Numerical continuation methods for marine ice-sheet systems with various friction laws

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Ice sheets are complex components of the climate system whose understanding is crucial in order to obtain robust predictions, in particular in context of the future sea-level rise. Marine regions, which are the areas in contact with the ocean, are of particular interest because they are non-linear systems. In particular, it has been previously shown that they exhibit turning-point bifurcations and hysteresis (Schoof, Ice sheet grounding line dynamics: Steady states, stability, and hysteresis, in *J. Geophys. Res.*, vol. 112, 2007). Mathematically, marine regions can be formulated as obstacle problems, in which the "obstacle" is the underlying bedrock.

Numerical continuation methods are great tools to study marine ice-sheet systems, as they allow to obtain the solutions associated with a range of parameter values, which naturally leads to bifurcation diagrams. In the glaciology literature, such methods have been used for a 1D geometry and with the so-called Weertman friction law (Mulder et al., Stochastic marine ice sheet variability, in *J. Fluid Mech.*, vol. 843, pp. 748-777, 2018). However, there is an interest in applying this kind of methods to more general configurations, in particular to 2D geometries and to more complex friction laws.

The main challenge for this extension is the presence of non-linear or non-smooth terms in the governing equations, which depends on the mathematical formulation of the contact problem and the friction laws used. In this presentation, we describe several continuation methods which can be applied to our problem, and we illustrate them on several configurations. Specifically, we introduce a novel constraint function that does not rely on the assumption that the solution curve is smooth, as opposed to the classical pseudo arc-length method. This constraint is based on variables that appear in a primal-dual formulation of the obstacle problem. We show that this continuation method is efficient and compatible with several friction laws which depend on both the velocity and the effective pressure between the ice and the bedrock.