Hybridisation of Discontinuous Galerkin Methods for Shock Capturing in Scale Resolving Simulations

Amaury Bilocq[1], Nayan Levaux[1], Vincent E. Terrapon[1] and Koen Hillewaert[1]
[1]A&M department, University of Liege, Belgium
E-mail: amaury.bilocq@uliege.be

High-order simulation methods like Discontinuous Galerkin (DG) have proven suitability for Direct Numerical Simulation (DNS) and (implicit) Large Eddy Simulations (LES) of subsonic flows. In supersonic conditions, shock waves may develop. The discontinuity over the shock cannot be captured by polynomial interpolation and, therefore, both convergence and stability of the simulation deteriorate as Gibbs oscillations develop. In the extreme case, these oscillations lead to unphysical solutions and the failure of the computation. Shock capturing methods usually add artificial viscosity to smooth the shock such that it can be safely represented. However, this reduces accuracy and negatively impacts the turbulent kinetic energy budget. It is therefore desirable to reduce its action to a minimum.

The instability of high-order methods, caused by under-interpolation and integration, can be mitigated by leveraging a discrete equivalent of the entropy and its evolution equation, which introduces a discrete bound for the solution. Most of these entropy-consistent schemes are based on the use of entropy variables and the "summation-by-parts" (SBP) theorem, leading to the analogue to the integration-by-parts theorem at the discrete level. The vast majority rely on nodes that coincide with the Gauss-Legendre-Lobatto (GLL) quadrature points (Gassner, J. Comput. Phys. 327, 2016). The presence of such nodes on the boundary greatly helps constructing SBP operators between two elements. However, the GLL quadrature has lower accuracy leading to the build-up of error. More recently, entropy stable schemes based on Gauss quadrature nodes, without points on the boundary, have been developed. While these methods improve greatly the accuracy of the solution, such SBP operators are numerically very costly since they introduce an "all-to-all" flux coupling between all degrees of freedom in the element (Chan, SIAM J. Sci. Comput. 41, 2019).

This work presents the implementation of a hybrid DG solver for shock capturing. To alleviate the computational cost associated with the entropy stable approach based on Gauss quadrature nodes, the entropy stable is only activated in cells where shock stabilization is necessary or where the turbulence is under-resolved. Everywhere else, a standard DG formulation is applied. This hybrid approach is compared to the artificial viscosity method and full entropy stable scheme on several test cases, such as the Shu-Osher test case and the inviscid strong vortex-shock wave interaction. The method shows the same improvement as the full entropy stable scheme over the artificial viscosity approach, but at a much lower computational cost.