Building Fast Stochastic Surrogate Models for Extracting RL Parameters of Wound Inductors Modeled Using FEM

Geoffrey Lossa[1], Olivier Deblecker[2], Zacharie De Grève[2], Christophe Geuzaine[3]
[1]Faculty of Engineering, General Physics Unit, University of Mons, Belgium
[2]Faculty of Engineering, Power Electrical Engineering Unit, UMons
[3]Dept. of Electrical Engineering and Computer Science, University of Liège, Belgium

In this work, fast stochastic surrogate models are derived for extracting RL parameters of wound inductors using the Finite Element method. To this end, the Representative Volume Element (RVE) technique is employed to convert the geometrical uncertainties (e.g. due to conductor positions in the winding window) into material uncertainties (complex permeability and conductivity). The dimensionality of the stochastic input space is in that way reduced, thereby allowing the use of the Polynomial Chaos Expansion (PCE) technique for building the stochastic surrogate.

Index Terms—**Homogenization, finite elements, representative volume element, polynomial chaos expansion, uncertainty modeling.**

I. INTRODUCTION

In many modeling problems, model inputs can be hard to obtain with an acceptable accuracy, so that simulation results may be difficultly interpretable. For instance, when modeling wound inductors with the finite element method, the position of the conductors in the winding window is usually not known precisely [1], and/or material parameter values can only be provided by the manufacturers with a limited accuracy (e.g. 20% for the magnetic permeability). In this context, it is crucial to take into account these uncertainties in the modeling process, using e.g. Monte Carlo simulations. This process can however become very time consuming, given the high number of evaluations needed to cover the whole probabilistic input space. In this context, stochastic surrogate models, such as Polynomial Chaos Expansions (PCE) can be employed to speed up the computations. However, PCE surrogates can only deal with a limited amount of input random variables, which is not appropriate in our case, considering that random variables can be associated with the position of each conductor in the winding window. This work proposes to reduce the dimensionality of the input space of the stochastic surrogate by using the Representative Volume Element (RVE) [2] technique, thereby allowing the use of lighter PCE surrogates. The method is demonstrated on the RL parameters extraction of wound inductors.

In section 2, we will present the methodology used in this hybrid approach, with the RVE and PCE concepts adapted to our problem. Section 3 will be devoted to the validation through simulation results, for which comparisons between the reference and surrogate models from different points of view (distributions of model outputs, CPU times, probability densities) will be useful. Finally, a conclusion will present advantages linked to this approach and the short-term prospects.

Manuscript received November 23, 2020; revised December 26, 2020.
Corresponding author: G. Lossa (email: geoffrey.lossa@umons.ac.be).

II. METHODOLOGY

In order to construct the PCE as a stochastic surrogate (for the propagation of geometric uncertainties) while avoiding the problem of *curse of dimensionality* (i.e. large increase in the number of evaluations of the deterministic FE model), we used in this work a technique which consists in reducing the dimension of the stochastic input space. This reduction is made possible thanks to the transformation of the geometric uncertainties (positions of the conductors in the winding window) into material uncertainties which are much smaller in number. The distributions of these equivalent properties will then be necessary for the construction of the PCE.

A. Representative Volume Element

The theory of RVE [2] consists in extracting from a finite volume of a heterogeneous medium, equivalent properties of the associated homogeneous medium in order to reduce computational loads. This implies that this representative volume must be small in order to be numerically analyzed. But it must also be large enough to represent the micro-structures without introducing macroscopic properties that do not exist. The starting hypothesis is to consider that the periodic structure repeats in space indefinitely. This can lead to deviations from the reference model.

The extraction of equivalent properties is done by decoupling the skin and proximity effects and by matching the complex power on the two equivalent models [3, 4] (fine and homogenized models). The conductivity, related to the skin effect, is deduced by imposing a net current I in the coil, whereas the reluctivity, related to the proximity effect, is deduced by imposing a unidirectional magnetic induction (B_x for instance) on the periodical structure.

The complex power absorbed by the RVE can be calculated by [4]

$$S = P + iQ = (\frac{l}{2}) \int_{\text{RVE}} (\boldsymbol{j}^2/\sigma + i\,\omega\,\nu_0\,\boldsymbol{b}^2)\,d\Omega, \qquad (1)$$

where P and Q are the active and reactive powers; l, $\boldsymbol{j}$ and $\boldsymbol{b}$ are the depth on the 3rd dimension (taken equal to 1m), the current density and the magnetic induction. In terms of global quantities, the skin and proximity effects may be taken into account by expressing the complex power in the following form [3]

$$S = P + iQ = (1/\sigma_{\text{skin}}\,\boldsymbol{J}^2/2 + i\,\omega\,\nu_{\text{prox}}\,\boldsymbol{b}_{av}^2/2)\Omega_{\text{RVE}}\,, \quad (2)$$

where σ_{skin}, ν_{prox} and $\boldsymbol{b}_{av}$ define the equivalent complex conductivity and reluctivity and the average induction in the homogeneous medium related to the RVE. $\boldsymbol{J}$ and Ω_{RVE} represent the current density and the homogenized volume of the RVE respectively. For practical reasons, this power can also be expressed as a function of an equivalent impedance Z_{skin} [4] which can be integrated into the homogenized model of the inductor by circuit coupling (see Fig. 1):

$$S = P + iQ = Z_{\text{skin}}\,I^2/2 + i\,\omega\,l\,A_c\,\nu_{\text{prox}}\,\boldsymbol{b}_{av}^2/2 \quad (3)$$

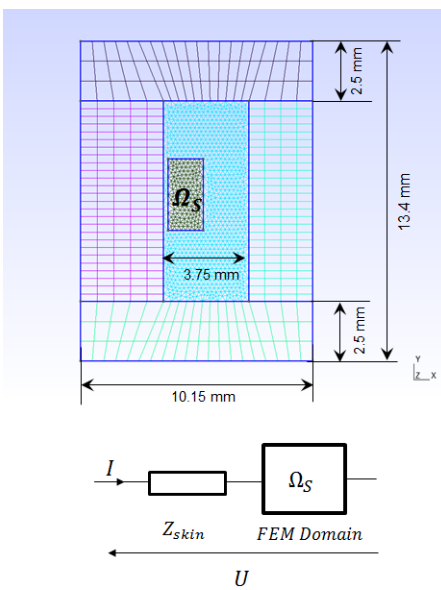


Fig. 1. Meshing and circuit coupling between Z_{skin} and the homogenized winding Ω_{S} (U and I being respectively the voltage imposed, as a constraint at circuit level, and the current weakly deduced from FEM).

A skin effect excitation can be obtained by imposing a unitary net current ($I = 1A$) in the conductors and homogeneous boundary conditions ($\boldsymbol{a} = 0$, through a zero net flux on the RVE, $\boldsymbol{b}_{av} = 0$). The proximity effect excitation of the RVE central cell can be obtained by imposing a unit magnetic induction ($\boldsymbol{b}_{av} = 1T$) through appropriate boundary conditions and a zero net current ($I = 0$) in the RVE conductors. From these two types of excitation, we can deduce the equivalent impedance (see Fig. 2) and reluctivity of the homogeneous medium constituting the RVE.

B. Size of the Representative Volume Element

The choice of the size of the RVE is a crucial step which comes into play in the accuracy of the PCE surrogate to represent the reference FE model over the whole range of frequencies. Reducing the systematic deviation may be done by increasing the size of the RVE. To make this choice, we compared the extracted parameters (reluctivity for instance, through FE models) of three different sizes of the RVE over the range of frequencies. Their geometry consists of one layer or two layers of conductors around a central cell or the one consisting of the whole winding window. Fig. 3 presents the current density maps, for different sizes of RVE, resulting from a proximity effect constraint through a magnetic induction (B_x= 1 T) along the x-axis.

By comparing the evolutions (see Fig. 4) of the extracted parameter and the computational loads generated by the three cases, our choice was oriented towards the RVE consisting of one layer of conductors around a central cell. This choice is explained by the fact that it presents the smallest of the quasi-periodic structures which can represent the geometric uncertainties in the winding window with weak edge effects (e.g. errors on outputs parameters linked to the homogenization process).

C. Polynomial chaos expansion

The PCE is the development $\mathcal{M}^{PC}(X)$ of a model response $Y = \mathcal{M}(X)$ in the space of random functions relating to the distributions of its random inputs X [5].

$$Y = \mathcal{M}(X) \approx \mathcal{M}^{PC}(X) = \sum_{\alpha\in\mathcal{A}} y_\alpha \varphi_\alpha(X), \quad (4)$$

where $\mathcal{A}$ is the truncated set of multi-indices of the PCE.

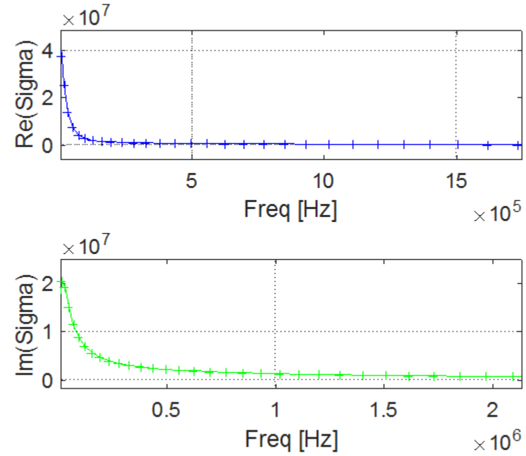


Fig. 2. Real and imaginary parts of conductivity extracted from RVE.

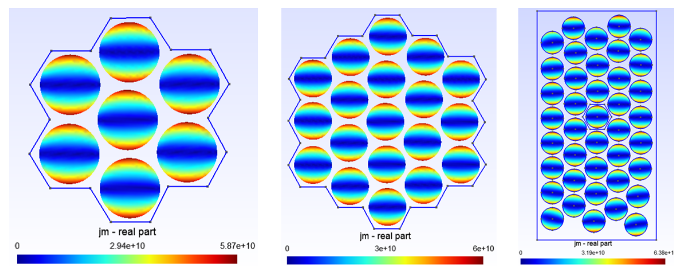


Fig. 3. Current density module j_m map for different sizes of RVE excited by a magnetic induction B_x= 1 T.

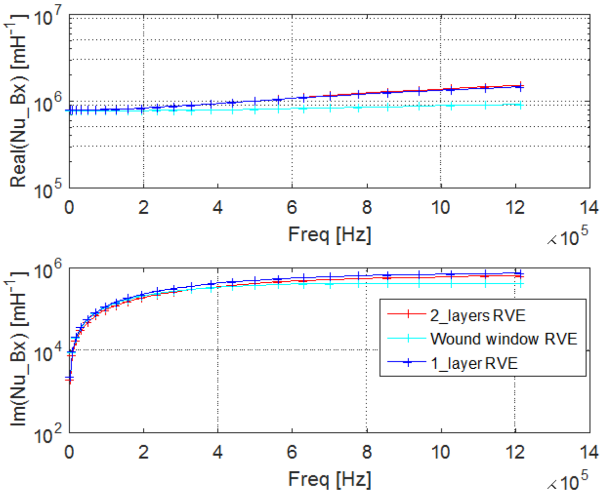


Fig. 4. Comparison of equivalent reluctivity extracted from RVEs of different sizes.

The computation of its coefficients y_α is carried out by different methods, including the most classic projection method [5], using multiple integrals. The number N of integration nodes depends on the dimension M of the random input and on the maximum degree p of the PCE according to the expression:

$$N = (p+1)^M \tag{5}$$

Hence this explosion of the number of evaluations of the numerical reference model which is often called *curse of dimensionality*.

To build the PCE substitute, we start from a sample of size N of the random inputs (called experimental design). The determination of the PCE coefficients then goes through the resolution of the following optimization problem:

$$\hat{y} = arg\ min\ \mathbb{E}[\left(y^T\varphi(X) - \mathcal{M}(X)\right)^2], \tag{6}$$

for which the solution is given by the ordinary least squares (OLS) method, namely

$$\hat{y} = (A^TA)^{-1}A^Ty, \tag{7}$$

where $A_{ij} = \varphi(X)_j(x^{(i)})$ contains the values taken by the polynomial basis at level of points of the experimental design and is called experimental matrix; $i = 1, \cdots, N; j = 0, \cdots, P-1$.

An important parameter for evaluating the quality of the constructed PCE is the relative empirical error, deduced from the experimental design. This kind of parameter can lead to over-fitting, since it decreases with the degree p of the PCE regardless of the size N of the experimental design. In order to overcome this problem, an adaptive technique for constructing sparse PCEs using an *a priori* cross validation error [5] was used in this work. This type of PCEs implies low order interactions between the random variables, which are by experience the most important in various numerical models (sparsity-of-effects principle). This was taken into account in the initial optimization problem (see equation (6)), by associating it with a penalty term of the form $\lambda||y||_1$, i.e. the problem:

$$\hat{y} = arg\ min_{y\in\mathbb{R}^{|\mathcal{A}|}}\ \mathbb{E}[(y^T\varphi(X) - \mathcal{M}(X))^2] + \lambda||y||_1, \tag{8}$$

where the regularization term $||\hat{y}||_1 = \sum_{\alpha\in\mathcal{A}} |y|_\alpha$ forces the minimization to favor low rank solutions. In this latter context, the least angle regression (LAR) algorithm is the one implemented in UQLab [6], a specific module for uncertainty quantification under Matlab.

From Table I, we can notice the accuracy with which we manage to fit the parameter R (from finite element method) at 1.2 MHz thanks to LAR strategy. For the same max degree, one observes a more sparse basis than with the OLS strategy.

TABLE I
COMPARING OLS AND LAR STRATEGIES FOR SURROGATE MODEL OF $R_{@1.2\mathrm{MHz}}$ PARAMETER

$R_{@1.2\mathrm{MHz}}$ [Ω]	Error	Max. Degree	Nonzero Coef.
OLS	0.4723	2	15
LAR	0.0406	2	8

III. SIMULATION RESULTS

The test case consists of a 40-turns inductor (of 5 layers) having a ferrite core without air gap. The conductors have a 0.315 mm diameter. The distributions of the RL parameters from reference and homogenized 2D models of the inductor are presented in Fig. 5. For this test case, we observe a very low dispersion of the inductance along the range of frequencies analyzed. This shows that at the magnetic level, the random aspect related to the positions of conductors in the winding window does not really have any influence. In other words, inductance keeps its deterministic aspect. This may be justified by the fact that the ferrite magnetic core, in which the large part of the magnetic flux circulates, does not introduce any uncertainty linked, for example, to its geometric dimensions or to its air gap thickness.

The construction of the PCE surrogate is based on the homogenized FE model (2D magnetostatic formulation) of the inductor as a deterministic model allowing the computation of PCE coefficients. Thus, the number of random variables would be considerably reduced compared to the initial one (two coordinates for each conductor in the winding window) relating to the brute-force FE model (with fine mesh and 2D ***a***-*v* magnetodynamics formulation).

To validate the proposed methodology, the comparison between the distributions of RL parameters extracted from the homogenized FE model and PCE surrogate is done through their histograms (see Fig. 6). We can observe a good agreement between them. This shows the ability of the PCE surrogate to mimic the behavior of the reference model in face of geometrical uncertainties linked to the positions of the conductors in the winding window. Another advantage of the PCE lies in the fact that predictions are available faster than for an expensive reference model. The following table shows how faster a stochastic analysis can be done using the PCE surrogate.

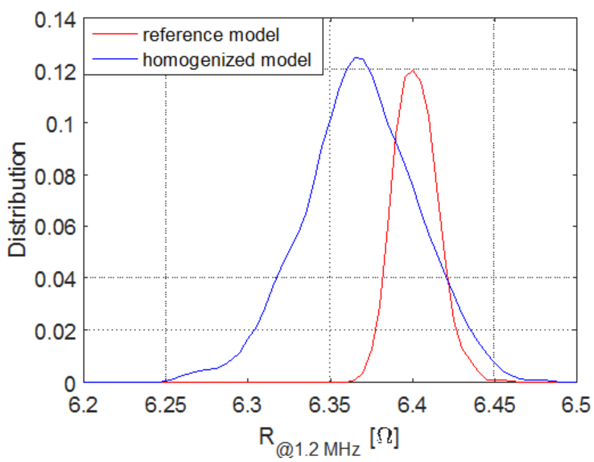
TABLE II
CPU TIME COMPARISON BETWEEN REFERENCE AND PCE MODELS*

CPU time [s]	Per iteration	370 MC iterations**
PCE surrogate	0.3075	0.9625
Brute-force model	21	7489
From RVE to homogenization	14	4754

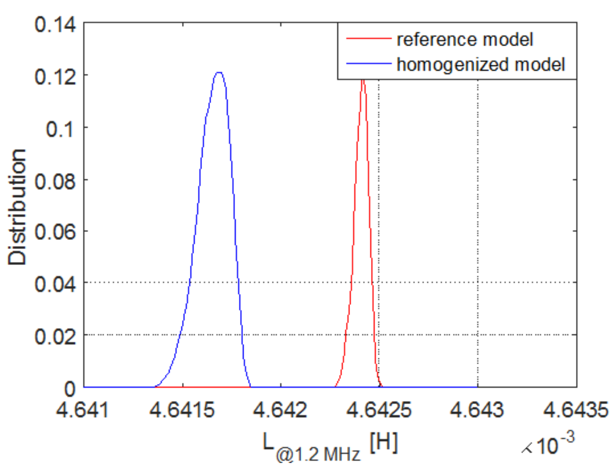
* from DC to 1.2 MHz; ** 370 Monte Carlo iterations

IV. Conclusion

In this work, we have shown how the dimension of the random input of a FE model for extraction of RL parameters of an inductor could be greatly reduced thanks to the transformation of geometrical uncertainties into material uncertainties, and then to build a surrogate model which requires fewer resources and less computation time. The advantage of this approach is rather in the reduction of the number of evaluations of the deterministic numerical model, for instance, in case of sensitivity analysis of the outputs (RL parameters of a wound inductor) of the numerical model faced with the variability of the random inputs. As a short-term perspective, we plan to conduct a sensitivity analysis of this FE model with respect to the geometrical uncertainties and also to the material uncertainties linked to the magnetic permeability and the air gap size of the magnetic core thanks to the use of PCE substitute and to the RVE theory.

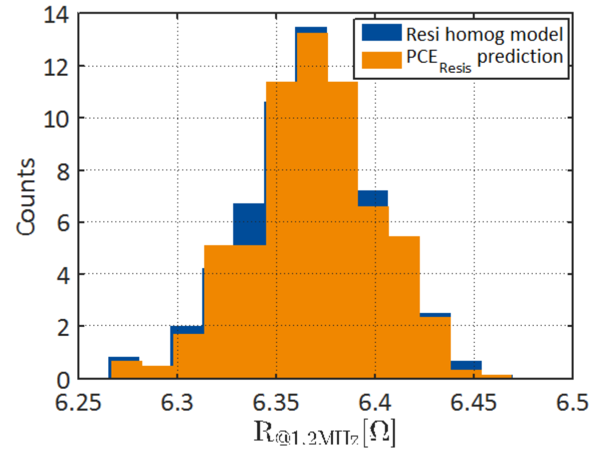


(a)

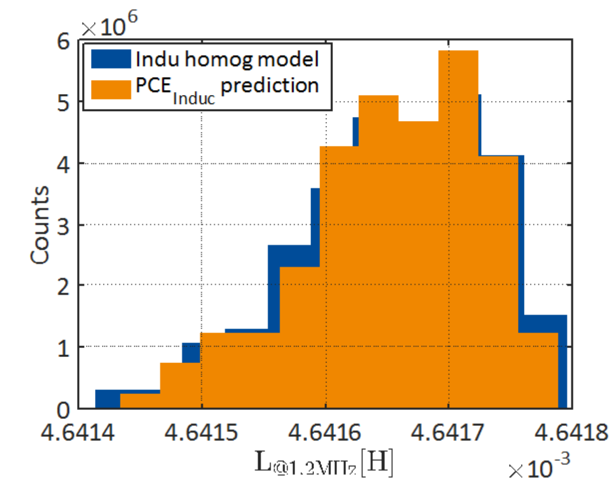


(b)

Fig. 5. Distribution of the RL parameter at 1.2 MHz, extracted from brute-force and homogenized models.



(a)



(b)

Fig. 6. Histograms of RL parameter (at 1.2 MHz) extracted from the homogenized model and the PCE prediction.

References

[1] G. Lossa, et al., "Influence of the Geometric Uncertainties on the RLC parameters of Wound Inductors Modeled Using the Finite Element Method," *IEEE Trans. Magn.,* vol. 53, no. 6, Jun.2017.

[2] F. Otto, A. Gloria, M. Duerinckx, "Characterization of fluctuations in stochastic homogenization," *Workshop on Uncertainties Quantification,* 3^{rd} GAMM AGUQ, Mar.2018.

[3] G. Meunier, et al., "Homogenization for Periodical Electromagnetic Structure: Which Formulation?," *IEEE Trans. Magn.,* vol. 46, no. 8, Aug.2010.

[4] R. Vazquez Sabriego, et al., "Time-Domain Homogenization of Windings in 3-D Finite Element Models," *IEEE Trans. Magn.,* vol. 44, no. 6, Jun.2008.

[5] S. Marelli, B. Sudret, "UQLab user manuel – Polynomial chaos expansions," *Chair of Risk, Safety and Uncertainty Quantification,* Report # UQLab-V1.3-104, ETH Zurich, Switzerland, 2019.

[6] S. Marelli, B. Sudret, "UQLab: A framework for uncertainty quantification in Matlab," *Int. Conf. on Vulnerability, Risk analysis and Management,* Proc. 2nd ICVRAM, Liverpool, UK, 2014, 2554-2563.