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Designing negative derivative feedback controller based on maximum damping and H_2 method

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Abstract

In this paper, a straightforward procedure is presented for optimal design of negative derivative feedback (NDF) controller with maximum damping and H_2 optimization method. NDF is a controller, which works as a band-pass filter, cutting off the control action far from the natural frequencies associated with the controlled modes and reducing spillover effect. Since it is a bandpass filter, it can effectively control the lower or higher frequency disturbances. It is also implementable on vibration mitigation applications with high performance. For this end, a simple one degree of freedom system is considered and afterward, the controller parameters are extracted dependent on closed loop damping. The H_2 method is used to calculate the optimal value of closed loop damping. The effect of changing the controller parameters on the system response are evaluated and discussed in detail. Also, the control effort for various closed loop damping has been calculated and compared with performance index of controller. A detailed comparison between performance and control effort are also presented. The results show high impact of NDF controller on vibration mitigation and its applicability to employ on various systems.

Keywords: negative derivative feedback (NDF), maximum damping, H_2 optimization, performance index, control effort, bandpass filter, vibration mitigation

(Some figures may appear in colour only in the online journal)

1. Introduction

In recent years, due to development of light structures with slightly damped characteristics, controlling the vibration of structures has become more and more substantial [1–3]. The most important issue for this structure is high amount of vibrations amplitudes near their natural frequencies, mainly due to the low damping ratio. A high level of vibration can easily cause a degradation of system performance and consequently decrease the structure health and lifetime.

In order to prevent this issue, vibration control approaches are developed to avoid high vibration levels actively and passively.

In past few years, active control and passive control methods have been developed widely. The active control has more impact and power in vibration reduction amount. Therefore, in any case, which is possible to utilize active control methods, researchers would have inclination to use it. Therefore, various types of active control methods have been developed and each method have its own benefits and drawbacks. The drawbacks of each controller can easily make it impossible to apply on many applications. Therefore, scientists are always looking for a new control logic with higher performances and lower restrictions of utilization on industrial applications.

Previously, many effective active techniques are introduced in order to damp structural vibrations. Some of popular ones are direct velocity feedback [4], integral force feedback [5], and positive position feedback (PPF) [6]. These methods were

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used extensively for increasing the damping of structural resonance. Among the mentioned methods, the PPF, as a second order low-pass filter, is one of the effective control techniques to be implemented on the plant having no high frequency roll-off. However, since it is a low-pass filter, as a consequence, the PPF effectively reduces the spillover on the higher modes, but it introduces a significant static error and worsen the system response at lower frequencies. This drawbacks of PPF makes it inappropriate for targeting high frequency modes. Because the static error will be applied to the system from zero frequency to the targeted mode at high frequency which is not desirable. Also, PPF is only effective on one mode and does not have any effects on a band of frequency.

To overcome the imperfections of these controller techniques, Cazzulani *et al* [7] proposed a new resonant control logic called as negative derivative feedback (NDF) control for the first time. The NDF is more robust than other logics with respect to the spillover effects and by acting as a pass-band filter, it cuts off both the higher and lower uncontrolled modes' contribution. NDF showed better performance with respect to all the other techniques, both in terms of achieved damping and robustness to low- and high-frequency problems. After that, Cola *et al* [8] proposed an algorithm for designing NDF by output feedback control optimal solution. The procedure proposed was complicated and hard to implement. Also, the optimal choice for designing NDF was not presented.

Syed [9] compared PPF and NDF performance on vibration control of a flexible arm featuring piezoelectric actuator. Based on extracted results, it was shown that NDF controller is more effective than PPF controller in terms of performance measures. Ripamonti and Cola [10], developed an adaptive NDF modal control algorithm-based output feedback control method and applied it on a carbon fiber plate. The proposed algorithm is very interesting, but it is very hard to apply on any system. Debattisti *et al* [11], evaluated distributed wireless-based control strategy through selective NDF algorithm. Also, one important factor in smart structures is evaluating sensor/actutaor distributions and segmentation on the structures which has been studied by many researchers as well [13–15].

In overall, the NDF controller is a new method which has not been studied deeply and also an optimal way for determining the controller parameters has not been presented. Therefore, in this study a careful consideration is taken in order to tune the parameters of the controller to get an optimal performance for a targeted mode without creating any undesired issues on other modes or frequency ranges.

In this article, a straightforward procedure is presented for designing the NDF controller. In order to illustrate the design methodology clearly, a simple one degree of freedom is considered. The systems displacement is considered to be controlled by a simple NDF controller. The performance index of system is extracted and based on maximum damping method. It is derived as a function of closed loop damping. Afterwards, in order to obtain an optimal value for closed loop damping, a H_2 optimization algorithm is used. After extracting optimal value for the closed loop damping, all of the NDF controller parameters are extracted accordingly. The results show high power of NDF controller which can easily damp a highly low damped mode. The effect of each parameter of controller (controller gain, damping ratio, cutoff frequency) on the performance index of the system are also evaluated. The stability of the system considering an NDF controller is also evaluated. The control effort ratio to disturbance input is also evaluated based on maximum damping and H_2 method as well. It has been shown that by the increase of the closed loop damping, the control effort ratio increases, as well.

2. Single degree of freedom system

In this section, a single one degree of freedom (SODF) system is considered for implementing the NDF controller. The system's mass is m and the stiffness of it is k and a displacement sensor is used to feedback the displacement of mass (figure 1). A disturbance force (F_d) is applied on the mass and the NDF controller is producing an action force F_a which is dependent on the system displacement. The important point here is the fact that NDF controller uses the system's displacement as an input.

The governing equation of motion of the system is presented below:

$$m\ddot{x}+kx=F_\mathrm{d}+F_\mathrm{a} \tag{1a}$$

$$F_\mathrm{a}=k_\mathrm{c}u \tag{1b}$$

where x is the mass displacement and u is the controller signal. The dynamics of NDF controller in time domain is presented in equation (2a). For more clarifications the frequency response function of the controller ($\frac{U}{X}$) is presented in equation (2b). The controller signal is magnified by the gain of k_c and applied on the actuator (equation (1b)):

$$\ddot{u}+2\xi_\mathrm{c}\omega_\mathrm{c}\dot{u}+\omega_\mathrm{c}^2u=-\omega_\mathrm{c}\dot{x} \tag{2a}$$

$$\frac{U}{X}=-\frac{\omega_\mathrm{c}s}{s^2+2\xi_\mathrm{c}\omega_\mathrm{c}s+\omega_\mathrm{c}^2} \tag{2b}$$

where ξ_c, ω_c and k_c damping ratio, cut-off frequency and gain of NDF controller, respectively. To extract a dimensionless equation, a transfer form of $\tau=\omega_0 t$ where $\omega_0=\sqrt{\frac{k}{m}}$, is considerate and the aforementioned equations becomes:

$$x''+x=f_\mathrm{d}+f_\mathrm{a} \tag{3a}$$

$$u''+2\xi\alpha u'+\alpha^2u=-\alpha x' \tag{3b}$$

where:

$$f_\mathrm{d}=\frac{F_\mathrm{d}}{k} \tag{4a}$$

$$f_\mathrm{a}=\frac{F_\mathrm{a}}{k}=\frac{k_\mathrm{c}}{k}u=\beta u \tag{4b}$$

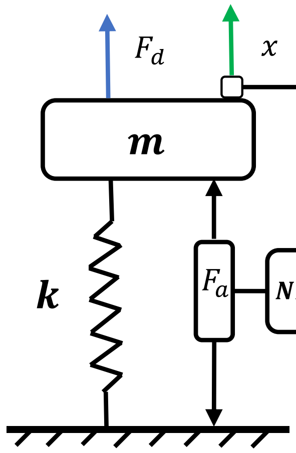


Figure 1. One degree of freedom system oscillator with an active NDF controller.

$$\alpha = \frac{\omega_c}{\omega_0} \tag{4c}$$

$$\beta = \frac{k_c}{k} \tag{4d}$$

$$\xi = \xi_c. \tag{4e}$$

After applying Laplace transform to equations (3*a*) and (3*b*), the performance index of the system in the closed loop configuration (the ratio of displacement to disturbance force) in Laplace domain is extracted and presented in below:

$$\frac{x}{f_d} = \frac{s^2 + 2\xi\alpha s + \alpha^2}{(s^2+1)(s^2+2\xi\alpha s+\alpha^2)+\alpha\beta s}. \tag{5}$$

The above equation represents the performance index of the NDF controller on the SDOF system and to increase the efficacy of a controller it is desired to minimize the value of the performance index, in order to have lower displacement when higher disturbances are applied to the system. The NDF controller should be designed in a way which minimizes the performance index of system.

2.1. Maximum damping method

For minimizing the performance index, first a maximum damping method is utilized. Maximum damping is a very effective method which ensures achievement of highest damping possible after closing the loop, by finding best candidates for locations of the controller poles. Based on the maximum damping method, in these kinds of systems, the best pole of NDF is at location where creates both of the closed-loops poles with an equal damping ratio. By considering this method and, the performance index of the closed loop system can be written as:

$$\frac{x}{f_d} = \frac{s^2 + 2\xi\alpha s + \alpha^2}{(s^2+2\eta\gamma s+\gamma^2)^2} \tag{6}$$

where $\gamma = \frac{\omega_f}{\omega_0}$, ω_f and η are closed-loop resonance frequency and damping of the system, respectively. For applying maximum damping to an SODF, the characteristic equation of them should be the same. This means that, by comparing the denominator of the fraction in equations (5) and (6) and equating the polynomial coefficient of them maximum damping method will be applied to SDOF system. This creates the following equations:

$$4\eta\gamma = 2\xi\alpha \tag{7a}$$

$$\left(4\eta^2+2\right)\gamma^2 = \alpha^2+1 \tag{7b}$$

$$4\eta\gamma^3 = (2\xi+\beta)\,\alpha \tag{7c}$$

$$\gamma^4 = \alpha^2. \tag{7d}$$

By considering these four equations, the NDF controller parameters can be defined based on the closed loop damping. From equation (7*d*), it can be easily concluded that

$$\gamma = \sqrt{\alpha}. \tag{8}$$

This equation implies that the resonance frequency of the closed loop system can be determined by the cutoff frequency of the controller. Also considering equations (7*a*) and (8), the damping ratio of the controller can be extracted:

$$\xi = \frac{2\eta}{\sqrt{\alpha}}. \tag{9}$$

By putting equation (8) into the equation (7*b*), the controller cutoff frequency parameter can be determined:

$$\alpha = \left(2\eta^2+1\right) - 2\eta\sqrt{\eta^2+1}\ (\text{For } \alpha<1) \tag{10a}$$

$$\alpha = \left(2\eta^2+1\right) + 2\eta\sqrt{\eta^2+1}\ (\text{For } \alpha>1). \tag{10b}$$

The parameter of α determines the distance between the controller cutoff frequency to targeted mode. Based on maximum damping method, this value is dependent on the closed loop damping value. Considering equations (7*c*)–(9), the controller gain parameter is extracted:

$$\beta = 4\eta\left(\sqrt{\alpha} - \frac{1}{\sqrt{\alpha}}\right). \tag{11}$$

So, in overall based on the maximum damping method, the NDF parameters are defined by equations (9)–(11). All of these parameters are dependent on the closed loop damping (η) and therefore these equations are rewritten dependent only closed loop damping in the below table.

So, all of the parameters of the controller are determined as a function of closed loop damping based on maximum damping method and presented in table 1. Having table 1

Table 1. Parameters of controller as a function of closed loop damping.

	$\alpha < 1$	$\alpha > 1$
α	$(2\eta^2+1) - 2\eta\sqrt{\eta^2+1}$	$(2\eta^2+1) + 2\eta\sqrt{\eta^2+1}$
ξ	$\frac{2\eta}{\sqrt{(2\eta^2+1)-2\eta\sqrt{\eta^2+1}}}$	$\frac{2\eta}{\sqrt{(2\eta^2+1)+2\eta\sqrt{\eta^2+1}}}$
β	$4\eta\left(\frac{2\eta^2-2\eta\sqrt{\eta^2+1}}{\sqrt{(2\eta^2+1)-2\eta\sqrt{\eta^2+1}}}\right)$	$4\eta\left(\frac{2\eta^2+2\eta\sqrt{\eta^2+1}}{\sqrt{(2\eta^2+1)+2\eta\sqrt{\eta^2+1}}}\right)$

and considering equations (4*c*), (4*e*) and (4*d*), all of parameters of equation (2*b*) is defined based on maximum damping method. All of these parameters are dependent on the closed loop damping value which should be determined optimally. In the next step an optimization method is deployed to determine the value of closed loop damping (and consequently the controller parameters) optimally.

2.2. H_2 optimization method

In previous section, the candidates for maximum damping method are extracted. These infinite number of candidates can be calculated for each value of closed loop damping from zero to one. In the next step an optimization method should be considered to find an optimal one among all of the candidates for the controller. The optimization method can affect the optimal value for closed loop damping significantly. Two methods can be considered for optimization which are H_2 method and H_∞ method. H_2 method minimizes the magnitude of performance index in all of the frequencies from zero to infinity in the optimization procedure, while H_∞ minimizes the magnitude of performance index only the natural frequency of plant. Even though H_∞ can lead to higher values for closed loop damping, it can come to the price of vibration magnification in the frequencies which was not considered in the process of optimization. This can easily cause lower phase margin or instability issues in the closed loop. Therefore, it is highly recommended to consider H_2 method in which all of frequencies (from zero to infinity) are considered in the process of optimization. This can guarantee the stability of the closed loop system with an acceptable amount closed loop damping.

In this section a H_2 method is used to determine the optimal value of the closed loop damping. The optimal value of the closed loop damping should be chosen in order to minimize the performance index of the system. For this purpose, first the performance index of the system is rewritten based on the closed loop damping and controller cutoff frequency parameter (α) (which is also dependent on closed loop damping (equation (10)). So, by putting equations (8) and (9) into equation (6), the performance index equation after applying maximum damping can be extracted as:

$$\frac{x}{f_d} = \frac{s^2+4\eta\sqrt{\alpha}s+\alpha^2}{\left(s^2+2\eta\sqrt{\alpha}s+\alpha\right)^2}. \tag{12}$$

There are several ways to minimize the above equation. However, in order to reduce undesired impacts of the control action for all frequencies, H_2 method is preferred. Based on the H_2 optimization method, the value of $\int_0^\infty \left|\frac{x}{f_d}\right|^2 d\Omega$ should be minimized. According to Crandall *et al* [12], for a general transfer function such as:

$$G(s) = \frac{b_3s^3+b_2s^2+b_1s+b_0}{a_4s^4+a_3s^3+a_2s^2+a_1s+a_0}. \tag{13}$$

The cost function for H_2 norm for this transfer function is:

$$\int_{-\infty}^{\infty} |G|^2 d\Omega = \pi\frac{N_1+N_2+N_3+N_4}{D_1} \tag{14}$$

where

$$N_1 = \frac{b_0^2}{a_0}(a_2a_3-a_1a_4) \tag{15a}$$

$$N_2 = a_3\left(b_1^2-2b_0b_2\right) \tag{15b}$$

$$N_3 = a_1\left(b_2^2-2b_1b_3\right) \tag{15c}$$

$$N_4 = \frac{b_3^2}{a_4}(a_1a_2-a_0a_3) \tag{15d}$$

$$D_1 = a_1(a_2a_3-a_1a_4)-a_0a_3^2. \tag{15e}$$

Based on these equations, the cost function of the system performance index (equation (12)) are extracted. The cost function is dependent on the closed loop damping ratio:

$$\mathrm{PI} = \pi\frac{\left(4\eta^2+1\right)\alpha^2+16\eta^2-2\alpha+1}{16\eta^3\alpha\sqrt{\alpha}}. \tag{16}$$

In order to find the optimal value for closed-loop damping, the cost function (after putting equation (10) into equation (16) is differentiated with respect to closed-loop damping (η) and equating the extracted equation to zero ($\frac{d(\mathrm{PI})}{d\eta}=0$) yields:

$$\begin{array}{cc} \alpha>1 & \alpha<1 \\ \eta_{\mathrm{opt}} = 0.41056 & \text{No optimal Value} \end{array}. \tag{17}$$

For the condition of cutoff frequency lower than targeted mode ($\alpha < 1$) the optimized value for closed loop damping is 0.410 56 and for the condition of cutoff frequency higher than targeted mode ($\alpha > 1$) there is no optimal value for closed-loop damping. These cost function for various closed-loop damping are plotted in the figure 2 for both conditions to show the results insightfully (figure 2).

By H_2 method, in the case of $\alpha < 1$ there is an optimal value for the closed-loop damping around 0.41056. However, in the case $\alpha > 1$ there is no optimal value for the closed-loop damping and in this condition as the η increases from zero to one, the cost function decreases dramatically. Therefore, for having a better performance it is inclined to consider the condition of $\alpha < 1$ which means that the cutoff frequency of controller lower than targeted mode. This has a benefit of damping the frequencies after the target mode as well.

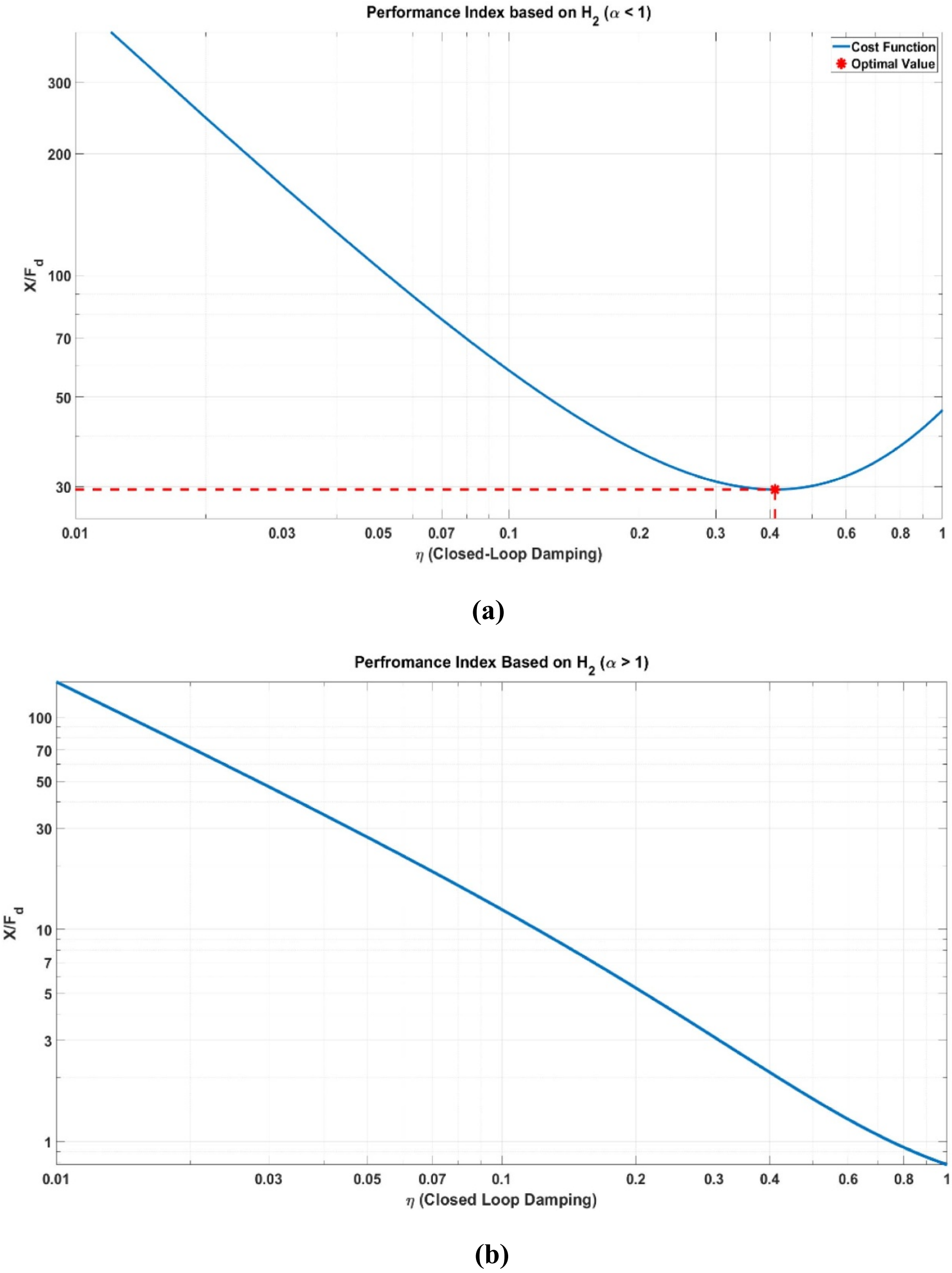


Figure 2. H_2 norm cost Function for various closed loop damping ratios (a) $\alpha < 1$ (b) $\alpha > 1$.

3. Stability

In this section, the stability of the system considering NDF controller is evaluated. Based on equation (5), the characteristic equation of the system is:

$$s^4 + 2\xi\alpha s^3 + \left(\alpha^2 + 1\right) s^2 + (2\xi + \beta)\,\alpha s + \alpha^2 = 0. \quad (18)$$

Based on Routh stability criterion, for the above system to be stable, the conditions should be meet is extracted and presented on table 2.

Table 2. Stability criteria for NDF.

$\alpha < 1$	$\alpha^2 - 1 < \frac{\beta}{2\xi} < 0$
$\alpha > 1$	$0 < \frac{\beta}{2\xi} < \alpha^2 - 1$

The results show that, higher gains or lower damping ratio of the controller can lead to instability. In order to avoid instability, increasing the gain of controller should be accompanied by increasing the damping ratio or in another word, decreasing the damping ratio should be accompanied by decreasing the gain of controller as well. Also, the effect of the cutoff frequency parameter (α) is not ignorable. In order to increase the stability of system, it is preferred to have higher distance between the cutoff frequency and the targeted mode. This means that choosing a cutoff frequency near to the targeted mode (before or after the mode) increases the possibility of instabilities. For this reason, usually NDF controller is cutting off the control action far from natural frequencies associated with controlled modes.

4. NDF controller for $\alpha < 1$

In this section, an NDF controller in the condition of $\alpha < 1$ is evaluated. For this purpose the effect of changing the closed-loop damping (η), controller damping ratio (ξ), controller gain (β), controller cutoff frequency (α) on the frequency response and root locus are investigated separately. A simple system with the specifications of $m = 1$ kg and $k = 1$ N m^{-1} is considered and the response of the system for different conditions are calculated and compared with each other in detail.

4.1. Effect of changing closed loop damping η

In the figure 3, the effect of the changing closed-loop damping (η) on the frequency response and root locus are evaluated, separately.

In the frequency response, it has been shown that increasing the value of closed-loop damping till the optimal value, increases the damping of structure. Increasing the closed-loop damping more than that, creates more vibration reduction around the targeted mode, however it also causes high vibration magnification at lower frequencies. Hence, increasing the value of η increases the vibration reduction amount around the natural frequency, but this comes to the price of vibration magnification at lower frequencies which is not desirable. On the other hand, decreasing the value of η, decreases the vibration reduction amount around the natural frequency and system damping. Therefore, there is a compromise between the vibration reduction around the natural frequency and vibration magnification at lower frequencies. The optimum choice is the optimal value of closed loop damping. The optimal magnitude of η has an impressive vibration reduction around natural frequency with less undesirable effect on lower frequencies.

The locations of NDF poles for various values of η are also presented in figures 3(b)–(d). In all of them, the closed loop poles are intersecting at one location and that is feature of maximum damping method. In the case of $\eta = \eta_{\mathrm{opt}}/2$ the NDF poles are imaginary and before the targeted mode. But in the case of $\eta = \eta_{\mathrm{opt}}$ and $\eta = 2 \times \eta_{\mathrm{opt}}$ the NDF poles are real in the horizontal axes. This shows the importance of the location of poles of NDF on the closed loop damping. In figure 3(d) root locus of the system with optimal η is presented.

4.2. Effect of changing controller damping ratio ξ

In the figure 4, the effect of the changing controller damping ratio (ξ) on the frequency response and root locus are presented separately.

The frequency response figure shows that increasing the value of ξ, decreases the effect of controller on vibration reduction amount around the natural frequency, slightly. On the other hand, decreasing the value of ξ increases the vibration magnification level around the natural frequency but it also creates a pick at lower frequencies which is unsettling. Another important point is the fact that the root locus figure shows that decreasing the value of ξ may lead the system to become close to instability. This fact was also shown in the table 2.

All of these results show that the controller damping ratio has a very high impact on the system response and should be chosen adequately. Otherwise, it can easily cause lower performances in the response or even instabilities. In another word, the NDF controller is very sensitive with the value of the damping ratio and this value can impact highly on the performance in closed loop.

4.3. Effect of changing controller gain β

In the figure 5, the effect of the changing controller damping gain (β) on the frequency response and root locus are shown separately.

The results shows clearly that, enlarging the value of β (controller gain) rises the amount of vibration reduction around the natural frequency. However, this causes a pick at lower frequency which is unfavorable. Also, the side effect of higher gain is the probability of instability. Therefore, it is not recommended to increase the gain higher than optimal gain. From the root locus plot it is obvious that increasing the gain higher than 40% of the optimal value, can cause instability. This phenomenon was discussed in table 2 as well.

On the other hand, reducing the value of β, dwindles the effect of controller and declines the vibration reduction amount around the natural frequency.

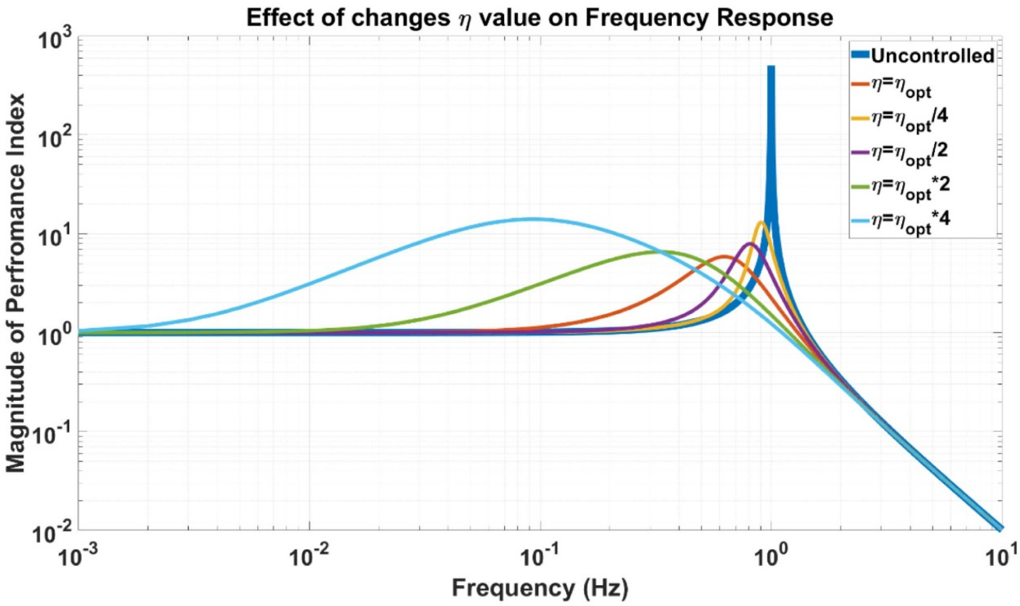
4.4. Effect of changing controller cut-off frequency (α)

In the figure 6, the effect of the altering controller cut-off frequency (α) on the frequency response and root locus are presented separately.

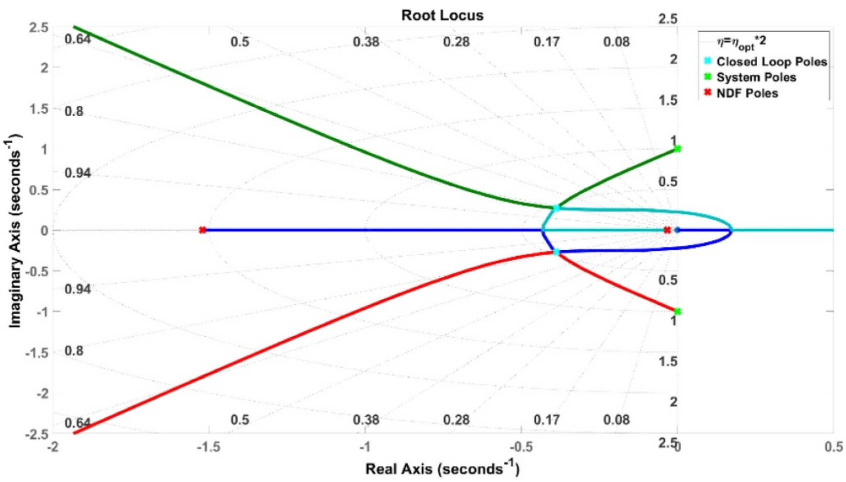
Figure 6(b) shows that declining the value of α (which represents controller cutoff frequency) drops the damping of the system and consequently decreases the amount of vibration reduction around the natural frequency.

On the other hand, enlarging the value of α (figure 6(c)), rises the system's damping and the vibration reduction level around the natural frequency. However, this comes with price of vibration magnification in the lower frequencies (figure 6(a)). Also, figure 6(c) shows that increasing the controller cutoff frequency parameter, can lead the system to close margins of instability.

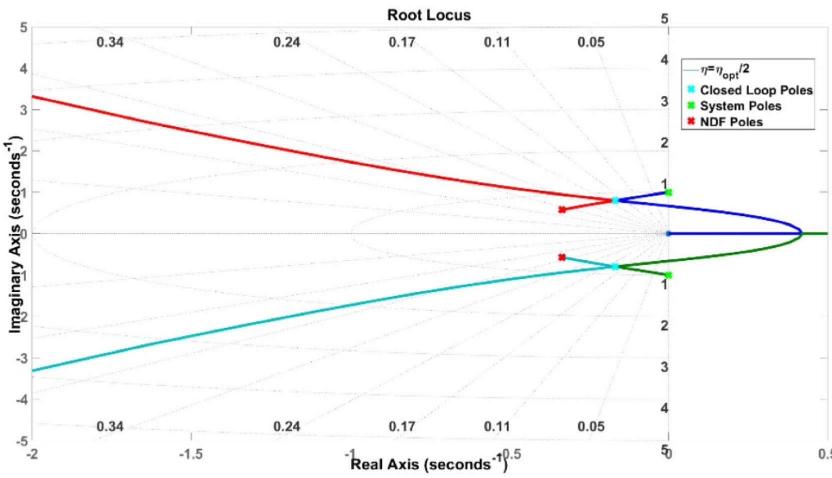
In the frequency response (figure 6(a)), all of the responses goes through one fixed point which considered as a fixed point of the system. This fixed point, has a lower frequency than targeted mode.



(a)

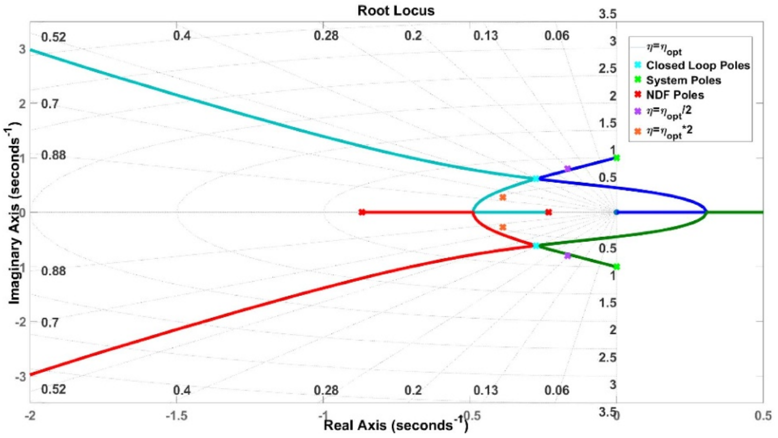


(b)



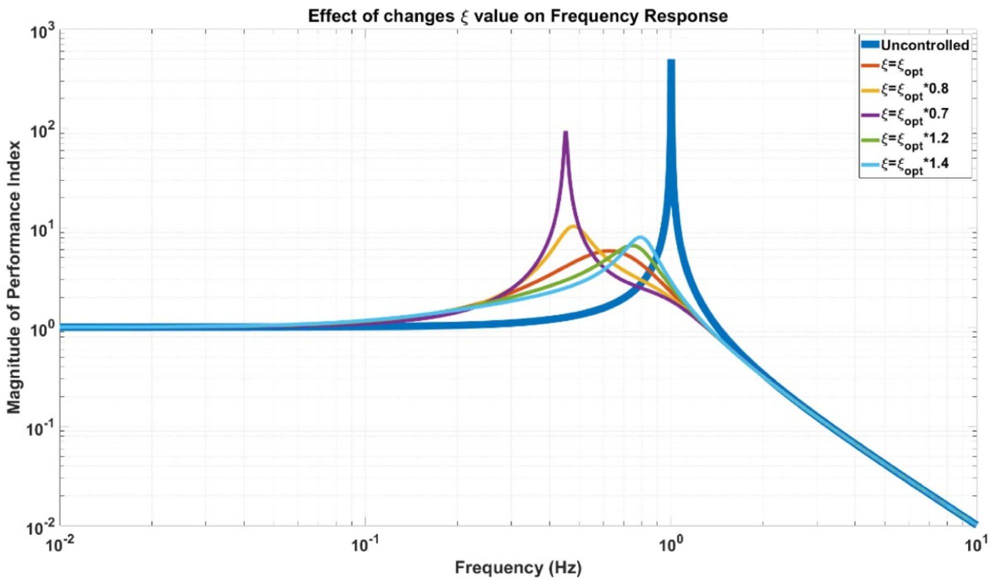
(c)

Figure 3. (a) Performance index for various values of η in the condition of $\alpha < 1$ and (b) root locus for $\eta = 2 \times \eta_{\text{opt}}$. (c) Root locus for $\eta = \eta_{\text{opt}}/2$. (d) Root locus for $\eta = \eta_{\text{opt}}$.

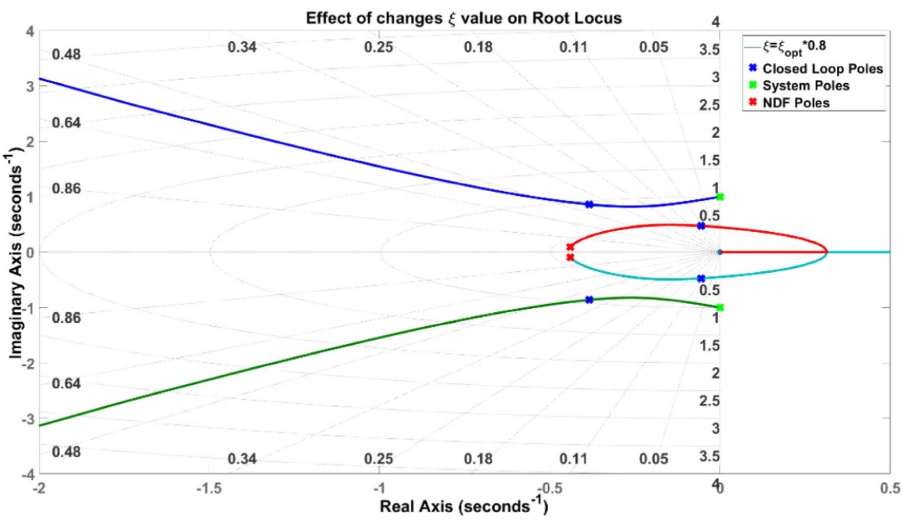


(d)

Figure 3. (Continued.)

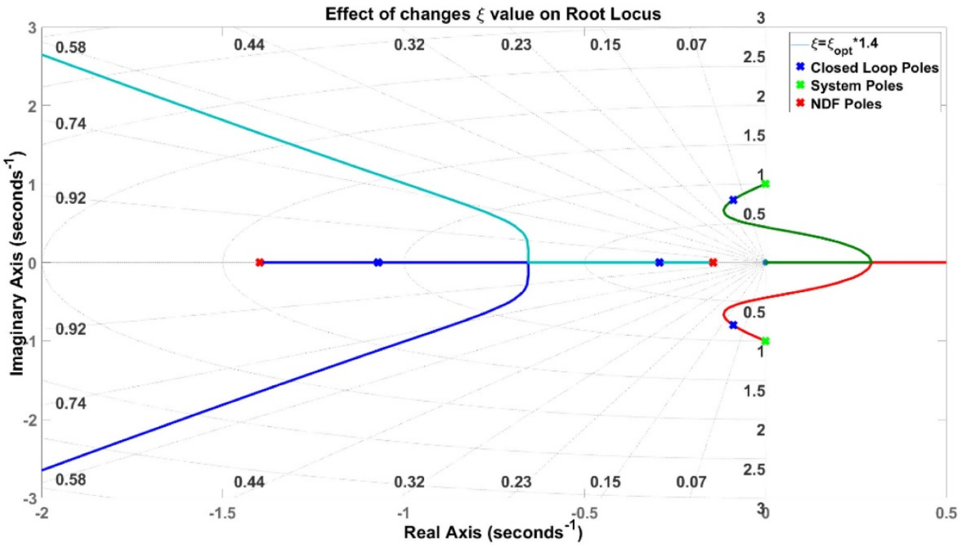


(a)

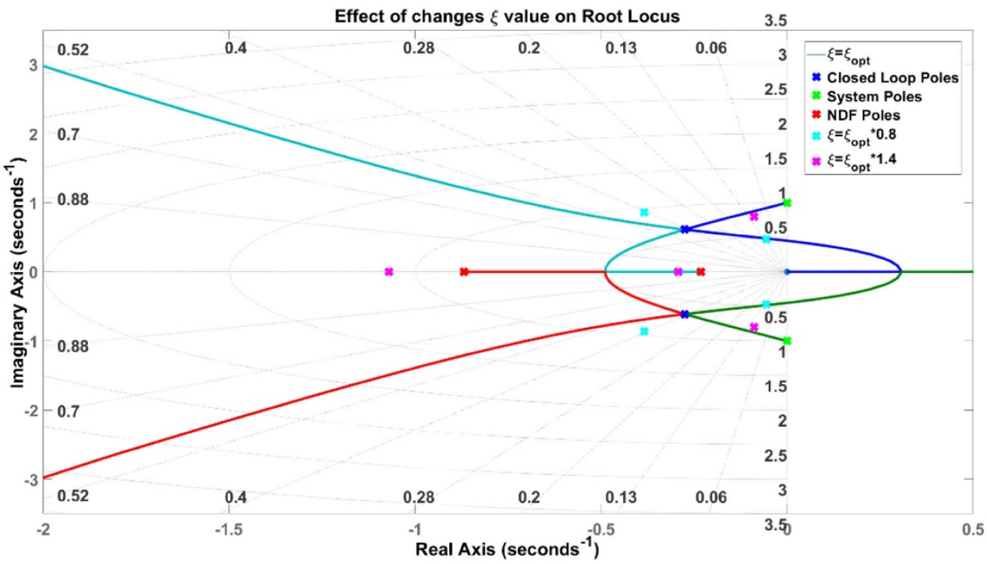


(b)

Figure 4. (a) Performance index for various values of ξ in the condition of $\alpha < 1$ and (b) root locus for $\xi = 0.8 \times \xi_{\text{opt}}$. (c) Root locus for $\xi = \xi_{\text{opt}} \times 1.4$. (d) Root locus for $\xi = \xi_{\text{opt}}$.



(c)



(d)

Figure 4. (Continued.)

5. NDF controller for $\alpha > 1$

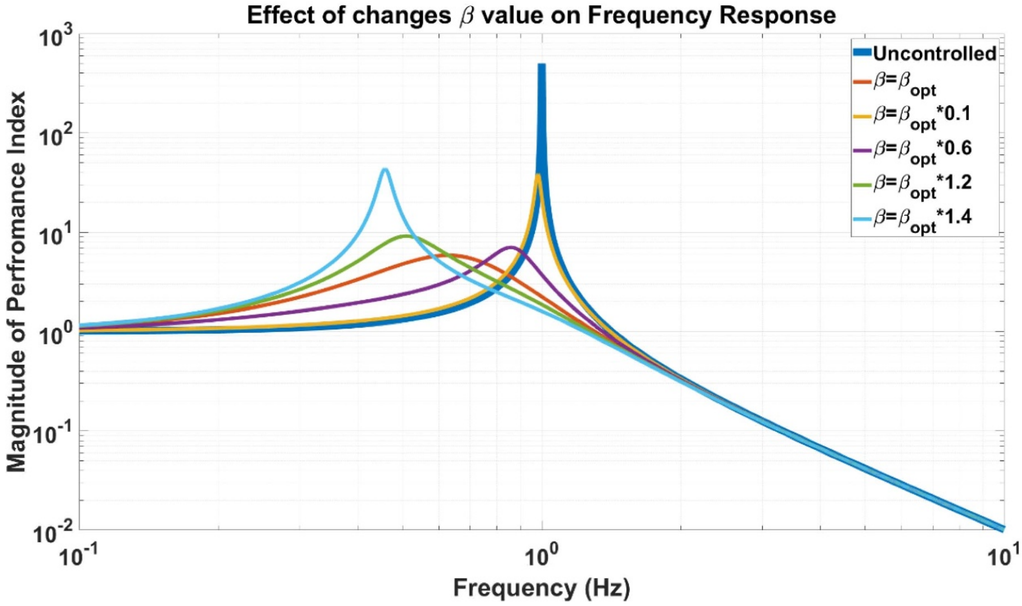
Similarly, in this section, the response of SDOF system with NDF controller in the condition of $\alpha > 1$ is evaluated in detail. For this end, the effect of changing the closed loop damping (η), controller damping ratio (ξ), controller gain (β), controller cutoff frequency (α) on the frequency response and rot locus are investigated separately. As before, a simple system with the specification of $m = 1$ kg and $k = 1$ N m^{-1} is considered.

5.1. Effect of changing closed loop damping η

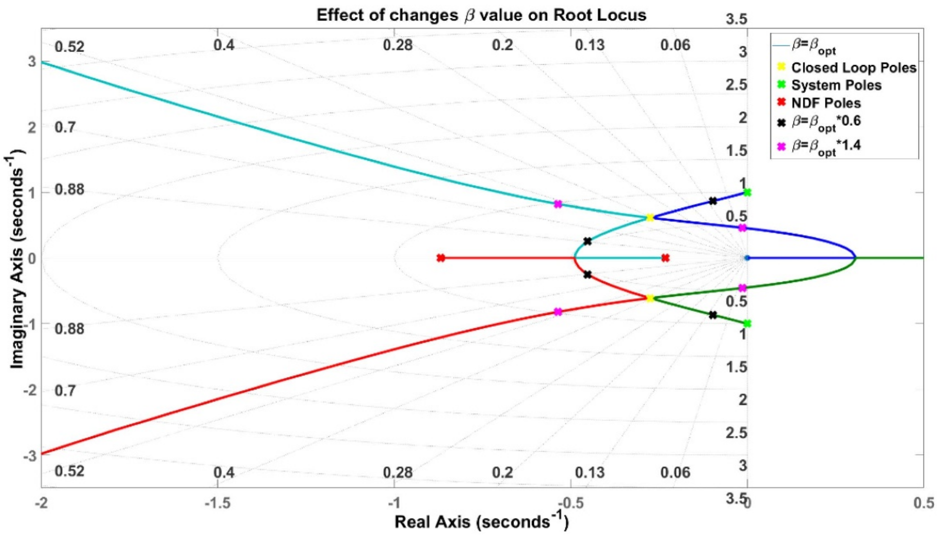
In the figure 7, the effect of the changing closed loop damping (η) on the frequency response and root locus are presented.

In the above figure, rising the value of η (from zero to one) escalates the amount of vibration reduction around the natural frequency dramatically and as it obvious in the root locus figure, for $\eta = 1$ all of the closed loop poles are located in the horizontal axes which has damping of 1. This magnitude is the highest possible damping value for a system.

Therefore, in overall, the NDF controller with the cutoff frequency location after the natural frequency ($\alpha > 1$) has more power on reducing vibration. However, on the other hand, it causes vibration magnification in the frequencies higher than targeted mode (figure 7(a)). This downside can create problems especially in the systems with higher number of modes. This means that in higher modes the vibration level will



(a)



(b)

Figure 5. (a) Performance index for various values of β in the condition of $\alpha < 1$ and (b) root locus for $\beta = \beta_{\mathrm{opt}}$ and closed loop poles for other values of β ($\beta_{\mathrm{opt}} \times 0.6$, $\beta_{\mathrm{opt}} \times 1.4$).

increases after applying this controller. This immense drawback inclines the designer to choose the condition of $\alpha < 1$ rather than $\alpha > 1$.

5.2. Effect of changing controller damping ratio ξ

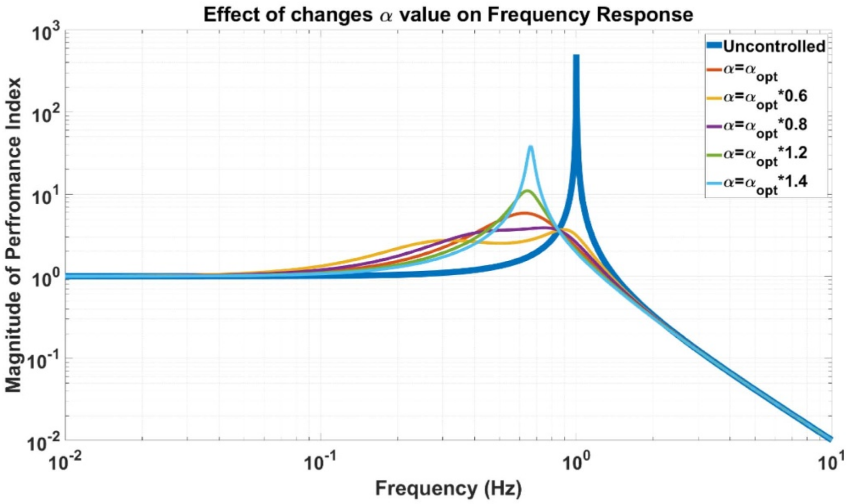
In the figure 8, the effect of the changing controller damping ratio (ξ) on the frequency response and root locus are described in detail.

In figure 8, declining the value of ξ rises the amount of vibration mitigation level around the natural frequency more. However, this causes vibration magnification at frequencies higher than natural frequency which is unsettling. Another important point is the fact that the root locus figure shows that decreasing the value of ξ may lead the system close to the instability. This fact was also mentioned in the table 2. On the other hand, rising the value of ξ, reduces the amount of vibration magnification in the frequencies after the mode. However, it decreases the performance of controller or the vibration reduction amount around the natural frequency.

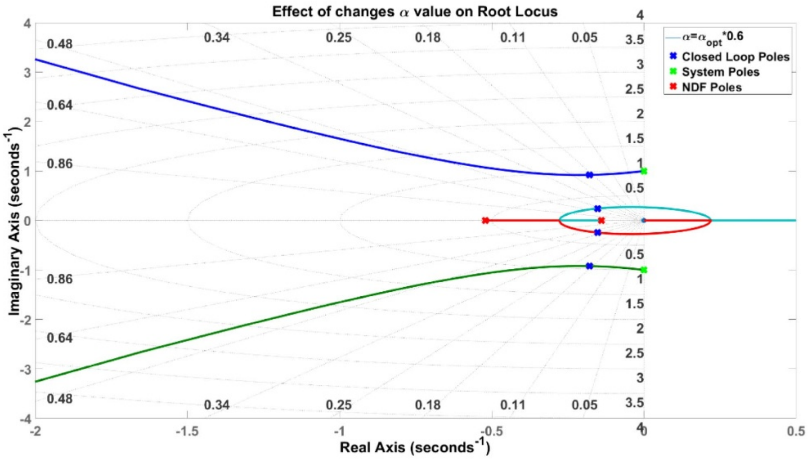
5.3. Effect of changing controller gain β

In the figure 9, the effect of the changing controller damping gain (β) on the frequency response and root locus are presented.

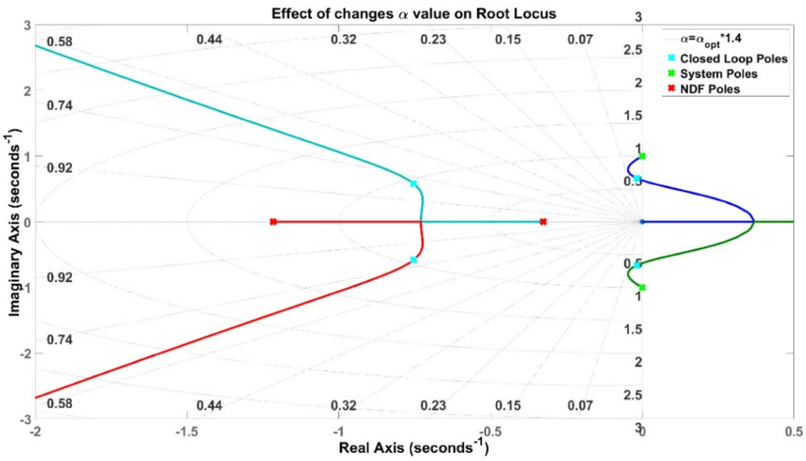
Clearly, changing the value of controller gain (β) affects the controller performance. Rising the value of β increases the



(a)

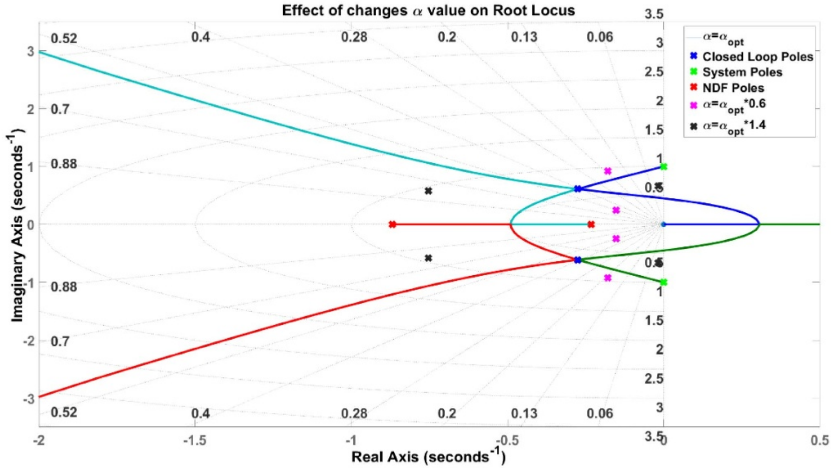


(b)



(c)

Figure 6. (a) Performance index for various values of α in the condition of $\alpha < 1$ and (b) root locus for $\alpha = 0.6 \times \alpha_{\text{opt}}$. (c) Root locus for $\alpha = \alpha_{\text{opt}} \times 1.4$. (d) Root locus for $\alpha = \alpha_{\text{opt}}$.



(d)

Figure 6. (Continued.)

amount of vibration reduction around the natural frequency more, but this cause vibration magnification at frequencies higher than natural frequency. Another important point is the fact that the root locus figure shows that increasing the value of β may lead the system to become close to instability. This fact is also shown in the table 2.

Moreover, decreasing the value of β, declines the closed loop damping of the system and hence the vibration reduction level around the natural frequency decreases enormously.

5.4. Effect of changing controller cut-off frequency (α)

In the figure 10, the effect of the changing controller damping gain (α) on the frequency response and root locus are evaluated.

The results show that increasing the value of α, diminishes the vibration reduction level around the natural frequency. Moreover, decreasing the value of α, increases the amount of vibration reduction around the natural frequency more. However, this creates vibration magnification at frequencies higher than natural frequency. Also, one interesting point in the frequency response is the fact that all of curves goes through one fixed point which considered as a fixed point of the system. This fixed point has a higher frequency than targeted mode. This fixed point is different than the one in the figure 6(a).

6. Control effort

In this section, the control effect of NDF controller applied to SDOF system is determined using maximum damping and H_2 method. One of the important factors of any controller is its needed effort in the presence of disturbances with various magnitudes. The best option for any controller is to damp higher vibration levels with lower controller effort. For this end, the ratio of actuator force to disturbance magnitude should be minimized. At first, the ratio of actuator force to disturbance magnitude for the SDOF system presented in the figure 1 is extracted and presented in below:

$$\frac{F_\mathrm{a}}{F_\mathrm{d}} = \frac{\frac{F_\mathrm{a}}{x}}{\frac{F_\mathrm{d}}{x}} = \frac{\frac{F_\mathrm{a}}{x}}{\frac{kf_\mathrm{d}}{x}} = \frac{\frac{\frac{F_\mathrm{a}}{k}}{x}}{\frac{f_\mathrm{d}}{x}} = -\frac{\alpha\beta s}{(s^2+1)(s^2+2\xi\alpha s+\alpha^2)+\alpha\beta s}. \tag{19}$$

Similar to the section 2.1, considering maximum damping method, in these kinds of systems, the best pole of NDF is at location where creates both of the closed-loops poles with an equal damping ratio. By considering this method, the control effort ratio can be written as:

$$\frac{F_\mathrm{a}}{F_\mathrm{d}} = -\frac{\alpha\beta s}{(s^2+2\eta\gamma s+\gamma^2)^2}. \tag{20}$$

By putting equation (8) in the equation (21), the below can be extracted:

$$\frac{F_\mathrm{a}}{F_\mathrm{d}} = -\frac{\alpha\beta s}{(s^2+2\eta\sqrt{\alpha}s+\alpha)^2}. \tag{21}$$

Equation (21), represents the control effort of the system after applying maximum damping method. The lower value control effort is recommended. Therefore, in order to find lower amount of the control effort H_2 optimization method is used. In this way, the value of $\int_{-\infty}^{\infty}\left|\frac{F_\mathrm{a}}{F_\mathrm{d}}\right|^2 \mathrm{d}\Omega$ should be minimized. For this purpose, considering equations (13)–(15), the cost function of the control effort (equation (21)) is extracted. The cost function is dependent on the closed loop damping ratio and it is presented in below:

$$\int_{-\infty}^{\infty}\left|\frac{F_\mathrm{a}}{F_\mathrm{d}}\right|^2 \mathrm{d}\Omega = \pi\frac{\beta^2}{16\eta^3\sqrt{\alpha}} = \frac{\pi\left(\alpha^2-2\alpha+1\right)}{\eta\sqrt{\alpha}} = \mathrm{CE} \tag{22}$$

where α,β is determined in table 1 for the both cases of $\alpha<1$ and $\alpha>1$. The control effort cost function value is minimized when $\frac{\mathrm{d(CE)}}{\mathrm{d}\eta}$ is equal to zero. However, this does not happen for neither for $\alpha<1$ nor for $\alpha>1$. The extracted cost function

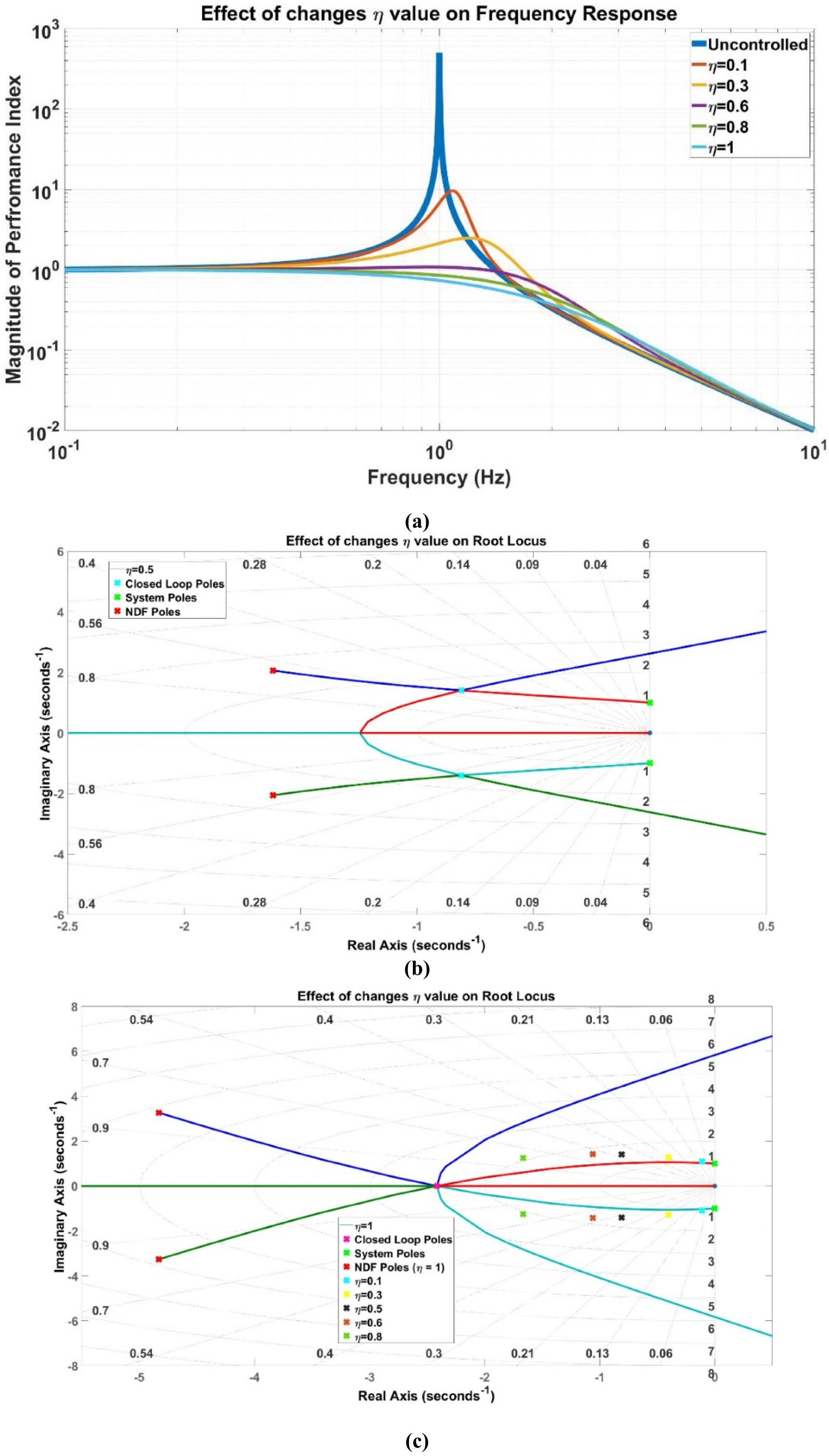


Figure 7. (a) Performance index for various values of η in the condition of $\alpha > 1$ and (b) root locus for $\eta = 0.5$. (c) Root locus for $\eta = 1$ and closed loop poles for other values of η (0.1, 0.3, 0.5, 0.6, 0.8).

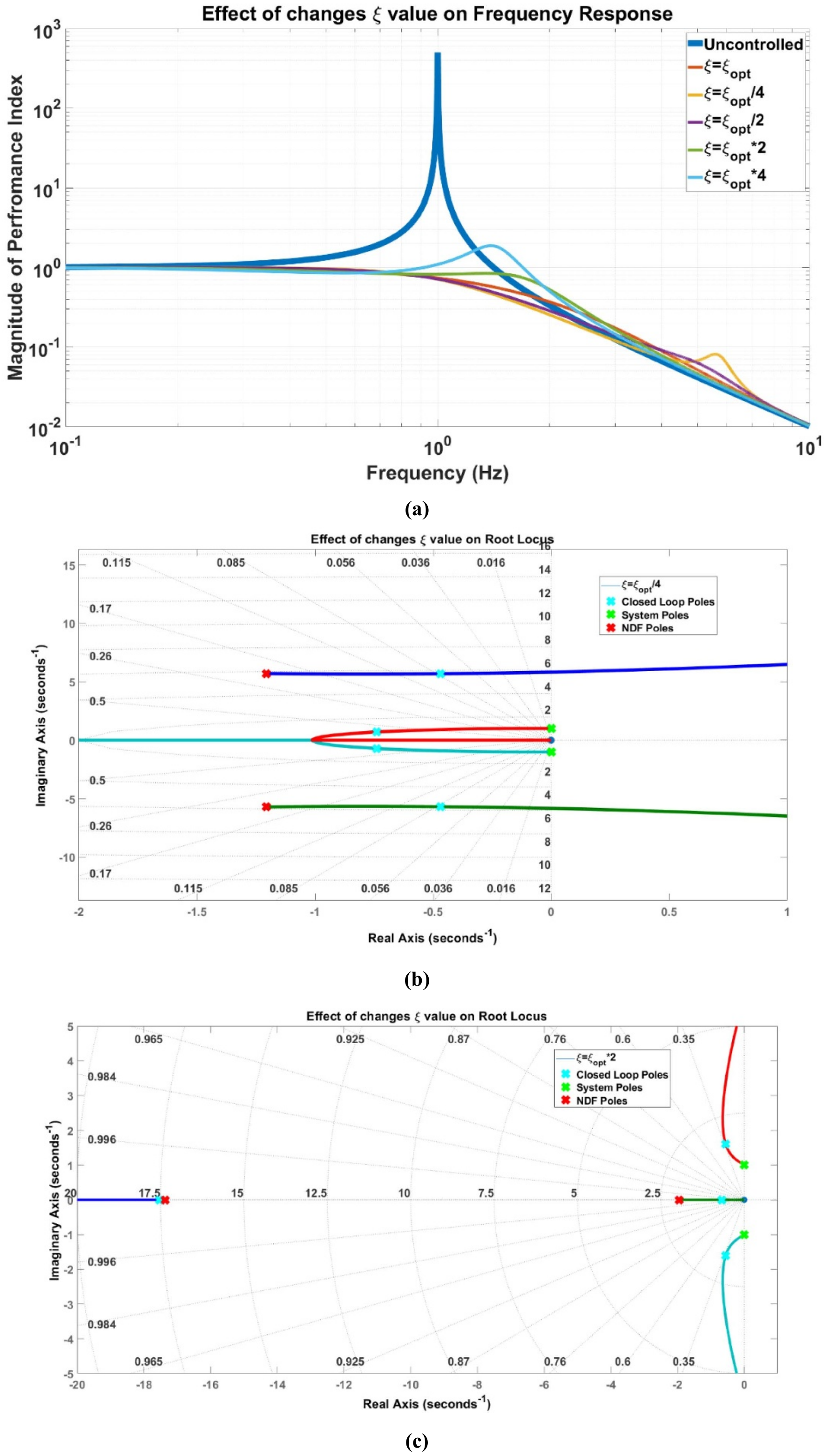
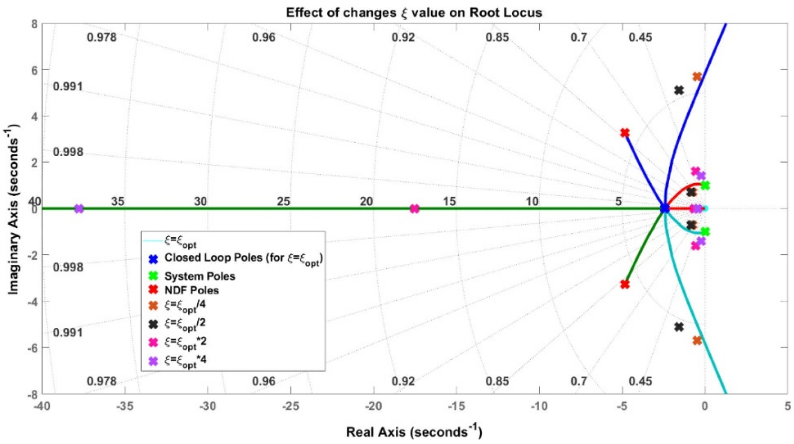
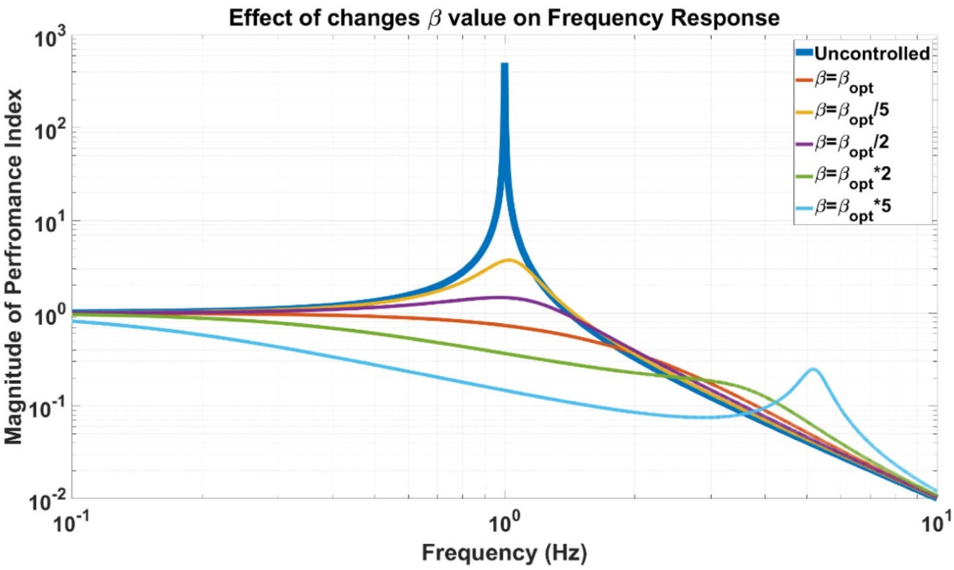


Figure 8. (a) Performance index for various values of ξ in the condition of $\alpha > 1$ and (b) root locus for $\xi = \xi_{opt}/4$. (c) Root locus for $\xi = \xi_{opt} \times 2$. (d) Root locus for $\xi = \xi_{opt}$ and closed loop poles for other values of ξ ($\xi_{opt}/4$, $\xi_{opt}/2$, $\xi_{opt} \times 2$, $\xi_{opt} \times 4$).

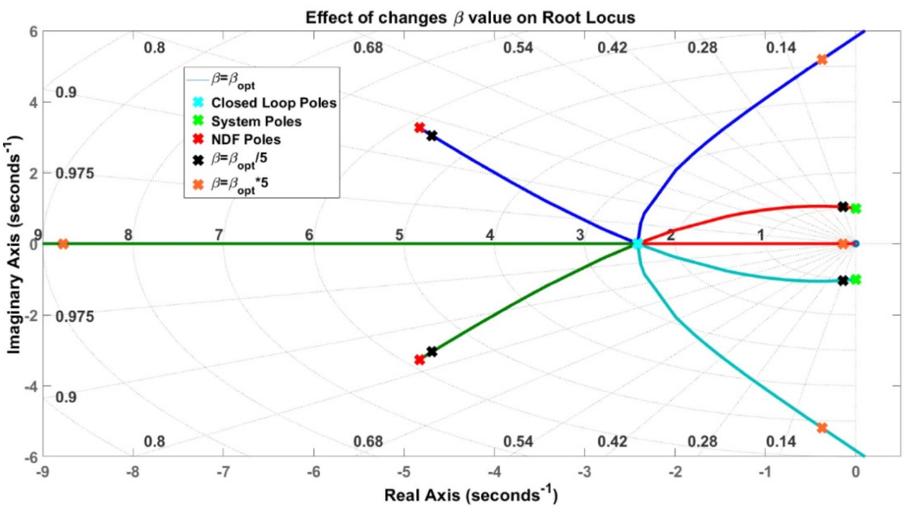


(d)

Figure 8. (Continued.)



(a)



(b)

Figure 9. (a) Performance index for various values of β in the condition of $\alpha > 1$. (b) Root locus for various values of β in the condition of $\alpha > 1$ and closed loop poles for other values of β ($\beta_{\text{opt}}/2$, $\beta_{\text{opt}} \times 5$).

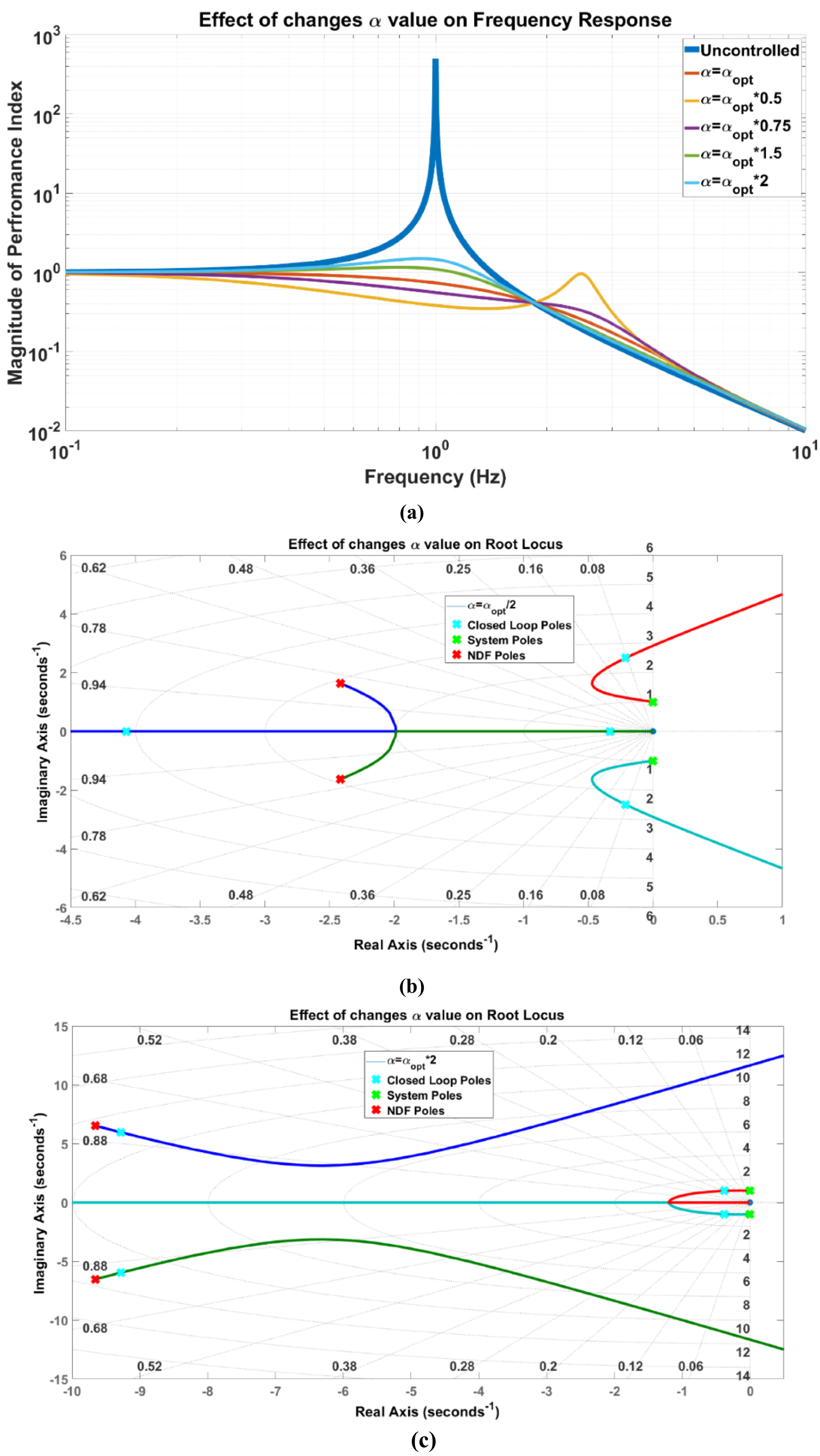
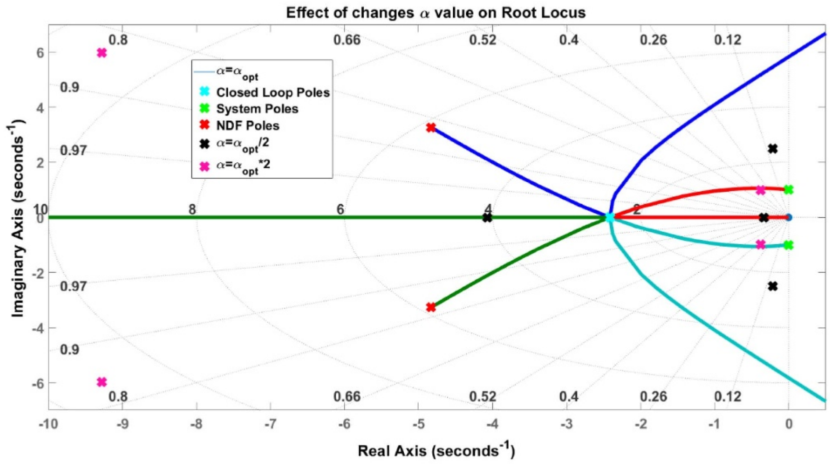


Figure 10. (a) Performance index for various values of α in the condition of $\alpha > 1$. (b) Root locus for $\alpha = \alpha_{\text{opt}}/2$. (c) Root locus for $\alpha = \alpha_{\text{opt}} \times 2$. (d) Root locus for $\alpha = \alpha_{\text{opt}}$ and closed loop poles for other values of α ($\alpha_{\text{opt}}/2$, $\alpha_{\text{opt}} \times 2$).

for various magnitude of η is presented in figure 13 for both cases of $\alpha < 1$ and $\alpha > 1$ (in figure 13).

Clearly, as the η increases from zero to one, the control effort increases as well for both cases of $\alpha < 1$ and $\alpha > 1$. This means that for lower actuation force, it is better to consider lower closed loop damping. However, this will effectively diminish the performance of the controller. It is interesting that the control effort for $\alpha < 1$ is always higher than



(d)

Figure 10. (Continued.)

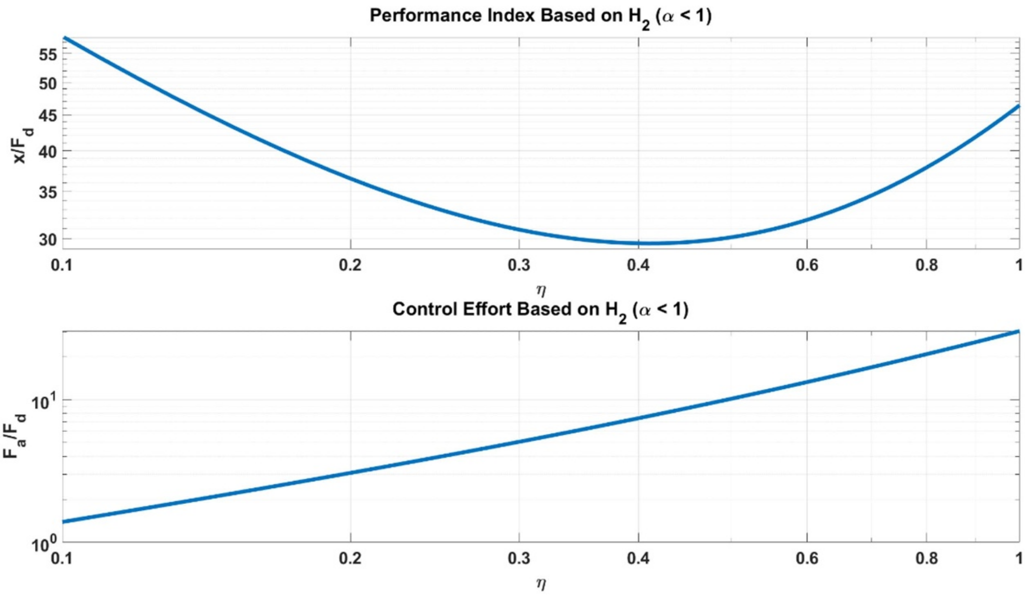


Figure 11. Performance index and control effort for various η in the case of $\alpha < 1$.

$\alpha > 1$. This means that for lower power consumption of the actuation it is better to consider $\alpha > 1$ rather than $\alpha < 1$.

In order to choose the value of closed loop damping wisely, the performance index of the system and control effort should be evaluated simultaneously. Therefore, the cost function of performance index and control effort for various value of closed loop damping in the case of $\alpha < 1$ is presented in figure 11.

In the case of $\alpha < 1$, if the control effort does not limit the experiment, then the optimal parameter of $\eta_{\text{opt}} = 0.41056$ is the best practical choice. However, if there are any limitations, lower amounts for η can be considered and that would definitely decrease the controller performance on the vibration reduction amount. Also, in the condition of $\alpha < 1$, the values higher than η_{opt} is not recommended at all. That is because of higher control effort accompanied by lower performance at the same time which both of them are unsettling.

Similarly, the cost function of performance index and control effort for various value of closed loop damping, in the case of $\alpha > 1$ is presented in the figure 12.

In the case of $\alpha > 1$, the control effort and performance index have the opposite directions in respect with η. As η enlarges from zero to one, the performance index of the system decreases dramatically. However, this leads to higher control efforts. This means as the η increase the control effort will also increase. Therefore, in this condition there is compromise between performance index and control effort. If the actuator can apply higher forces, then higher performances will definitely be reachable, but in practice there are always limited actuation force.

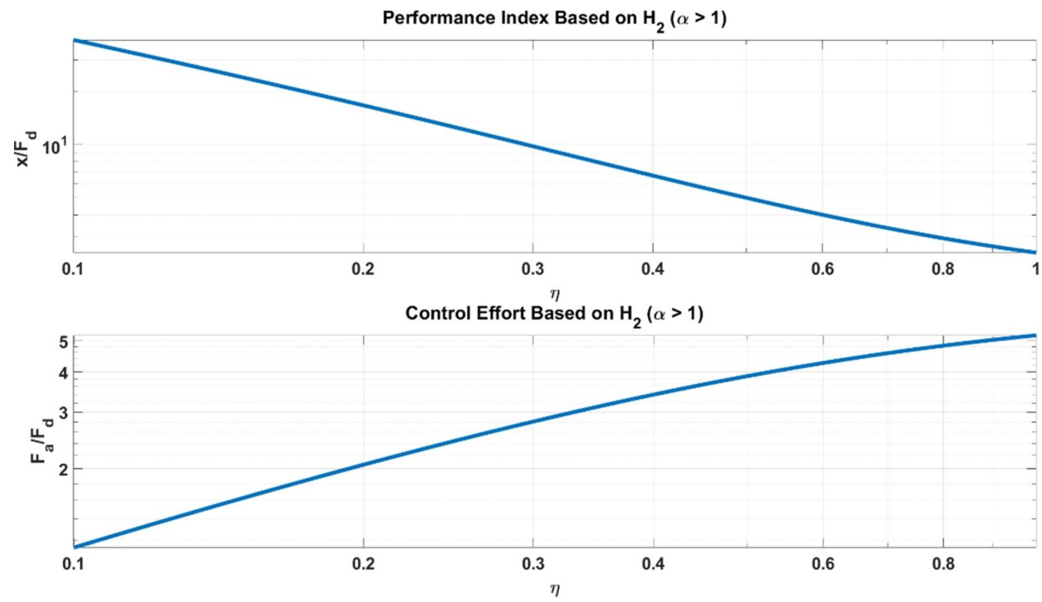


Figure 12. Performance index and control effort for various η in the case of $\alpha > 1$.

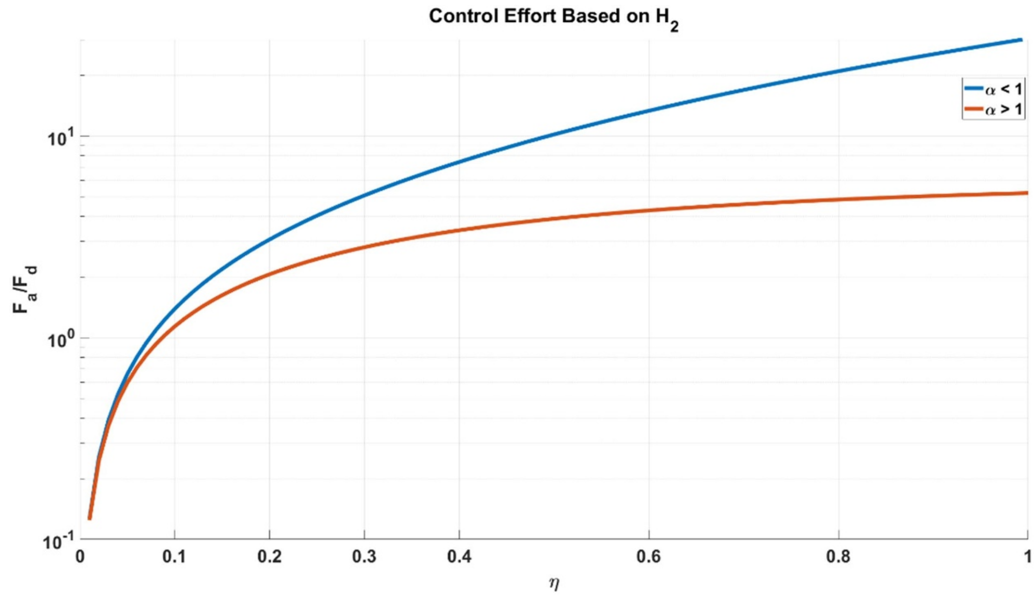


Figure 13. The control effort cost function for various η in the case of $\alpha < 1$ and $\alpha > 1$.

7. Optimal choices

In overall in this article, two options are presented for designing the NDF controller which are based on the location of the cutoff frequency in regards with the targeted mode. The first option is choosing the cutoff frequency location before the targeted mode ($\alpha < 1$), and the second one is choosing the cutoff frequency location after the targeted mode ($\alpha > 1$).

The control effort for the case of $\alpha > 1$ is always lower than the case of $\alpha < 1$ (figure 13). Also, the performance index of the system when $\alpha > 1$, can be even much lower than the optimal value of the performance index of the system when $\alpha < 1$ (figure 2). These beneficial features may seem very tempting at the first glance. However, the drawback of choosing $\alpha > 1$, is the vibration magnifications after the targeted mode (figure 7(a)). This means that if there are some modes after the target mode in the system, the level of vibration in those mode will enlarge which is a big obstacle in choosing $\alpha > 1$. On the other hand, if the designer chooses $\alpha < 1$ with the optimal value of η, not only the performance of the controller on the targeted mode will be impressive, but also in higher modes of the system, there will be an appealing vibration reduction as well (figure 3(a)). This superior feature makes the condition of $\alpha < 1$ more appropriate and logical

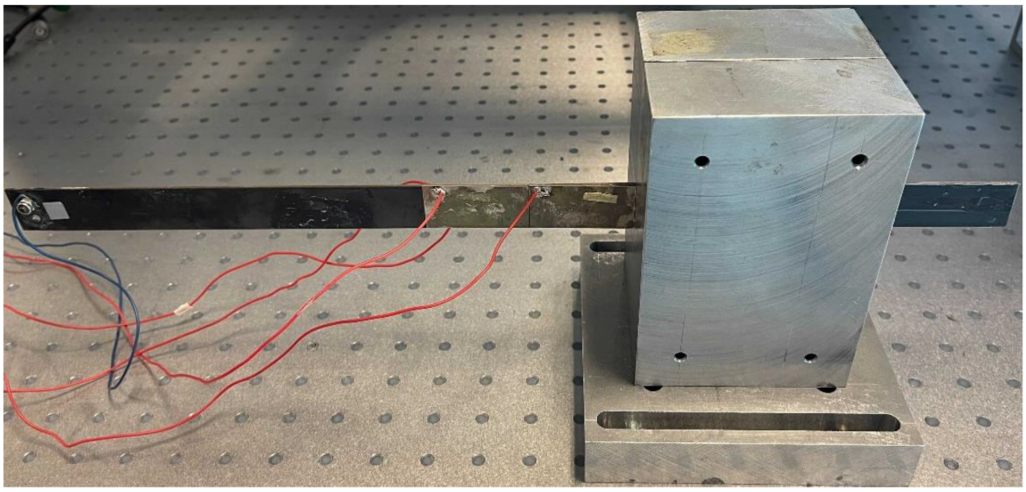


Figure 14. Experiment setup, cantilever beam with two pairs of piezoelectric patches.

choice. Therefore, it is strongly recommended to choose the cutoff frequency before the targeted mode with the presented optimal value of closed loop damping. In this way, not only the vibration mitigation level is impressive in the targeted mode, but also in close modes after the targeted mode, vibration level will decrease marginally.

There some cases that the choice of $\alpha > 1$ can be logical as well. When the frequency of disturbance input of the system is limited to bandwidth, in which only the targeted mode presents, then the choice of $\alpha > 1$ would be more appealing. Because not only this can increase the performance of the controller but also the control effort can be decreased impactfully.

8. Experimental validation

In this section, in order to validate the presented method, an experiment on a smart structure has been performed. For this purpose, a beam with two pairs of piezoelectric patches is used. One of pairs of piezoelectric patches is used as a controlling unit and the other one is used to measure the performance index of beam in controlled and uncontrolled condition. The experimental setup is presented in the figure 14. In figure 14, the second pair is used to close the loop and the first pair is used for performance index measurement. This means that in the first pair of patches, a chirp signal from 0 Hz to 200 Hz is injected to one patch as an excitation (or disturbance) signal and the displacement of beam is measured in the other patch. The length, width and thickness of this beam, which is made by steal, are 255 mm, 30 mm, and 3 mm respectively. Piezo 1, 2, 3 and 4 are standard PIC-225 patches, glued on each side of the beam. The length, width and thickness of patches are 50 mm, 30 mm, 0.75 mm respectively. The first pair is located next to the clamped boundary condition and the second pair is located after the first pair on the beam. One side of all patches are connected to the beam and the beam is connected to ground. This means that one electrode of all patches is connected to ground. The other electrode (or side) of each patch is connected to cable which is connected to the Microlabbox. MicroLab-Box is used to inject excitation signal to the actuator and measure the sensor signal form the setup at a sampling frequency of 10 kHz. Signal conditioner is also utilized during the experiment on the sensor output of piezoelectric patches for omitting sensor noise.

Table 3. Controller's parameters.

ξ_c	**1.02**
ω_c	101.5
β	285.7

The first pair of piezoelectric patches is used for measuring the performance index and the second pair is used for controlling the structure's displacement. The first mode of beam has natural frequency of 142 rad s^{-1} and it has damping of 0.0097 and this mode is targeted to damp by NDF. Based on the presented method an NDF controller is designed the damp the first bending mode of the beam. The controller's parameters are presented in table 3. The results of this experiment is presented in figure 15.

Obviously, the result shows that the first bending mode of beam is completely damped by using NDF controller. This easily shows the effectiveness and power of NDF controller. The second point is the fact that the second mode of the beam is slightly damped as well, even though the controller was designed to damp the first mode. This shows that the NDF can be effective with the modes close to the targeted mode. The third point which is worth to mention is the fact that the controller has no side effect like vibration magnification in other frequencies which is appealing for damping a certain band of frequency. This is because of the fact that NDF is working as a

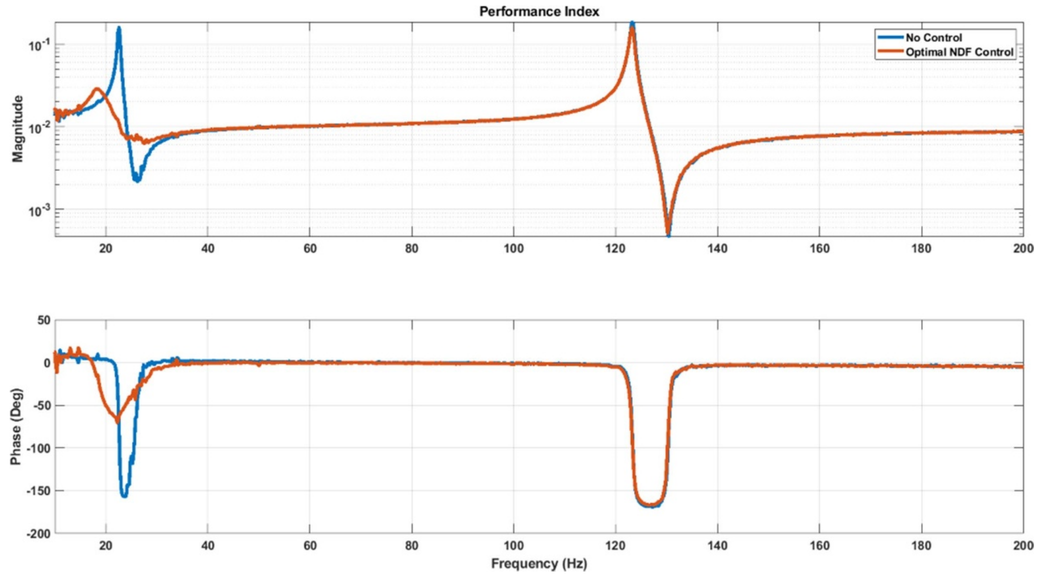


Figure 15. Performance index of beam in controlled and uncontrolled condition.

bandpass filter and H_2 method is used for tuning the controller parameters.

9. Conclusion

In this article, a direct method is presented to design NDF controller optimally. For this purpose, maximum damping method and H_2 optimization method is utilized to tune the parameters of the NDF controller for SODF system. NDF is a controller which works as a band-pass filter, cutting off the control action far from the natural frequencies associated with the controlled modes and reducing the spillover effect. Since it is a bandpass filter, it can effectively control the lower and higher frequency. The results shows that the NDF controller can easily reduce vibration of targeted mode effectively. It is recommended that the cutoff frequency of the controller is chosen before the targeted mode and in this case, the controller can be effective for a band of frequency. The effect of changing the NDFs parameters of the on the system response are evaluated in detail. Also, the control effort for various disturbance input has been calculated and compared with performance of controller. The designed aim is to have higher performance with lower control effort.

Data availability statement

No new data were created or analyzed in this study.

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