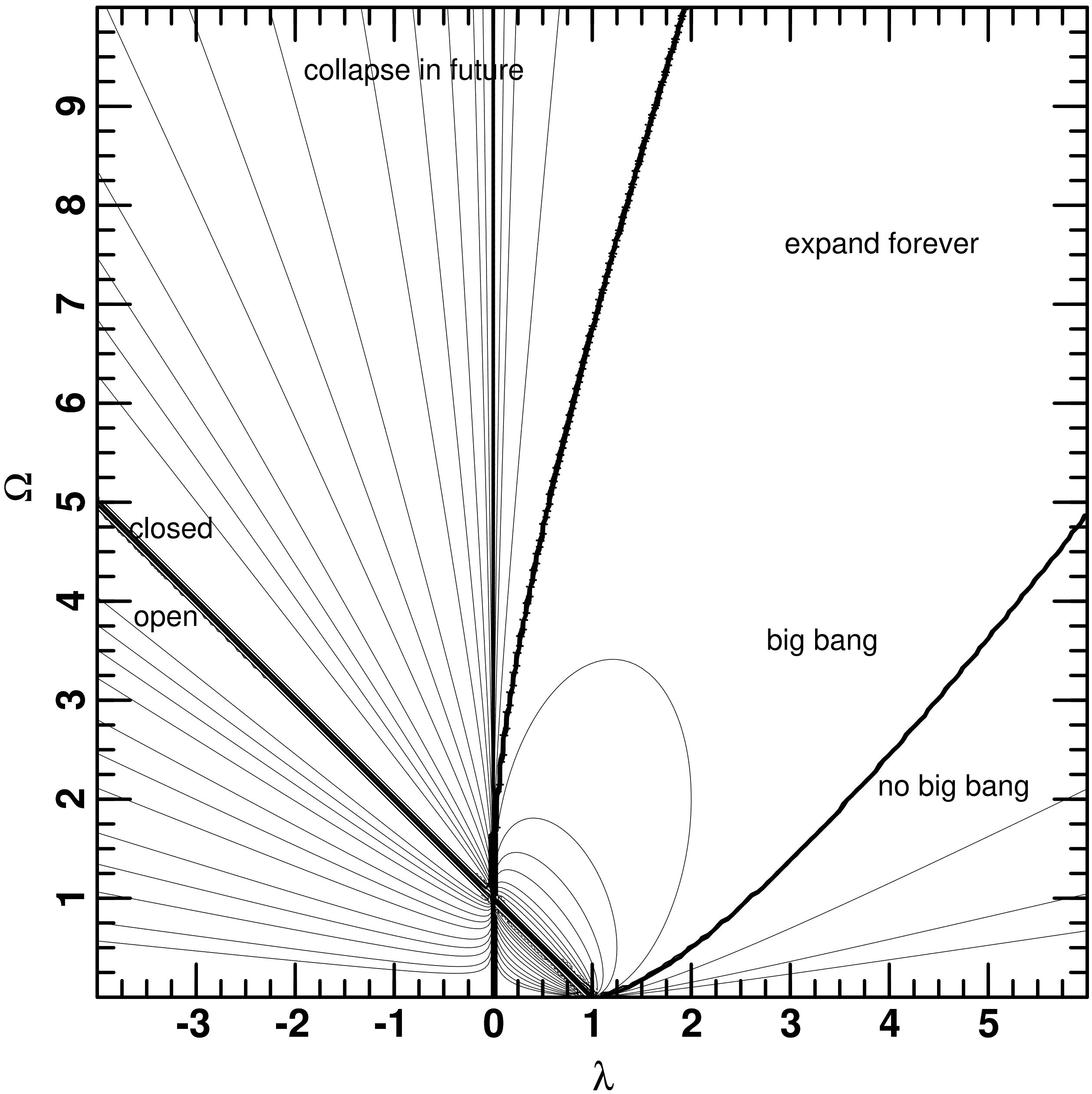

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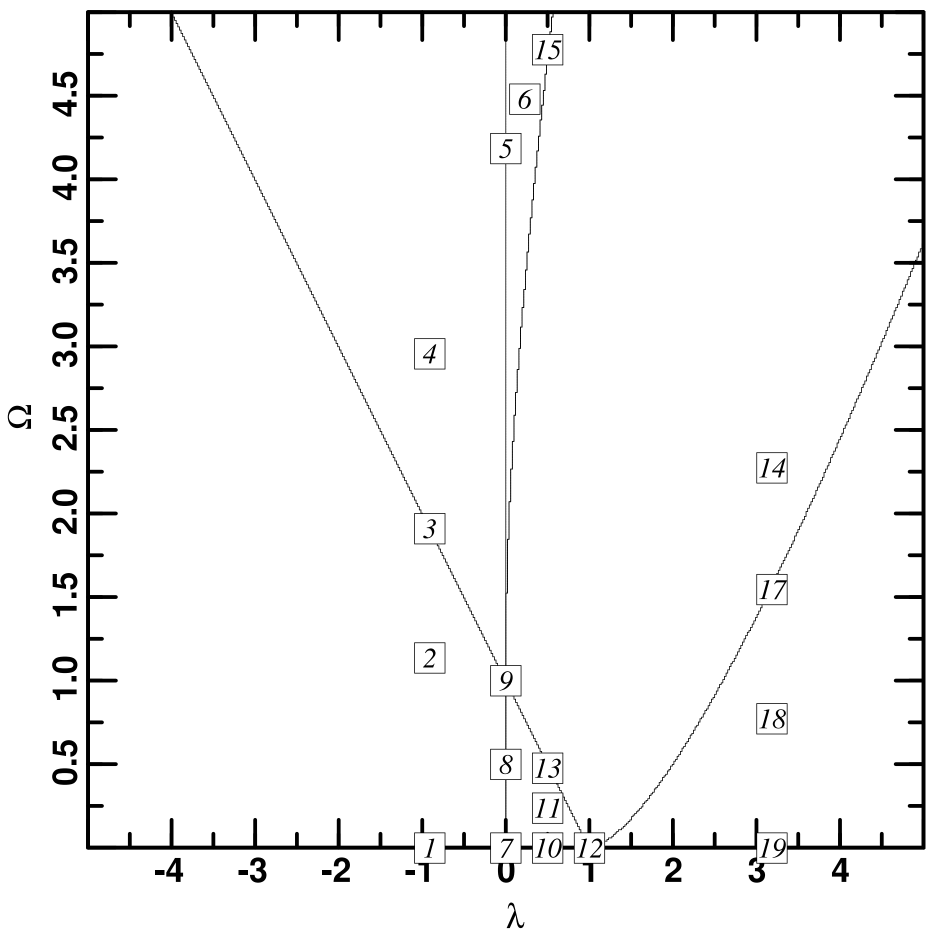

The 19 types of FRW models with (at most) dust and a cosmological constant

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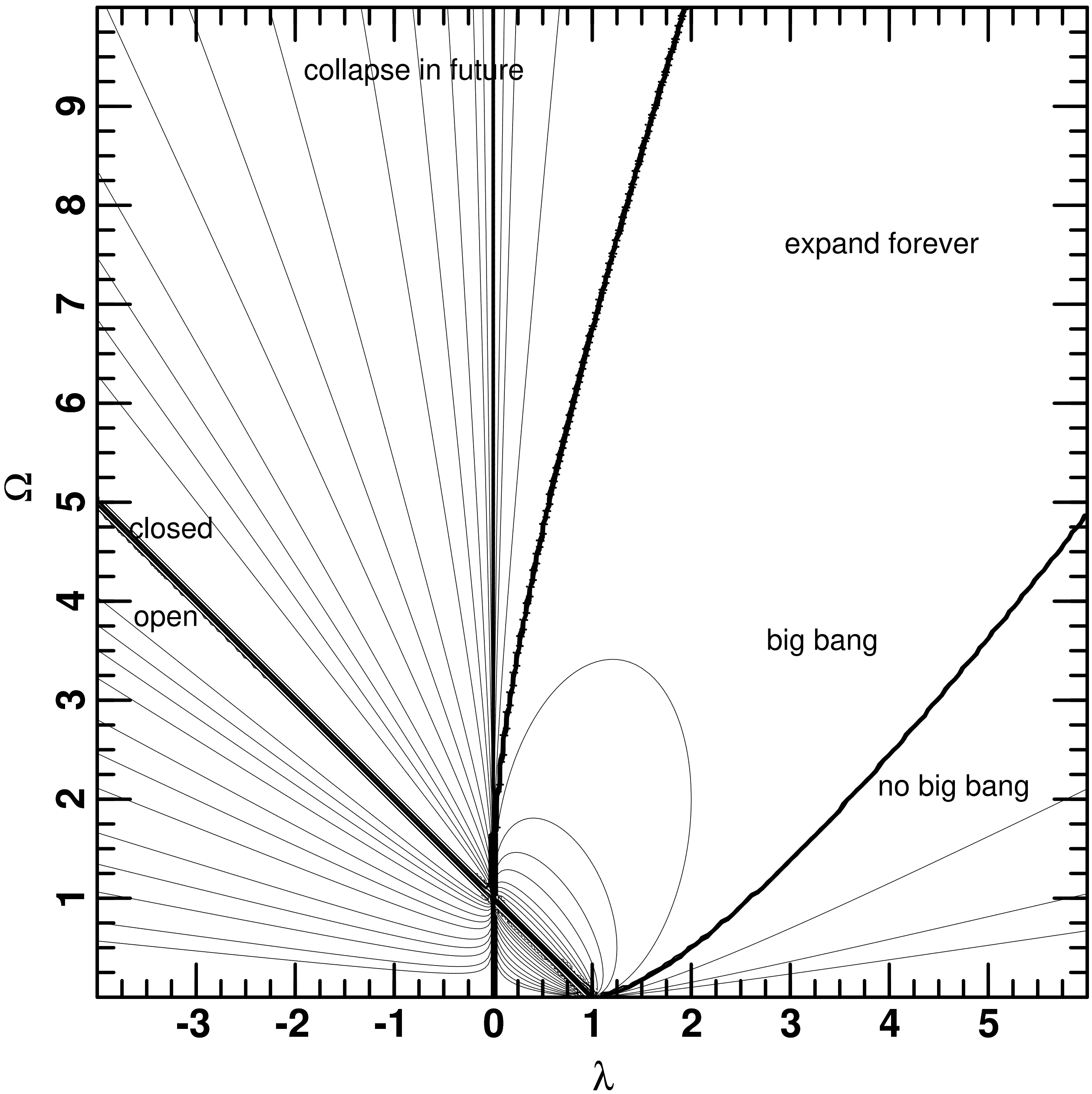

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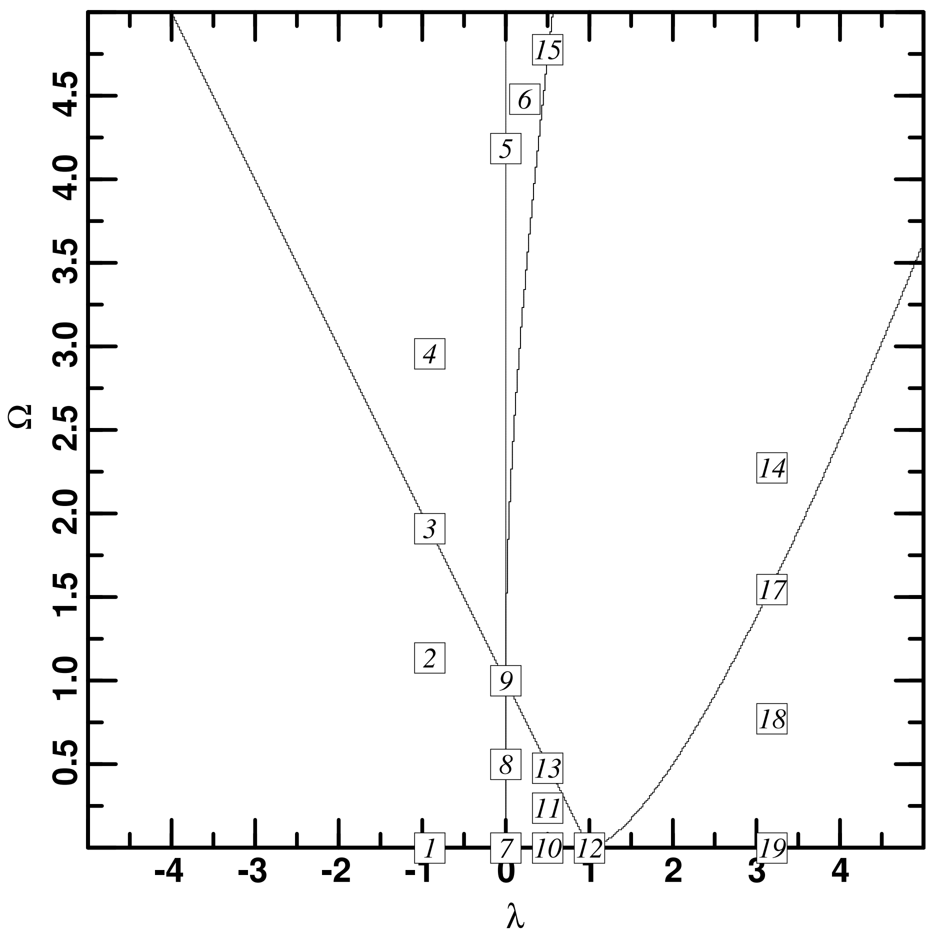

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