

# A pyramidal classification of ST relationship models

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**Abstract.** Although intensive works have been devoted to spatiotemporal modeling during the last two decades, there are only a few models dedicated to relationships between two (or more) moving objects. This paper proposes a classification framework of spatiotemporal relationship models based on the ways spatiotemporal histories are analyzed. First, we discuss model conditions of application. Then we propose a classification of spatiotemporal relationship models based on a pyramidal representation of the quantity of extracted information, i.e. the nature and the complexity of provided information, from spatiotemporal histories. This classification aims helping users to choose adequate models for their purposes. The proposed spatiotemporal histories analysis method could lead to the development of new spatiotemporal relationship models.

**Keywords.** Spatiotemporal relationships, spatiotemporal reasoning, spatiotemporal information, spatiotemporal history, temporal spaces, spatiotemporal clustering.

## Introduction

Although intensive works have been devoted to spatiotemporal databases, spatiotemporal data model, physical storage structures and spatiotemporal representation during the last two decades, there are only a few models which focus on relationships between moving objects. Spatio-temporal relationship models give information about the relationships between two (or more) moving objects in time. First developments were based on the combination of spatial and temporal relationship logics. Recently, new models have been developed on the basis of spatio-temporal shape descriptions. These shapes are obtained from object's movement over time. In this paper, we propose a classification framework of spatiotemporal relationship models. The aim is to help users selecting the most adequate models for their purposes (e.g. epidemiology, crime mapping, robot navigation, collisions analysis...).

Evolution of objects can be rather complex. Objects life and movement imply changes in spatial and temporal dimensions [1]. It is widely admitted that spatiotemporal evolutions are realized with temporal spaces [2]. Temporal spaces are spaces which represent spatial dimensions and time together. Each dimension is mapped to one system axis. 3D moving objects could not be represented in temporal

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spaces because it would imply a representation of a 4D space. However, from a formal point of view, this concept is still valid. Moving objects create spatiotemporal shapes or spatiotemporal histories representing their position over time [3]. In this paper, we concentrate on moving point models. Consequently created spatio-temporal shapes will be points, lines or segments. These representations are commonly called spatiotemporal histories.

Spatiotemporal relationships between two objects can be seen as the direct or indirect description of a relationship between two spatiotemporal histories. Indeed, most of the spatiotemporal relationship models can be described in temporal space frameworks. They focus on different spatiotemporal history properties such as connections, distances, slopes and projections on spatial or temporal axis... We will classify spatiotemporal relationship models depending on these kinds of properties. The proposed methodology is still valid when dealing with moving objects in 3D space.

Complete object's life is often more complex than simple movement. Entire object's existence could not be represented with a continuous spatiotemporal history. Indeed, there is always a period of time where the object did not exist yet and will not exist anymore. Moreover, during the analysis, objects could leave the analyzed zone or not be visible for operator for a while. This implies holes in spatiotemporal histories; a key in a pocket is still existent but is not visible, a soccer player who is not on the field exists but is not present, he could replace someone and then be present just for a certain period, an employee who is in vacation is still existent but not present for the company, etc. In [4], a general representation of object's life evolution has been introduced. It formalizes concepts of *existence / non existence* and *presence / non presence*. This representation is useful to search the co-occurrence zone between objects where others spatiotemporal relationship models can not be used. Indeed, most of existing spatiotemporal relationship models between two objects could only be applied when objects share a common period of both existence and presence. Some of them are even restricted to disjoint objects.

The rest of the paper is structured as follows. First, we give a short reminder of selected spatiotemporal relationship models. Then we describe what we call the co-occurrence zone of objects life evolution. Using a collision example between two objects, each analyzed model is represented in temporal space framework. Finally, a pyramidal classification is proposed and we conclude.

## 1. Overview of spatiotemporal reasoning models

The aim of this section is to briefly recall some important aspects of models studied in this paper; it is not a comprehensive state-of-the-art on spatiotemporal relationship models. We consider being spatiotemporal relationship models, models which describe relationships between two objects over space and time.

Qualitative Trajectory Calculus (QTC) proposed by Van de Weghe [5-7] has been widely introduced to enable comparisons between points at different instants qualitatively. Quantitative values are represented as three qualitative values (-, 0, +). Different types of QTC have been introduced. In its basic form, QTC compare the position of two objects at different time points. The movement of one object (k) with respect to the second object (l) is studied by comparing the distance between l at the current time point t and k at the time immediately before the current time point  $t^-$  with the distance between l at t and k at the time point immediately after the current time point  $t^+$ . The reverse reasoning is done for l, i.e. the movement of l with respect to k is studied by comparing the distance between k at t and l at  $t^-$ , with the distance between k at t and l at  $t^+$ . Some other developments have been done taking into account the speed and acceleration of objects. This leads to a complete but complex calculus describing in detail the movement of two disjoint objects. Be aware that QTC can only be applied when two objects are both existent and present and when objects are disjoint during all the analyzed period. In this paper, we consider only the basic form of QTC.

The double cross calculus (DCC) introduced by Freksa [8, 9] describes the movement of two objects with respect to each other between two instants of time. Vectors represent objects movement. A vector is sketched between the two positions of the first object at the two different time points. An orthogonal line is drawn through this vector from starting point to ending point. This leads to a division of space in 15 zones (six areas, seven locations on lines and two points). Each zone corresponds to a spatio-temporal relationship between the reference system created and the second object. The reference frame of this representation is fixed on one of two objects. This model is able to describe cognitive considerations such as 'moving on the left', 'moving towards', 'moving along'... Firstly developed as a spatial relationship models, it describes moving points at different times, we could consequently consider it as a spatiotemporal relationship model.

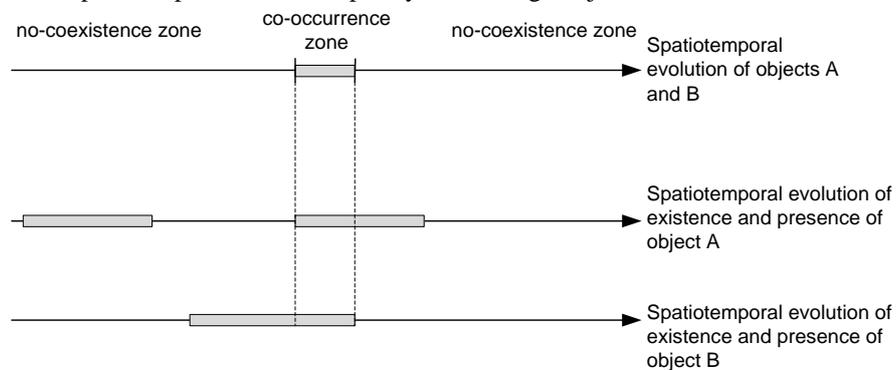
The relative representation of trajectories in geographical space defined by Noyon and Claramunt [10, 11] propose to represent trajectories the way they are perceived by observer and provide complementary view of absolute reference system commonly used. This model is valid for points, lines and regions. In the framework of this paper, we are focusing only on point features. The relationships are based on two basic primitives which are the relative position and the relative velocity. These two properties are combined and provide complete reasoning calculus.

Last model, the spatio-temporal generalized model (STGM), has been proposed by Hallot and Billen [4, 14]. This spatiotemporal topological relationship model aims to extract information from spatiotemporal histories. It uses topological relationships applied to spatiotemporal histories. They propose a set of 25 spatiotemporal relationships for moving points. This model allows a quick identifying of the objects which can be analyzed deeper with other models such as QTC, Double Cross...

Although projective relationships defined by Billen [12-13] have not been yet presented as spatiotemporal relationship models, we wish to point out here some interesting properties for our analysis. Projective relationships are based on projective geometry. This geometry can be seen as an extension of topological geometry. Projective relationships are able to describe advanced connection between objects refining boundaries connection between objects. Such relationships could be substituted by the topological relationships in the STGM.

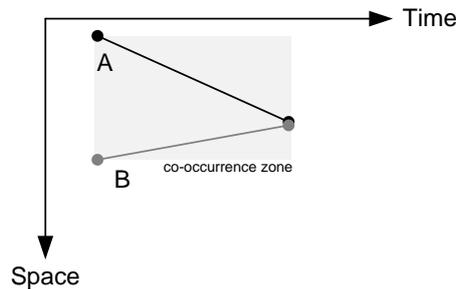
## 2. Spatiotemporal relationship models in temporal spaces

Objects do not have an infinite existence. There is always an interval of time where objects did not exist before their utility/existence/life and another period after that. Considering huge data collections over long periods of time would imply to deal sometimes (often) with non co-existent objects. Some of them will not exist anymore during existence of other ones and vice versa. Most of spatiotemporal relationship models can only deal with coexistent objects. Thus, spatiotemporal relationship cannot be defined between them. Considering all the complexity of spatiotemporal information, it is first necessary to select a co-occurrence zone of both analyzed objects. In a previous work [4], we proposed a representation of spatiotemporal information allowing considering relationships between non contemporary or non both visible spatially objects. In this representation, an object which is not existent yet or not existent anymore is considered as a “*non existing object*”. If during the analysis, an object leave the analyzed zone or is not visible but still existent, it will be considered as “*non present object*”. Based on these two properties and general topology, 13 general relationships between objects are defined. This representation helps selecting quickly co-occurrence periods when objects both exist and are present. This co-occurrence zone can be symbolized for two objects as in figure 1. Be aware that this zone combines temporal and spatial considerations; it is not limited to a temporal selection. Spatiotemporal evolution line is divided in three parts. First one is the no-coexistence zone; middle is the co-occurrence zone which means that two objects are both present and existent; and finally again a no-coexistence zone. In the no-coexistence zone objects are either not contemporary or not spatially present. It is worth to mention that there can be more than one co-occurrence zone. Indeed, if one object disappears for a while during the analysis, most of spatiotemporal relationship models could not describe spatiotemporal relationships anymore though objects are still existent.



**Figure 1.** Representation of co-occurrence zone of spatiotemporal evolution of object A and object B.

First step of analysis is then to select co-occurrence zones in which all models can describe spatiotemporal relationships. We wish to point out here that most of the time co-occurrence zone is very limited in regards of entire object evolution. The proposed classification of spatiotemporal relationship models is based on the quantity of information extracted from the co-occurrence zone of spatiotemporal object evolution and on the implicit properties used to describe spatiotemporal histories. The quantity of extracted spatiotemporal information from spatiotemporal history means the complexity of possible analysis which can be performed using a particular model, e. g. a model only focusing on collision detection extracts less information than a model dealing with direction, speed, acceleration... To have a clear representation of spatiotemporal histories description, we use an example of two objects moving to each other as represented in figure 2. The temporal space representation shows that object A is moving faster than object B. The two objects meet together at the end of their co-occurrence zone. The representation is limited to a 2D temporal space. Indeed, the relationships between the points are the same in 1D space or more. Moreover, most of spatiotemporal relationship models only use the relative distance between objects. This kind of distance is usefully represented with a 1D space.



**Figure 2.** 2D Temporal space representation of spatiotemporal histories of object A and B.

The geometric shape of spatiotemporal histories can be analyzed by different ways. This is the basis of our proposed classification. Models are not presented in the paper according to particular order. They will be classified further with a pyramidal representation of the information quantity they provide.

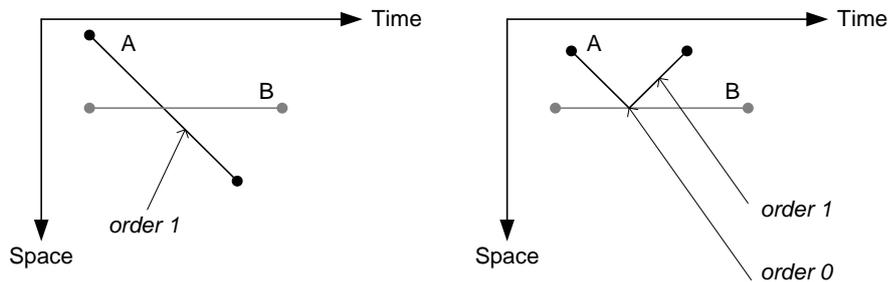
First model is the spatiotemporal generalized model (STGM). It only focuses on the topological connection between spatiotemporal histories. The analysis gives a topological relationship between two lines (see fig. 3). Although this model do not give complete information about movement of object A and B. It gives principal information about the connection of two objects. Information about collision is known. This model does not change if dealing with moving points in a 3D environment. Applications such as epidemiology, information transmission can be realized with this model.



Semantic interpretation :  
*Two objects meet at their born or at their death.*

**Figure 3.** Topological intersection matrix pattern, topological relationships of spatiotemporal histories of A and B and semantic interpretation of the relationships [4].

Projective space analysis gives more detailed information about connections between spatiotemporal histories. It can differentiate the two situations presented in figure 4. In a topological point of view, there is no difference between the two spatiotemporal situations presented. However, projective relationships differentiate the spatio-temporal history of A in the right part of figure 4. The orders which compose the geometrical shape in a projective point of view are different. The angle is of order 0 although right segment parts are of order 1 (see [13] for further explanation about object's point order).

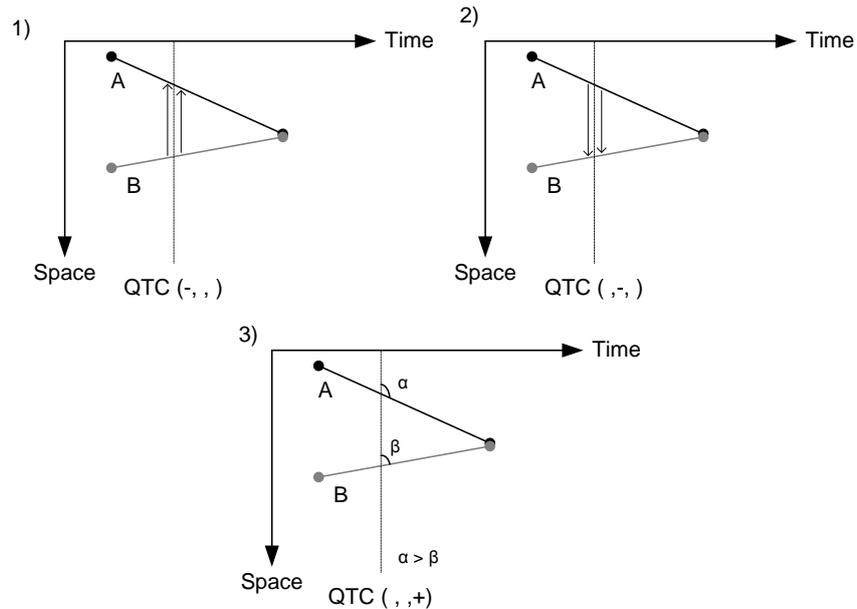


**Figure 4.** Two spatiotemporal situations in temporal space framework. On the left, object A meets B without trajectory change. On the right, A meets B and changes of direction. Arrows shows the order of represented shapes in projective geometry (see [13] for more details).

This model's extension offers more information than with topological relationships only i.e. direction changes and particular connection can be retrieved from spatiotemporal analysis. This extension of STGM has not been formalized yet.

Although qualitative trajectory calculus (QTC) model has not been defined in terms of spatiotemporal history analysis, its basic form can be easily represented in temporal space framework. For two points moving in 2D or 3D space, the representation uses a 2D temporal space to symbolize the minimum distance between points over time. Figure 5 shows the two qualitative values used in QTC and their interpretation in temporal spaces. The distance evaluation between time point immediately before and immediately after QTC time is easy. Part 1 and 2 shows these two distances' comparison with arrows. If first distance is higher than the second, qualitative value is -, if they are equal then the qualitative value is 0 and it is + if the second is higher than the first. QTC may also provide information about the relative

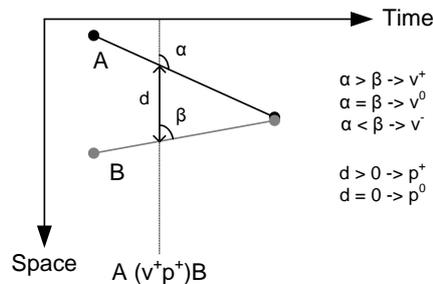
speeds of moving objects. This could be represented in temporal spaces as well. The last QTC value (part 3 in figure 5) compares the slopes  $\alpha$  and  $\beta$  respectively representing the speed of object A and B. The qualitative value is + if  $\alpha > \beta$ , 0 if they are equal and - if  $\alpha < \beta$ .



**Figure 5.** Three temporal spaces representing the three qualitative values used in QTC. Part 1 shows the distance between A and B. Part 2 shows the distance between B and A. Part 3 compares the slope (velocity) between A and B.

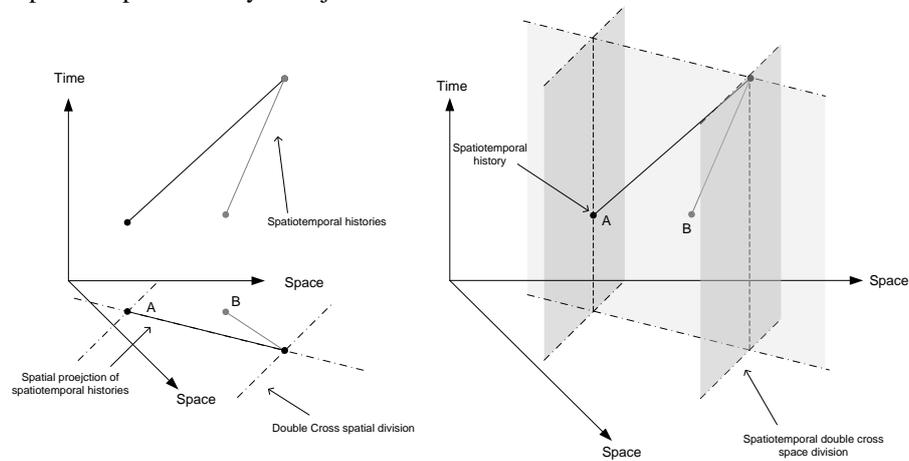
QTC analysis can be summarized as a qualitative distance and slope analysis of spatiotemporal histories. This analysis provides more information than topological or projective ones on spatiotemporal histories. However, QTC analysis has to be repeated many times to give complete information about spatiotemporal situation while others give relationship about complete situation.

Relative representation of trajectories provides similar analysis of spatiotemporal histories than QTC. Although this model is based on a relative reference system view, it can be represented through temporal space. Indeed, primitives used to define this model are the relative position and the relative speed. These information are available on temporal space representation. Figure 6 shows the relative representation of trajectories of the collision example.



**Figure 6.** Temporal space representation of relative representation of trajectories. Analysis is limited to moving points.

Double Cross Calculus (DCC) could not be represented in 2D temporal space. Indeed it is not based on the distance between analyzed objects. 3D temporal space is necessary to represent double cross calculus. This one keeps all the 2D spatial information. In terms of spatiotemporal histories, DCC analyzes the projection of spatiotemporal histories on spatial plane and divides space with projection into several zones. Relationships give information about object membership to spatial division. Figure 7 (left) shows spatial projection of previous collision example and double cross spatial clustering. The right part gives an interpretation of clustering of spatiotemporal space with double cross (which becomes double plane crosses). Created planes are based on the spatiotemporal history of object A.



**Figure 7.** Temporal space representation of Double Cross Calculus. On the left, classic spatial projection with spatial division. On the right, temporal space interpretation of spatiotemporal space division.

In terms of spatiotemporal history analysis, the Double Cross Calculus can be seen as an analysis of spatiotemporal space division. It gives information about the memberships of one object to defined zones. It would not be possible to represent this division for objects moving in a 3D space, although the method continues to be valid.

### 3. Pyramidal classification of spatiotemporal relationship models

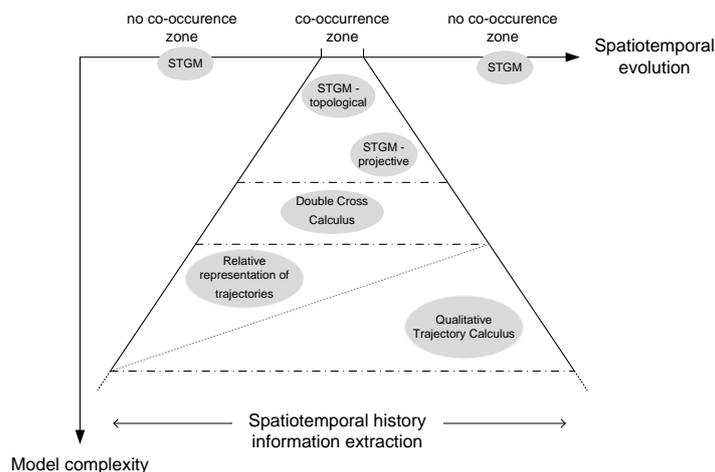
Previous analysis can be summarized through a pyramidal classification (see figure 8). The classification is based on the quantity of extracted information from spatiotemporal history analysis. The top of the pyramid is limited to co-occurrence zone. As explained before, presented models are limited to space and time zone in which analyzed objects are both existent, present and, for some models, disjoint. Models can be organized following their analysis of spatiotemporal histories.

First ones, Topological and Projective models focus on geometrical connection between spatiotemporal histories. Proposed spatiotemporal relationships describe entire object spatiotemporal configurations. They are easy to use and are valid when dealing with objects in a 3D environment. However, they do not give information about directions, velocity... This is why they are on the top of the pyramid. They can deal

with applications requiring only connections and meetings such as crime mapping, epidemiology...

Second group is the double cross calculus which works with division of spatiotemporal space. It qualifies the membership of one object to several zones. This model offers information about directions, orientations. Deeper analysis can be realized to derive information about velocity. This models' group deals with collisions and directions. They can be used for basic robot navigation, movement analysis, movement pattern recognition...

The last group of models is placed on the same pyramid stage because they are both based on qualitative distance and slope analysis. They give information about velocity, distance between objects. In this paper, QTC have been analyzed in its basic form. Some of their extensions combine kinds of double cross calculus and acceleration considerations. Relative representation of trajectories can offer complementary information from the absolute reference representation. These models offer more complex information for robot navigation, air-traffic control...



**Figure 8.** Pyramidal classification of spatiotemporal relationship models based on spatiotemporal history information extraction.

#### 4. Conclusion

Spatiotemporal relationship models provide information about spatial and temporal relationships between two objects. Most of them are qualitatively based. Each spatiotemporal relationship model provides diverse information about spatiotemporal configurations. Our first analysis shows that most of the spatiotemporal relationship models can only be applied during a small subset of object life evolution. Indeed, most of spatiotemporal relationship models are available when objects are both existent, present, and even sometimes disjoint. During an object's life there are several periods of time when objects are neither contemporary nor spatially visible. More general models exist and can help to select specific spatial and temporal zones in which more detailed spatiotemporal relationship models can be used.

A classification method is applied to the following selected models: the qualitative trajectory calculus, the relative representation of trajectories, the double cross calculus, topological and projective spatiotemporal relationship models. Although all these models are not entirely defined in terms of spatiotemporal relationships, they all provide information about relationships between two objects in regards of space and time. Models are classified on the basis of the kind of geometric analysis that could be applied to spatiotemporal histories in a temporal space. Topological and projective relationship models focus on spatiotemporal histories connections, double cross calculus highlights the division of spatiotemporal space and the QTC and relative representation of trajectories deals with the qualitative distance of temporal slices and the qualitative slopes of spatiotemporal histories. Based on this analysis, models are classified in a pyramidal representation. This pyramidal representation can be useful to help users to select the most appropriate models considering their needs. If they have to concentrate on collision, meeting between objects without taking into account the speed and direction, models such as the topological or projective analysis are sufficient. For example, in epidemiology, getting information about meeting between contagious and healthy person might be enough. Models such as QTC offer more information; it supports deeper analysis taking into account speed, direction, acceleration. Applications such as robot navigation, accident re-forming and air-traffic control can be performed with the relative representation of trajectories model.

The presented classification methodology allows classification, comparison and selection of spatiotemporal relationship models. We also believe that it could be a design framework for new prospective spatiotemporal relationship models.

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