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Global Malmquist and cost Malmquist indexes for group comparison

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Abstract
The Malmquist index (MI) has demonstrated its usefulness in comparing the performances of Decision Making Units (DMUs) performances. The global MI (GMI) has been suggested as a means to overcome three drawbacks of the MI: non-circularity, disparate measurements, and infeasibility. Recently, it has appeared that the MI can also be used to compare groups of DMUs. While this new function of the index has also increased its usefulness, it presents the same drawbacks as the MI. In this paper, we define the global counterpart of the MI for group contexts. We also consider the case where DMUs have an economic optimization behavior by proposing a global cost MI (GCMI). The GCMI requires the observation of the input prices. As it may represent a strong assumption, we propose solutions. These two novel indexes equip the practitioners with a new toolkit. We illustrate the usefulness of our new indexes with the Chinese energy sector.

Keywords Global cost Malmquist index · Global Malmquist index · Groups · Circularity · Infeasibility · China

1 Introduction

Caves et al. (1982) introduced the Malmquist index (MI; named after (Malmquist 1953), to measure relative productivity performance changes of Decision Making Units (DMUs; such as firms, plants, utilities, countries, regions) over several periods of time.[1] Two main reasons explain the important attention given to this index for both theoretical and practical works.[2] On the one hand, as the MI is based on a technical formulation of efficiency, it only requires the observation of the input and the output data. On the other hand, as shown by Färe et al. (1994), the MI can be decomposed into different components (such as efficiency change and technical change) to better understand the causes of performance changes between DMUs over time.

The MI, however, also presents certain less desirable features or even drawbacks. Firstly, it is based on a technical formulation of efficiency, whereas a structural approach is often more accurate. Next, the MI is not circular: there is no established relationships between MIs for consecutive periods. Afterwards, the MI averages two possibly disparate measurements of performance change. Finally, when relying on Data Envelopment Analysis (DEA)-based linear programs to compute the technical efficiency scores, infeasibility may occur.[3]

Fortunately, solutions have been proposed to overcome these four drawbacks. Firstly, Maniadakis and Thanassoulis (2004) suggested an MI-based index when DMUs are cost minimizers. They named this the cost Malmquist index (CMI).[4] At this point, it is worth noting that the CMI, contrary to the MI, requires the observation of the input prices (see Section 3.6 for more discussion). Next, Xue and

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[1] We notice that some authors, such as O'Donnell (2012) and Peyrache (2014), have questioned the purpose of the MI; namely whether it really measures productivity performance change and under what conditions. This debate is beyond the scope of this paper.

[2] See, for example, for extensions: Chen (2003), Chen and Ali (2003), Zelenyuk (2006), Yu (2007), Kao (2010), Portela and Thanassoulis (2010), Wang and Lan (2011), Kao and Hwang (2014), Mayer and Zelenyuk (2014), Fuentes and Lillo-Banuls (2016), Asmild et al. (2017), Kao (2017), and Kevork et al. (2017).

[3] DEA, after Charnes et al. (1978), is an approach to productive efficiency measurement. DEA is intrinsically nonparametric, which means that it does not require a parametric/functional specification of the production technology. Typically, a DMU's efficiency can be computed by simple linear programs. Refer to Cooper et al. (2004), Cooper et al. (2007), Fried et al. (2008), and Cook and Seiford (2009) for reviews. See Section 3.5 for the DEA-based linear programs in the group context.

[4] See, for example, for extensions: Yang and Huang (2009), Huang and Juo (2015), Walheer (2018b), and Zhu et al. (2017).

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Harker (2002) give conditions to avoid infeasibility issues when relying on DEA-based linear programs. Finally, Färe and Grosskopf (1996) defined sufficient conditions for the MI to satisfy circularity and average same measurements of performance change.

In many settings, DMUs are split into different groups depending on, for example, their ownership, geographical localization, economic infrastructure, resource endowments, social environment, operational settings, etc. For these contexts, Camanho and Dyson (2006) and Thanassoulis et al. (2015), respectively, defined an MI and an CMI tailored to evaluate the performance difference between (two) groups.[5] Their indexes mirror the initial MI of Caves et al. (1982) and the CMI of Maniadakis and Thanassoulis (2004) with the particularity of comparing groups of DMUs over one period of time instead of DMUs over several periods of time.

As a result, their new indexes share the same advantages, i.e., they are easy to compute and can be decomposed into different components; as well as the disadvantages, i.e., non-circularity, average possibly disparate measurements of performance change, and sensitivity to infeasibility issues. Non-circularity, in this context, implies the lack of stable performance ranking between the groups. That is, there is no relationship between the indexes capturing the differences in performances between groups. Next, the chosen group for the technology may have an impact on the performance difference results. That is, the two ratios forming the MI and the CMI may give different measures of performance change. Finally, when relying on DEA for computing the efficiency scores, infeasibility may occur. This happens, for example, when a group over- or underperforms. All in all, while these drawbacks are important for the MI and the CMI used to compare DMUs over periods of time, they are probably even more important in the contexts of comparing groups over one period of time.

To date, no solution has been proposed to overcome all these issues for the MI and the CMI in group contexts. In this paper, we propose two new indexes: a global MI (GMI) and a global cost MI (GCMI) to compare groups. These indexes present several attractive features: they are circular, give only one measurement of performance difference, and are immune to infeasibility issues. At this point, it is fair to highlight that there is also a price to pay: rely on a fixed reference technology and use a common price. We apply our methodology to the case of the energy sector in China. A particularity of our application is that we use the firm-level database of the National Bureau of Statistics in China: the Chinese Annual Survey of Industrial Firms Database. This database offers a unique opportunity to study the energy sector in China from a microeconomic point of view. The sector has been playing a major role in boosting rapid Chinese industrialization and urbanization. As such, measuring the performances of the sector is of crucial importance for the managers of the firms, but also for policy makers and regulators.

The rest of the paper is structured as follows. In Section 2, we present a brief discussion about circularity of MIs. In Section 3, we define the GMI and the GCMI for group contexts. In Section 4, we use our new indexes to measure the performances of firms in the Chinese energy sector. In Section 5, we present our conclusions.

2 Circular Malmquist indexes

As[6] mentioned in the Introduction, one of the less desirable features or even drawbacks of the MI, as defined by Caves et al. (1982), is that it does not satisfy the circularity property. In this Section, we define circularity, and discuss whether this is a major problem and what solutions are available.

Intuitively, circularly is observed when there is a 'circle relationship' between indexes of different time periods (Frisch (1936)). For example, when comparing DMUs over three time periods t, u, and v, circularly is satisfied if the following is correct: $I_{tu} \times I_{uv} = I_{tv}$, for all t, u, v. Of course, circularity is only important when three or more time periods are considered. In fact, this is a simple solution to avoid this drawback: compare performance changes between two time periods. A direct consequence of the non-circular feature of the MI is that there is no clear relationship between the results of different time periods. This implies, for instance, that it is difficult to establish a stable ranking of the time periods. Stability has here to be understood as obtaining the same ranking for the time periods for a fixed sample using the indexes. An important question is whether it is possible to make the MI circular ? Many researchers—such as Berg et al. (1992), Chambers and Färe (1994), Färe and Grosskopf (1996), Althin (2001), Shestalova (2003), Pastor and Lovell (2005, 2007), Afsharian and Ahn (2015), and Diewert and Fox (2017)—have tried to answer this question by suggesting different approaches. Intuitively, to obtain a fixed ranking something has to be fixed over time.[7] A common point of all these approaches is that extra assumptions are needed to ensure circularity. Whether these additional assumptions make sense or are too restrictive

[5] The technique has been extended in two directions; Aparicio et al. (2017) for unbalanced panels, and Walheer (2018a) for multi-output DMUs.

[6] This Section has been added on the request of an anonymous referee. We thank the referee for challenging us.

[7] Note that the problem become even more complex when the numbers of DMUs change over time. This is why most of the empirical works using the MI use a balanced panel dataset.

may be a subjective question and depend on the empirical context. In fact, it is a balance between accepting additional assumptions to make the MI circular or continuing with a non-circular MI.

A first theoretical result, due to Färe and Grosskopf (1996), provides a sufficient and not a necessary condition to ensure circularity of the MI. It has been demonstrated that the MI is circular when the technological change over time shifts the entire frontier by the same amount. They name this property Hicks neutrality. In practice, there is no reason for it to be correct and, in fact, it often represents a restrictive assumption about the production process. Building on this first result, more practical approaches not assuming Hick neutrality ensued. We may regroup these techniques into two main procedures: the first one is devoted to constructing a common or fixed reference technology, whereas the second alternative consists in resorting to fixed weights.

At this point, it is fair to highlight that whatever the selected procedure it represents an important price to pay to impose circularity (Sickles and Zelenyuk, 2019). This probably explains why most empirical works use the initial MI as defined by Caves, Christensen and Diewert (1982), and, hence, implicitly accepts its non-circular feature (and the other drawbacks presented above in the Introduction). This, though, does not imply that defining MIs that satisfy circularity is irrelevant; at least, from a theoretical point of view.

When relying on a common reference technology, an MI-based index that has received much attention in light of its extensions and applications is the global MI (GMI) introduced by Pastor and Lovell (2005).[8] The GMI has the particularity of using data of all DMUs in all periods to define the technology. As a result, the GMI is circular, gives one measurement of performance change, and is immune to infeasibility issues. In other words, the GMI solves three of the four issues of the MI. The last issue has been solved by Tohidi et al. (2012) who have extended the GMI to the case of cost minimizing DMUs. Their index is naturally named the global cost Malmquist index (GCMI).[9]

Instead of using the MI to compare DMUs over time, it can be used to compare DMUs in different groups, as suggested by Camanho and Dyson (2006) and followers. In that context, circularity is defined in a similar fashion as before. For example, when comparing three groups of DMUs a, b, and c, we obtain circularity if the following is satisfied: $I_{ab} \times I_{bc} = I_{ac}$, for all a, b, c. Clearly, there is no reason that the MI is now circular when comparing groups while it was not when comparing time periods. Again, whether this is a problem is a subjective question. Nevertheless, in group contexts, it is worth noting that non-circularity implies the impossibility to establish a stable ranking between group performances, which is probably a desirable feature when using a group comparison tool. As for the initial use of the MI, three solutions are possible: to compare only two groups, to accept the non-circularity feature, and to select additional assumptions in order to obtain a circular MI. In the latter case, Camanho and Dyson (2006) have proposed to rely on specific fixed weights.

In this paper, we suggest an alternative approach: defining a global technology to make the MI circular in group contexts. By doing so, we define two new indexes: a GMI and a GCMI to compare groups. The new indexes mirror the GMI of Pastor and Lovell (2005) and the GCMI of Tohidi et al. (2012), but are tailored to compare groups over one period of time instead of DMUs over several periods of time. As such, the new indexes share the virtues of the initial definitions of the GMI and GCMI. That is, they are circular, give only one measurement of performance difference, and are immune to infeasibility issues. They also share the disadvantages such as relying on a fixed reference technology and using a common price. As with the initial definition of the GCMI of Tohidi et al. (2012), our GCMI for group contexts also requires the observation of the input prices. As this assumption may be restricted for some settings and applications, we propose solutions when the input prices are partially or not observed.

3 Methodology

We develop a global Malmquist index (*GMI*) and a global cost Malmquist index (*GCMI*) to compare groups of DMUs. In what follows, we assume that we observe n DMUs split into two groups: A and B. We restrict our attention to two groups for simplicity. It is fairly easy to extend to the case of more than two groups. We assume that they are n_A DMUs in group A and $\boldsymbol{n_B}$ DMUs in group B. Clearly, the following holds true: $n_A + n_B = n$. In group A, each DMU t ϵ $\{1,\ldots, n_A\}$ produces Q outputs, captured by $\mathbf{y}_t^A \in \mathbb{R}_+^Q$, using P inputs, captured by $\mathbf{x}_t^A \in \mathbb{R}_+^P$ at price $\mathbf{w}_t^A \in \mathbb{R}_+^P$. Similarly, in group B, each DMU t ϵ $\{1,\ldots, n_B\}$ produces $\mathbf{y}_t^B \in \mathbb{R}_+^Q$ using $\mathbf{x}_t^B \in \mathbb{R}_+^P$ at price $\mathbf{w}_t^B \in \mathbb{R}_+^P$.

Before defining our concepts of *GMI* and *GCMI* we briefly review the MI of Camanho and Dyson (2006) and the CMI of Thanassoulis et al. (2015). This allows us to better position our contribution among the existing approaches. Next, we define our notions of *GMI* and *GCMI*.

[8] See, for example, for extensions and applications: Oh (2010), Oh and Lee (2010), Pastor et al. (2011), Wang et al. (2012), Afsharian and Ahn (2015), and Oh and Lee (2017). The GMI is named global since it is based on a global technology; in our context, it is the technology that envelops all group-specific technology sets (see (12)).

[9] See, for example, for extensions, Tohidi and Razavyan (2013), Huang and Juo (2015), and Cho and Wang (2017).

Afterwards, we compare and explain how to decompose our indexes. Finally, we explain how to compute the indexes in practice when prices are available or not.

3.1 Malmquist and cost Malmquist indexes for group comparison

We define the technology in terms of input requirement sets. For example, for DMU t in group A, it is given by:

$$I_t^A\left(\mathbf{y}_t^A\right) = \left\{\mathbf{x}^A \in \mathbb{R}_+^P \middle| \mathbf{x}^A \text{ can produce } \mathbf{y}_t^A\right\}. \tag{1}$$

$I_t^A\left(\mathbf{y}_t^A\right)$ contains the inputs $\mathbf{x}^A$ that can produce the output quantities $\mathbf{y}_t^A$. We assume that these sets are nested, monotone, convex, and satisfy constant returns-to-scale. Those axioms are very general and common to many popular performance indexes. Two important remarks have to be made at this stage. One, monotonicity and convexity are not required to define cost-based performance index (see, for example, Varian (1984) and Tulkens (1993) for discussion). Two, the new indexes can also be defined when assuming variable returns-to-scale; in that case, an extra component, conceptually similar to the one suggested by Ray and Desli (1997) for the MI of Caves et al. (1982), capturing scale efficiency change will be present in the decomposition. For the sake of simplicity, we rely on constant returns-to-scale in the following.

The first step to define the MI for group contexts is the concept of (input) distance function. Note that we could alternatively define the MI for group comparison in terms of technical efficiency à la Debreu (1951)-Farrell (1957), as technical efficiency is the reciprocal of the (input) distance function. For example, it is given for DMU t in group B taking group A as the referent group for the technology by:

$$D_t^{A,B} := D_t^{A,B}\left(\mathbf{y}_t^B, \mathbf{x}_t^B\right) = \sup\left\{\theta \middle| \left(\frac{\mathbf{x}_t^B}{\theta}\right) \in I_t^A\left(\mathbf{y}_t^B\right)\right\}. \tag{2}$$

$D_t^{A,B}$ is an adapted version of the Shephard's (1953, 1970) distance function in the group context. We have that $D_t^{A,B} \geq 1$. $D_t^{A,B}$ is interpreted as the reverse of the maximal equiproportionate/radial input reduction of DMU t in group B that still allows for producing the output quantity $\mathbf{y}_t^B$ given the technology of group A. As such, the subscript A on the distance input function refers to the referent group for the technology, while the subscript B indicates the group of the evaluated DMU.

Building on the concept of distance function, Camanho and Dyson (2006) define the MI to compare group B to group A as follows:

$$MI = \left[MI^A \times MI^B\right]^{1/2}, \tag{3}$$

where

$$MI^A = \left(\frac{\left[\prod_{t=1}^{n_B} D_t^{A,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} D_t^{A,A}\right]^{1/n_A}}\right)^{-1} \text{ and } MI^B = \left(\frac{\left[\prod_{t=1}^{n_B} D_t^{B,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} D_t^{B,A}\right]^{1/n_A}}\right)^{-1}. \tag{4}$$

Both MI^A and MI^B compare the performance of group B to group A. The only difference between the two indexes is the referent group for the technology: group A for MI^A and group B for MI^B. $MI^A > 1$ indicates that the DMUs in group B are, on average, better (technical) performers than those in group A. A value smaller than 1 indicates the opposite, while a value of 1 means that the two groups, on average, have similar performances. The interpretation of MI^B is analogous. To avoid an arbitrary choice of the reference group, a commonly agreed procedure in this context (see Färe et al. (1994)) is to take the geometric average of the two components. As a result, MI captures the (technical) performance of DMUs in group B with respect to those in group A, and this is irrespective of the chosen technology, i.e., the referent group. A value greater than 1 induces greater average performances of DMUs in group B than those in group A while a value smaller indicators the converse.

Contrary to the MI, the CMI is designed to take the cost minimization behavior of the DMUs into consideration. Therefore, the first step is to define the notion of cost efficiency in the group comparison context. For example, cost efficiency for DMU t in group B taking group A as the referent group for the technology is given by:

$$CE_t^{A,B} := CE_t^{A,B}\left(\mathbf{y}_t^B, \mathbf{x}_t^B, \mathbf{w}_t^B\right) = \frac{C_t^{A,B}}{\mathbf{w}_t^{B\prime}\mathbf{x}_t^B}, \tag{5}$$

where

$$C_t^{A,B} := C_t^{A,B}\left(\mathbf{y}_t^B, \mathbf{w}_t^B\right) = \min_{\mathbf{x}^B \in I_t^A\left(\mathbf{y}_t^B\right)} \mathbf{w}_t^{B\prime}\mathbf{x}^B. \tag{6}$$

$C_t^{A,B}$ gives the minimal cost to produce the output quantity $\mathbf{y}_t^B$ given the input prices $\mathbf{w}_t^B$ and the technology of group A. By construction, minimal cost cannot exceed actual cost: $C_t^{A,B} \leq \mathbf{w}_t^{B\prime}\mathbf{x}_t^B$. If equality holds, outputs are produced with minimal cost. If equality does not hold, it reflects potential cost savings. As a consequence, it implies that $CE_t^{A,B}$ is situated between 0 and 1, with 1 indicating that outputs are produced efficiently, i.e., with minimal cost. A lower value reflects greater cost inefficiency and thus potential cost savings. Note that $CE_t^{A,B}$ is only an adapted version of Farrell's (1957) cost efficiency in the group context.

Thanassoulis et al. (2015) define the CMI for comparing group B to group A as follows:

$$CMI = \left[CMI^A \times CMI^B\right]^{1/2}, \tag{7}$$

where

$$CMI^A = \frac{\left[\prod_{t=1}^{n_B} CE_t^{A,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} CE_t^{A,A}\right]^{1/n_A}} \text{ and } CMI^B = \frac{\left[\prod_{t=1}^{n_B} CE_t^{B,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} CE_t^{B,A}\right]^{1/n_A}}. \tag{8}$$

The same comments made for *MI* apply for *CMI*. The only difference between the two indexes is that *CMI* is designed to take the cost optimization behavior of the DMUs into account. $CMI > 1$ induces greater cost-performances of DMUs in group B than those in group A, $CMI < 1$ implies the opposite, and $CMI = 1$ reveals similar cost-performances.

We end this part by showing that *MI* and *CMI* can be connected by introducing the notion of allocative Malmquist index (*AMI*):

$$CMI = MI \times AMI. \tag{9}$$

Allocative efficiency, introduced by Farrell (1957), measures inefficiency due to non-optimal allocation of inputs (with respect to the cost optimization behavior). In our group context, it is defined, for example for DMU t in group B taking group A as the referent group for the technology, by the product of the cost efficiency measurement and the distance function: $CE_t^{A,B} \times D_t^{A,B}$. Building on the notion of allocative efficiency, we naturally define *AMI* as follows:

$$AMI = \left[AMI^A \times AMI^B\right]^{1/2}, \tag{10}$$

where

$$AMI^A = \frac{\left[\prod_{t=1}^{n_B} CE_t^{A,B} \times D_t^{A,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} CE_t^{A,A} \times D_t^{A,A}\right]^{1/n_A}} \text{ and } AMI^B = \frac{\left[\prod_{t=1}^{n_B} CE_t^{B,B} \times D_t^{B,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} CE_t^{B,A} \times D_t^{B,A}\right]^{1/n_A}}. \tag{11}$$

AMI^A and AMI^B compare the performance of group B to group A taking, respectively, group A and group B as the referent group for the technology. $AMI > 1 (< 1)$ indicates that the DMUs in group B, on average, are more (less) allocative efficient than those in group A. A value of 1 means that the two groups, on average, have similar allocative performances. Allocative-type indexes can be found in, for example, Maniadakis and Thanassoulis (2004), Yang and Huang (2009), Huang and Juo (2015), Walheer (2018b), and Zhu et al. (2017).

3.2 Global Malmquist and cost Malmquist indexes for group comparison

The first step to define the GMI for the group contexts is the concept of global technology. Intuitively, the global technology is defined for all groups, and thus remains unchanged across groups. As a result, for the global technology, the membership of the DMUs to a specific group does not matter. Let us define $\mathbf{y}_t \in \mathbb{R}_+^Q$, $\mathbf{x}_t \in \mathbb{R}_+^P$, and $\mathbf{w}_t \in \mathbb{R}_+^P$, as the output, input and input price vectors of DMU $t \in \{1,\ldots, n\}$, respectively.

The global input requirement set is defined for DMU t as follows:

$$GI_t(\mathbf{y}_t) = \left\{I_t^A(\mathbf{y}_t) \bigcup I_t^B(\mathbf{y}_t)\right\}. \tag{12}$$

In words, the global technology is obtained as an envelopment of the group-specific technology sets. At this point, it should be clear that the envelopment could be assumed as convex or non-convex. It will not change the definition of the concepts, but only the practical implementation (see Section 3.5). Allowing for convexity or not of the envelopment is a choice left to the practitioners. Of course, this choice clearly depends on the setting and/or the application considered. We remark that the debate about imposing convexity or not is not only specific to *GMI* and *GCMI*, but also holds true for many concepts in the efficiency literature. See, for example, Deprins et al. (1984), Kerstens and Vanden Eeckaut (1999), Podinovski (2004a, b), and Leleu (2009), Huang et al. (2013), and Afsharian and Ahn (2015).

Based on the global set, we can define our notion of global (input) distance function in the group context, given for DMU t by:

$$GD_t := GD_t(\mathbf{y}_t, \mathbf{x}_t) = \sup\left\{\theta | \left(\frac{\mathbf{x}_t}{\theta}\right) \in GI_t(\mathbf{y}_t)\right\}. \tag{13}$$

GD_t is the reverse of the maximal equiproportionate/radial input reduction of DMU t that still allows for producing the output quantity $\mathbf{y}_t$ given the global technology. By construction $GD_t \geq 1$, with $GD_t = 1$ reflects technicallly efficient behaviour, and $GD_t > 1$ means that potential input savings are possible.

Using the concept of global distance function, we define the GMI in the group context as follows:

$$GMI = \left(\frac{\left[\prod_{t=1}^{n_B} GD_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} GD_t\right]^{1/n_A}}\right)^{-1} \tag{14}$$

An initial observation is that *GMI*, contrary to *MI*, is not based on the geometric average. As such, it provides a

single measure of performance differences. It thus avoids the difficult task of choosing a referent group for the technology, and the potential issue of different performance measurements of the two components (i.e. MI^A and MI^B for *MI*). Next, *GMI* is circular. That is, there is a relationship between *GMI*s when there are several groups. Circularity also implies that *GMI* fulfils transitivity, or in other words, there is a stable ranking between the groups. Note that this is not guaranteed with the *MI* (see Section 3.3 for more discussion). Finally, *GMI* is immune to potential infeasibility issues (see Section 3.5 for more discussion). *GMI* captures the (average) technical performances between DMUs in group *B* and in group *A*. $GMI > 1$ induces a greater performance of DMUs in group *B* than those in group *A*, $GMI < 1$ implies the opposite, and $GMI = 1$ reveals similar performances.

To define the GCMI for group contexts, we first have to introduce the extra notion of global input prices, denoted by $\mathbf{w}^G \in \mathbb{R}^P_+$. These prices are common across groups and can be seen, in microeconomic contexts, as benchmark or referent prices (Kuosmanen et al. (2006)), and in macroeconomic contexts, as the country, region, or sector prices. Different options are available at this stage. The choice is left to the practitioners, and clearly depends on the application considered. For example, we may assume that all DMUs face the same input prices. That is, $\mathbf{w}^G = \mathbf{w}_t$, for $t = 1,\ldots, n$. This strategy is used, for example, in Färe and Zelenyuk (2003, 2007), Zelenyuk (2006, 2016), Cherchye et al. (2015, 2016), Walheer (2018a, 2018b, 2018d), and Färe and Karagiannis (2017). Another option is to define the global input prices as a function of the DMU-specific input prices. That is, $\mathbf{w}^G = F(\mathbf{w}_1,\ldots, \mathbf{w}_n)$. An example, discussed in Tohidi et al. (2012) in a context of comparing DMUs, is to rely on a weighting average: $\mathbf{w}^G = \sum_{t=1}^n \omega_t \mathbf{w}_t$ (with $\sum_{t=1}^n \omega_t = 1$ and $\omega_t \geq 0, \forall t$).

Building on our notions of global technology and global input prices, we define the global minimal cost for DMU *t* as follows:

$$GC_t := GC_t\left(\mathbf{y}_t, \mathbf{w}^G\right) = \min_{\mathbf{x} \in GI_t(\mathbf{y}_t)} \mathbf{w}^{G\prime}\mathbf{x}. \tag{15}$$

GC_t defines the minimal cost to produce $\mathbf{y}_t$ given the global technology and the global input prices. Also, $GC_t \leq \mathbf{w}^{G\prime}\mathbf{x}_t$ for every DMU *t*. If they coincide, $\mathbf{y}_t$ is produced with minimal cost with respect to the global setting (i.e., technology and prices). If they do not, it reflects potential cost savings.

Naturally, the concept of global cost efficiency is defined for DMU *t* as follows:

$$GCE_t := GCE_t\left(\mathbf{y}_t, \mathbf{x}_t, \mathbf{w}^G\right) = \frac{GC_t}{\mathbf{w}^{G\prime}\mathbf{x}_t}. \tag{16}$$

GCE_t is studied between 0 and 1. $GCE_t = 1$ implies that DMU *t* has a cost efficient behaviour, while a lower value induces larger cost inefficiency behaviour with respect to the global setting.

We define our concept of *GCMI* as follows:

$$GCMI = \frac{\left[\prod_{t=1}^{n_B} GCE_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} GCE_t\right]^{1/n_A}}. \tag{17}$$

GCMI shares the same virtues as those of *GMI*: it provides a single measure of cost-performance between groups *B* and *A*, avoids to make a choice for the referent group for the technology, and is circular. *GCMI* captures the (average) cost-performances between DMUs in group *B* and in group *A*. $GCMI > 1(<1)$ induces a greater (smaller) cost-performance of DMUs in group *B* than those in group *A*.

3.3 Comparison

We first define two extra notions to ease our discussion of the comparison between our global indexes and the indexes of Camanho and Dyson (2006) and Thanassoulis et al. (2015): the best practice (technical) gap *BPG* and the best practice cost gap *BPCG*. For example, they are defined for group *B* taking group *A* as the referent group as:

$$BPG^{A,B} = \left(\frac{\left[\prod_{t=1}^{n_B} GD_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_B} D_t^{A,B}\right]^{1/n_B}}\right)^{-1} \text{ and } BPCG^{A,B} = \frac{\left[\prod_{t=1}^{n_B} GCE_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_B} CE_t^{A,B}\right]^{1/n_B}}. \tag{18}$$

$BPG^{A,B}$ gives the (average) best practice gap of the DMUs in group *B* between the global technology and the technology of group *A*. When $BPG^{A,B} = 1$, it means that, on average, the distance functions are the same with respect to both technologies; that is, there is no gap. The smaller the ratio, the greater the gap. $BPCG^{A,B}$ has to be interpreted in the same way, but in cost terms.

To compare *GMI* and *MI*, we make use of a ratio $\frac{GMI}{MI}$. It can be shown that this ratio corresponds to the geometric average of the ratio of benchmark technology gaps (the proof is given in Appendix B):

$$\frac{GMI}{MI} = \left[\frac{GMI}{MI^A}\frac{GMI}{MI^B}\right]^{1/2} = \left[\frac{BPG^{A,B}}{BPG^{A,A}} \times \frac{BPG^{B,B}}{BPG^{B,A}}\right]^{1/2}. \tag{19}$$

An initial observation is that, in general, *GMI* is different *MI*, and there is no established ranking between the two indexes (that is $GMI \lesseqgtr MI$). *GMI* corresponds to *MI*, i.e., $\frac{GMI}{MI} = 1$, when $GMI = MI^A = MI^B$. This happens when the chosen group for the technology is irrelevant. In that case, we have that the global and the group-specific technologies correspond. This implies that the distance functions with respect to the global and group-specific technologies are equal. As such, the best practice gaps are equal to 1, making

the geometric average of the ratio of the best practice gaps equals to 1.

A similar decomposition can be obtained when comparing *GCMI* and *CMI* (the proof is given in Appendix B):

$$\frac{GCMI}{CMI} = \left[\frac{GCMI}{CMI^A}\frac{GCMI}{CMI^B}\right]^{1/2} = \left[\frac{BPCG^{A,B}}{BPCG^{A,A}} \times \frac{BPCG^{B,B}}{BPCG^{B,A}}\right]^{1/2}. \tag{20}$$

We conclude that $\frac{GCMI}{CMI} = 1$, when $GCMI = CMI^A = CMI^B$. As for the relationship between *GMI* and *MI*, this requires the global and the group-specific technologies to correspond, but also that the global and group-specific inputs coincide. When these two conditions are satisfied, we have that the global and group-specific cost efficiency measurements are similar, making the best practice cost gaps equal to unity. In that case, the geometric average of the ratio of best practice technology gaps equals 1. Clearly, these two assumptions are rather restrictive in practice, making *GCMI* different from *CMI*.

3.4 Decomposition

A desirable feature of the initial MI of Caves et al. (1982) is that it can be decomposed into different sources (Färe et al. (1994)). Attractively, our two new indexes keep this desirable feature. In fact, *GMI* and *GCMI* can be decomposed into two parts as follows (the proofs are given in Appendix B):

$$GMI = TED \times BPD, \tag{21}$$

$$GCMI = CED \times BPCD, \tag{22}$$

where $TED = \left(\frac{\left[\prod_{t=1}^{n_B} D_t^{B,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} D_t^{A,A}\right]^{1/n_A}}\right)^{-1}$; $CED = \frac{\left[\prod_{t=1}^{n_B} CE_t^{B,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} CE_t^{A,A}\right]^{1/n_A}}$; $BPD = \frac{BPG^{B,B}}{BPG^{A,A}}$; and $BPCD = \frac{BPCG^{B,B}}{BPCG^{A,A}}$.

TED measures the difference in (technical) efficiency spreads between group *B* and group *A*. This component is present in the decomposition of *MI* (see Camanho and Dyson (2006)). It is interpreted as the ratio of mean distance of the DMUs in Group *B* from their own frontier to that of the DMUs in Group *A* from their own frontier. When $TED > 1$, it indicates that the efficiency spread is smaller for DMUs in group *B* than for those in group *A*. $TED < 1$ shows the opposite, while $TED = 1$ implies similar efficiency spread for both groups. Two important remarks should be made about *TED*. First, this ratio does not capture relative (technical) performances of units in each group as the referent boundaries differ between the numerator and the denominator. Second, we may interpret *TED* as a comparison of structural efficiency of two industries (see Farrell (1957) and Camanho and Dyson (2006)). In a similar vein, *CED* defines the difference in cost efficiency spreads between group *B* and group *A*. A value larger (smaller) than 1 indicates that the there is greater (smaller) consistency in cost efficiency levels in DMUs of group *B* than in those of group *A*. This component is also present in the decomposition of *CMI* (see Thanassoulis et al. (2015)).

The distinguishing components of the proposed decompositions are thus captured by *BPD* and *BPCD*. *BPD* is the difference in best practice (technical) gaps when comparing group *B* to group *A*. $BPD = 1$ when $BPG^{B,B} = BPG^{A,A}$. This occurs when DMUs in group *B* are, on average, at the same distance from the global technology than DMUs in group *A*. The closer DMUs in group *B* are from the global technology (i.e., the farther DMUs in group *A* are), the larger *BPD* is. Therefore, $BPD > 1$ (<1) means that DMUs in group *B* have, on average, greater (smaller) best practice in technical terms than those in group *A*. *BPCD* is interpreted in an analogous manner, but in cost terms.

As a final remark, we point out that the advantages of *GMI* and *GCMI* discussed previously, i.e. circularity and single measurement of performance differences between groups *B* and *A*, also hold true for *TED*, *CED*, *BPD*, and *BPCD*.

An important point is to relate *GMI* and *GCMI*. We can obtain a connection similar to the one discussed for *MI* and *CMI* in (9). Firstly, note that *CED* and *BPCD* can be decomposed into two parts as follows (the proofs are given in Appendix B):

$$CED = TED \times AED, \tag{23}$$

$$BPCD = BPD \times BPAD, \tag{24}$$

where $AED = \frac{\left[\prod_{t=1}^{n_B} CE_t^{B,B} \times D_t^{B,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} CE_t^{A,A} \times D_t^{A,A}\right]^{1/n_A}}$; and $BPAD = \frac{\frac{\left[\prod_{t=1}^{n_B} GCE_t \times GD_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_B} CE_t^{B,B} \times D_t^{B,B}\right]^{1/n_B}}}{\frac{\left[\prod_{t=1}^{n_A} GCE_t \times GD_t\right]^{1/n_A}}{\left[\prod_{t=1}^{n_A} CE_t^{A,A} \times D_t^{A,A}\right]^{1/n_A}}}$.

AED captures the difference in allocative efficiency spreads between group *B* and group *A*. This component relates the differences in (technical) efficiency and cost efficiency spreads (see Thanassoulis et al. (2015)). A value larger than 1 indicates that there is greater consistency in allocative efficiency levels in DMUs of group *B* than in those of group *A*. In other words, it implies that the input prices are better aligned with the mix of inputs used (with respect to the cost minimization behavior) by DMUs in Group *B* rather than in Group *A*. Next, *BPAD* is a new component and captures the difference in best practice allocative gaps when comparing group ***B*** to group *A*. Note that $GCE_t \times GD_t$ defines the notion of global allocative

efficiency of DMU t in the group context based on the global technology and the global input prices.

To interpret *BPAD*, let us first define the notion of best practice allocative gap. It is given, for example, for group B taking group A as the referent group for the technology:

$$BPAG^{A,B} = \left(\frac{\left[\prod_{t=1}^{n_B} GCE_t \times GD_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_B} CE_t^{A,B} \times D_t^{A,B}\right]^{1/n_B}} \right)^{-1}. \tag{25}$$

$BPAG^{A,B}$ gives us a best practice allocative gap of group B between the global technology and the technology of group A. When $BPAG^{A,B} = 1$, it means that, on average, the allocative efficiency measurements are similar with respect to both technologies. That is, there is no allocative gap. A smaller ratio induces a greater gap. Using this concept, we can equivalently rewrite *BPAD* as follows:

$$BPAD = \frac{BPAG^{B,B}}{BPAG^{A,A}} \tag{26}$$

As such, *BPAD* mirrors *BPD* but in allocative terms. $BPAD > 1$ means that DMUs in group B, on average, have greater allocative efficiency with respect to the global technology than DMUs in group A. $BPAD < 1$ reflects the opposite, while $BPAD = 1$ means similar gaps between the two groups. In other words, *BPAD* captures the change in the distance between technical and cost frontiers between group B and group A.

Combining (21) and (22) with (23) and (24), we obtain a connection between *GMI* and *GCMI*:

$$\begin{aligned} GCMI &= CED \times BPCD, \\ &= (TED \times AED) \times (BPD \times BPAD), \\ &= (TED \times BPD) \times AED \times BPAD, \\ &= GMI \times (AED \times BPAD), \\ &= GMI \times GAMI. \end{aligned} \tag{27}$$

GAMI stands for the global allocative Malmquist productivy index for group contexts. It measures the (average) allocative performances between DMUs in group B and in group A. $GAMI > 1$ induces greater allocative performances of DMUs in group $\boldsymbol{B}$ than those in group A, $GAMI < 1$ implies the opposite, and $GAMI = 1$ reveals similar allocative performances. *GAMI* has the same desirable properties than *GMI* and *GCMI*: it is circular and provides a single measure of allocative performance differences. Formally, it is defined as follows:

$$GAMI = \frac{\left[\prod_{t=1}^{n_B} GCE_t \times GD_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} GCE_t \times GD_t\right]^{1/n_A}}. \tag{28}$$

Also, it can be shown that *GAMI* can be decomposed into two parts (the proof is given in Appendix B):

$$GAMI = AED \times BPAD, \tag{29}$$

and that it is related to *AMI* as follows (the proof is given in Appendix B):

$$\frac{GAMI}{AMI} = \left[\frac{GAMI}{AMI^A}\frac{GAMI}{AMI^B}\right]^{1/2} = \left[\frac{BPAG^{A,B}}{BPAG^{A,A}} \times \frac{BPAG^{B,B}}{BPAG^{B,A}}\right]^{1/2}. \tag{30}$$

As *GAMI* depends on both cost efficiency and distance function, we need both conditions to have equality. That is, it requires that the global and the group-specific technologies correspond, but also that the global and group-specific inputs coincide. When those two conditions are satisfied, the global and group-specific cost efficiency measurements and distance functions are similar, making the best practice allocative gaps and the differences in best practice allocative gaps equal to unity. As such, *GAMI*=*AMI* for those cases.

All in all, *GMI* and *GCMI* are equal when *GAMI*=1. This happens when there is no inappropriate allocation of inputs at the global level (with respect to the cost minimization behavior). This occurs when the input prices are the most favourable (i.e., the shadow prices). See Section 3.6 for more discussion on the relationships between indexes and components.

3.5 Practical implementation

We rely on Data Envelopment Analysis (DEA, after Charnes et al. (1978)) to compute the distance functions and the cost efficiency scores. The two main advantages of using DEA in this context are, one, no functional form assumption is required about the production process (in particular for the input requirement sets), and, two, linear programs can be used to evaluate the indicators. Also, note that when relying on DEA, interesting results when input prices are partially or not observed are found (see Section 3.6). At this point, it is worth noting that the proposed linear programs do not suffer from infeasibility issues. This is one advantage of the global setting (this is not guaranteed for *MI* and *CMI*).

Before presenting the linear programs, we first explain how to practically capture returns-to-scale assumption when relying on input requirement sets (instead of on production possibility sets, as in the initial DEA models of Charnes et al. (1978)). In fact, as shown by Petersen (1990) and Bogetoft (1996), it suffices to define a factor that scales the outputs up or down to make two DMUs comparable.

Extending their initial definition to our group context, we obtain the following factor for $s \in \{1,\ldots, n\}$:

$$\alpha_s := \alpha_s(\mathbf{y}_t) = \inf\left\{\alpha \in \mathbb{R}_0^+ | \alpha\, \mathbf{y}_s \geq \mathbf{y}_t\right\}. \tag{31}$$

α_s gives the factor by which $\mathbf{y}_s$ has be scaled to be comparable with $\mathbf{y}_t$ (Note that when such factor cannot be found, it suffices to set $\alpha_s = +\infty$). The restricted value for the factor α_s captures the chosen returns-to-scale assumption. As such, $\mathbb{R}_0^+$ is for constant returns-to-scale. Other returns-to-scale assumptions are easily implemented by replacing $\mathbb{R}_0^+$ by (0,1], [1,∞), {1} for the decreasing, increasing and variable returns-to-scale, respectively.

Let us start with the linear programs for *GCMI*. To evaluate that index, it suffices to compute for every DMU t operating at $(\mathbf{y}_t, \mathbf{x}_t)$ with input prices $\mathbf{w}^G$ the following linear program (LP-1):

$$\begin{aligned} GCE_t = & \max_{GC_t \in \mathbb{R}_+} \frac{GC_t}{\mathbf{w}^{G'}\mathbf{x}_t} \\ \text{s.t. } & GC_t \leq \mathbf{w}^{G'}(\alpha_s \mathbf{x}_s) \text{for all } s \in \{1, \ldots, n\} : (\alpha_s \mathbf{y}_s) \geq \mathbf{y}_t. \end{aligned}$$

In words, the constraint picks the minimal cost GC_t when comparing the evaluated DMU t to the dominating DMUs (i.e., DMUs that produce more outputs than $\mathbf{y}_t$, when rescaled by the factor α_s). Therefore, if the minimal cost GC_t correspond to the actual cost $\mathbf{w}^{G'}\mathbf{x}_t$, the program will give a cost score of 1. Otherwise, the cost score will be smaller than 1. It could seem counter-intuitive to maximize a cost function. In fact, the maximization selects the most favorable minimal cost (this spirit is called the benefit of the doubt; see, for example, Cherchye et al. (2007), Cherchye et al. (2013), Cherchye et al. (2016), and Walheer (2018a, b, d)). In the technical formulation (see **(LP-3)** and **(LP-4)**), the benefit of the doubt spirit is captured by the multipliers.

For the decomposition of *GCMI*, four additional cost efficiency measurements have to be evaluated for every DMU t in both groups: $CE_t^{A,A}$, $CE_t^{A,B}$, $CE_t^{B,A}$, and $CE_t^{B,B}$. For example, $CE_t^{A,B}$ for every DMU t in group B operating at $\left(\mathbf{y}_t^B, \mathbf{x}_t^B\right)$ with input prices $\mathbf{w}_t^B$ is obtained as follows (LP-2):

$$\begin{aligned} CE_t^{A,B} = & \max_{C_t^{A,B} \in \mathbb{R}_+} \frac{C_t^{A,B}}{\mathbf{w}_t^{B'}\mathbf{x}_t^B} \\ \text{s.t. } & C_t^{A,B} \leq \mathbf{w}_t^{B'}\left(\alpha_s^{A,B}\mathbf{x}_s^A\right) \text{for all } s \in \{1, \ldots, n_A\} : \left(\alpha_s^{A,B}\mathbf{y}_s^A\right) \geq \mathbf{y}_t^B, \end{aligned}$$

where for $s \in \{1, \ldots, n_A\}$, $\alpha_s^{A,B} := \alpha_s^{A,B}\left(\mathbf{y}_t^B\right) = \inf\left\{\alpha \in \mathbb{R}_0^+ | \alpha \mathbf{y}_s^A \geq \mathbf{y}_t^B\right\}$ is the rescale factor when comparing DMU s in group A to DMU t in group B. In (LP-2), the constraint picks the minimal cost $C_t^{A,B}$ when comparing the evaluated DMU t in group B to the dominating DMUs in group A. $CE_t^{A,A}$, $CE_t^{B,A}$, and $CE_t^{B,B}$ are easily obtained by changing A to B and/or B to A in (LP-2).

Next, for *GMI*, it suffices to compute for every DMU t operating at $(\mathbf{y}_t, \mathbf{x}_t)$ the following linear program (LP-3):

$$\begin{aligned} (GD_t)^{-1} = & \min_{\theta_t, \lambda_s} \theta_t \\ & \sum_s \lambda_s(\alpha_s \mathbf{x}_s) \leq \theta_t \mathbf{x}_t \text{ for all } s \in \{1, \ldots, n\} : (\alpha_s \mathbf{y}_s) \geq \mathbf{y}_t, \\ & \forall s : \lambda_s \geq 0. \end{aligned}$$

For the decomposition of *GMI*, four distance functions have to be evaluated for each DMU t in both groups: $D_t^{A,A}$, $D_t^{A,B}$, $D_t^{B,A}$, and $D_t^{B,B}$. For example, $D_t^{A,B}$ for every DMU t in group B operating at $(\mathbf{y}_t^B, \mathbf{x}_t^B)$ is obtained as follows (LP-4):

$$\begin{aligned} (D_t^{A,B})^{-1} = & \min_{\theta_t^B, \lambda_s^A} \theta_t^B \\ & \sum_s \lambda_s^A\left(\alpha_s^{A,B}\mathbf{x}_s^A\right) \leq \theta_t \mathbf{x}_t^B \text{ for all } s \in \{1, \ldots, n_A\} : \left(\alpha_s^{A,B}\mathbf{y}_s^A\right) \geq \mathbf{y}_t^B, \\ & \forall s : \lambda_s^A \geq 0. \end{aligned}$$

$D_t^{A,A}$, $D_t^{B,A}$, and $D_t^{B,B}$ are obtained by changing A to B and/or B to A in (LP-4). As a final remark, it is possible to define the program when not assuming that the global and/or the group-specific input requirements are convex. For the sake of compactness, we do not explicitly give those programs here.

3.6 Observation of the group-specific and global input prices

In practice, the group-specific and/or the global inputs could be difficult to observe or to rely on. For these cases, *GMI* (and its decomposition) can be computed as it does not depend on the input prices, but this is not the case for *GCMI* and for its decomposition. In particular, the global input prices are required to compute *GCMI*, and both the global and group-specific input prices are needed for its decomposition. For these cases, the dependence of *GCMI* to the input prices clearly represents an important drawback. In this last part, we propose a solution to overcome this issue. Therefore, it means that it is still possible to evaluate *GCMI* even if no input price data are available. That is, it is still possible to consider the cost minimization behavior of the DMUs when defining the index even if no input price data are available.

Different strategies can be used to overcome the issue of unknown input prices at both levels. We choose to rely on the benefit of the doubt approach (see, for example, Cherchye et al. (2007), Cherchye et al. (2013), Cherchye et al. (2016), and Walheer (2018a)). The main advantage of this method is its easy and intuitive use. Moreover, for our context, when relying on this strategy we end with interesting relationships between the indexes and their decomposition (Table 1). Alternative methods include multi-objective programming (see, for example, Zimmermann (1978), Charnes et al. (1989), Kao and Chan (2013), Tohidi and Razavyan (2013), Kao et al. (2014), and Despotis et al. (2016)), the law of one price

Table 1 Linear programs and relationships: a summary

Group-specific input prices	Global input prices: Observed		Global input prices: Not observed	
	LPs	Relationships	LPs	Relationships
Observed	(LP-1)	–	(LP-6)	AED=1/BPAD, GAMI=1,
	(LP-2)		(LP-2)	*GCMI=GMI*
Not observed	(LP-1)	AED=1,	(LP-6)	*AED*=1, *TED=CED*,
	(LP-5)	*TED=CED*,	(LP-5)	*BPAD*=1, *BPD=BPCD*,
		GAMI=BPAD		*GAMI*=1, *GCMI=GMI*

(Kuosmanen et al. (2006), and Fang and Li (2015)), or industry-based approaches (Li and Ng (1995), Ylvinger (2000), Färe and Zelenyuk (2003, 2007), Walheer (2016a, b), Zelenyuk (2006, 2016), and Färe and Karagiannis (2017)). These approaches make extra assumptions on either the technologies or the input prices, or are more complex (e.g. non-linearity), while the benefit of the doubt approach gives full flexibility and requires solving linear programs.

At this point, we remark that when partial information on either the global input prices or the group-specific input prices is available, they can be incorporated in (LP-5) and (LP-6). For example, bounds for the prices could be observed, or the relationship between the group-specific and the global inputs prices could be known. Incorporating these types of additional information gives the advantage of increasing the realism of the computed input prices. See our application in Section 4 for an example.

Let us start with the case when the group-specific prices, i.e., $\mathbf{w}_t^B$ for $t \in \{1, \ldots, n_B\}$ and/or $\mathbf{w}_t^A$ $t \in \{1, \ldots, n_A\}$, are not observed. In that case, we suggest choosing the prices that maximize the minimal costs. Or, in other words, we evaluate the DMUs in the best possible way, which gives the benefit of the doubt in the absence of true price information. For example when the input prices are not observed for group B, $CE_t^{A,B}$ can be evaluated for every DMU t operating at $\left(\mathbf{y}_t^B, \mathbf{x}_t^B\right)$ by the following program (NLP-1):

$$
\begin{aligned}
CE_t^{A,B} = & \max_{\substack{C_t^{A,B} \in \mathbb{R}_+ \\ \mathbf{w}_t^B \in \mathbb{R}_+^P}} \frac{C_t^{A,B}}{\mathbf{w}_t^{B\prime} \mathbf{x}_t^B} \\
\text{s.t. } & C_t^{A,B} \leq \mathbf{w}_t^{B\prime}\left(\alpha_s^{A,B} \mathbf{x}_s^A\right) \text{ for all } s \in \{1, \ldots, n_A\} : \\
& \left(\alpha_s^{A,B} \mathbf{y}_s^A\right) \geq \mathbf{y}_t^B.
\end{aligned}
$$

(NLP-1) looks very similar to (LP-2). The only difference is that in (NLP-1), the input prices are variables, while in (LP-2), they are observed. As such, variables appear at both the numerator and denominator of the objective function in (NLP-1). As a result, (NLP-1) is a non-linear program. We can make (NLP-1) linear by setting the denominator $(\mathbf{w}_t^{B\prime} \mathbf{x}_t^B)$ equals 1. This transformation, introduced by Charnes and Cooper (1962), has been made popular by Charnes et al. (1978) for nonparametric efficiency methods. The equivalent linear program is thus given by (LP-5):

$$
\begin{aligned}
CE_t^{A,B} = & \max_{\substack{C_t^{A,B} \in \mathbb{R}_+ \\ \mathbf{w}_t^B \in \mathbb{R}_+}} C_t^{A,B} \\
\text{s.t. } & C_t^{A,B} \leq \mathbf{w}_t^{B\prime}\left(\alpha_s^{A,B} \mathbf{x}_s^A\right) \text{ for all } s \in \{1, \ldots, n_B\} : \left(\alpha_s^{A,B} \mathbf{y}_s^A\right) \geq \mathbf{y}_t^B, \\
& \mathbf{w}_t^{B\prime} \mathbf{x}_t^B = 1.
\end{aligned}
$$

Besides the group-specific input prices, the global input prices $\mathbf{w}^G$ might also be unobserved. As done previously for the group-specific input prices, we choose the group-specific input prices that maximize the minimal costs. The program to evaluate GCE_t for every DMU t operating at $(\mathbf{y}_t, \mathbf{x}_t)$ is given as follows (NLP-2):

$$
\begin{aligned}
GCE_t = & \max_{\substack{GC_t \in \mathbb{R}_+ \\ \mathbf{w}^G \in \mathbb{R}_+^P}} \frac{GC_t}{\mathbf{w}^{G\prime} \mathbf{x}_t} \\
\text{s.t. } & GC_t \leq \mathbf{w}^{G\prime}(\alpha_s \mathbf{x}_s) \text{ for all } s \in \{1, \ldots, n\} : (\alpha_s \mathbf{y}_s) \geq \mathbf{y}_t.
\end{aligned}
$$

We can make (NLP-2) linear by using the same strategy as for (NLP-1). That is, we set $\mathbf{w}^{G\prime} \mathbf{x}_t = 1$. In other words, we make use of Charnes and Cooper's transformation tailored to the global context. The equivalent linear program is thus given by (LP-6):

$$
\begin{aligned}
GCE_t = & \max_{\substack{GC_t \in \mathbb{R}_+ \\ \mathbf{w}^G \in \mathbb{R}_+^P}} GC_t \\
\text{s.t. } & GC_t \leq \mathbf{w}^{G\prime}(\alpha_s \mathbf{x}_s) \text{ for all } s \in \{1, \ldots, n\} : (\alpha_s \mathbf{y}_s) \geq \mathbf{y}_t, \\
& \mathbf{w}^{G\prime} \mathbf{x}_t = 1.
\end{aligned}
$$

We propose a summary in Table 1 presenting which programs use to evaluate *GCMI* depending whether the group-specific and/or the global input prices are observed or not. In this table, we also present the relationships between the indexes and their components for the different cases.

As discussed previously, allocative efficiency measures inefficiency due to non-optimal allocation of inputs (with respect to the cost optimization behavior). When relying on the most favorable group-specific input prices, no non-optimal allocation of inputs occurs at that level. As a result, we have that there is no allocative inefficiency at the group level. In other words, the allocative efficiency spreads in group B coincide with the spread in group A, making $AED = 1$. It follows that efficiency change based on technical and cost efficiency coincides, i.e., $TED = CED$. Also, when $AED = 1$, we have by construction that *GAMI* corresponds to *BPAD*.

Table 2 Group specification

Group	Number				Total
	Total	Public	Private	Foreign	output
Gas	559	200	178	141	95,914,589
Heat	683	299	307	36	42,501,312
Other	127	41	39	31	12,097,320
Nuclear	6	5	0	1	23,624,421
Hydro	1642	716	711	56	103,307,173
Thermal	11,716	783	6919	3751	1,529,870,706

Next, when relying on the most favorable global input prices, using a similar reasoning to the one done previously for the group-specific input prices, it implies that no allocative inefficient behavior is present at that level. As a result, we have that $GAMI = 1$, making $GCMI = GMI$ and $AED = 1/BPAD$. That is, both definitions of the global indexes coincide. It gives an interpretation of *GMI* as a shadow *GCMI*, as it is defined with the most favorable (or shadow) global input prices. Also, it is intuitive that the equality between *GMI* and *GCMI* only depends on the global input prices (and thus not on the group-specific input prices) as *GCMI* is defined on the basis of the global setting and not on the group-specific setting.

Finally, when we rely on both most favourable global and group-specific input prices, we combine all the previous results: $AED = 1$, $TED = CED$, $GAMI = 1$, $GMI = GCMI$ and $AED = 1/BPAD$. Therefore, we additionally have that $BPAD = 1$ and $BPD = BPCD$.

4 Application

We apply our technique to the case of the energy sector in China. Rapid industrialization and urbanization in China have been accompanied by significant energy consumption. As such, measuring the performances of this sector is of crucial importance for managers, policy makers, and regulators. Malmquist and related indexes have been widely used for this task. See for example, for recent works, Zhang et al. (2011), Zhang and Choi (2013), Fang et al. (2015), Zhang et al. (2015), Emrouznejad and Yang (2016a, 2016b), and therein references.

A particularity of our analysis is that we rely on firm-level data. Indeed, we make use of the Chinese Annual Survey of Industrial Firms Database of the National Bureau of Statistics in China. Therefore, this represents a unique opportunity to study the energy sector in China. Data are available annually between 1998 and 2007. We choose to concentrate our investigation on 2007 since it is the most recent time period.

We consider a very general production process: firms use labor (measured by the annual average employed persons) and capital (measured by the annual average balance of current assets, in 1000 RMB) to produce output (measured in 1000 RMB).[10] We choose to measure output in currency term to make the comparison between different types of energy possible, as they are measured in different units. This is also the choice made by the National Bureau of Statistics in China. Also, we assume that the firms are cost minimizers. We believe that choosing a cost minimizing behavior is more reasonable than, for example, a profit maximizing behavior, as the output side of the production process is rather exogenous to the plant (in the short run at least). Therefore, given a certain output production, firms can try to set the costs at their minimal level.

To present our empirical application, we first define the groups. Next, we present the results of the global indexes. Finally, we compare and decompose our global indexes.

4.1 Group selection

We naturally split firms with regard to their energy type: gas, heat, nuclear, hydro, thermal (steam), and other (e.g., wind, solar). The number of firms per group and the total energy production is given in Table 2. There we also make a distinction regarding the ownership. Note that collective firms are included in the private firms, and the Foreign category includes both foreign partnerships and foreign-owned firms.

The largest group, in terms of number of firms and total output production is *Thermal*. More than half of the firms in that group are private, while more than one fourth are foreign. Next, the *Gas* and *Hydro* groups have similar total output production, but there are three times as many firms in the *Hydro* group. The distribution of firms between the three categories is more or less uniform for the *Gas* group, while there are around 700 private and public firms for the *Hydro* group. The *Heat* group has a total production equal to less than half of the *Gas* group production, but contains more firms. There are only six nuclear firms, meaning that those firms have the largest average production (see Table 10 in Appendix A). The smallest total production is for the *Other* groups, but this group is not negligible. This also reveals the lesser importance of alternative energy production; as for example, less polluting production processes.

In Table 10, given in Appendix A, we present the descriptive statistics of the two inputs and the output per group. These descriptive statistics reveal that the size of the firms vary (sometimes importantly) within groups. In other words, they indicate the potential presence of outliers in our sample. The impacts on the index results could be huge since outliers may disproportionately and misleadingly influence the evaluation of the firms' performance. To overcome this potential problem, we make our results

[10] See, for example, Walheer (2018c) for more discussion on how to model energy firms.

robust to the presence of outliers by using the order$-m$ (where m can be viewed as a trimming parameter) and the order$-\alpha$ (analogous to traditional quantile functions) procedures (see, for example, Daraio and Simar (2007) for discussion). Also, using these procedures ensures that our results are not sensitive to potential data measurement issues (see, for example, Brandt et al. (2012, 2014) for more discussion about the challenges of using the Chinese Annual Survey of Industrial Firms Database).

4.2 Results

Unfortunately, no data are provided for the input prices in the Chinese Annual Survey of Industrial Firms Database. As such, we make use of (LP-5) to compute the group-level cost efficiency measurements. Nevertheless, the total wage payable is provided. As such, we use these data to construct bounds for the labor prices. In particular, we divide the wage payable by the number of employees, and take a 50% centered interval to obtain lower and upper bounds (we choose 50% to give enough flexibility to the linear programs).[11] For the global input prices, we prefer not to specify any relationship; as such, we make use of (LP-6) to compute the global cost efficiency measurements. For the distance functions, we use (LP-3) and (LP-4). At this point, we remark that no infeasibility issues have been detected in the computation process. This is one the advantages of our global indexes. The results for *GCMI*, *GMI*, and *GAMI* are shown in Table 3.

The results have to be interpreted as follows: for example, $GCMI = 1.07$ captures the (average) cost performance between the firms in *Gas* and *Heat*; that is, it means that the firms in the *Gas* group present, on average, greater cost-performance than those in the *Heat* group. Also, it is worth noting that we obtain *GCMI* between *Heat* and *Gas* as the reverse of *GCMI* between *Gas* and *Heat*: $\frac{1}{1.07} = 0.93$ (see (14)), and it indicates that *Heat* has a worse cost-performance than *Gas*.

Thermal is the best performing group for both the cost and technical perspectives (for the allocative perspective, improvements could be done). This group contains the largest number of firms, and produces the highest share of energy in China. Next, *Gas* performs better than all the other groups, but it is not the case for *Hydro*, which presents the worst performance. These two groups share the second place in terms of energy production. Finally, *Nuclear* presents rather bad performances in terms of cost and technical indexes, but is better in terms of allocative index.

[11] Note that if we do not use any bounds for the prices, we obtain the same results for *GCMI* and *GMI*. That is, in this case, *GMI* is interpreted as a shadow *GCMI*. It implies also that $GAMI = 1$ for all groups when relying on the best input prices (see Section 3.6 for more details).

An attractive feature of Malmquist indexes is that they can be decomposed into different components giving the option to better understand the performance differences observed. As shown in Section 3.4, this is also the case for our global indexes. *GCMI* can be decomposed into two parts: *CED*, capturing difference in within-group cost efficiency spreads, and *BPCD*, capturing difference in best practice cost gaps. In a similar vein, *GMI* can be decomposed into two parts: *TED*, capturing difference in within-group technical efficiency spreads, and *BPD*, capturing difference in best practice technical gaps; and *GAMI* can be decomposed into two parts: *AED*, capturing difference in within-group allocative efficiency spreads, and *BPAD*, capturing best practice allocative gaps. The decompositions are given in Tables 4, 5, and 6, respectively.

The decompositions reveal that the best performances of *Thermal* are mainly due to a smaller efficiency spread in that group with respect to the other groups. For *Gas*, the better performances are mainly explained by better best practice. The worst performances of *Hydro* are due to either greater spread or worse best practice depending on which group is the comparison partner. The relatively bad performances of *Nuclear* are explained by very large spreads, but the best practice indicators reveal good performances for this group.

4.3 Circularity

One of the advantages of our indexes is their circularity. In other words, they provide a stable ranking of the groups. For example, *GCMI* between *Gas* and *Heat* can be obtained as multiplying *GCMI* between *Gas* and *Hydro* and *GCMI* between *Hydro* and *Heat*: $1.13 \times \frac{1}{1.05} = 1.07$ (Table 3). Transitivity is also confirmed. For example, $GCMI = 1.07$ between *Gas* and *Heat* and $GCMI = 1.05$ between *Heat* and *Hydro*, implies that the index should indicate that *Gas* presents better cost-performance than *Hydro*. This is the case as *GCMI* between *Gas* and *Hydro* is 1.13 (Table 3).

Let us compare our results based on *GCMI* with those obtained using *CMI* of Thanassoulis et al. (2015). Clearly, similar comparisons could be conducted between *GMI* and *MI* of Camanho and Dyson (2006), and between *GAMI* and *AMI* defined in Section 3.1. The results for *CMI* are displayed in Table 7.

Before interpreting the results, we point out that several cases of infeasibility occur when computing *CMI*. Indeed, contrary to our global indexes, *CMI* is sensitive to infeasibility issues. For these cases, we set the cost efficiency scores and the distance functions to 1. An initial observation is that the results for *CMI* are rather different from those obtained with *GCMI*. Next, *CMI* is not circular. For example, $CMI = 0.65$ between *Gas* and *Heat* and $CMI = 1.26$ between *Heat* and *Hydro*, but *CMI* between *Gas* and *Hydro* is 0.89, which is different to $0.65 \times 1.26 = 0.82$. Note that, for our application, transitivity is fulfilled for *CMI*.

Table 3 GCMI, GMI, and GAMI results

GCMI	*Gas*	*Heat*	*Other*	*Nuclear*	*Hydro*	*Thermal*
Gas	1	1.07	1.05	1.13	1.13	0.51
Heat		1	0.98	1.05	1.05	0.91
Other			1	1.08	1.08	0.93
Nuclear				1	1.00	0.45
Hydro					1	0.45
Thermal						1
GMI	*Gas*	*Heat*	*Other*	*Nuclear*	*Hydro*	*Thermal*
Gas	1	1.03	1.03	1.32	1.10	0.58
Heat		1	1.00	1.27	1.07	0.98
Other			1	1.28	1.07	0.98
Nuclear				1	0.84	0.44
Hydro					1	0.52
Thermal						1
GAMI	*Gas*	*Heat*	*Other*	*Nuclear*	*Hydro*	*Thermal*
Gas	1	1.04	1.02	0.86	1.02	0.89
Heat		1	0.98	0.83	0.99	0.93
Other			1	0.84	1.01	0.95
Nuclear				1	1.19	1.03
Hydro					1	0.86
Thermal						1

Table 4 Decomposition of GCMI

CED	*Gas*	*Heat*	*Other*	*Nuclear*	*Hydro*	*Thermal*
Gas	1	0.45	0.86	0.31	0.58	0.43
Heat		1	1.89	0.68	1.27	0.95
Other			1	0.36	0.67	0.50
Nuclear				1	1.87	1.39
Hydro					1	0.74
Thermal						1
BPCD	*Gas*	*Heat*	*Other*	*Nuclear*	*Hydro*	*Thermal*
Gas	1	2.37	1.22	3.65	1.96	1.19
Heat		1	0.52	1.54	0.83	0.96
Other			1	2.99	1.60	1.86
Nuclear				1	0.54	0.32
Hydro					1	0.61
Thermal						1

Finally, the last issue when relying on *CMI* is that it could average two disparate measurements of performance differences. In Table 8, we present the two components: CMI^A and CMI^B (see Section 3.1, and in particular (8)).

CMI^A and CMI^B are interpreted in a similar manner; the only difference between these two indexes is the referent group for the technology: group A for CMI^A and group B for CMI^B. As such, the referent group has an impact when both indexes give opposite rankings. This occurs, for example, when comparing *Heat* and *Thermal*. In that particular case, $CMI^A = 0.91$ implying that *Thermal* performs better than *Heat*, while $CMI^B = 1.08$ implying the opposite. Note that the problem of choosing a referent group is dropped when relying on the global indexes.

We end this part by relating *GCMI* and *CMI*. One way is to make use, as explained in Section 3.3, of the ratio $\frac{GCMI}{CMI}$. These ratios are given in Table 9. When these ratios are equal (or at least close enough) to 1, we conclude that the chosen referent group has no impact on the ranking. Clearly, for our application, the difference between the two approaches is quite important.

5 Conclusion

Since its initial definition, the Malmquist index (MI) has demonstrated its usefulness as a practical decision support tool. Initially, the MI was used for contexts of several Decision Making Units (DMUs) observed for more than

Table 5 Decomposition of GMI

TED	*Gas*	*Heat*	*Other*	*Nuclear*	*Hydro*	*Thermal*
Gas	1	0.51	0.89	0.37	0.62	0.50
Heat		1	1.75	0.74	1.22	0.99
Other			1	0.42	0.70	0.57
Nuclear				1	1.65	1.35
Hydro					1	0.81
Thermal						1
BPD	*Gas*	*Heat*	*Other*	*Nuclear*	*Hydro*	*Thermal*
Gas	1	2.04	1.16	3.53	1.79	1.15
Heat		1	0.57	1.73	0.88	0.99
Other			1	3.03	1.54	1.73
Nuclear				1	0.51	0.32
Hydro					1	0.64
Thermal						1

Table 6 Decomposition of GAMI

AED	*Gas*	*Heat*	*Other*	*Nuclear*	*Hydro*	*Thermal*
Gas	1	0.89	0.97	0.83	0.94	0.86
Heat		1	1.08	0.93	1.05	0.96
Other			1	0.86	0.97	0.88
Nuclear				1	1.13	1.03
Hydro					1	0.91
Thermal						1
BPAD	*Gas*	*Heat*	*Other*	*Nuclear*	*Hydro*	*Thermal*
Gas	1	1.16	1.05	1.03	1.09	1.03
Heat		1	0.91	0.89	0.94	0.97
Other			1	0.98	1.04	1.07
Nuclear				1	1.06	1.00
Hydro					1	0.95
Thermal						1

Table 7 CMI results

CMI	*Gas*	*Heat*	*Other*	*Nuclear*	*Hydro*	*Thermal*
Gas	1	0.65	1.56	0.34	0.89	0.71
Heat		1	2.13	0.83	1.26	0.99
Other			1	0.53	0.63	0.50
Nuclear				1	1.60	1.35
Hydro					1	0.85
Thermal						1

two periods. Recently, it has been shown that the MI can also be used in contexts of comparing groups of DMUs for one period of time. In these contexts, the MI, therefore, is used to compare group performances, instead of comparing DMU performances.

The popularity of the MI is explained by two main reasons: only the input and output data are required, and the MI can be decomposed into several components; but it also presents important drawbacks: it is based on a technical formation of efficiency; it is not circular; it averages two possibly disparate measurements of performance change; and infeasibility may occur in the computation process. Recently, attention has been given to propose solutions to overcome these drawbacks in the contexts of DMUs. In particular, the concept of Global MI (GMI) has been introduced. The GMI keeps the same advantages as those of the MI, without present these drawbacks. Nevertheless, there is also a price to pay: rely on a fixed reference technology and use a common price

In this paper, we extend the concept of GMI to the case of groups of DMUs. That is, the new proposed index is circular, gives only one measure of performance difference, is immune to infeasibility issues, requires only the observation of the inputs and outputs, and can be decomposed in different components. We also consider the case when DMUs have an economic optimization behavior by proposing the concept of Global cost Malmquist index (GCMI) for the group contexts. Contrary to the GMI, the GCMI requires the observation of the input prices. As it may represent a strong assumption for some settings/applications, we propose solutions when the input prices are partially or not observed.

Table 8 CMI^A and CMI^B results

CMI^A	*Gas*	*Heat*	*Other*	*Nuclear*	*Hydro*	*Thermal*
Gas	1	0.57	1.49	0.38	0.81	0.70
Heat		1	1.96	0.68	1.24	0.91
Other			1	0.36	0.62	0.41
Nuclear				1	1.35	1.30
Hydro					1	0.75
Thermal						1
CMI^B	*Gas*	*Heat*	*Other*	*Nuclear*	*Hydro*	*Thermal*
Gas	1	0.75	1.63	0.31	0.98	0.72
Heat		1	2.32	1.01	1.27	1.08
Other			1	0.78	0.64	0.60
Nuclear				1	1.88	1.40
Hydro					1	0.97
Thermal						1

Table 9 Comparison

$\frac{GCMI}{CMI}$	*Gas*	*Heat*	*Other*	*Nuclear*	*Hydro*	*Thermal*
Gas	1	0.61	1.48	0.30	0.79	1.39
Heat		1	2.18	0.79	1.19	1.09
Other			1	0.49	0.58	0.53
Nuclear				1	1.60	3.00
Hydro					1	1.89
Thermal						1

We illustrate our new indexes with the case of the energy sector in China. MI, and related indexes, have been widely used to evaluate the performances of this sector, as rapid industrialization and urbanization in China have made this sector of crucial importance for managers, policy makers and regulators. A particularity of our analysis is that we rely on firm-level data using the Chinese Annual Survey of Industrial Firms Database of the National Bureau of Statistics in China. We found that cost reductions are possible, but that the reasons are different for each type of energy.

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Compliance with ethical standards

Conflict of interest The author declares no competing interests.

6 Appendix A

Table 10

Table 10 Output and inputs: descriptive statistics

Group	Statistics	Output	Labour	Capital
Gas	min	1638	7	90
	mean	171,582	267	87,712
	median	40,802	90	20,511
	max	5,873,353	4775	2,187,281
	std	455,418	532	209,668
Heat	min	727	3	−11,706
	mean	62,227	199	58,211
	median	26,650	108	16,893
	max	1,287,954	4320	2,579,683
	std	110,689	362	156,537
Other	min	1500	4	15
	mean	95,254	67	56,071
	median	33,854	39	19,571
	max	3,979,634	452	1,712,800
	std	360,333	73	166,193
Nuclear	min	740,492	10	1,145,279
	mean	3,937,404	954	3,444,331
	median	4,091,597	1143	2,109,562
	max	6,727,616	1726	7,327,116
	std	2,046,670	770	2,669,585
Hydro	min	40	1	0
	mean	62,915	164	47,261
	median	15,646	75	10,970
	max	16,153,496	10,634	10,848,232
	std	433,225	381	300,018
Thermal	min	10	1	0
	mean	130,580	260	50,981
	median	23,756	110	8494
	max	18,825,752	62,378	13,561,304
	std	518,861	778	235,969

7 Appendix B

Proof of Eq. (19):

$$\frac{GMI}{MI}=\left[\frac{GMI}{MI^{A}}\frac{GMI}{MI^{B}}\right]^{1/2},$$

$$=\left[\frac{\left(\frac{\left[\prod_{t=1}^{n_B} GD_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} GD_t\right]^{1/n_A}}\right)^{-1}\left(\frac{\left[\prod_{t=1}^{n_B} GD_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} GD_t\right]^{1/n_A}}\right)^{-1}}{\left(\frac{\left[\prod_{t=1}^{n_B} D_t^{A,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} D_t^{A,A}\right]^{1/n_A}}\right)^{-1}\left(\frac{\left[\prod_{t=1}^{n_B} D_t^{B,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} D_t^{B,A}\right]^{1/n_A}}\right)^{-1}}\right]^{1/2},$$

$$=\left[\frac{\left(\frac{\left[\prod_{t=1}^{n_B} GD_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_B} D_t^{A,B}\right]^{1/n_B}}\right)^{-1}\left(\frac{\left[\prod_{t=1}^{n_B} GD_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_B} D_t^{B,B}\right]^{1/n_B}}\right)^{-1}}{\left(\frac{\left[\prod_{t=1}^{n_A} GD_t\right]^{1/n_A}}{\left[\prod_{t=1}^{n_A} D_t^{A,A}\right]^{1/n_A}}\right)^{-1}\left(\frac{\left[\prod_{t=1}^{n_A} GD_t\right]^{1/n_A}}{\left[\prod_{t=1}^{n_A} D_t^{B,A}\right]^{1/n_A}}\right)^{-1}}\right]^{1/2},$$

$$=\left[\frac{BPG^{A,B}}{BPG^{A,A}}\times\frac{BPG^{B,B}}{BPG^{B,A}}\right]^{1/2}.$$

Proof of Eq. (20):

$$\frac{GCMI}{CMI}=\left[\frac{GCMI}{CMI^{A}}\frac{GCMI}{CMI^{B}}\right]^{1/2},$$

$$=\left[\frac{\frac{\left[\prod_{t=1}^{n_B} GCE_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} GCE_t\right]^{1/n_A}}\frac{\left[\prod_{t=1}^{n_B} GCE_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} GCE_t\right]^{1/n_A}}}{\frac{\left[\prod_{t=1}^{n_B} CE_t^{A,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} CE_t^{A,A}\right]^{1/n_A}}\frac{\left[\prod_{t=1}^{n_B} CE_t^{B,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} CE_t^{B,A}\right]^{1/n_A}}}\right]^{1/2},$$

$$=\left[\frac{\frac{\left[\prod_{t=1}^{n_B} GCE_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_B} CE_t^{A,B}\right]^{1/n_B}}\frac{\left[\prod_{t=1}^{n_B} GCE_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_B} CE_t^{B,B}\right]^{1/n_B}}}{\frac{\left[\prod_{t=1}^{n_A} GCE_t\right]^{1/n_A}}{\left[\prod_{t=1}^{n_A} CE_t^{A,A}\right]^{1/n_A}}\frac{\left[\prod_{t=1}^{n_A} GCE_t\right]^{1/n_A}}{\left[\prod_{t=1}^{n_A} CE_t^{B,A}\right]^{1/n_A}}}\right]^{1/2},$$

$$=\left[\frac{BPCG^{A,B}}{BPCG^{A,A}}\times\frac{BPCG^{B,B}}{BPCG^{B,A}}\right]^{1/2}.$$

Proof of Eq. (21):

$$GMI=\left(\frac{\left[\prod_{t=1}^{n_B} GD_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} GD_t\right]^{1/n_A}}\right)^{-1},$$

$$=\left(\frac{\left[\prod_{t=1}^{n_B} GD_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} GD_t\right]^{1/n_A}}\times\frac{\left[\prod_{t=1}^{n_A} D_t^{A,A}\right]^{1/n_A}}{\left[\prod_{t=1}^{n_A} D_t^{A,A}\right]^{1/n_A}}\times\frac{\left[\prod_{t=1}^{n_B} D_t^{B,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_B} D_t^{B,B}\right]^{1/n_B}}\right)^{-1},$$

$$=\left(\frac{\left[\prod_{t=1}^{n_B} D_t^{B,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} D_t^{A,A}\right]^{1/n_A}}\right)^{-1}\times\left(\frac{\left[\prod_{t=1}^{n_A} D_t^{A,A}\right]^{1/n_A}}{\left[\prod_{t=1}^{n_A} GD_t\right]^{1/n_A}}\times\frac{\left[\prod_{t=1}^{n_B} GD_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_B} D_t^{B,B}\right]^{1/n_B}}\right)^{-1},$$

$$=\left(\frac{\left[\prod_{t=1}^{n_B} D_t^{B,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} D_t^{A,A}\right]^{1/n_A}}\right)^{-1}\times\frac{\left(\frac{\left[\prod_{t=1}^{n_B} GD_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_B} D_t^{B,B}\right]^{1/n_B}}\right)^{-1}}{\left(\frac{\left[\prod_{t=1}^{n_A} GD_t\right]^{1/n_A}}{\left[\prod_{t=1}^{n_A} D_t^{A,A}\right]^{1/n_A}}\right)^{-1}},$$

$$=\left(\frac{\left[\prod_{t=1}^{n_B} D_t^{B,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} D_t^{A,A}\right]^{1/n_A}}\right)^{-1}\times\frac{BPG^{B,B}}{BPG^{A,A}},$$

$$=TED\times BPD.$$

Proof of Eq. (22):

$$GCMI=\frac{\left[\prod_{t=1}^{n_B} GCE_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} GCE_t\right]^{1/n_A}},$$

$$=\frac{\left[\prod_{t=1}^{n_B} GCE_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} GCE_t\right]^{1/n_A}}\times\frac{\left[\prod_{t=1}^{n_A} CE_t^{A,A}\right]^{1/n_A}}{\left[\prod_{t=1}^{n_A} CE_t^{A,A}\right]^{1/n_A}}\times\frac{\left[\prod_{t=1}^{n_B} CE_t^{B,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_B} CE_t^{B,B}\right]^{1/n_B}},$$

$$=\frac{\left[\prod_{t=1}^{n_B} CE_t^{B,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} CE_t^{A,A}\right]^{1/n_A}}\times\frac{\left[\prod_{t=1}^{n_A} CE_t^{A,A}\right]^{1/n_A}}{\left[\prod_{t=1}^{n_A} GCE_t\right]^{1/n_A}}\times\frac{\left[\prod_{t=1}^{n_B} GCE_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_B} CE_t^{B,B}\right]^{1/n_B}},$$

$$=\frac{\left[\prod_{t=1}^{n_B} CE_t^{B,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} CE_t^{A,A}\right]^{1/n_A}}\times\frac{\frac{\left[\prod_{t=1}^{n_B} GCE_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_B} CE_t^{B,B}\right]^{1/n_B}}}{\frac{\left[\prod_{t=1}^{n_A} GCE_t\right]^{1/n_A}}{\left[\prod_{t=1}^{n_A} CE_t^{A,A}\right]^{1/n_A}}},$$

$$=\frac{\left[\prod_{t=1}^{n_B} CE_t^{B,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} CE_t^{A,A}\right]^{1/n_A}}\times\frac{BPCG^{B,B}}{BPCG^{A,A}},$$

$$=CED\times BPCD.$$

Proof of Eq. (23):

$$CED=\frac{\left[\prod_{t=1}^{n_B} CE_t^{B,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} CE_t^{A,A}\right]^{1/n_A}},$$

$$=\frac{\left[\prod_{t=1}^{n_B} CE_t^{B,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} CE_t^{A,A}\right]^{1/n_A}}\times\frac{\left[\prod_{t=1}^{n_A} D_t^{A,A}\right]^{1/n_A}}{\left[\prod_{t=1}^{n_A} D_t^{A,A}\right]^{1/n_A}}\times\frac{\left[\prod_{t=1}^{n_B} D_t^{B,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_B} D_t^{B,B}\right]^{1/n_B}},$$

$$= \left(\frac{\left[\prod_{t=1}^{n_B} D_t^{B,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} D_t^{A,A}\right]^{1/n_A}} \right)^{-1} \times \frac{\left[\prod_{t=1}^{n_B} CE_t^{B,B} \times D_t^{B,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} CE_t^{A,A} \times D_t^{A,A}\right]^{1/n_A}},$$

$$= TED \times AED.$$

Proof of Eq. (24):

$$BPCD = \frac{\frac{\left[\prod_{t=1}^{n_B} GCE_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_B} CE_t^{B,B}\right]^{1/n_B}}}{\frac{\left[\prod_{t=1}^{n_A} GCE_t\right]^{1/n_A}}{\left[\prod_{t=1}^{n_A} CE_t^{A,A}\right]^{1/n_A}}},$$

$$= \frac{\left[\prod_{t=1}^{n_B} GCE_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_B} CE_t^{B,B}\right]^{1/n_B}} \times \frac{\left[\prod_{t=1}^{n_B} D_t^{B,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} D_t^{A,A}\right]^{1/n_A}} \times \frac{\left[\prod_{t=1}^{n_B} GD_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} GD_t\right]^{1/n_A}},$$

$$= \frac{\left(\frac{\left[\prod_{t=1}^{n_B} GD_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_B} D_t^{B,B}\right]^{1/n_B}} \right)^{-1}}{\left(\frac{\left[\prod_{t=1}^{n_A} GD_t\right]^{1/n_A}}{\left[\prod_{t=1}^{n_A} D_t^{A,A}\right]^{1/n_A}} \right)^{-1}} \times \frac{\left[\prod_{t=1}^{n_B} GCE_t \times GD_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_B} CE_t^{B,B} \times D_t^{B,B}\right]^{1/n_B}} \Big/ \frac{\left[\prod_{t=1}^{n_A} GCE_t \times GD_t\right]^{1/n_A}}{\left[\prod_{t=1}^{n_A} CE_t^{A,A} \times D_t^{A,A}\right]^{1/n_A}},$$

$$= \frac{BPG^{B,B}}{BPG^{A,B}} \times \frac{\left[\prod_{t=1}^{n_B} GCE_t \times GD_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_B} CE_t^{B,B} \times D_t^{B,B}\right]^{1/n_B}} \Big/ \frac{\left[\prod_{t=1}^{n_A} GCE_t \times GD_t\right]^{1/n_A}}{\left[\prod_{t=1}^{n_A} CE_t^{A,A} \times D_t^{A,A}\right]^{1/n_A}},$$

$$= BPD \times BPAD.$$

Proof of Eq. (29):

$$GAMI = \frac{\left[\prod_{t=1}^{n_B} GCE_t \times GD_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} GCE_t \times GD_t\right]^{1/n_A}},$$

$$= \frac{\left[\prod_{t=1}^{n_B} GCE_t \times GD_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} GCE_t \times GD_t\right]^{1/n_A}} \times \frac{\left[\prod_{t=1}^{n_A} CE_t^{A,A} \times D_t^{A,A}\right]^{1/n_A}}{\left[\prod_{t=1}^{n_A} CE_t^{A,A} \times D_t^{A,A}\right]^{1/n_A}} \times \frac{\left[\prod_{t=1}^{n_B} CE_t^{B,B} \times D_t^{B,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_B} CE_t^{B,B} \times D_t^{B,B}\right]^{1/n_B}},$$

$$= \frac{\left[\prod_{t=1}^{n_B} CE_t^{B,B} \times D_t^{B,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} CE_t^{A,A} \times D_t^{A,A}\right]^{1/n_A}} \times \frac{\left[\prod_{t=1}^{n_A} CE_t^{A,A} \times D_t^{A,A}\right]^{1/n_A}}{\left[\prod_{t=1}^{n_A} GCE_t \times GD_t\right]^{1/n_A}} \times \frac{\left[\prod_{t=1}^{n_B} GCE_t \times GD_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_B} CE_t^{B,B} \times D_t^{B,B}\right]^{1/n_B}},$$

$$= \frac{\left[\prod_{t=1}^{n_B} CE_t^{B,B} \times D_t^{B,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} CE_t^{A,A} \times D_t^{A,A}\right]^{1/n_A}} \times \frac{\frac{\left[\prod_{t=1}^{n_B} GCE_t \times GD_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_B} CE_t^{B,B} \times D_t^{B,B}\right]^{1/n_B}}}{\frac{\left[\prod_{t=1}^{n_A} GCE_t \times GD_t\right]^{1/n_A}}{\left[\prod_{t=1}^{n_A} CE_t^{A,A} \times D_t^{A,A}\right]^{1/n_A}}},$$

$$= \frac{\left[\prod_{t=1}^{n_B} CE_t^{B,B} \times D_t^{B,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} CE_t^{A,A} \times D_t^{A,A}\right]^{1/n_A}} \times \frac{BPAG^{B,B}}{BPAG^{A,A}},$$

$$= AED \times BPAD.$$

Proof of Eq. (30):

$$\frac{GAMI}{AMI} = \left[\frac{GAMI}{AMI^A} \frac{GAMI}{AMI^B} \right]^{1/2},$$

$$= \left[\frac{\frac{\left[\prod_{t=1}^{n_B} GCE_t \times GD_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} GCE_t \times GD_t\right]^{1/n_A}}}{\frac{\left[\prod_{t=1}^{n_B} CE_t^{A,B} \times D_t^{A,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} CE_t^{A,A} \times D_t^{A,A}\right]^{1/n_A}}} \frac{\frac{\left[\prod_{t=1}^{n_B} GCE_t \times GD_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} GCE_t \times GD_t\right]^{1/n_A}}}{\frac{\left[\prod_{t=1}^{n_B} CE_t^{B,B} \times D_t^{B,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} CE_t^{B,A} \times D_t^{B,A}\right]^{1/n_A}}} \right]^{1/2},$$

$$= \left[\frac{\frac{\left[\prod_{t=1}^{n_B} GCE_t \times GD_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_B} CE_t^{A,B} \times D_t^{A,B}\right]^{1/n_B}}}{\frac{\left[\prod_{t=1}^{n_A} GCE_t \times GD_t\right]^{1/n_A}}{\left[\prod_{t=1}^{n_A} CE_t^{A,A} \times D_t^{A,A}\right]^{1/n_A}}} \frac{\frac{\left[\prod_{t=1}^{n_B} GCE_t \times GD_t\right]^{1/n_B}}{\left[\prod_{t=1}^{n_B} CE_t^{B,B} \times D_t^{B,B}\right]^{1/n_B}}}{\frac{\left[\prod_{t=1}^{n_A} GCE_t \times GD_t\right]^{1/n_A}}{\left[\prod_{t=1}^{n_A} CE_t^{B,A} \times D_t^{B,A}\right]^{1/n_A}}} \right]^{1/2},$$

$$= \left[\frac{BPAG^{A,B}}{BPAG^{A,A}} \times \frac{BPAG^{B,B}}{BPAG^{B,A}} \right]^{1/2}$$

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