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Fractional derivatives model of aeroelastic derivatives of bridge decks

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ABSTRACT: In a linear setting aeroelastic forces of bridge decks are expressed by means of so-called aeroelastic derivatives. These frequency-dependent functions are typically measured in dedicated wind tunnel experiments or sometimes also by means of numerical simulations. For several reasons, it is necessary to model, interpolate or smooth the experimentally measured coefficients. Jones' method and its extensions consist in approximating the aeroelastic derivatives with rational fractions. In this paper, another family of models is presented. It generalizes Jones' approximation by considering fractional derivatives. Both the new and existing models are fitted to experimental data. It is shown that better fitting can be obtained with the proposed model.

Keywords: Scanlan coefficients, Jones' approximation, indicial function, wind tunnel, flutter

1. INTRODUCTION

Long span bridges are very sensitive to wind. Therefore, the determination of the aerodynamic forces plays an important role in the design of these structures. These forces can be expressed through six state variables and eighteen aerodynamic parameters called Scanlan coefficients or flutter derivatives [Tamura Y., Kareem A. (2013)]. These coefficients are generally obtained experimentally. The fitting of these coefficients can be carried out by using spline interpolations. This is mainly used in the frequency domain in order to determine the critical wind speed. More advanced studies such as buffeting including non-linear time domain analysis require a mathematical model for the flutter derivatives. Jones' approximation has been commonly applied thanks to its formulation with integer exponents in the frequency domain. These integer exponents result in indicial functions expressed by means of exponential functions in the time domain, which allows structural analysis by means of standard integration techniques for dynamics systems, upon introduction of augmented aerodynamic states.

An extension to Jones' approximation is proposed in this paper based on fractional exponents. In literature, rational exponents models have already been used. Swinney has demonstrated their effectiveness in approximating the aeroelastic behaviour of an airfoil section in freestream flow [Swinney David V. (1989)]. He was able to perform a better fitting of Theodorsen's function, inter alia, with a 2-parameter model than Jones' approximation with 4 parameters.

It is legitimate to wonder why, from a physical point of view, the flutter derivatives would be modelled by non-integer powers. This could be explained by the fact that turbulence develops in the very near vicinity of a bridge deck, even when it comes to estimating flutter, and that the Kolmogorov cascade explains a spectral exponent of -5/3 at high frequency.

This work presents an extension of Swinney's fractional derivative model, which is also found to be a generalization of Jones' model when the exponents are integers. Although the proposed model can be used in the framework of a linear time invariant modelling of the aeroelastic system, this extended abstract is limited to showing the fitting of the model parameters to some experimental data.

2. PROBLEM FORMULATION

The proposed model is the following:

$$C(k) = F(k) + iG(k) = a_0 - \sum_{j=1}^{n} \frac{a_j (ik)^{\alpha_j}}{b_j + (ik)^{\beta_j}} \tag{1}$$

where $k = \frac{\omega B}{2U}$ is the reduced frequency of oscillations, B is the deck width, U is the wind speed, $i = \sqrt{-1}$, $\{a_j, b_j\} \in \mathbb{R}$ and $\{\alpha_j, \beta_j\} \in \mathbb{R}_0^+$. Our model consists of a constant term a_0 and a sum of four-parameter terms. The particularity compared to the Jones function is that we added fractional exponents, which results in fractional derivatives in time domain.

This model is used to fit Scanlan coefficients H^* and A^* (or even P^*). The fitting consists in minimizing a cost function [Caracoglia L., Jones N.P. (2003)], defined as

$$\min \sum_l^M R_l^2 = \min \sum_l^M \left\{ \left[F_{Lh}(k_l) - \frac{2k_l [H_1^*(2k_l)]}{\hat{C}_L} \right]^2 + \left[G_{Lh}(k_l) - \frac{-2k_l [H_4^*(2k_l)]}{\hat{C}_L} \right]^2 \right\} \tag{2}$$

for H_1^* and H_4^* (and similar expression for other pairs of derivatives) where $l = 1, \dots, M$ are the sample points of each derivative and $\hat{C}_L$ is the first derivative of the static coefficient at $\alpha = 0$. This objective function is slightly different from that used in [Caracoglia L., Jones N.P. (2003)]. It is justified by that fact the variable k multiplies Scanlan coefficients in the loads formula, and an unbiased objective function shall therefore directly involve $F(k)$ and $G(k)$.

It is readily seen that the proposed model generalizes both the Jones and Swinney functions. Indeed, by setting $\alpha_j = \beta_j = 1$, the generalized Jones approximation is obtained:

$$C(k) = F(k) + iG(k) = a_0 - \sum_{j=1}^{n} \frac{a_j (ik)}{b_j + (ik)}. \tag{3}$$

By setting, $n = 1$, $\alpha_1 = \beta_1 = \alpha$, $a_0 = 1$, $a_1 = \frac{1}{2}$ and $b_1 = \frac{1}{2a}$, Swinney's function for the flat plate is obtained:

$$C(k) = F(k) + iG(k) = \frac{1 + a(ik)^\alpha}{1 + 2a(ik)^\alpha}. \tag{4}$$

3. ILLUSTRATIONS

Swinney performed the fitting of Theodorsen's function with a fractional order polynomial function composed of only two parameters a and α as shown in Equation (4). By taking $a = 2.19$ and $\alpha = 5/6$, his function provides better accuracy than Jones' approximation with four parameters [Swinney David V. (1989)]. Driven by this successful result for the flat plate, we illustrate the use of the proposed generalized model to fit bridge deck derivatives. Application concerns the Deer-Isle Sedgewick Bridge [Caracoglia L., Jones N.P. (2003)]. The fitting of $H_{1,\dots,4}^*$ and $A_{1,\dots,4}^*$ has been carried out. In order to provide a fair comparison between our function and Jones' function, five and nine parameters have been taken into account for both models. The values of these parameters can be found in Table 1 for the Jones' function and in Table 2 for the fractional derivatives function.

On Figure 1, the fitting for each Scanlan coefficient is shown. It can be seen that the fractional derivatives model is able to capture the behaviour of the bridge properly. By comparing both functions for the same number of parameters, the fractional derivatives model has always a lower residual. This observation stands also for the fractional derivatives function with 5 parameters (only one term, n=1) compared with the Jones function with 9 parameters (i.e. keep n=4 terms in the model). Moreover, ill-conditioning has been observed with Jones' function when more than 2 or 3 terms are considered. This is shown in Table 1 when two a_j are almost equal and opposite sign while their corresponding b_j are almost equal too.

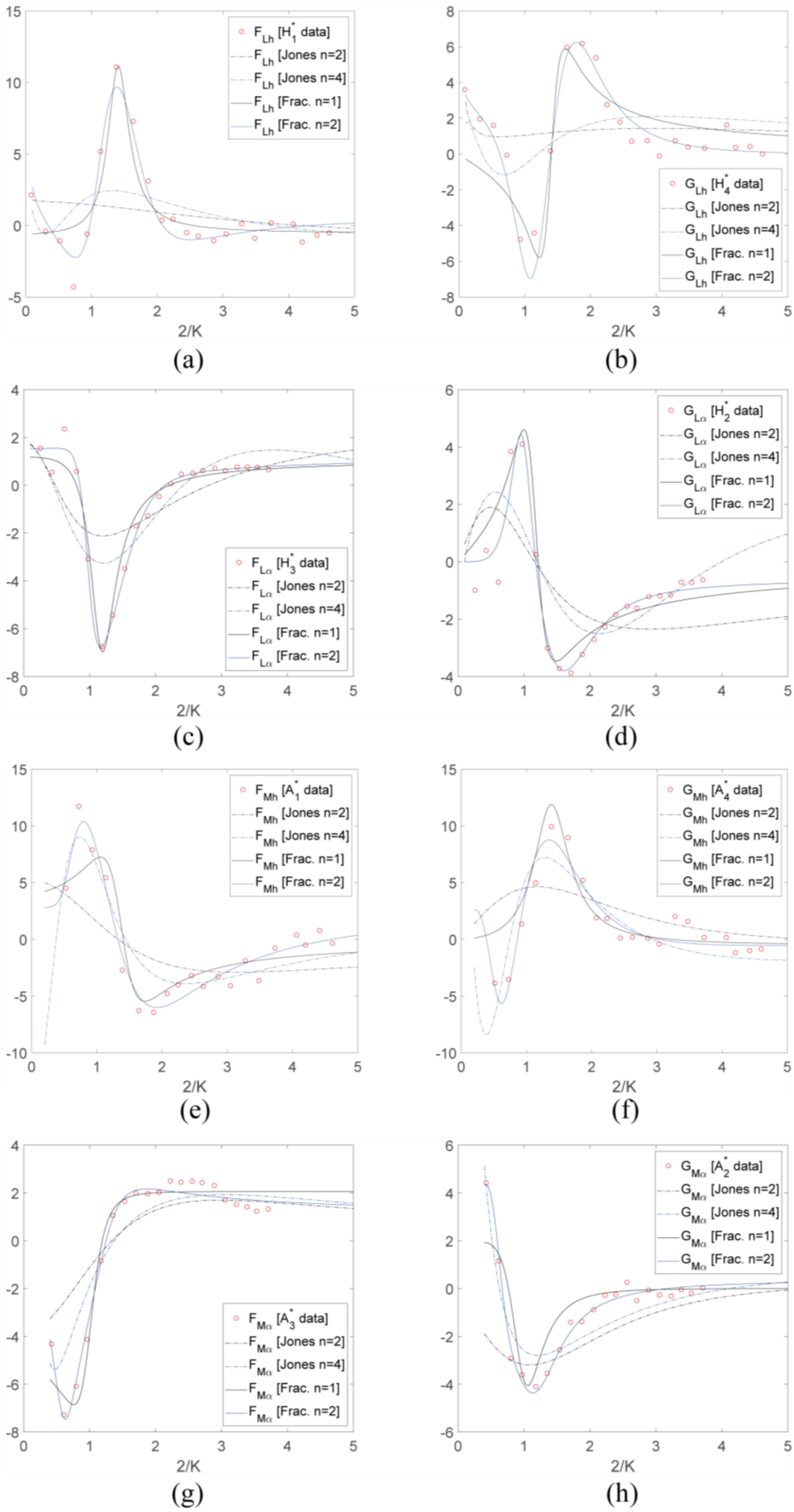

Figure 1. Fitting of Scanlan coefficients for the Jones function and the fractional derivatives function: Jones' approximation with 2 or 4 terms, and the fractional derivative model with 1 or 2 terms.

Table 1. Fitting with the Jones function for n = 2 (upper half) and n = 4 (lower half)

IF	Residual	a_0	a_1	b_1	a_2	b_2	a_3	b_3	a_4	b_4
Φ_{Lh}	354.20	-0.989	-2.676	0.312	-63.91	223.7	/	/	/	/
$\Phi_{L\alpha}$	120.04	2.449	828.6	0.796	-827.9	0.804	/	/	/	/
Φ_{Mh}	436.18	-1.103	590.7	0.458	-597.0	0.465	/	/	/	/
$\Phi_{M\alpha}$	129.95	0.558	838.8	0.500	-834.4	0.497	/	/	/	/
Φ_{Lh}	294.41	-1.476	308.8	0.815	-307.4	0.792	-5.535	5.708	0.4303	-4.3138
$\Phi_{L\alpha}$	88.62	-2.245	-198.0	0.609	359.4	0.494	-165.5	0.388	423.29	5823.7
Φ_{Mh}	147.98	1.356	1232.4	1.093	-2447	0.937	1231.5	0.819	9044.2	-5634.6
$\Phi_{M\alpha}$	39.66	0.424	18.61	1.265	2315	11.19	-4.222	0.2615	-2328.6	10.9659

Table 2. Fitting with the fractional derivatives function for n = 1 (upper half) and n = 2 (lower half)

IF	Residual	a_0	a_1	α_1	b_1	β_1	a_2	α_2	b_2	β_2
Φ_{Lh}	104.53	-0.6247	-2.204	0.923	0.533	1.836	/	/	/	/
$\Phi_{L\alpha}$	21.17	1.182	2.446	0.788	0.749	1.775	/	/	/	/
Φ_{Mh}	146.1	4.052	2.933	0.077	0.553	1.735	/	/	/	/
$\Phi_{M\alpha}$	26.10	2.063	9.354	4.908	0.973	5.067	/	/	/	/
Φ_{Lh}	28.01	0.6061	9.896	1.685	0.766	1.618	-8.973	1.807	0.422	1.602
$\Phi_{L\alpha}$	7.441	1.536	-0.674	0.171	0.092	4.813	8.610	0.225	0.946	5.222
Φ_{Mh}	48.47	2.373	143.93	1.399	7.807	4.441	20.744	1.816	-0.335	2.898
$\Phi_{M\alpha}$	3.622	-0.645	-1.775	0.122	0.723	1.601	11.198	2.398	-4.134	3.171

4. CONCLUSION

In this paper, a fractional derivatives function has been proposed in order to fit the Scanlan coefficients $H^*_{1,\dots,4}$ and $A^*_{1,\dots,4}$. This function has been compared with Jones' function which has been widely used up to now. By considering less parameters than the Jones function, our model performed a high-quality fitting. Therefore, the fractional derivatives model is able catch behaviours of bridge decks that are very sensitive to wind actions such as the Deer-Isle Sedgewick Bridge.

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