Non-Markovian Quantum Dynamics in Strongly Coupled Multimode Cavities Conditioned on Continuous Measurement

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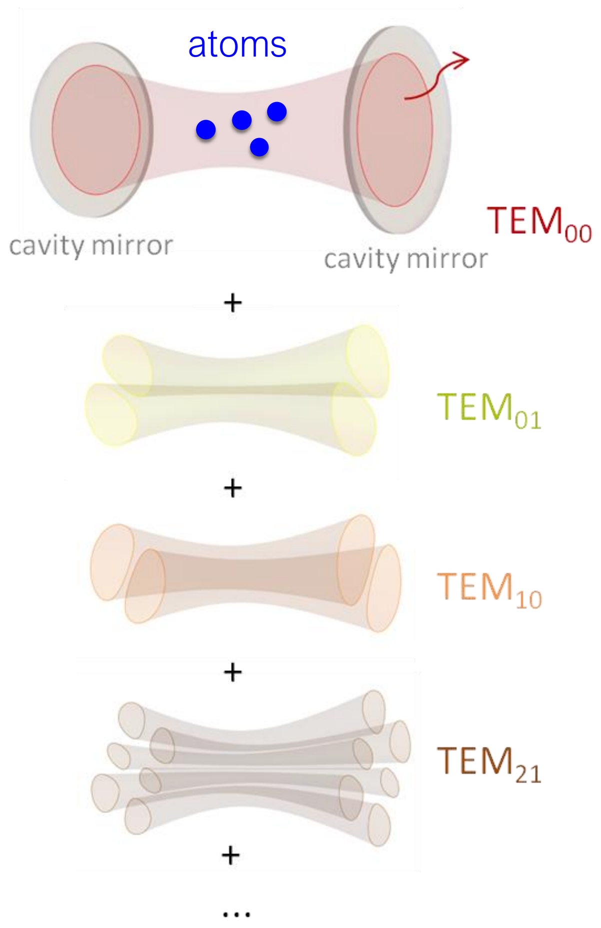
Atoms in (multi)-mode cavities

Motivation

- Ideal platforms to study many-body physics out-of-equilibrium
- Cavity modes mediate interactions between atoms, allowing for the realisation of various spin models.

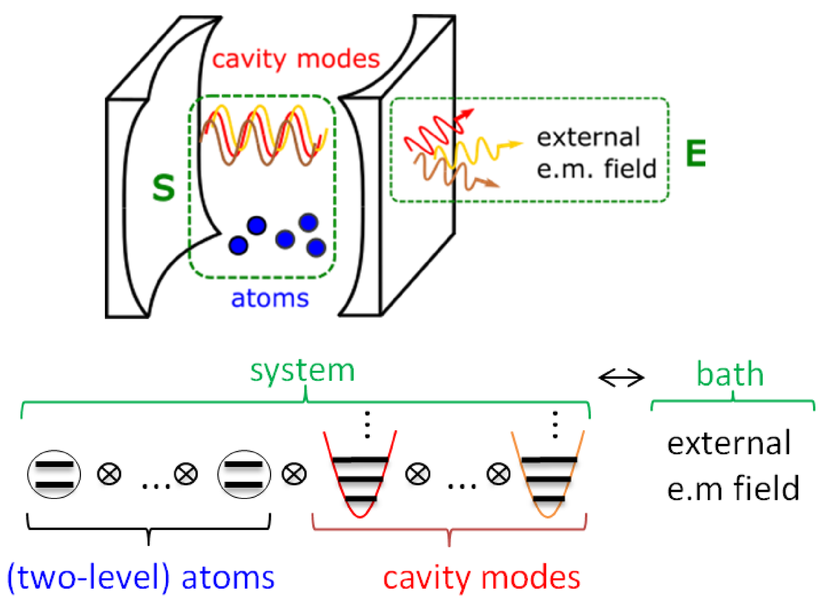
Single-mode cavity → all-to-all interactions
Multi-mode cavity → richer interactions
[Kollár, NJP **17**, 043012 (2015)]
[Vaidya, PRX **8**, 011002 (2018)]



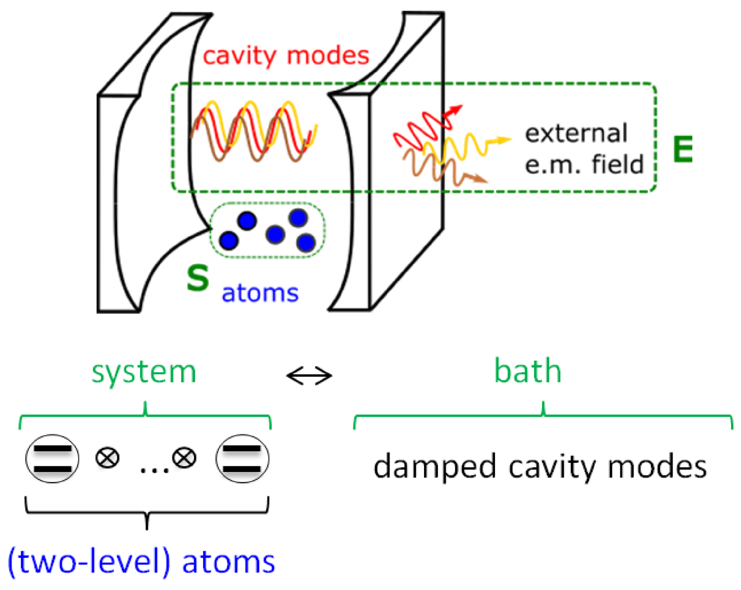
Applications :
- Quantum liquid crystalline states [Nat. Phys. **5**, 845 (2009)]
- Frustration and glassiness [PRL **107**, 277201 (2011)]
- Dynamical gauge fields [PRL **118**, 045302 (2017)]
- Associative memories [PRX 11, 021048 (2021)]
- NP hard problems (N-queen) [Quantum **3**, 149 (2019)]
- Scrambling [PRL **123**, 130601 (2019)]
- ...

Theoretical challenges

Two open system descriptions



(+) standard Markovian bath
(-) large system Hilbert space



(+) smaller system Hilbert space
(-) more complicated bath
(bath = damped harmonic oscillators)

→Need for atom-only descriptions, which are usually non-Markovian

Example: Single-mode driven-dissipative (Z_2) Dicke model

« Atoms+cavity » dynamics governed by

$$\partial_t \rho = -i[H_{\mathrm{Dicke}}, \rho] + \kappa \mathcal{L}[\hat{a}]$$

$$H_{\mathrm{Dicke}} = \omega_0 \hat{S}^z + \omega \hat{a}^\dagger \hat{a} + 2g\left(\hat{a} + \hat{a}^\dagger\right)\hat{S}^x$$

What is the atom-only description that captures the superradiant phase transition [FD et al., PRA **99**, 033845 (2019)] ?

M.E. Derivation using: • Born approx. : $\rho_{\mathrm{tot}}(t) \approx \rho(t) \otimes \rho_B$

• 2[nd] perturbation theory :

$$\dot{\rho} = -\int_0^{t\,\to\,\infty} dt' \mathrm{Tr}_B\left([H_1(t), [H_1(t'), \rho_{\mathrm{tot}}(t' \to t)]]\right)$$ Markov approx.

with int. Hamilt. $H_1(t) = gX(t)[S^+(t) + S^-(t)]$

$$X(t) = a(t) + a^\dagger(t)$$
$$S^\pm(t) = S^\pm e^{\pm i\omega_0 t}$$

• Bath (i.e. cavity) corr. function: $\mathrm{Tr}_B(X(\tau)X(0)\rho_B) = e^{-i\omega\tau - \kappa|\tau|}$

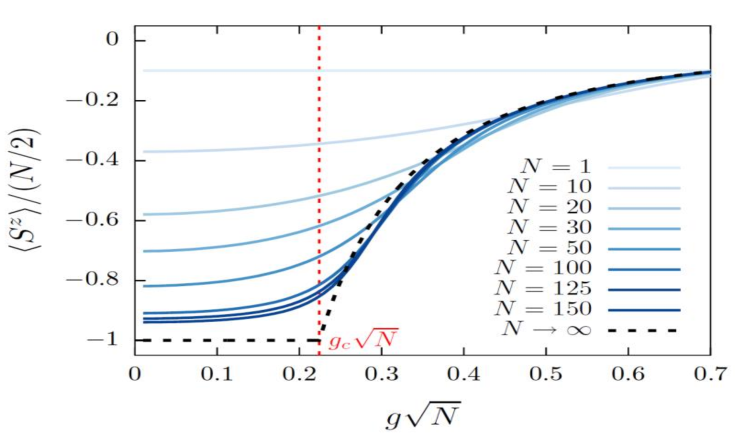
→ 2[nd] order Redfield master eq. (atoms only):

$$\begin{aligned}\dot{\rho} = &- i\,[\omega_0 S^z, \rho] - \\ &\{Q_+\left(S^+S^+\rho - S^+\rho S^+\right) + Q_-\left(S^+S^-\rho - S^-\rho S^+\right) \\ &Q_+\left(S^-S^+\rho - S^+\rho S^-\right) + Q_-\left(S^-S^-\rho - S^-\rho S^-\right) \\ &Q_-^*\left(\rho S^+S^+ - S^+\rho S^+\right) + Q_-^*\left(\rho S^+S^- - S^-\rho S^+\right) \\ &Q_+^*\left(\rho S^-S^+ - S^+\rho S^-\right) + Q_+^*\left(\rho S^-S^- - S^-\rho S^-\right)\}\end{aligned}$$

$$Q_\pm = \frac{g^2}{\kappa + i(\omega \pm \omega_0)}$$

Can we perform two additional (standard) approximations ?

- secular approximation (kill terms with two S^+ or two S^-) ? → No !
- large-detuning limit (set $Q_\pm = Q$) → No !



Secular approx.	Large detuning approx.	→ Superradiant transition ?
Yes	Yes	→No
Yes	No	→No
No	Yes	→No
No	No	→ Yes

→ correct description via a 2[nd] order Redfield master eq.
(i.e., a Born-Markov master eq. **without** the secular approx.)

Non-Markovian dynamics conditioned on measurement

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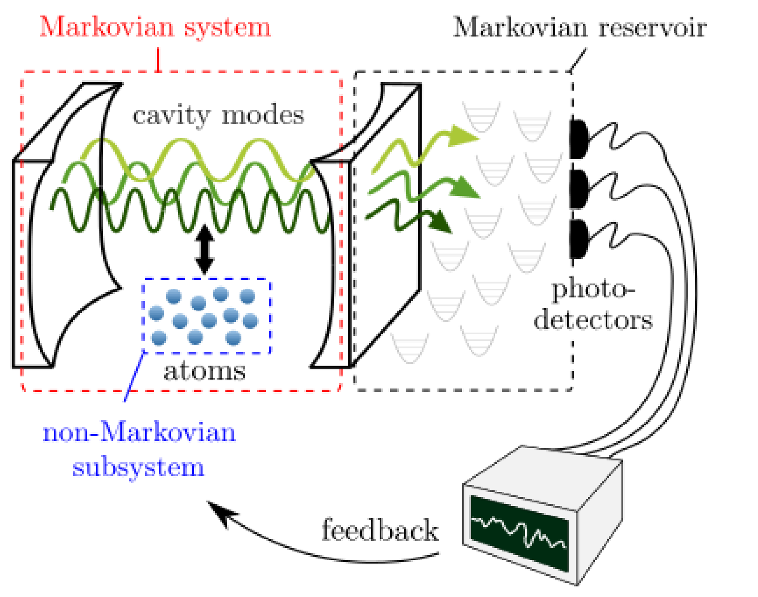
- What can we learn about a non-Markovian system such as atoms in cavity (or circuit) QED via output light measurement ?



Single-mode case: homodyne measurement of the output light

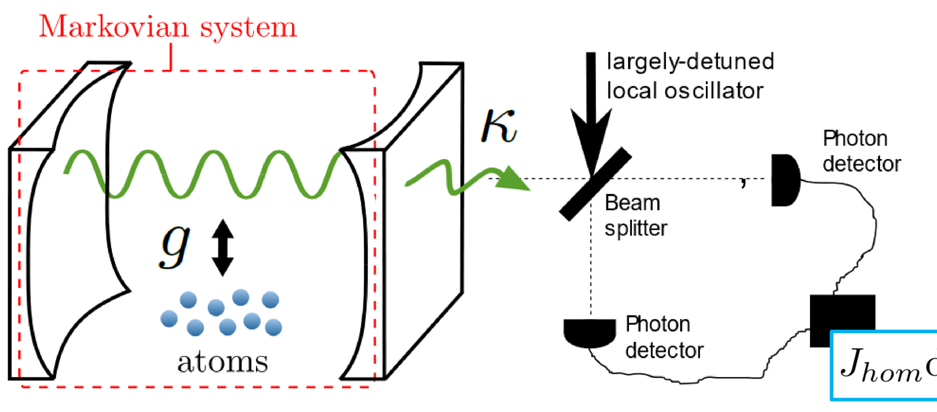
$H_{AC} = H_A + \Delta a^\dagger a + g(aL^\dagger + a^\dagger L)$

« atoms + cavity » Markovian stochastic Schrödinger eq.

$$\mathrm{d}|\psi_{AC}\rangle = \Big(-\mathrm{i}H_{AC} - \kappa a^\dagger a + \kappa\langle a + a^\dagger\rangle a\Big)|\psi_{AC}\rangle \mathrm{d}t + \sqrt{2\kappa}a|\psi_{AC}\rangle \mathrm{d}W + |\psi_{AC}\rangle \mathrm{d}\mathcal{N}.$$

$$\mathrm{d}\mathcal{N} = -\tfrac{\kappa}{4}(\langle a + a^\dagger\rangle)^2 \mathrm{d}t - \tfrac{1}{2}\sqrt{2\kappa}\langle a + a^\dagger\rangle \mathrm{d}W$$

Wiener increment: $E[\mathrm{d}W] = 0 \quad \mathrm{d}W^2 = \mathrm{d}t$

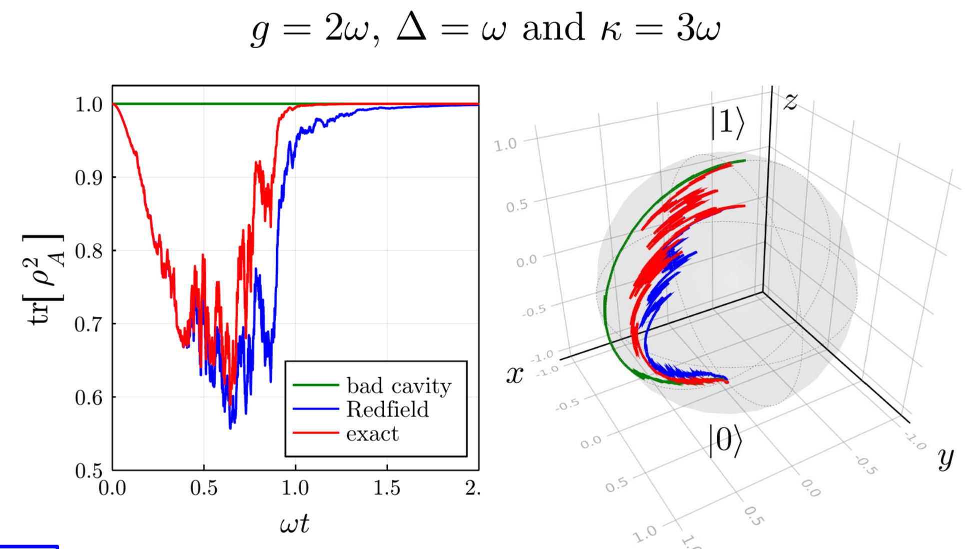


$J_{hom}\mathrm{d}t = \sqrt{2\kappa}\langle a + a^\dagger\rangle \mathrm{d}t + \mathrm{d}W$ Homodyne current

Different atom-only descriptions:

- Example for the Jaynes-Cummings model

$$H_A = \frac{\omega}{2}\sigma_z, \qquad L = \sigma_-$$



Bad cavity limit ($\kappa \gg \Delta, \omega$)

$$\mathrm{d}\rho_A \approx -\mathrm{i}[H_A, \rho_A]\mathrm{d}t + \frac{g^2}{\kappa}\left(2L\rho_A L^\dagger - \{L^\dagger L, \rho_A\}\right)\mathrm{d}t + \frac{\mathrm{i}g\sqrt{2}}{\sqrt{\kappa}}\left((L - \langle L\rangle)\rho_A + \rho_A(L^\dagger - \langle L^\dagger\rangle)\right)\mathrm{d}W\,.$$

Redfield case (weak coupling approximation)

$$\mathrm{d}\rho_A \approx \Big(-\mathrm{i}[H_A, \rho_A] + [\bar{L}\rho_A, L^\dagger] + [L, \rho_A\bar{L}^\dagger]\Big)\mathrm{d}t + \frac{\mathrm{i}}{g}\sqrt{2\kappa}\Big(\left(\rho_A(\bar{L}^\dagger - \langle\bar{L}^\dagger\rangle) - (\bar{L} - \langle\bar{L}\rangle)\rho_A\right)\Big)\mathrm{d}W. \qquad \bar{L}(t) = \int_0^t \mathrm{d}s g^2 e^{-(\mathrm{i}\Delta + \kappa)(t-s)} e^{-\mathrm{i}H_A s} L e^{\mathrm{i}H_A s}$$

Exact cHEOM (conditioned Hierarchical Equations Of Motion)

$$\begin{aligned}\mathrm{d}\rho_A^{(n,m)} = &\Big(-i[H_A, \rho_A^{(n,m)}] - [(n-m)i\Delta + (m+n)\kappa]\,\rho_A^{(n,m)} \\ &+ g^2\left(nL\rho_A^{(n-1,m)} + m\rho_A^{(n,m-1)}L^\dagger\right) + \left[\rho_A^{(n+1,m)}, L^\dagger\right] + \left[L, \rho_A^{(n,m+1)}\right]\Big)\mathrm{d}t \\ &+ \Big(-\sqrt{2\kappa}\langle X\rangle\rho_A^{(n,m)} + \frac{i}{g}\sqrt{2\kappa}\left(\rho_A^{(n,m+1)} - \rho_A^{(n+1,m)}\right)\Big)\mathrm{d}W,\end{aligned}$$

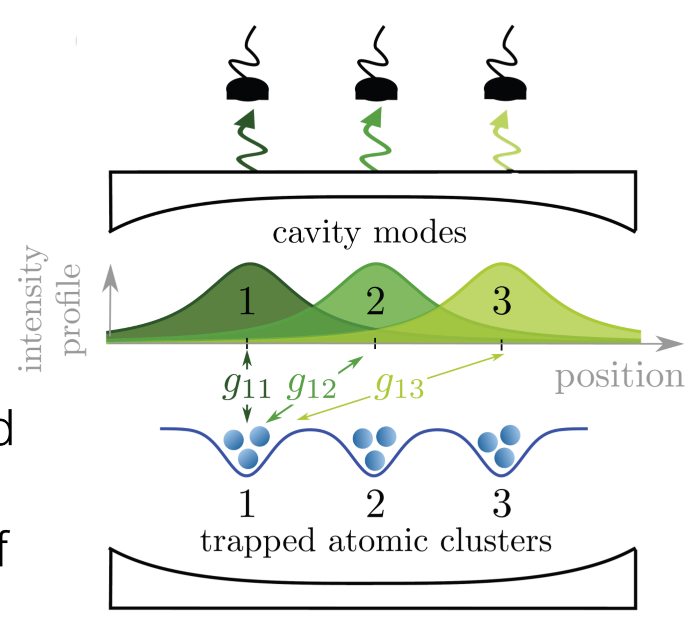
→ Exact conditioned atomic dynamics described by a mixed state

Multi-mode case: Information gain from output measurement

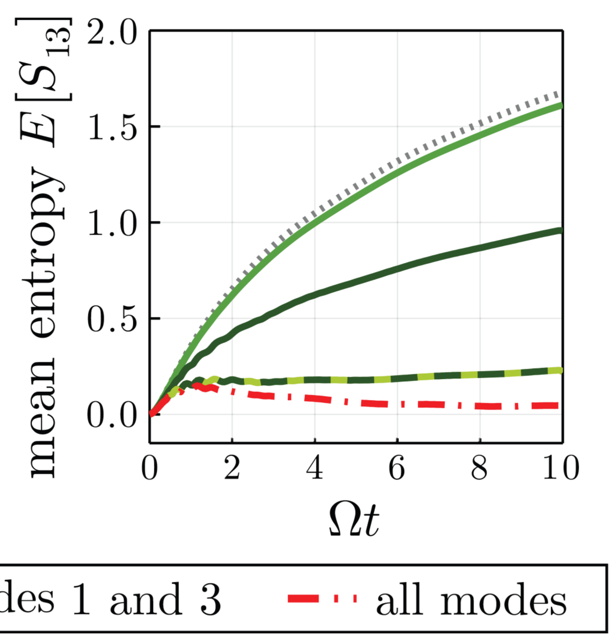
$$H = \Omega\sum_{i=1}^{N_{\mathrm{clusters}}} J_i^z + \Delta\sum_{k=1}^{N_{\mathrm{modes}}} a_k^\dagger a_k + \sum_{ik} g_{ik} J_i^x (a_k + a_k^\dagger)$$

$\Delta = \Omega/2$
$\kappa = 2\Omega$

$$g = \begin{pmatrix} 0.4 & 0.115 & 0.003 \\ 0.115 & 0.4 & 0.115 \\ 0.003 & 0.115 & 0.4 \end{pmatrix}\Omega$$



The atomic dynamics and the information gain depend on the number of monitored modes:



- Von Neumann entropy : $S[\rho_A] = -\mathrm{tr}\rho_A \ln\rho_A$
- Average information gain in a single experimental run : $S[E[\rho_A]] - E[S[\rho_A]]$
- $E[S_{13}]$: average entropy of the reduced state of the atomic clusters 1 and 3

→ Information gain and entanglement controlled by monitoring

Conclusion & Outlook

- Need for atom-only descriptions beyond standard approx. in cavity (or circuit) QED [Damanet *et al.*, PRA **99**, 033845 (2019); Palacino & Keeling, PRR **3**, 032016 (2021)]
- Derivation of an exact stochastic atom-only description with a measurement interpretation (cHEOM), with applications to e.g. quantum feedback control beyond Markov [Link *et al.*, PRX Quantum **3**, 020348 (2022)]
- **Outlook**: e.g. combining cHEOM with MPO to capture conditioned non-Markovian many-body dynamics (unconditioned case: see [Flannigan *et al.*, PRL **128**, 063601 (2022)])

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