

Groundwater modeling: methodological and conceptual choices

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References:

Dassargues A., 2018. Hydrogeology: groundwater science and engineering, 472p. Taylor & Francis CRC press (Chapter 12) Dassargues A. 2020. Hydrogéologie appliquée: science et ingénierie des eaux souterraines, 512p. Dunod (Chapitre 12)

Groundwater modeling: methodological and



conceptual choices

- ► Terminology
- General methodology
- Conceptual model
 - Time scale
 - Spatial scale, extension and dimensionality
 - Initial values
 - Boundaries
 - Parsimony vs complexity
 - Software choice and numerical model main characteristics
 - Modeling errors
- Data needs
- Implementation, calibration and sensitivity
- Evaluation/reporting
- References



A model ?

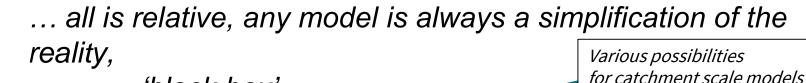
• a model is a tool for simulating reality in a simplified form

(Wang and Anderson 1982)

- a mathematical description of the physical reality can already be considered as a mathematical model
- a mathematical model can be solved or computed analytically or numerically
- 'Any type of modeling includes subjective decisions and simplifying assumptions because the true complexity of a natural system is never fully represented and data about properties and variables include uncertainties' (Fienen 2013)



Black-box model: a set of mathematical equations is developed by empirical or statistical fitting of parameters to reproduce historical records of the main variable ('data driven' model) (Anderson et al. 2015)



- 'black box'
- 'grey box'

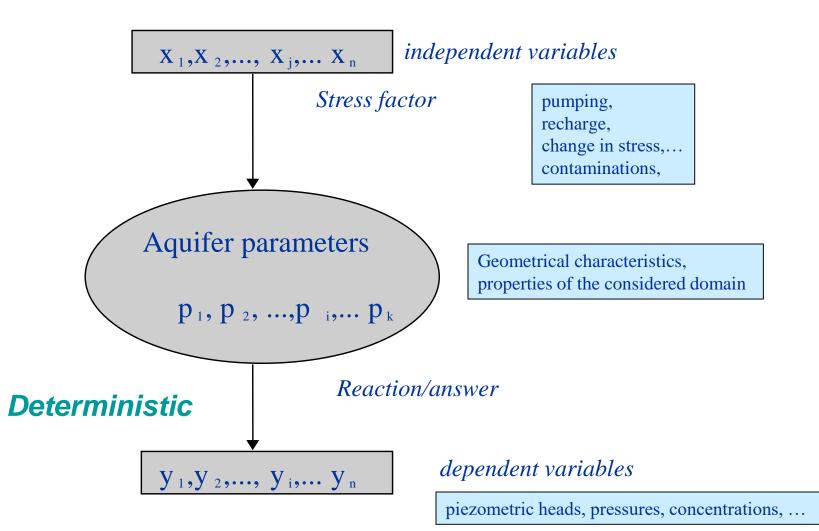
(Brouyère et al. 2011)

- physically based but not spatially distributed
- spatially distributed but not physically based
- spatially distributed and physically consistent

the lack of precision in the representation of reality strongly depends on the scale at which the problem is considered (Dassargues 1998)

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Stochastic/probabilistic using Monte Carlo multiple simulations, the same schema can be used with multiple equally likely sets of parameters, independent variables, and dependent variables. (Konikow and Mercer, 1988, Dassargues 2018)



Deterministic models versus Stochastic/Probabilistic models:

Deterministic Model: the answer (reaction) of the simulated system, under a set of considered stress factors, is unique and defined in a pure deterministic process (even if the new simulated scenario is out of the stress range of the calibration)

Stochastic/Probabilistic Model: in addition, the possible uncertainties on the parameters, on the initial conditions, on the BC's, ...

- combined resolution (can be very heavy)
- most often, n resolutions of n equiprobable cases, and then statistics for estimating results dispersion and confidence intervals

➡ allows to take into account 'soft-data'



...more about stochastic modeling

Sources of uncertainty are multiple and of different types:

1) associated to subjective conceptual choices made to simplify the reality into a model

(Cooley 2004, Rojas et al. 2008, Wildemeersch et al. 2014 and many others)

2) embedded in parameters data uncertainty

(de Marsily et al. 2005, Brunner et al. 2012 and many others)

3) highly parameterized models, where parameters value determination represents an ill-posed problem (among others: Carrera and Neuman 1986a, Moore and Doherty 2005, Hill and Tiedeman 2007, Beven 2009)

4) from initial and boundary conditions

...more about stochastic modeling



For predictions, the uncertainty of the stress factors linked to each simulated scenario can be integrated

> (e.g. Rojas et al. 2010c, Sulis et al. 2012, Goderniaux et al. 2015 and many others)

A formal stochastic formulation in the partial differential equations for flow and solute transport can be used

(see many books, among others: Dagan 1989, Gelhar 1993, Kitadinis 1997, Zhang 2002, Rubin 2003)

In practice, the most commonly-used : Monte Carlo simulations with multiple equally-likely realizations of the model parameter sets that are conditioned on the existing data

(e.g. Vecchia and Cooley 1987, Deutsch and Journel 1998, Huysmans and Dassargues 2006, Dassargues et al. 2006, Tonkin et al. 2007 and many others)

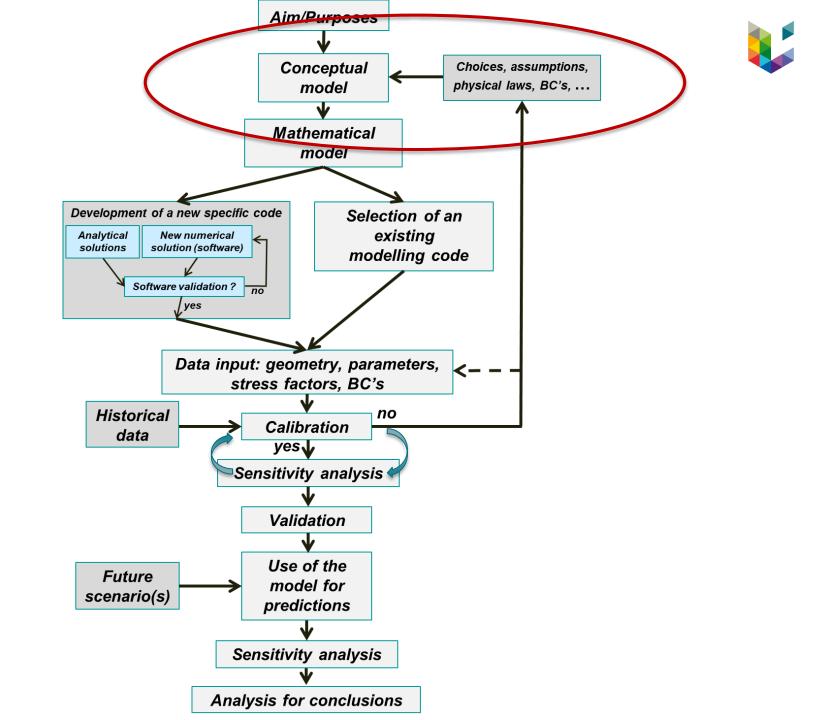
Multiple simulations multiple responses statistically treated assuming (most often) Gaussian behavior

- results in statistical distributions
- probability distribution for each response based on the statistical distribution of data (including 2^{ary} data, parameters
- and stress factors) (Huysmans et al. 2008, Huysmans and Dassargues 2009, 2011) 8

Different steps of a groundwater numerical model :

- clear definition of the final aim
- conceptual model
- mathematical model
- numerical model, development or choice of an existing code
- data input
- calibration and then validation
- 🕈 🖕 sensitivity analysis

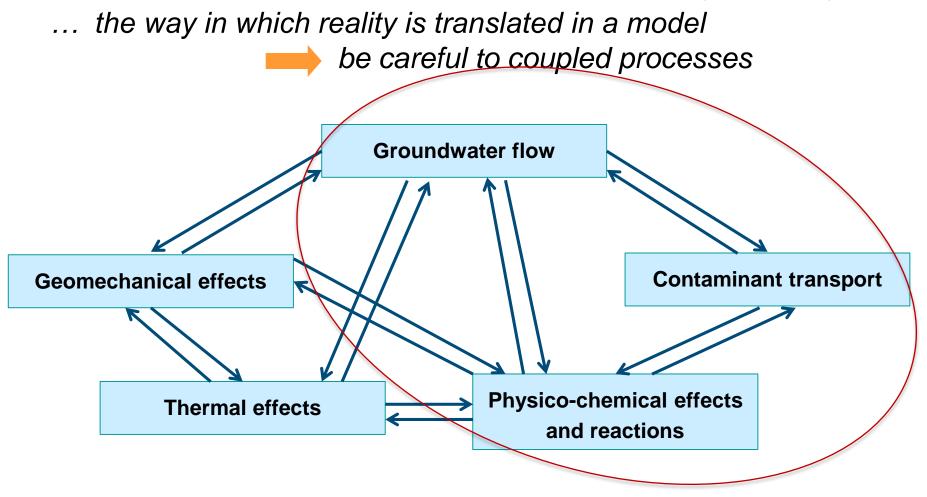
- application (use) of the model
- results analysis with regards to the initial question
- redaction of a report



General methodology: conceptual model



(Anderson et al. 2015)



(Rosbjerg and Madsen 2005, Dassargues 2018, Dassargues 2020)



fundamental step where the main assumptions of the modelling are chosen

- steady state or transient analysis
- scale level
- model dimensionality: 1D, 2D vertical, 2D horizontal, quasi-3D, 3D
- boundary geometry and location, boundary conditions
- geological media (porous / fissured / double porosity / ...)
- homogeneity/heterogeneity, isotropy/anisotropy, properties changes in function of time
- *initial conditions within the domain*

... 'poorly justified assumptions can potentially discredit an entire groundwater model'

. . .



- Steady state

- it does not exist in the reality
- $\triangle Res = 0$ and $Q_{in} = Q_{out}$
- when piezometric heads and fluxes can be considered as relatively stable
- when transient data are lacking (first guess, ...)
- with data allowing to deduce a 'mean behaviour' of the system : $R_{mean}, Q_{mean}, H_{mean}...$
- for starting with a problem, before going to transient conditions
- adopted for simplification, considering extreme conditions and being on the 'security side'

can be difficult to converge when data are not realistic or when non linearities are not considered

→ transient simulation with constant conditions + time step increasing

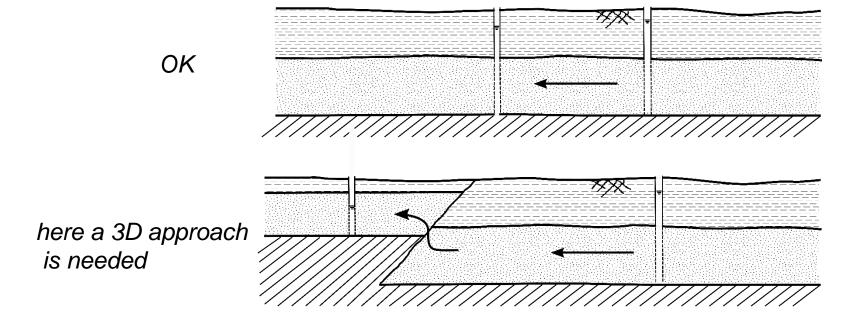


- Transient simulation

- requires generally more data
- takes more CPU time
- sometimes needed in function of the context
 - transient character of the gw flow conditions
 - transient transport (it is generally the case) on a supposed steady gw flow

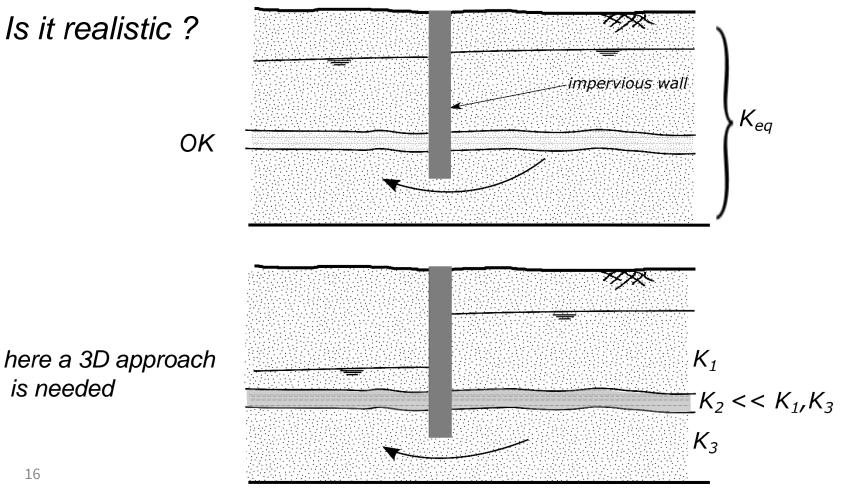
- 1D, ... (often in the partially saturated zone)
- 2D horizontal models:

groundwater flow considered as mostly horizontal Is it realistic ?

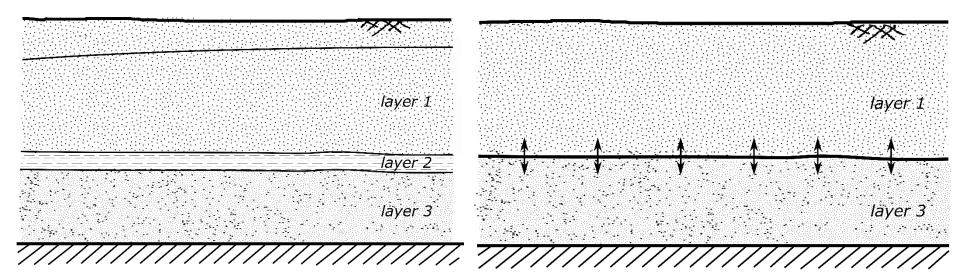


2D horizontal models:

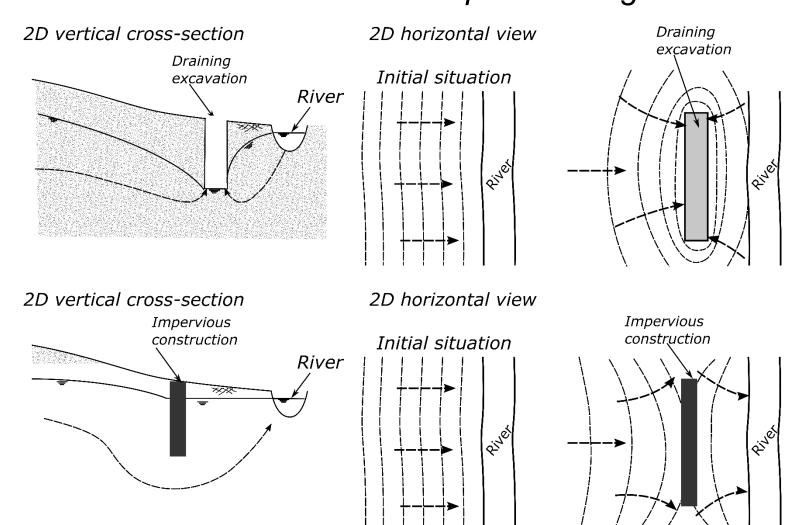
groundwater flow considered as mostly horizontal



- pseudo-3D or quasi-3D
 - multi-layers system with 2D gw flow in each of them
 - strictly vertical flow in aquitards calculated by applying the Darcy's law



• 2D vertical models: OK but gw flow not in the considered plane is neglected





- Initial conditions: initial values of the main variable (generally piezometric head h) in each node of the mesh
 - 1st used values for a steady state computation (1st approximation)
 - influence the convergence process and the CPU time for reaching the steady state equilibrium
 - if the convergence is not ideal, results can be affected
 - actual initial state of the system at time to for starting a transient simulation
 - if h_i are not consistent with BC's and stress factors, then
 Δh calculated can be completely strange
 - very often: starting with a steady state and continuing with a transient simulation



Boundary conditions: to be discussed

see next chapters about groundwater flow simulations and solute transport simulations including various interactions

(e.g. river groundwater interactions)

(Goderniaux et al. 2009)



(Hill 2006, Gómez-Hernández 2006, Wildemeersch 2012)

Parsimony or complexity: merits and pitfalls

- any process-based model becomes complex and remains uncertain
- complexity could be considered through the use of stochastic approaches conditioned on the available data

(Beven and Freer 2001, Gómez-Hernández 2006, Beven and Binley 1992, Hoeting et al. 1999, Neuman 2003, Rojas et al. 2008 and 2010a)

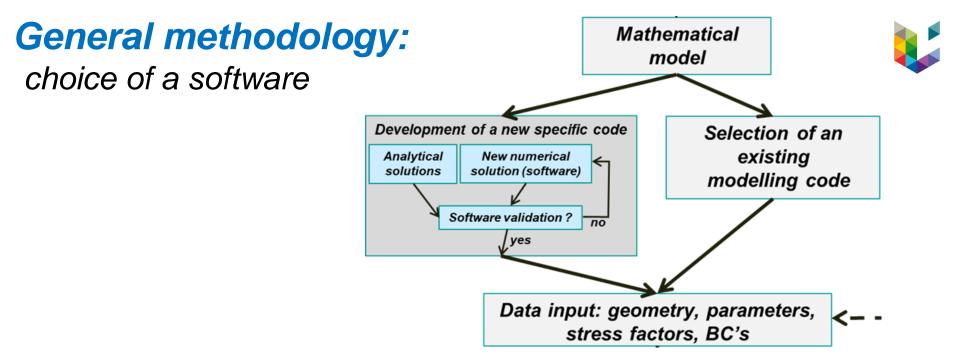
- complexity could be introduced in a stepwise fashion, from simple to complex
 each chosen hypothesis can be tested
- preserve refutability and transparency

modelled processes remain understandable

(Ward 2005, Schwartz et al. 2017, Kurtz et al. 2017)

 to determine if a simple model provides reliable results, its results should be compared to results from a more complex one

(de Marsily et al. 2005)



- if a new code is developed: it must be validated for the same kind of processes
- choose your code in function of your conceptual model
- many existing codes for different purposes

Do not use a hammer to drive a screw or do not use a screwdriver to drive a nail !



numerical models main characteristics

- study area represented by a mesh of elements or cells to which nodal points (or nodes) are associated
- in those subdomains (cells, elements, volumes) the medium is assumed homogeneous
- the continuous variable by a discrete variable (the solution will be found at discrete points of the spatio-temporal domain)
- a finer spatial discretization means a better approximation of the solution
- partial differential equations are replaced by a system of algebraic equations
- the state variables are the unknown
- a solution obtained for each specified set of parameter values

^{• • • •}



numerical models main characteristics (2)

- iterative procedures more efficient than direct matrix inversion methods
- solution = values at discrete locations in the simulated domain generated from the spatial discretization
- if transient problem, the time scale is also discretized in time steps
- solution at the n discrete nodes and for all time steps, then interpolations at any location in space and time

(Wang and Anderson 1982)



numerical models main characteristics (3)

For an iterative solution,

Convergence = computed values converge towards the exact values, in particular when the spacing between nodes is decreasing

Stability = the numerical errors (truncation + roundoff) should not increase in the solution computation within one time step or from a time step to the next ones

(Volume, mass or energy) **conservation** is preserved (i.e. the numerical solutions must preserve and satisfy balance equations at the local as at the global scales)

(Bear and Cheng 2010, Diersch 2014)



numerical models main characteristics (4)

Physical consistency is dependent on the conceptual choices to simplify the reality for an efficient modelling

Numerical consistency is ensured if truncation errors tend to zero for decreasing mesh increments and time steps

Accuracy = describing the (lowest as possible) modeling errors (truncation and roundoff errors + conceptual and calibration errors)

Resolution = the smallest increment or decrement of the considered variable value that can be calculated by the model

(Paniconi and Putti 2015)

REV concept = considered volume of geological medium for quantifiying properties at the appropriate scale (by averaged equivalent values)

(Bachmat et Bear 1986, Bear et Verruijt 1987, de Marsily, 1986, Dagan, 1989)



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a very useful concept that implicitly assumes a continuum and a porous medium (Molz 2017)

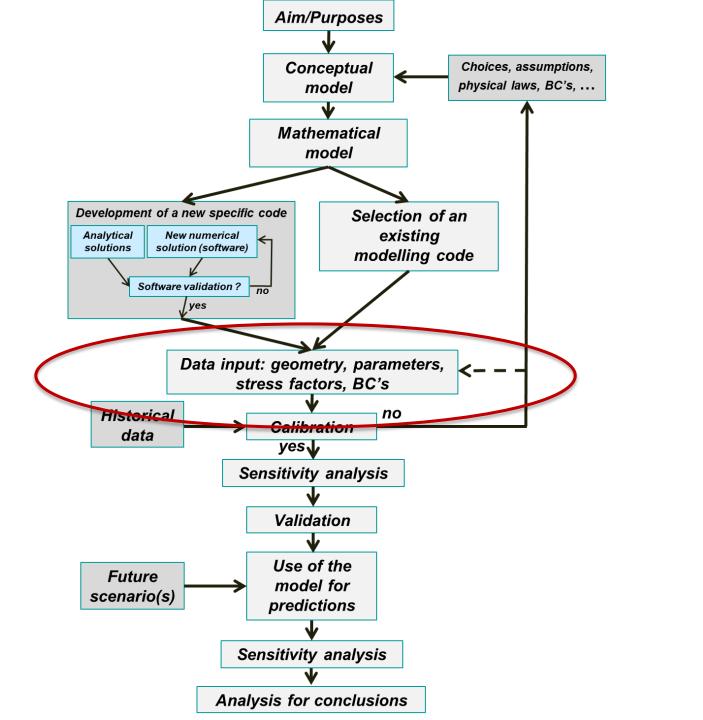
General methodology: modeling errors



- conceptual errors (linked to main conceptual choices, systematic)
- approximation errors (linked to the chosen spatial/temporal resolution)
- numerical errors (linked to the numerical method adopted for solving the system of equation, truncation and roundoff errors, ...)
- measurement errors (implicitly introduced during the calibration process, see next section)

Scale issue: measurement scale is

very different than model scale







4 kinds of data:

- 1D, 2D or 3D geometry of the modelled zone (geology, topography, hydrology, concerned problem, scale, ...)
- values for the properties (parameters) playing a role in the modeled processes (i.e. for gw flow: K and S_s or T and S, for solute transport n_e, a_L, a_T, R, ...)
- stress factors applied on the modelled domain (i.e. for gw flow: recharge, pumping, injections, for solute transport mass injection or removal)
- historical (measured) data concerning the main problem variable (i.e. for gw flow: piezometric heads, for solute transport: concentrations) or its first derivative (i.e. for gw flow: flow rates or fluxes, for solute transport advective or dispersive mass fluxes) ... distributed data in the domain that will be used for calibration (or inverse modeling) procedure



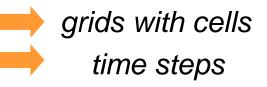
model implementation discretisation, parameters, stress-factors and historical data

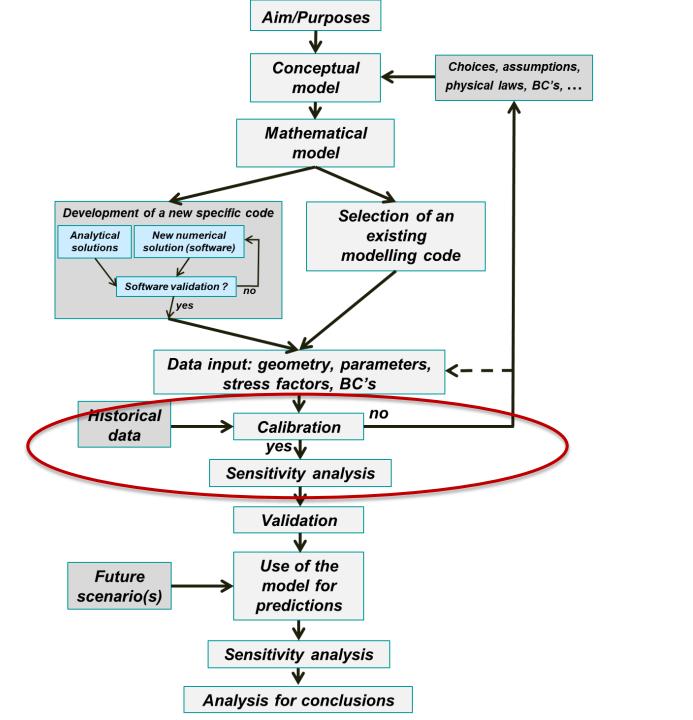
Conceptual model



translated in a usable form for modelling:

- Spatial discretisation
- Time discretisation
- Boundary Conditions (BC's)
- Sink /source terms
- Initial values for the main variable
- Initial values for possible useful other state variables





General methodology: model calibration



Change (adaptation) of the parameters values and distribution ... for a better simulation of the reality ... this reality is considered as represented by

historical data sets

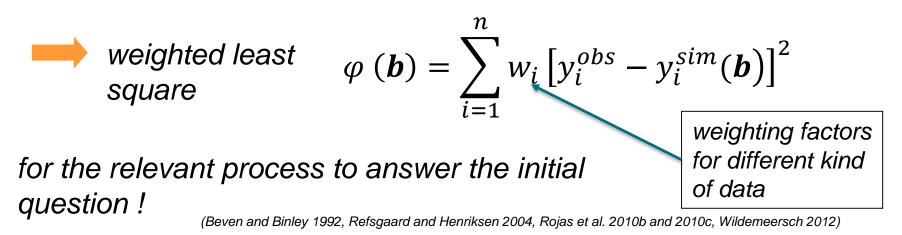
How to quantify objectively the good fit ?

accounting for the discrepancies between observed and computed values of the main variables and/or one or more of their derived variables

Different steps :

- Objective function formulation (be careful: any objective function is subjective !)
- Sensitivity analysis
- Change in parameters values (inverse problem);
- Validation using another data set (most often another time
- ³² period, for transient modelling)

General metodology: performance criteria for calibration



If the aim is to simulate the baseflow evolution in a watershed:

$$\varphi_{NS}(\boldsymbol{b}) = 1 - \frac{\sum_{t=1}^{nt} [q_t^{obs} - q_t^{sim}(\boldsymbol{b})]^2}{\sum_{t=1}^{nt} [q_t^{obs} - \mu^{obs}]^2} \in]-\infty, 1]$$

(Nash and Sutcliffe 1970, Wildemeersch 2012)

General methodology: model calibration = inverse modeling



could be helpful to gain a full understanding of the physical behavior of the simulated system

- manual trial-and-error procedure
- automatically non-linear regression methods = inverse modelling

more efficient to produce useful statistics

- main issues : the non-uniqueness of the solution
- introduce prior information on the parameter values to avoid as far as possible an ill-posed inversion

(Carrera et al. 2005, Hill and Tiedeman 2007, Carrera and Neuman 1986b)

General methodology: sensitivity analysis = calibration tool



the amount the simulated value would change given a change in the parameter value

simple sensitivities

the amount the simulated value would change given a 1% change in the parameter value

- dimensionless scaled sensitivities (dss)
- composite scaled sensitivities (css)

the importance of observations as a whole to a single parameter

(Hill 1992, Anderman et al. 1996, Hill et al. 1998, Hill and Tiedeman 2007)

(example in Goderniaux et al. 2015)

- calculated using inverse modeling codes as PEST and UCODE
- + possible use of pilote points (Doherty 2005, Skahill and Doherty 2003 & 2005, Poeter et al. 2005)
- + parameter correlation coefficients (example in Batlle Aguilar et al. 2009)
- + USE of other data (e.g. temperature,

geophysical data)

the degree of correlation between couple of parameters and/or stress factors

(Klepikova et al. 2016)

(Rentier et al. 2002)

General methodology: evaluation & reporting



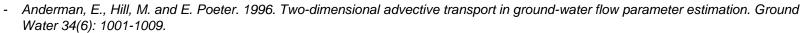
very important to analyse and evaluate the reliability of model results and adopted conceptual choices with regards to the <u>question to be answered</u>...

Reporting

modelling study realised step by step ... these steps must be described in the final report to establish clearly the reliability of the results despite the simplifying assumptions of the conceptual model

the reader must be able to understand the justification of the conceptual choices and the rigour of the followed approach

References (1)



- Anderson, M.P., Woessner, W.W. and R.J. Hunt. 2015. Applied groundwater modeling Simulation of flow and advective transport. Academic Press Elsevier.
- Bachmat, Y. and J. Bear. 1986. Macroscopic modelling of transport phenomena in porous media, part 1: The continuum approach. Transport in Porous media (1) 213-240.
- Batlle-Aguilar, J., Brouyère, S., Dassargues, A., Morasch, B., Hunkeler, D., Hohener, P., Diels, L., Vanbroekhoeven, K., Seuntjens, P. and H.Halen. 2009. Benzene dispersion and natural attenuation in an alluvial aquifer with strong interactions with surface water. Journal of Hydrology 361: 305-317.
- Bear, J. and A.H.D. Cheng. 2010. Modeling groundwater flow and contaminant transport. Springer.
- Bear, J. and A. Verruijt. 1987. Modeling groundwater flow and pollution. Dordrecht: Reidel Publishing Company.
- Beven, K.J. 2009. Environmental modelling: an uncertain future? An introduction to techniques for uncertainty estimation in environmental prediction. Routledge.
- Beven, K. and A.M. Binley. 1992. The future of distributed models: model calibration and uncertainty prediction. Hydrol. Process 6: 279-298.
- Beven, K. and J. Freer. 2001. Equifinality, data assimilation, and uncertainty estimation in mechanistic modelling of complex environmental systems. Journal of Hydrology 249: 11-29.
- Brouyère, S., Wildemeersch, S., Orban, Ph., Leroy, M., Couturier, J. and A. Dassargues. 2011. The Hybrid Finite-Element Mixing-Cell method: a candidate for modelling groundwater flow and transport in karst systems, In Proc. H2Karst, 9th Conference on Limestone Hydrogeology, Bertrand C., Carry N., Mudry J., Pronk M. & Zwahlen F. (Eds), 79-82.
- Brunner, P., Doherty, J. and C.T. Simmons. 2012. Uncertainty assessment and implications for data acquisition in support of integrated hydrologic models. Water Resources Research 48: W07513.
- Carrera, J. Alcolea, A., Medina, A. Hidalgo, J. and L. Slooten. 2005. Inverse problem in hydrogeology. Hydrogeology Journal 13(1): 206-222.
- Carrera, J. and S.P. Neuman. 1986a. Estimation of aquifer parameters under transient and steady state conditions: 1. Maximum likelihood method incorporating prior information. Water Resources Research 22(2): 199-210.
- Carrera, J. and S.P. Neuman. 1986b. Estimation of aquifer parameters under transient and steady state conditions: 2. Uniqueness, stability, and solution algorithms. Water Resources Research 22(2): 211-227.
- Cooley, R.L. 2004. A theory for modeling groundwater flow in heterogeneous media. USGS Professional Paper 1679.
- Dagan, G. 1989. Flow and transport in porous formations, New York: Springer.

References (2)



- Dassargues, A. 1998. Application of groundwater models in karstic aquifers, In Karst Hydrology, Eds. Ch. Leibundgut, J. Gunn and A. Dassargues, IAHS Publication 247: 7-14.
- Dassargues A. 2020. Hydrogéologie appliquée : science et ingénierie des eaux souterraines, 512p. Dunod. Paris.
- Dassargues A., 2018. Hydrogeology: groundwater science and engineering, 472p. Taylor & Francis CRC press, Boca Raton.
- Dassargues, A., Radu, J.P. and R. Charlier. 1988. Finite elements modelling of a large water table aquifer in transient conditions. Advances in Water Resources 11(2): 58-66.
- Dassargues, A., Rentier, C. and M. Huysmans. 2006. Reducing the uncertainty of hydrogeological parameters by co-conditional simulations: lessons from practical applications in aquifers and in low permeability layers. In Calibration and Reliability in Groundwater Modelling: From Uncertainty to Decision Making, IAHS Publication 304: 3-9.
- de Marsily, G. 1986. Quantitative hydrogeology : groundwater hydrology for engineers. Academic Press.
- de Marsily, G., Delay, F., Gonçalvès, J., Renard. Ph., Teles, V. and S. Violette. 2005. Dealing with spatial heterogeneity. Hydrogeology Journal 13: 161-183.
- Deutsch, C.V. and A.G. Journel. 1998. GSLIB geostatistical software library and user's guide. New-York :Oxford University Press.
- Diersch, H-J.G. 2014. Feflow Finite element modeling of flow, mass and heat transport in porous and fractured media. Springer.
- Doherty, J. 2003. Ground water model calibration using pilot points and regularization. Ground Water 41(2): 170-177.
- Doherty, J. 2005. PEST Model-independent parameter estimation User manual 5th Edition. Watermark Numerical Computing.
- Fienen, M.N. 2013. We speak for the Data. Groundwater 51(2): 157.
- Gelhar, L.W. 1993. Stochastic subsurface hydrology. Prentice Hall.
- Goderniaux, P., Brouyère, S., Fowler, H.J., Blenkinsop, S., Therrien, R., Orban, Ph. and A. Dassargues. 2009. Large scale surface subsurface hydrological model to assess climate change impacts on groundwater reserves. Journal of Hydrology 373: 122-138.
- Goderniaux, P., Wildemeersch, S., Brouyère, S., Therrien, R. and A. Dassargues. 2015. Uncertainty of climate change impact on groundwater reserves. Journal of Hydrology 528: 108-121.
- Gómez-Hernández, J.J. 2006. Complexity. Ground Water 44(6): 782-785.
- Hill, M. 1992. A computer program (MODFLOWP) for estimating parameters of a transient, three-dimensional, ground-water flow model using nonlinear regression. Open-File Report 91-484, USGS.
- Hill, M. 2006. The practical use of simplicity in developing ground water models. Ground Water 44(6): 775-781.
- Hill, M., Cooley, R. and D. Pollock. 1998. A controlled experiment in ground-water flow model calibration uinsg nonlinear regression. Ground Water 44(6): 775-781.
- Hill, M.C. and C.R. Tiedeman. 2007. Effective groundwater model calibration: With analysis of data, sensitivities, predictions, and uncertainty. John Wiley & Sons.

References (3)



- Hoeting, J., Madigan, D., Raftery, A. and C. Volinsky. 1999. Bayesian model averaging: a tutorial. Statistical Science 14(4): 382-417.
- Huysmans, M. and A. Dassargues. 2006. Stochastic analysis of the effect of spatial variability of diffusion parameters on radionuclide transport in a low permeability clay layer. Hydrogeology Journal 14: 1094-1106.
- Huysmans, M., Peeters, L., Moermans, G. and A. Dassargues. 2008. Relating small-scale sedimentary structures and permeability in a cross-bedded aquifer. Journal of Hydrology 361: 41-51.
- Huysmans, M. and A. Dassargues. 2009. Application of multiple-point geostatistics on modelling groundwater flow and transport in a cross-bedded aquifer. Hydrogeology Journal 17(8): 1901-1911.
- Huysmans, M. and A. Dassargues. 2011. Direct multiple-point geostatistical simulation of edge properties for modeling thin irregularlyshaped surfaces. Mathematical Geosciences 43(5): 521-536.
- Kitadinis, P.K. 1997. Introduction to geostatistics: application in hydrogeology. Cambridge University Press.
- Klepikova, M., Wildemeersch, S., Jamin, P., Orban, P., Hermans, T., Nguyen, F., Brouyere, S. and A. Dassargues. 2016. Heat tracer test in an alluvial aquifer: field experiment and inverse modelling. Journal of Hydrology 540: 812-823.
- Konikow, L.F. and J.M Mercer. 1988. Groundwater flow and transport modelling. Journal of Hydrology 100(2): 379-409.
- Kurtz, W., Lapin, A., Schilling, O.S., Tang, Q., Schiller, E., Braun, T., Hunkeler, D., Vereecken, H., Sudicky, E., Kropf, P., Franssen, H-J. H. and P. Brunner. 2017. Integrating hydrological modelling, data assimilation and cloud computing for real-time management of water resources. Environmental Modelling & Software 93: 418-435.
- Molz III, F.J. 2017. The development of groundwater modelling: The end of an era. Groundwater 55(1): 1.
- Moore, C. and J. Doherty. 2005. Role of the calibration process in reducing model predictive error. Water Resources Research 41(5): W05020.
- Nash, J.E. and J.V. Sutcliffe. 1970. River flow forecasting through conceptual models part I A discussion of principles. Journal of Hydrology 10(3): 282–290.
- Neuman, S. 2003. Maximum likelihood Bayesian averaging of uncertain model predictions. Stochastic Environmental Research and Risk Assessment 17(5): 291-305.
- Paniconi, C. and M. Putti. 2015. Physically based modeling in catchment hydrology at 50: Survey and outlook, Water Resources Research 51 :7090-7129.
- Peeters, L.J.M. 2017. Assumption hunting in groundwater modeling: Find assumptions before they find you, Groundwater: doi:10.1111/gwat.12565
- Poeter, E., Hill, M., Banta, E. and S. Mehl. 2005. UCODE_2005 and six other computer codes for universal sensitivity analysis, calibration and uncertainty evaluation. Techniques and Methods 6-A11. USGS
- Refsgaard, J.C. and H.J. Henriksen. 2004. Modelling guidelines terminology and guiding principles. Advances in Water Resources 27: 71-82.
- Rentier, C., Bouyère, S. and A. Dassargues. 2002. Integrating geophysical and tracer test data for accurate solute transport modelling in heterogeneous porous media. In Groundwater Quality 2001. Eds. S.F. Thornton and S.E. Oswald, IAHS Publication 275: 3-10.

References (4)



- Rojas, R., Feyen, L. and A. Dassargues. 2008. Conceptual model uncertainty in groundwater modeling: Combining generalized likelihood uncertainty estimation and Bayesian model averaging. Water Resources Research 44: W12418
- Rojas, R., Batelaan, O., Feyen, L. and A. Dassargues, A. 2010a. Assessment of conceptual model uncertainty for the regional aquifer Pampa del Tamarugal North Chile. Hydrol. Earth Syst. Sci. 14: 171-192.
- Rojas, R., Feyen, L., Batelaan, O. and A. Dassargues. 2010b. On the value of conditioning data to reduce conceptual model uncertainty in groundwater modelling. Water Resources Research 46(8): W08520.
- Rojas, R., Kahundeb, S., Peeters, L., Batelaan, O. and A. Dassargues. 2010c. Application of a multi-model approach to account for conceptual model and scenario uncertainties in groundwater modelling. Journal of Hydrology 394: 416-435.
- Rosbjerg, D. and H. Madsen. 2005. Concept of hydrologic modelling. In: Encyclopedia of Hydrological Sciences, M.G. Anderson (Ed.), John Wiley & Sons.
- Rubin, Y. 2003. Applied stochastic hydrogeology. New York: Oxford University Press.
- Schwartz, F.W., Liu, G., Aggarwal, P. and C.M. Schwartz. 2017. Naïve simplicity: The overlooked piece of the complexity-simplicity paradigm. Groundwater : doi:10.1111/gwat.12570
- Sulis, M., Paniconi, C., Marrocu, M., Huard, D. and D. Chaumont. 2012. Hydrologic response to multimodel climate output using a physically based model of groundwater/surface water interactions. Water Resources Research 48: W12510.
- Tonkin, M.J., Tiedeman, C.R. Ely, M.D. and M.C. Hill. 2007. OPR-PPR, a computer program for assessing data importance to model predictions using linear statistics. USGS, Techniques and MethodsTM-6E2.
- Vecchia, A.V. and R.L. Cooley. 1987. Simultaneous confidence and prediction intervals for nonlinear regression models with application to a groundwater flow model. Water Resources Research 23(7): 1237-1250.
- Wang, H.F. and M.P. Anderson. 1982. Introduction to groundwater modelling: finite difference and finite element methods, San Diego (CA): Academic Press.
- Ward, D. 2005. The simplicity cycle: Simplicity and complexity in design. Defense Acquisition, Technology, and Logistics 34(6): 18-21.
- Wildemeersch, S. 2012. Assessing the impacts of technical and structure choices on groundwater model performance using a complex synthetic case. PhD diss., University of Liège. Belgium.
- Wildemeersch, S., Goderniaux, P., Orban, P., Brouyère, S. and A. Dassargues. 2014. Assessing the effects of spatial discretization on large-scale flow model performance and prediction uncertainty. Journal of Hydrology 510: 10-25.
- Zhang, D. 2002. Stochastic methods for flow in porous media. San Diego (CA): Academic Press.