Reduced Order Modeling of Cyclically Symmetric Bladed Disks with Geometric and Contact Nonlinearities

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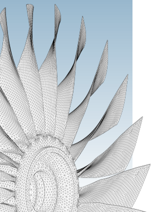
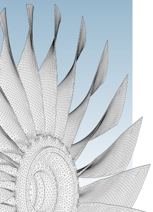
POLYTECHNIQUE MONTRÉAL
UNIVERSITÉ D'INGÉNIERIE

LIÈGE université

Outline



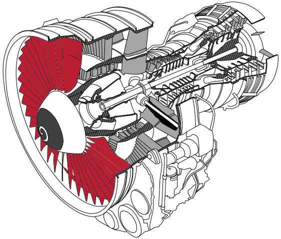


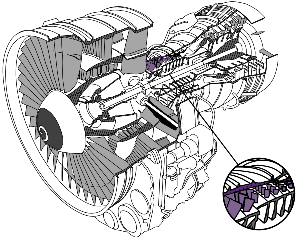
Environmental constraints

By 2050...

- 75% reduction in CO_2
- 90% reduction in NOx
- 65% reduction of noise

Consequences on bladed disks design





- Designing lighter and more flexible blades
- Geometric nonlinearities

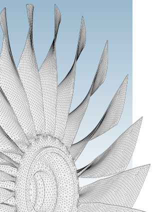
- Reducing clearances between the rotating blades and the casing
- Contact nonlinearities

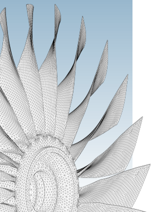
Bladed disks dynamics fundamentally nonlinear

Numerical modeling

Full order model

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} + \mathbf{g}_{\mathrm{nl}}(\mathbf{u}) = \mathbf{f}_e(t) + \mathbf{f}_c(\mathbf{u}, \dot{\mathbf{u}})$$





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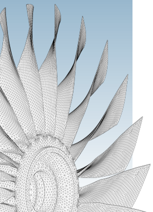
$$\mathbf{u} = \mathbf{\Phi}\mathbf{q}$$

Reduced order model

$$\tilde{\mathbf{M}}\ddot{\mathbf{q}} + \tilde{\mathbf{C}}\dot{\mathbf{q}} + \tilde{\mathbf{K}}\mathbf{q} + \tilde{\mathbf{g}}_{\text{nl}}(\mathbf{q}) = \tilde{\mathbf{f}}_e(t) + \tilde{\mathbf{f}}_c(\mathbf{q}, \dot{\mathbf{q}})$$

- Projection basis $\mathbf{\Phi}$?
- Reduced nonlinear internal forces $\tilde{\mathbf{g}}_{\text{nl}}$?
- Treatment of contact in the reduced space $\tilde{\mathbf{f}}_c(\mathbf{q}, \dot{\mathbf{q}})$?

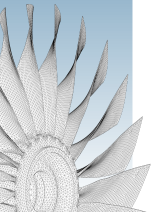


Objectives

Previous work[1]

- Development of a methodology to study the contact interactions of a single rotating blade with geometric nonlinearities
- Validation on an industrial compressor blade model

[1] E. Delhez et al. *Journal of Sound and Vibration* (2021). doi: 10.1016/j.jsv.2021.116037.



Objectives

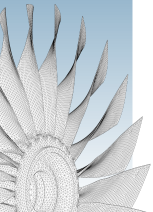
Previous work[1]

- Development of a methodology to study the contact interactions of a single rotating blade with geometric nonlinearities
- Validation on an industrial compressor blade model

This presentation

- In-depth contact analyses to characterize the influence of geometric nonlinearities
- Generalization of the methodology to full bladed disks

[1] E. Delhez et al. *Journal of Sound and Vibration* (2021). doi: 10.1016/j.jsv.2021.116037.



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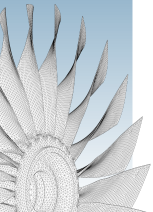

Methodology

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$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} + \mathbf{g}_{\mathrm{nl}}(\mathbf{u}) = \mathbf{f}_e(t) + \mathbf{f}_c(\mathbf{u}, \dot{\mathbf{u}})$$

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Reduced order model

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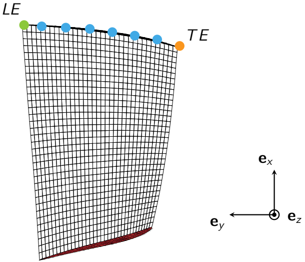
- **Projection basis**: Craig-Bampton modes and a selection of their modal derivatives[2]
- **Reduced nonlinear internal forces**: evaluation with the stiffness evaluation procedure (STEP)[3]
- **Contact**: explicit central finite difference time integration scheme with Lagrange multipliers[4]

[2] L. Wu et al. *Proceedings of the 27th International Conference on Noise and Vibration Engineering.* Leuven (Belgium), 2016.
[3] A. Muravyov et al. *Computers & Structures* (2003). doi: 10.1016/s0045-7949(03)00145-7.
[4] N. J. Carpenter et al. *International Journal for Numerical Methods in Engineering* (1991). doi: 10.1002/nme.1620320107.

Test case

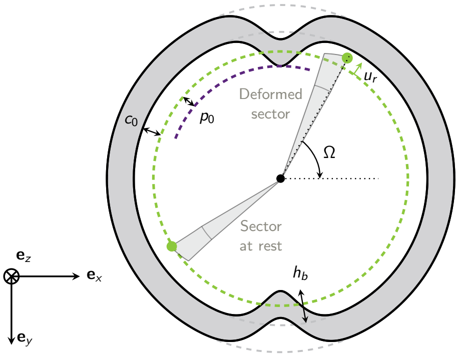
- NASA rotor 37 blade (transonic compressor blade) clamped at its root[5]
- Open and industrial test case
- 8 boundary nodes distributed between *LE* and *TE* (contact interface)
- Reduction basis: 189 modes = 24 static modes + 15 fixed interface linear normal modes + 150 modal derivatives



[5] L. Reid et al. Technical report. NASA TP 1337, 1978. doi: 10.1049/iet-gtd.2015.0403.

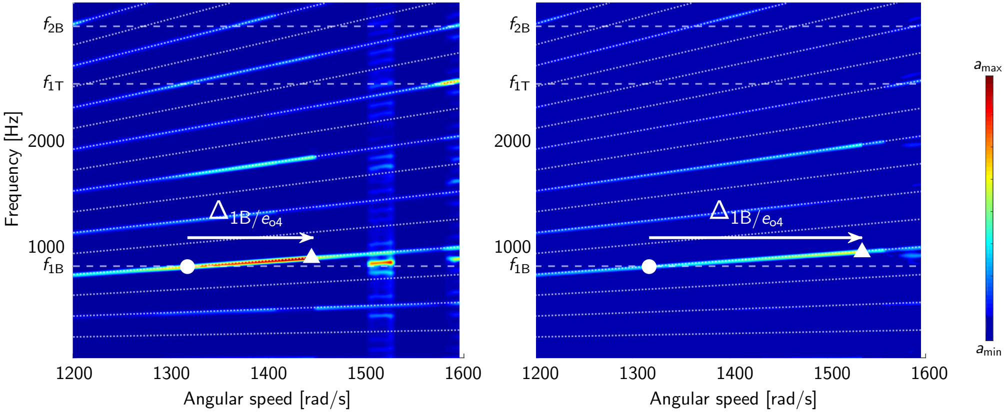
Contact scenario

- Blade rotating at a constant speed Ω around $\mathbf{e}_z$
- Direct contact with rigid casing – sliding friction
- Contact initiated by deformation of the casing with two lobes
- No aerodynamic loading, no gyroscopic or centrifugal effects, no thermal effects



Interaction maps of the radial displacement at *LE*

Interaction between the first bending mode (1B) and the fourth engine order (e_{o4})

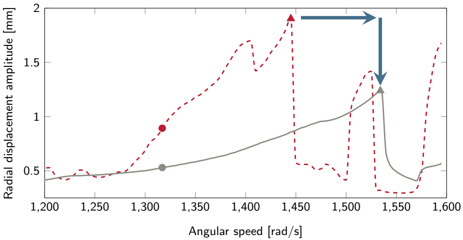


(a) Without geometric nonlinearities.

(b) With geometric nonlinearities.

▸ **Interaction maps, predicted linear (●) and nonlinear (▲) resonances.**

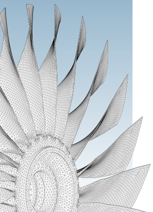
Nonlinear frequency response curve

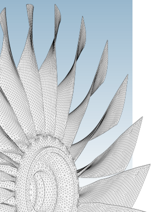


▶ **NFRC without (– –) and with (—) geometric nonlinearities, predicted linear (●) and nonlinear (▲) resonances.**

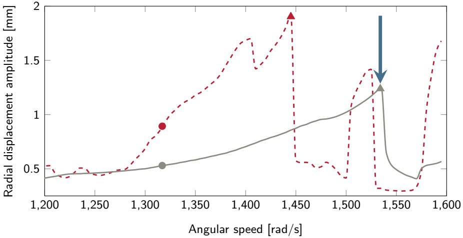
Observations

- Contact stiffening
- Amplitude jumps
- Influence of geometric nonlinearities
  - Smoother interactions
  - Additional contact stiffening





Nonlinear frequency response curve



- **NFRC without (– –) and with (—) geometric nonlinearities, predicted linear (●) and nonlinear (▲) resonances.**

Observations

- Contact stiffening
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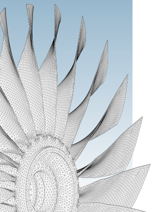
  - **Smoother interactions?**
  - Additional contact stiffening



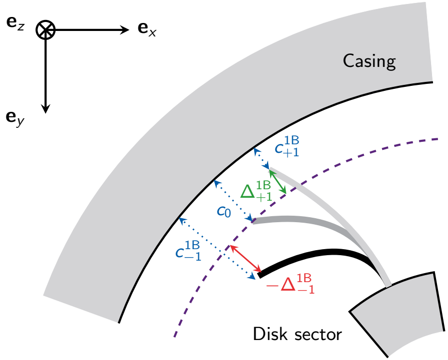
Clearance consumption

Definition

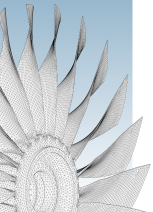
- Evolution of the clearance between the blade and the casing when the blade vibrates along 1 mode

$$\Delta(\delta) = c_0 - c(\delta)$$

- Possible key parameter for the design of blades subjected to contact interactions[6]



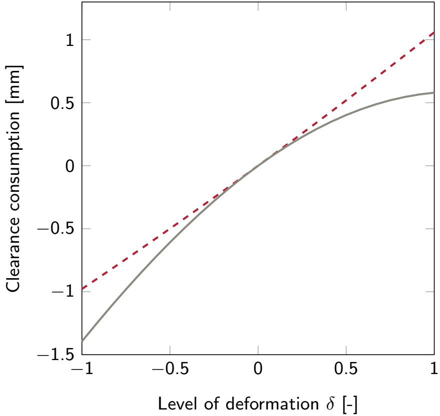
[6] A. Batailly et al. *Proceedings of the ASME Turbo Expo* (2016). doi: 10.1115/gt2016-56721.



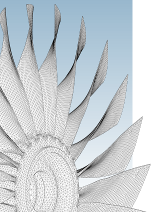
Clearance consumption

Observations

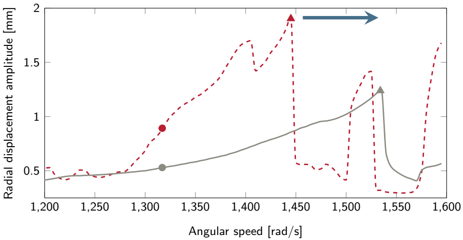
- Reduced clearance consumption with geometric nonlinearities
- Justify that the blade with geometric nonlinearities features lower vibration response to contact
- Linear model valid for $\delta \in [-0.25, 0.2]$



▶ **Clearance consumption at *LE* without (– –) and with (——) geometric nonlinearities.**



Nonlinear frequency response curve



- **NFRC without (– –) and with (——) geometric nonlinearities, predicted linear (●) and nonlinear (▲) resonances.**

Observations

- Contact stiffening
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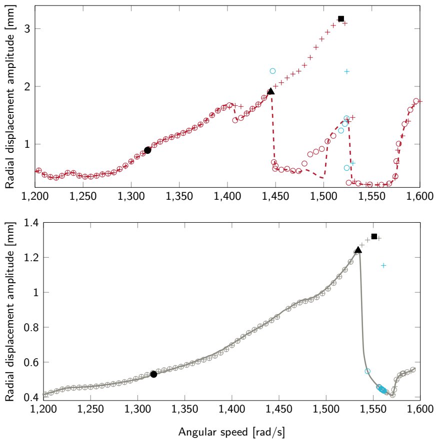
- Influence of geometric nonlinearities
  - Smoother interactions
  - **Additional contact stiffening?**

Nonlinear frequency response curve with continuation



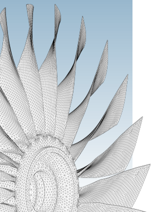
- **NFRC without (– –/——) and with continuation (+/ o), without geometric nonlinearities (above) and with geometric nonlinearities (below).**

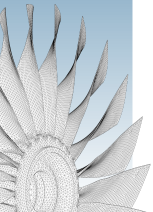
Numerical procedure

- NFRC built with a sequential continuation procedure
- Upward (+) and downward (o) angular speed sweeps

Observations

- Without continuation, nonlinear resonance (■) not correctly captured
- Contact stiffening similar with and without geometric nonlinearities





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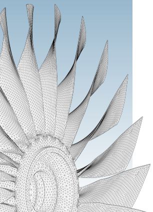

Generalization of the methodology with CMS techniques

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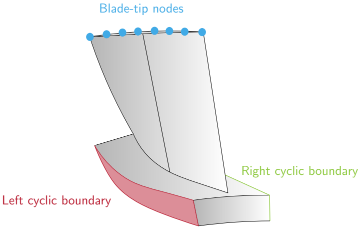
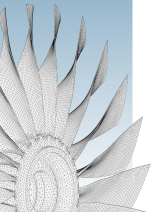


- **Projection basis**: for each sector, Craig-Bampton modes and a selection of their modal derivatives + second reduction of the cyclic boundary



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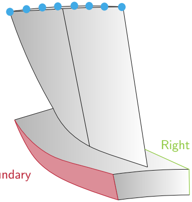
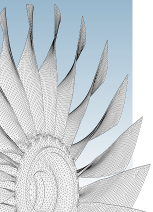


- **Projection basis**: for each sector, Craig-Bampton modes and a selection of their modal derivatives + second reduction of the cyclic boundary
- **Reduced nonlinear internal forces**: STEP, assumption of linear coupling between the sectors



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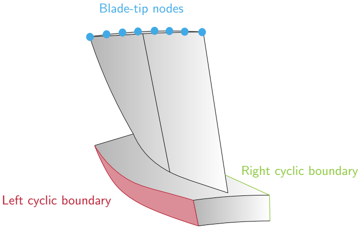

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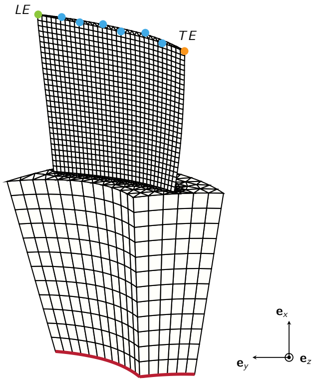
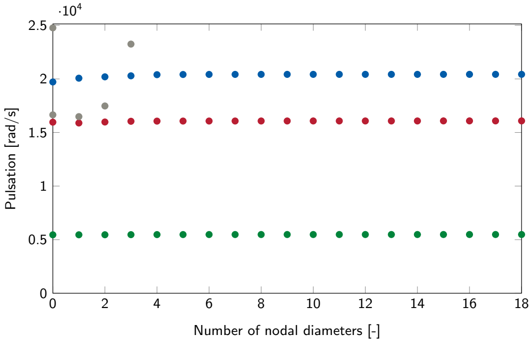
- **Contact**: explicit central finite difference time integration scheme with Lagrange multipliers

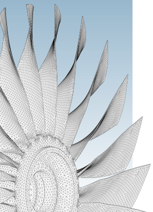
Test case



- NASA rotor 37 bladed disk with 36 sectors
- 133,605 degrees-of-freedom per sector
- Sectors clamped at disk lower surface

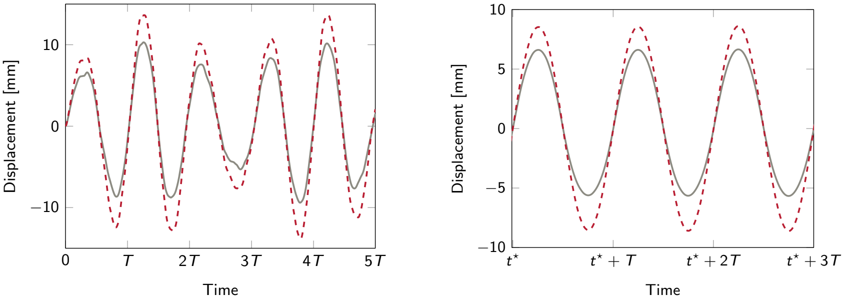


▶ **SAFE diagram.**



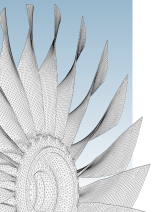
Verification without contact

- Reduction basis: per sector, 3 static modes (LE) + 10 fixed interface linear normal modes + 10 modal derivatives + 3 modes for the second projection (total: 972 modes)
- Blade excited by a harmonic excitation of amplitude $A = 400$ N and pulsation $\omega = 4{,}500$ rad/s in the $\mathbf{e}_z$ direction



(a) Transient regime.
(b) Steady-state regime.

- **Reference linear (– –) and nonlinear (——) solutions, reduced order model nonlinear solution (– –).**



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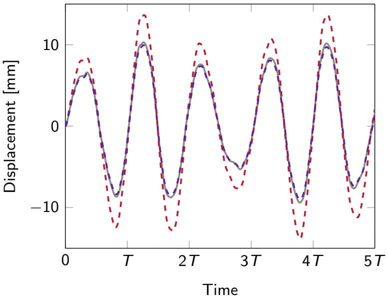
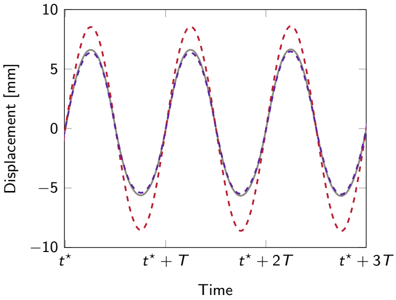
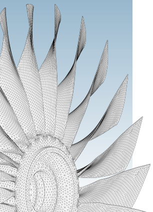


(a) Transient regime.

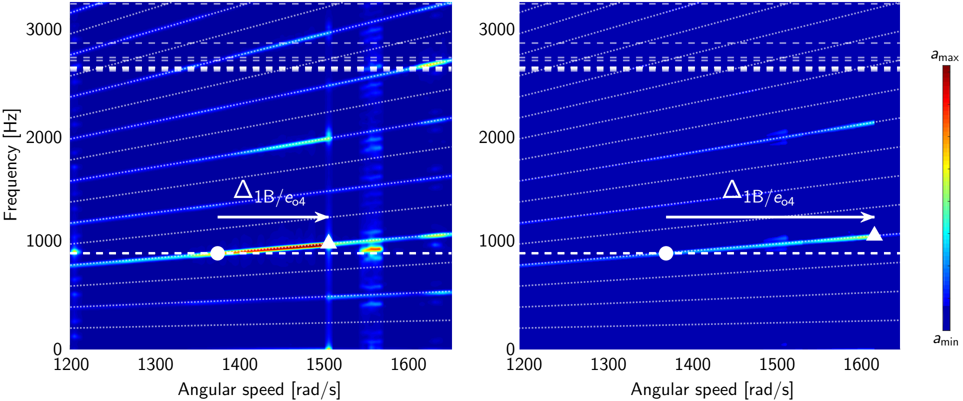


(b) Steady-state regime.

- **Reference linear (– –) and nonlinear (—) solutions, reduced order model nonlinear solution (– –).**



Interaction maps of the radial displacement at *LE*

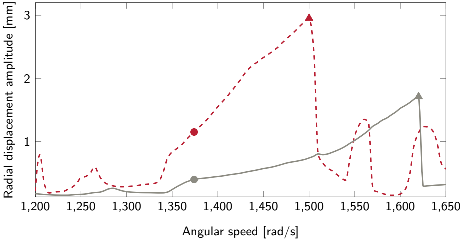


(a) Without geometric nonlinearities.

(b) With geometric nonlinearities.

- **Interaction maps, predicted linear (●) and nonlinear (▲) resonances.**

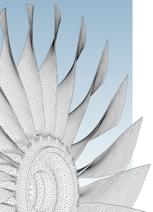
Nonlinear frequency response curve



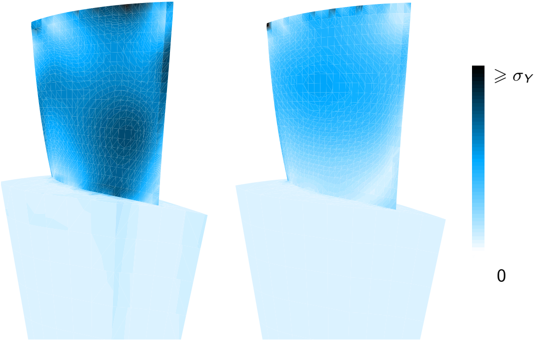
▶ **NFRC without (– –) and with (—) geometric nonlinearities, predicted linear (●) and nonlinear (▲) resonances.**

Observations

- Contact stiffening
- Amplitude jumps
- Influence of geometric nonlinearities
  - Smoother interactions (see clearance consumption analysis)
  - 'Additional contact stiffening' (continuation procedure required for accurate quantification)



Von Mises stress fields



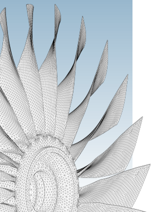
(a) Without geometric nonlinearities. (b) With geometric nonlinearities.

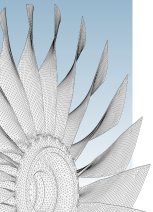
- **Von Mises stress fields at the resonance.**

Comparison

- Zones of maximal stresses not at the same locations
- Non-negligible stresses in the disk for the case without geometric nonlinearities
- Smaller stresses predicted with geometric nonlinearities (in line with predicted displacements)

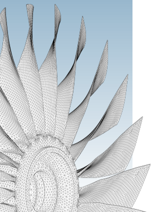
Outline





Conclusion

- Methodology to study the rubbing interactions of full bladed disks with geometric nonlinearities
  - Projection basis: Craig-Bampton reduction basis and selection of their modal derivatives + second reduction of cyclic boundary
  - Geometric nonlinearities: STEP
  - Contact nonlinearities: Lagrange multipliers
- Reduced order models are an efficient alternative to full order models
- Influence of geometric nonlinearities not negligible
- Parametric reduced order models can be built to account for gyroscopic and centrifugal effects
- Methodology also compatible with the introduction of mistuning



Conclusion

- Methodology to study the rubbing interactions of full bladed disks with geometric nonlinearities
  - Projection basis: Craig-Bampton reduction basis and selection of their modal derivatives + second reduction of cyclic boundary
  - Geometric nonlinearities: STEP
  - Contact nonlinearities: Lagrange multipliers
- Reduced order models are an efficient alternative to full order models
- Influence of geometric nonlinearities not negligible
- Parametric reduced order models can be built to account for gyroscopic and centrifugal effects
- Methodology also compatible with the introduction of mistuning

Thank you for your attention