



# New consistent set of design equations for equal-leg angle sections and members

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## Abstract

Angles profiles have been used since the very beginning of the steel construction due to their easy production, transportation, and ability to be connected. However, they exhibit some specific features that clearly distinguish them from other types of common sections, as they are monosymmetric sections with very small constants in both torsion and warping, their bending capacity and radius of gyration around the weak axis are substantially lower compared to the strong axis ones, their legs are possibly susceptible to local buckling, their plastic resistances are significantly higher than their elastic ones and finally, due to the eccentric connection in one leg, they are also subjected to bending when used as single members.

The above-listed features confirm that existing common design rules for other mostly doubly symmetric type of sections cannot safely cover angle sections, what inevitably leads to the need for development of specific design provisions for angles. Therefore, extensive experimental, analytical, and numerical studies have been conducted to propose a complete and duly validated set of design rules covering all aspects of their design (classification, cross-section and member resistance). The proposed rules will be included in the forthcoming version of EN1993-3.

## Keywords

Angle profiles, Equal-leg angles, Design rules, Classification, Stability, Buckling.

## 1 Introduction

Angle profiles are extensively used in lattice towers and masts for telecommunication purposes or electric power transmission, as well as in a wide range of civil engineering applications such as buildings and bridges; they are also used to strengthen existing structures. Their easy production and transportation, together with their excellent connectivity, make these profiles very attractive for the designers. However, equal-leg angle profiles considered here, exhibit some properties that clearly distinguish them from other common steel sections: (i) they are open profiles with very small section constants in both torsion and warping, (ii) they are monosymmetric sections, (iii) their bending capacity and radius of gyration around the weak axis are substantially lower around strong axis, (iv) their legs are prone to local buckling, (v) their plastic resistances are substantially higher than their elastic ones and (vi) due to the eccentric connection in one leg, they are subjected to bending in addition to axial force when used as single members. All these features explain that existing design rules for other, mostly doubly symmetric sections cannot safely cover angles, what unavoidably leads to the need for the development of specific

design provisions for these particular profiles.

Facing the lack of unified consistent rules for angles, Eurocode 3 has adopted a case-by-case approach, embedding individual rules and recommendations in various parts of it; in contrast with the American Code which includes in a single document, AISC 2000 [1], all rules concerning angle design. More specifically, EN1993-1-1 [2], as well its forthcoming version prEN 1993-1-1 [3], provide rules for cross-section classification and general design recommendations for the verification of the stability of members. EN1993-3-1 [4] gives specific rules for the buckling resistance of angle members used in towers, when connected eccentrically with bolts in one leg, while EN 1993-1-5 [5] presents rules for buckling resistance of class-4 angle sections prone to local buckling. Another European specification, the CENELEC standard EN 50341-1 [6] provides specific rules for the verification of lattice towers and its constituting parts. But for some aspects, the EN 50341-1 design methods for angle sections may diverge from the rules provided in the Eurocodes. Additionally, even if this is not strictly prevented, the application of EN 50341-1 to S460 steel grades remains questionable. It is also remarkable that amongst all these normative documents, a number of inconsistencies have been identified,

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concerning different aspects of design. For instance, for classification to compression, four different width-to-thickness ratios can be evaluated for the same class limit, depending to the referred document, while for class 4 cross-section, all these documents are referring to EN 1993-1-5 for the determination of the effective area even though in the most of them, the proposed class limits are in contradiction with it!

Extensive research has been carried out to study the behaviour of angle sections. Vayas et al. [7] give the inelastic capacity of angle sections to combined axial forces and biaxial bending. Trahair [8] examines angle section beams subjected to uniform eccentric transverse loading and gives the section capacity under combined shear, bending and torsion. Kettler et al. [9] highlight the importance of the end support conditions in their numerical study of rolled angles loaded in compression through bolted connections in one leg and in their comparisons with experimental results and with the provisions of the European standards. An innovative research work carried by Dinis et al. [10], led to the proposal of novel and more rational design rules based on the Direct Strength Method (DSM); this design approach has been extended in Ref. [11] to hot-rolled sections, applicable to short-to-intermediate angle columns under ideal support conditions subjected to compression. Bezas et al. [12] report on experimental and numerical studies of large rolled angle profiles from high strength steel (HSS) and give an extended literature review on relevant experimental investigations on angles. Extensive experimental investigations have been also carried out by various authors [13],[14].

In the frame of the European RFCS-funded project ANGELHY [15], existing European specifications on rolled equal angle sections have been reviewed, extensive experimental, analytical and numerical studies have been conducted and a complete set of design rules covering all aspects of design have been developed and duly validated. They include cross-section classification, cross-section design to individual and combined internal forces and moments, for all ranges of responses (plastic, elastic-plastic or elastic), as well as corresponding rules for member design. The proposed design rules will be included as Annex F in the latest version of prEN 1993-3 [16]. The aim of the paper is to discuss the proposed rules and compare them with the existing codes; their analytical and numerical validations are detailed in Ref. [17], [19], and thus, will not be addressed here.

## 2 Classification of equal-leg angle sections

Cross-section classification is of importance for the selection of the analysis and design procedures to employ: plastic, elastic or elastic with due account for local buckling. According to EN 1993-1-1, clause 5.5.2 (4) classification should be done for the compression parts of the cross-section. A strict application of this rule requires a separate classification of the cross-section for each combination of applied forces and moments. Since this rule is unpractical for design, a simpler approach is proposed here, where the cross-section is classified separately for compression, strong axis and weak axis bending. For the latter, the cross-section class may be different for positive or negative moments due to the monosymmetric shape of the profile, that may lead to different classes when the tip is in compression or in tension. It should be also noticed that even in the latest draft of the forthcoming version of EN 1993-1-1 that is available to the authors, namely prEN 1993-1-1 [3], no modification is contemplated regarding the classification of the cross-sections. Furthermore, the classification boundaries from class 3 to 4 are determined through the slenderness of the compression leg, even if in EN1993-1-1 there is a specific criterion for angles in compression (Table 5.2, sheet 3 of 3) which is based

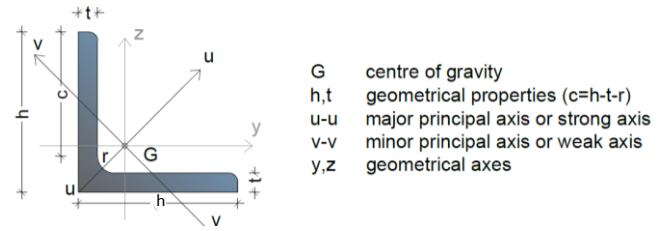
on the torsional instability mode, but as it has been shown in ANGELHY project it is not relevant.

Parametrical numerical investigations on short column subjected to compression, strong axis bending, and weak axis bending have been conducted in view of deriving appropriate classification criteria [17]. In all the numerical simulations, the applied load increasing up to failure. All the limits determined numerically have then been validated analytically, always following the main principles of Eurocode 3.

**Table 1** Maximum width-to-thickness ratios for compression parts of equal leg angle sections

	Section in compression	Section in strong axis bending $M_u$	Section in weak axis bending $M_v$ – tip in compression	Section in weak axis bending $M_v$ – tip in tension
<b>Class 1 – 2</b>	---	$\frac{c}{t} \leq 16 \varepsilon$	$\frac{c}{t} \leq 14 \varepsilon$	$\frac{c}{t} \leq 30 \varepsilon$
<b>Class 3</b>	$\frac{c}{t} \leq 13,9 \varepsilon$	$\frac{c}{t} \leq 26,3 \varepsilon$	$\frac{c}{t} \leq 26,9 \varepsilon$	---

where  $\varepsilon = \sqrt{235/f_y [\frac{N}{mm^2}]}$



**Figure 1** Notations for geometrical properties and principal axes

The complete set of the proposed duly validated classification criteria is summarized in Table 1. It may be seen that, unlike in the current Eurocodes, the same geometric parameters,  $c$  and  $t$ , are used for all cross-section loading situations (see Figure 1 for the notations). Comparisons of the proposed limits and the current provisions of EC3 are shown in Figure 2, Figure 3 and Figure 4 for cross section subjected to compression, strong and weak axis bending respectively.

## 3 Cross-section design resistances

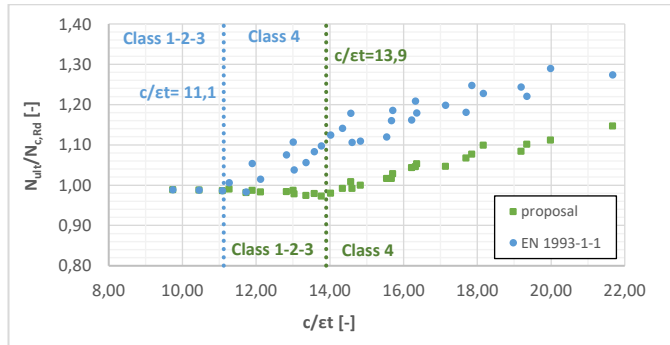
In the following paragraphs, formulae for the evaluation of the cross-section design resistance of equal leg angles are presented. Their validation has been achieved through the same numerical results obtained for the classification. The notations for the material properties, safety factors and other properties follow those given in EN 1993-1-1, and therefore no further definitions are given here, unless it is necessary. Furthermore, based on the results obtained in the course of classification, a linear transition between plastic and elastic bending resistances has been adopted. This smooth transition has already been proposed and validated for double symmetric cross-sections in the SEMI-COMP European funded project [18] and will be adopted for these sections in the forthcoming new version of Eurocode 3 Part 1-1 (prEN1993-1-1).

### 3.1 Cross-section resistance to compression

The cross-section design resistance for axial compression may be determined according to the provisions of EN 1993-1-1. For class 4 cross-sections, the relative slenderness used to calculate the effective area may be determined according to EN 1993-1-5 using the modified slenderness:

$$\bar{\lambda}_p = \frac{\bar{b}/t}{18,6\epsilon} = \frac{c/t}{18,6\epsilon} \quad (1)$$

In order to be in line with the proposed classification limits, the geometric property  $\bar{b}$  must be defined differently than in the current Eurocode provisions. Consequently, the statement of EN 1993-1-5, §4.4(2) that  $\bar{b} = h$  for equal leg angles should be replaced for this type of section by  $\bar{b} = c$ . This constitutes the only difference between the current proposal and the existing Eurocode 3 provisions.



**Figure 2** Cross-section resistance to compression. Ratio between numerical results and design compression resistance obtained from the current proposal and Eurocode 3 vs.  $c/\epsilon t$  ratio

Figure 2 illustrates the ratio between the numerically determined cross-section resistance ( $N_{ult}$ ), and the design resistance ( $N_{c,Rd}$ ), versus the  $c/\epsilon t$  ratio. The design resistances have been evaluated firstly with the current proposal and then based on the existing provisions of EN 1993-1-1 in combination with EN 1993-1-5 for class-4 sections. The vertical dot lines represent the class-3 limit as suggested in the paper ( $c/t \leq 13,9\epsilon$ ) and the one calculated by Eurocode 3 provisions ( $c/t \leq 11,1\epsilon$ ). This limit has been determined analytically by considering the risk of local plate buckling resistance of the leg according to EN 1993-1-5. It can be easily seen that the proposed model is less conservative for class-4 profiles. The benefits of the new classification criteria is also clear.

### 3.2 Cross-section resistance to strong axis bending

The design resistance of equal leg angle sections for bending about the major axis may be determined with:

$$M_{u,Rd} = W_u \frac{f_y}{\gamma_{M0}} \quad (2)$$

The section modulus  $W_u$  depends on the class of the cross-section and may be determined by:

$$W_u = \alpha_{i,u} W_{el,u} \quad (3)$$

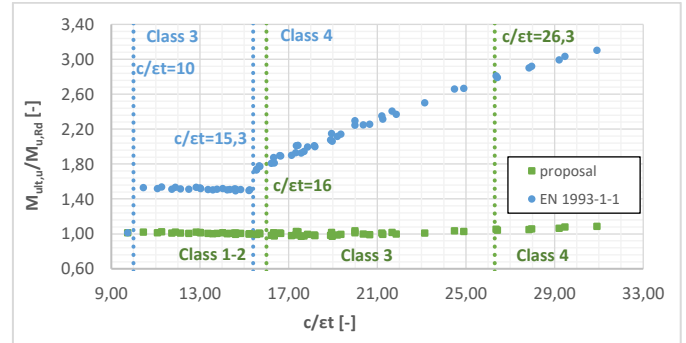
where:

$$\begin{aligned} \alpha_{2,u} &= 1,5 && \text{for class 1 or 2} \\ \alpha_{3,u} &= \left[ 1 + 0,5 \cdot \left( \frac{26,3\epsilon - c/t}{26,3\epsilon - 16\epsilon} \right) \right] && \text{for class 3} \\ \alpha_{4,u} &= W_{eff,u} / W_{el,u} = \rho_u^2 && \text{for class 4} \end{aligned}$$

The reduction factor  $\rho_u$  should be determined with equation (5) using the relative slenderness  $\bar{\lambda}_p$  obtained with:

$$\bar{\lambda}_p = \frac{c/t}{35,58\epsilon} \quad (4)$$

$$\rho_u = \frac{\bar{\lambda}_p - 0,188}{\bar{\lambda}_p^2} \quad (5)$$



**Figure 3** Cross-section resistance to strong axis bending. Ratio between numerical results and design compression resistance obtained from the current proposal and Eurocode 3 vs.  $c/\epsilon t$  ratio

Figure 3 illustrates the ratio between the numerically determined cross-section resistance ( $M_{ult,u}$ ), and the design resistance ( $M_{u,Rd}$ ), versus the  $c/\epsilon t$  ratio. The design resistances have been evaluated both with the current proposal and the existing provisions of EN 1993-1-1. The vertical dot lines represent the class limits as suggested in the paper (i.e.  $c/t \leq 16\epsilon$  for class 2 to 3 and  $c/t \leq 26,3\epsilon$  for class 3 to 4) and as calculated by EN 1993-1-1 provisions (i.e.  $c/t \leq 10\epsilon$  for class 2 to 3 and  $c/t \leq 15,3\epsilon$  for class 3 to 4). The benefits and the improvements brought from the proposals (classification and cross-section resistance) on the design of the cross-sections can clearly be observed.

### 3.3 Cross-section resistance to weak axis bending

The design resistance of equal leg angle sections for bending about the minor axis may be determined with:

$$M_{v,Rd} = W_v \frac{f_y}{\gamma_{M0}} \quad (6)$$

The section modulus  $W_v$  depends on the class of the cross-section and may be determined by:

$$W_v = \alpha_{i,v} W_{el,v} \quad (7)$$

where:

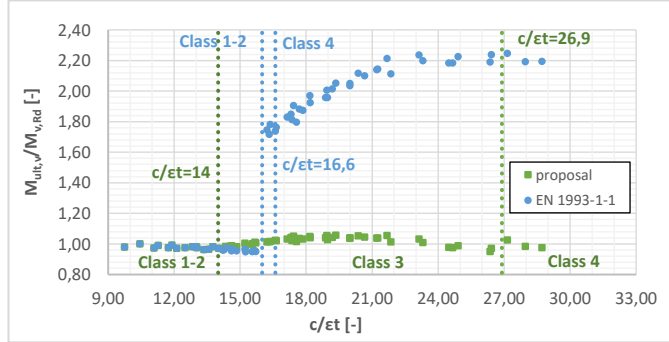
$$\begin{aligned} \alpha_{2,v} &= W_{pl,v} / W_{el,v} && \text{for class 1 or 2} \\ \alpha_{3,v} &= \left[ 1 + \left( \frac{26,9\epsilon - c/t}{26,9\epsilon - 14\epsilon} \right) \cdot \left( \frac{W_{pl,v}}{W_{el,v}} - 1 \right) \right] && \text{for class 3} \\ \alpha_{4,v} &= W_{eff,v} / W_{el,v} = 0,94 \cdot \rho_v^2 && \text{for class 4} \end{aligned}$$

The reduction factor  $\rho_v$  should be determined with equation (5) using the relative slenderness  $\bar{\lambda}_p$  obtained with:

$$\bar{\lambda}_p = \frac{c/t}{36,48\epsilon} \quad (8)$$

Figure 4 illustrates the ratio between the numerically determined cross-section resistance ( $M_{ult,v}$ ), and the design resistance ( $M_{v,Rd}$ ), versus the  $c/\epsilon t$  ratio. The design resistances have been evaluated

both with the current proposal and the existing provisions of EN 1993-1-1. The vertical dot lines represent the class limits as suggested in the paper (i.e.  $c/t \leq 14\epsilon$  for class 2 to 3 and  $c/t \leq 26,9\epsilon$  for class 3 to 4) and as calculated by EN 1993-1-1 provisions (i.e.  $c/t \leq 16,6\epsilon$  for class 2 to 3 and  $c/t \leq 15,9\epsilon$  for class 3 to 4; the profiles in between 15,9 and 16,6 are treated as class 4 sections for the calculations so as to face this inconsistency of the existing code provisions). The benefits and the improvements brought from the current proposals (classification and cross-section resistance) on the design of the sections may again be observed.



**Figure 4** Cross-section resistance to weak axis bending– tip in compression. Ratio between numerical results and design compression resistance obtained from the current proposal and Eurocode 3 vs.  $c/t$  ratio

#### 4 Buckling resistance of members

In the following, formulae for the design resistance and stability of equal-leg angle members as derived during the ANGELHY project are presented, discussed and compared with the Eurocode 3 provisions. The notations for the material properties, safety factors and other symbols follow again those given in EN 1993-1-1. In absence of the availability of analytical solutions for general application, they constitute engineering approximations based on following considerations:

- they should correctly represent the mechanical behavior and failure modes of cross-sections and members;
- they should cover stability checks under combined loading conditions, including compression and bending;
- they should smoothly adapt to specific cases, such as individual loadings and failure modes;
- they should be closely connected to analytical expressions for simple cases;
- they should remain simple and not too much overloaded with additional factors as a result of a calibration procedure to allow transparency and easy “by hand” control checks;
- they should easily adapt to the format of the existing Eurocode 3 rules, especially those of prEN 1993-1-1 that are applicable for other types of cross-sections and do not cover rolled angles.

The validation of the proposed formulae for the prediction of the carrying capacity of members with equal leg angle sections is based on comparisons with the results of numerical simulations and experimental tests of the member response for a wide range of parameters. The profile sizes, the member lengths and the steel grades of the samples have been selected in order to collect a large number of samples with properties that are commonly used in steel towers, but this is not limiting the range of application of the rules they can be easily used in other types of structures too. Representative cases are given here after, while details and more information about the validation process can be found in Ref. [19].

#### 4.1 Buckling resistance to compression

For members of equal leg angle section, flexural buckling should be checked according to the provisions given in prEN 1993-1-1, while flexural-torsional buckling may be disregarded. A pure torsional mode cannot be obtained for a centrally loaded angle column as explained in Ref. [20].

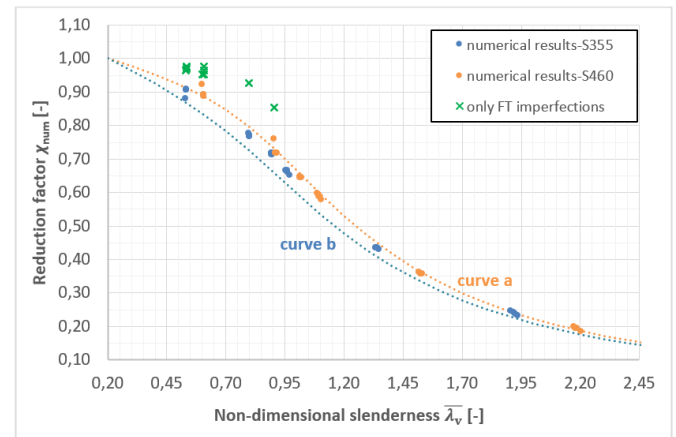
For class 4 cross-sections, the relative slenderness used to calculate the effective area may be determined according to EN 1993-1-5 using the modified slenderness:

$$\bar{\lambda}_p = \sqrt{\chi_{\min} \frac{c/t}{18,6\epsilon}} \quad (9)$$

where  $\chi_{\min}$  is the minimum of the reduction factors obtained for flexural buckling about the minor axis  $\chi_v$  and for flexural buckling about the major axis  $\chi_u$ .

It may be seen that the local plate slenderness  $\bar{\lambda}_p$  (eq. 9) depends in the proposed design rules not only on the geometrical and material properties of the cross-section ( $c/t$ -ratio and correspondingly  $\epsilon$ ) but also on the overall buckling reduction factor  $\chi_{\min}$ . This means that  $\bar{\lambda}_p$  decreases, as the overall slenderness of the angle member increases. As a result, the proposed design rules, in line with Ref. [21], account correctly for the fact that, in concentrically compressed angle members, the effects of local buckling diminish as the length of the member, and therefore the relevant global slenderness, increases.

Figure 5 illustrates the numerical results compared with the reference buckling curves a and b as they are reported in EN 1993-1-1. The buckling reduction factor  $\chi_{\text{num}}$  of the numerically tested samples has been evaluated as  $\chi_{\text{num}} = N_{\text{ult}}/N_{\text{pl}}$ . For the samples with a flexural-torsional eigenmode, two cases are distinguished in Figure 5; the numerical results reported with blue/orange points have been evaluated by using  $N_{\text{ult}}$  obtained by using an initial imperfection with a deformation shape similar to the first flexural instability mode, while the results presented with green points using the relevant buckling mode, i.e. the flexural-torsional one.

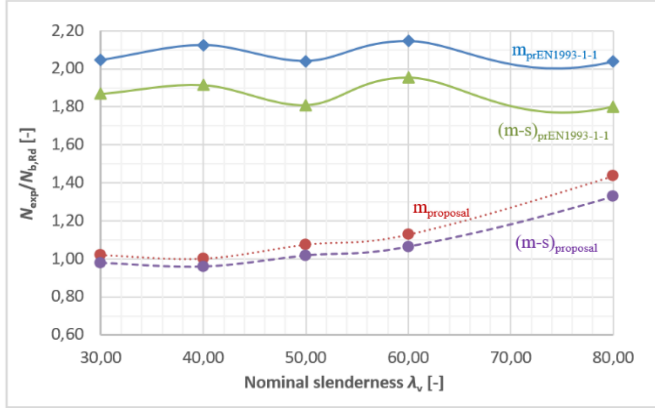


**Figure 5** Comparison of numerical results with buckling curves of prEN 1993-1-1

According to Ref. [3], the obtained numerical results for the S355 steel grade should be compared with curve b while for S460 with curve a. It can be easily observed that all the results referring to curve b are above the curve, while the results referring to curve a are in line, above or just a bit lower, which is acceptable given the 2% deviation that is considered in the numerical analyses (see Ref. [19] for more details). Regarding the results obtained using only an

equivalent imperfection based on the 1<sup>st</sup> eigenmode (i.e. the flexural-torsional one), it is obvious that they are much higher even when compared with curve a. Through this comparison, it can be easily observed that the slenderness should be calculated using only the minimum elastic critical force for the flexural buckling mode as reported in the paper.

The design formula is also checked against tests; at Tsinghua University, 66 tests were carried out on axially loaded columns from equal angle sections that were hinged at both extremities and reported in Ref. [13]. The cross-sections ranged from L 125x125x8 to L 200x200x14 and the material was high strength steel (HSS) S420. All the profiles were classified in class 4.



**Figure 6** Ratio between experimental and analytical loads for the Tsinghua University tests, mean values

Figure 6 presents the mean value (m) of the ratio between the experimental load resistances and the ones predicted by prEN 1993-1-1 and by the current proposal, as well as the mean minus one standard deviation value (m-s). There are only very few results where the analytical solution predicts lower values, with a maximal deviation of 5% in two tests, while the (mean - s) value of all tests with nominal slenderness 30 and 40 is 0,8% and 3,2% respectively. This is completely acceptable, considering that strain hardening was neglected in the analyses. It may be also seen that the current proposal gives a better prediction for the column capacity compared to the Eurocode 3. The conservative character of prEN 1993-1-1 may be explained by the fact that the non-dimensional slenderness is determined from the relevant buckling mode (in this experimental campaign the most were flexural-torsional ones), while as proposed, they should be based only on the flexural mode.

**Table 2** Comparisons for the centrally loaded ULIège tests

Specimen label	$\bar{\lambda}_v$ [-]	Class	$N_{exp}/N_{Rd,prEN1993-1-1}$ [-]	$N_{exp}/N_{Rd,prop}$ [-]
Sp11	1,31	1	1,13	1,13
Sp13	1,56	1	1,04	1,04
Sp15	1,81	1	1,04	1,04
Sp21	1,12	4	1,12	1,06
Sp23	1,30	4	1,02	0,99
Sp25	1,48	4	1,08	1,05

At the University of Liège, 12 compression tests were carried out on axially loaded pin-ended columns with or without eccentricity in the frame of the ANGELHY project; they are reported in Ref.[12]. Amongst them, 6 were loaded at the centre of gravity. The cross-sections were L 150x150x18 and L 200x200x16 with three

different lengths. The material was high strength steel (HSS) S420 and S460. The load was introduced through supports that correspond to fully hinged boundary conditions, allowing free rotation in- and out-of-plane.

The ratio between the experimental load and the buckling resistance from prEN 1993-1-1 as well as the resistance determined by the current proposal of the tests without eccentricity, are reported in Table 2. As for all centrally loaded specimens, the first instability mode was a flexural one, the value for the non-dimensional slenderness is the same for both procedures. Therefore, the difference of the ratios reported in Table 2 are due to the way to incorporate local buckling, which differs in both procedures.

#### 4.2 Lateral torsional buckling resistance to strong axis bending

Lateral torsional buckling of angle sections subjected to major axis bending should be checked according to EN 1993-1-1, considering a linear transition between plastic and elastic bending resistances (see section 3.2). For equal leg angle sections, the reduction factor for lateral torsional buckling  $\chi_{LT}$  may be obtained with:

$$\chi_{LT} = \frac{1}{\Phi_{LT}^2 + \sqrt{\Phi_{LT}^2 - \bar{\lambda}_{LT}^2}} \quad \text{but} \quad \begin{cases} \chi_{LT} \leq 1,0 \\ \chi_{LT} \leq 1/\bar{\lambda}_{LT}^2 \end{cases} \quad (10)$$

with:

$$\Phi_{LT} = 0,5[1 + 0,21(\bar{\lambda}_{LT} - 0,4) + \bar{\lambda}_{LT}^2] \quad (11)$$

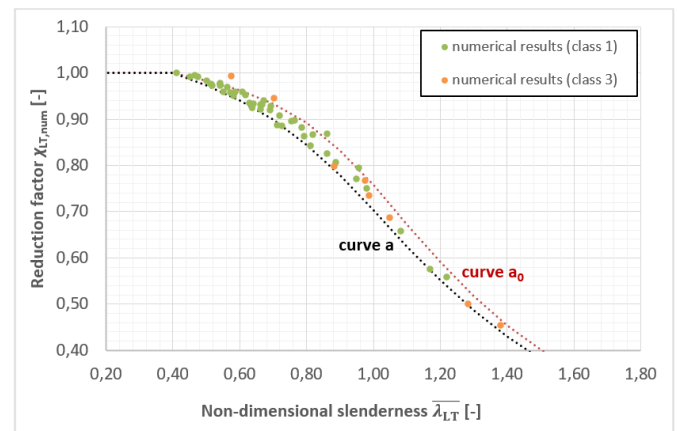
$$\bar{\lambda}_{LT} = \sqrt{\frac{\alpha_{LT} W_{el,y}}{M_{cr}}} \quad (12)$$

Following in essence the limitations of EN 1993-1-1, lateral torsional buckling may be ignored ( $\chi_{LT} = 1,0$ ) when  $\bar{\lambda}_{LT} \leq 0,4$  or  $\frac{M_{u,Ed}}{M_{cr}} \leq 0,16$ .

Additionally, for class 4 cross-sections, the relative slenderness may be determined using the modified slenderness:

$$\bar{\lambda}_p = \sqrt{\chi_{LT}} \frac{c/t}{35,58\epsilon} \quad (13)$$

where  $\chi_{LT}$  is the reduction factor obtained for lateral-torsional buckling.



**Figure 7** Comparison of numerical results with buckling curves for LTB of EN 1993-1-1

Figure 7 illustrates the numerical results compared with the buckling curves a and a<sub>0</sub> for LTB as they are defined by eq. (6.57) of

EN 1993-1-1. The reduction factor for lateral-torsional buckling  $\chi_{LT,num}$  of the numerical samples has been evaluated by the equation  $\chi_{LT,num} = M_{ult,u}/W_{uf,y}$  and the slenderness by using eq.12.

It is seen in the graph that all the results are above curve a, while most of them are below curve a<sub>0</sub>; this validates the proposed buckling curve for LTB of angle sections. It can be also observed that the resistance of some class-3 profiles is above curve a<sub>0</sub>. This could be explained by the fact that these cross-sections are classified as class 3 but with a  $c/\epsilon t$  ratio quite close to the class-2 limit, and so they are treated as class 3 sections while in reality, they reach their plastic resistance. On the contrary, due to the integration of the SEMI-COMP aspects (where a smooth transition between cross-section classes 2 and 3 is allowed), a profile classified as Class 3, but very close to Class 2, should be characterized by a section resistance close to  $M_{pl}$ . To set this clear, one should have in mind that a profile with a  $c/\epsilon t$  approximately equal to 16, could have a ratio  $M_{ult,u}/M_{pl}$  from 0,95 to 1,0. This justifies the small increased value of the numerical results. It should be also noticed that for higher  $c/\epsilon t$  ratios, the results conform to curve a.

### 4.3 Buckling resistance to bending and axial compression

The proposal check requires to fulfil two conditions for buckling around one or the other principal axis for hot rolled angle members subjected to compression and biaxial bending.

– strong axis check:

$$\left( \frac{N_{Ed}}{N_{bu,Rd}} + k_{uu} \frac{M_{u,Ed}}{M_{u,Rd}} \right)^{\xi} + k_{uv} \frac{M_{v,Ed}}{M_{v,Rd}} \leq 1 \quad (14)$$

– weak axis check:

$$\left( \frac{N_{Ed}}{N_{bv,Rd}} + k_{vu} \frac{M_{u,Ed}}{M_{u,Rd}} \right)^{\xi} + k_{vv} \frac{M_{v,Ed}}{M_{v,Rd}} \leq 1 \quad (15)$$

where:

$N_{Ed}$ ,  $M_{u,Ed}$  and  $M_{v,Ed}$  are the design values of the compression force, and the maximum bending moment acting along the member about the u-u axis and v-v axis, respectively.

$N_{bu,Rd}$ ,  $N_{bv,Rd}$ ,  $M_{u,Rd}$  and  $M_{v,Rd}$  are the design resistance of the axial force and of the maximum bending moments acting along the member about the u-u axis and v-v axis, respectively.

$k_{uu}$ ,  $k_{uv}$ ,  $k_{vu}$ ,  $k_{vv}$  are interaction factors according to Table 3.

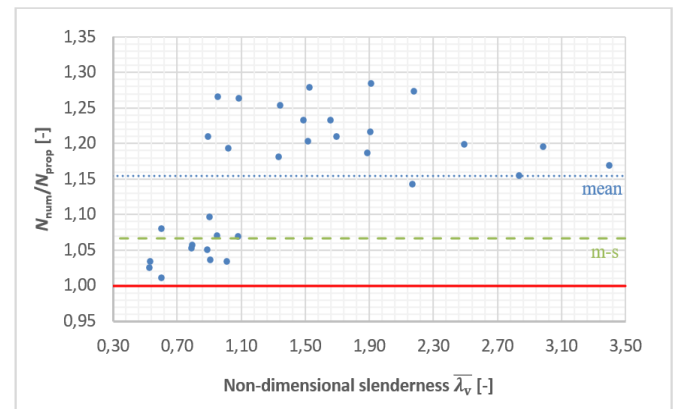
**Table 3** Interaction factors and exponent  $\xi$  for equations (14) and (15)

Interaction factors	$k_{uu} = \frac{C_u}{1 - \frac{N_{Ed}}{N_{cr,u}}}$	$k_{uv} = C_v$
	$k_{vu} = C_u$	$k_{vv} = \frac{C_v}{1 - \frac{N_{Ed}}{N_{cr,v}}}$
Equivalent uniform moment factor $C_i$	$-1 \leq \psi_i = \frac{M_{2i}}{M_{1i}} \leq 1$	$C_i = 0,6 + 0,4\psi_i \geq 0,4$
Exponent $\xi$	$c/t \leq 16\epsilon:$ $16\epsilon < c/t < 26,3\epsilon:$ $c/t > 26,3\epsilon$	$\xi = 2$ $\xi = \left[ 1 + \left( \frac{26,3\epsilon - c/t}{26,3\epsilon - 16\epsilon} \right) \right]$ $\xi = 1$

Obviously, in the vast majority of cases the weak axis check

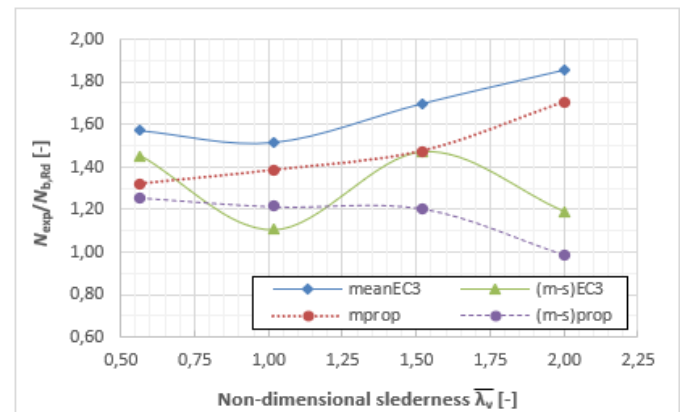
dominates, given the big difference of the angle radii of gyration between the two principal axes. This is true, unless the bracing conditions are such that the member is practically restrained in respect to weak axis buckling, or the load eccentricity that results a bending moment  $M_{u,Ed}$  around the strong axis is very large.

Figure 8 illustrates the ratio between numerical and analytical resistance loads according to the weak axis slenderness  $\bar{\lambda}_v$ . The analytical load is determined as the maximum load that satisfies both eqs. (14) and (15), without safety factors. The test points represent besides the slenderness, different load eccentricities and, therefore, different contributions of the compression or bending terms to the design equations. Accordingly, different analytical-to-numerical ratios are achieved, for the same slenderness. The weak axis check was the critical one and the members buckle towards weak axis. The mean value of the ratio  $N_{num}/N_{anal}$  is equal to 1,15 with a standard deviation of 8,77%. It can be seen that the analytical approach for the combined resistance is validated through the numerical results.



**Figure 8** Ratio between numerical and analytical loads for N+M<sub>u</sub>+M<sub>v</sub>

At the National Technical University of Athens, 33 compression tests were carried out on axially loaded pin-ended columns with or without eccentricity and reported in Ref. [14]. The cross-sections were equal-leg angles L 70x70x7 and the material S275. The profile was classified by EN 1993-1-1 in class 1. The load was introduced through supports that correspond to spherical hinged boundary conditions. The experimental results are compared with the resistance formulae of the current proposal, as well as with the resistance formulae for members as described in prEN 1993-1-1 (index EC3 in the graph), equations (8.88)-(8.89), in combinations with annex C where interaction factors for mono-symmetric sections are given.



**Figure 9** Ratio between experimental and analytical loads for the NTUA tests, mean values

Figure 9 presents the mean value ( $m$ ) of the ratio between the experimental loads and the analytical loads as determined by the above methods, and the mean minus one standard deviation value ( $m-s$ ). It can be seen that the current proposal gives a better prediction for the column capacity compared to the existing version of prEN 1993-1-1 and is always on the safe side.

## 5 Conclusions

Angle profiles belong to the most common structural steel shapes used in construction. As monosymmetric sections with negligible torsion and warping stiffness, they distinguish themselves from other common steel shapes and require specific rules for their design. Current European regulations provide rules of rather limited application, scattered in various codes and often exhibit significant lack of consistency. In the paper, a full set of design rules for angles is provided which may directly lead to the revision of present existing Eurocode provisions. These rules are written in the Eurocode format and will be included in the forthcoming version of prEN 1993-1-1 in Annex F. Their main features are as follows:

- The rules are generic for the referred profiles.
- They are simple to apply and are derived from basic rules of the stability theory.
- They remove inconsistencies of existing specifications.
- They cover all cross-section classes and allow a smooth transition between them, removing any artificial stepwise prediction of resistance.
- They represent the mechanical behavior and failure modes of cross-sections and members.
- They account directly for the presence of applied moments resulting from the connection eccentricities.

All the design rules were validated by extensive numerical analyses and numerous experimental tests. Experimental results were also compared to existing code provisions. It was shown that the proposed method predicts well the member capacity and may be used as an alternative to existing code provisions.

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