Use of Stochastic Linearization Technique on Controlled Structures with Nonlinear Fluid Viscous Dampers

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Abstract. The study of civil structures under any type of excitation generally requires the resolution of a second order differential equation, with mainly terms that define the structural characteristics and the excitation properties. This manuscript uses the probability analysis i.e. Stochastic Methods in order to achieve a simplified design method for additional damping devices i.e. Fluid Viscous Dampers. The control of structures with additional damping devices has been widely investigated in the last decade, however, the use of nonlinear viscous dampers is still ambiguous and their design can be time-consuming, often solved by a trial-and-error procedure. Considering a nonlinear additional damping device for a linear structure can be challenging for engineers, especially when seeking for simplicity i.e. frequency domain methods. Thus, statistical linearization technique (SLT) is of utmost importance for probabilistic analysis of nonlinear structural dynamics problem. A simplification of the analysis of an SDOF system equipped with such devices is achieved using spectral analysis and SLT to determine an equivalent damping ratio. An example is presented along the manuscript with proper validations performed with Monte Carlo simulation.

Key words: Stochastic Methods, Fluid Viscous Dampers, Spectral Analysis, Nonlinear Behavior, Monte Carlo, Equivalent Damping.

1. Introduction

Existing design methodologies rely on simplified concepts in which, both the loading and the structural strength are random processes (in dynamics) or variables. The use of stochastic analysis allows to consider all involved parameters in terms of their statistical i.e. probabilistic representation (Preumont, 2013).

Civil structures are generally designed to resist moderate to mild excitations; these structures could sustain significant deformations through their functionality, limiting structural displacement and peak accelerations to satisfactory levels is the main concern. Two key options are available to limit structural vibrations: increase structural size and material quantity to enhance the lateral stiffness, or increasing the amount of damping by means of additional devices (Duflot & al., 2008).

Fluid Viscous Dampers (FV dampers) are passive energy dissipation devices. They transform the input energy to heat, and considerably reduce the inter-story drift and column bending moments of the controlled structure (Constantinou & al., 1993; Soong & al., 2014). Their behavior, characterized by a force-velocity relationship, may be highly nonlinear.

It is a common practice nowadays to use Stochastic Linearization Technique (SLT) in the case of structures equipped with nonlinear devices under a stochastic loading. Since it enables to transform the nonlinear damping (or stiffness) to an equivalent one.

This manuscript treats the issue of nonlinear damping in linear structures using stochastic methods. The framework is limited to a deterministic Single Degree-Of-Freedom (SDOF) structure, equipped with a nonlinear FV damper, while the excitation is presented as a stationary stochastic process e.g. wind. In order to determine an equivalent damping ratio, the SLT is applied. It also allows extending the use of spectral analysis; results are validated with Monte Carlo simulations.

2. Fluid Viscous Dampers

The input energy diffused to a structure when subjected to a dynamic excitation is absorbed and dissipated. By means of the structural inherent damping, that combines the strength, deformability and flexibility, which transforms this energy to kinetic and potential energy. Additional damping devices are increasingly used in order to enhance this structural ability, and push the limits for the structural innovation.

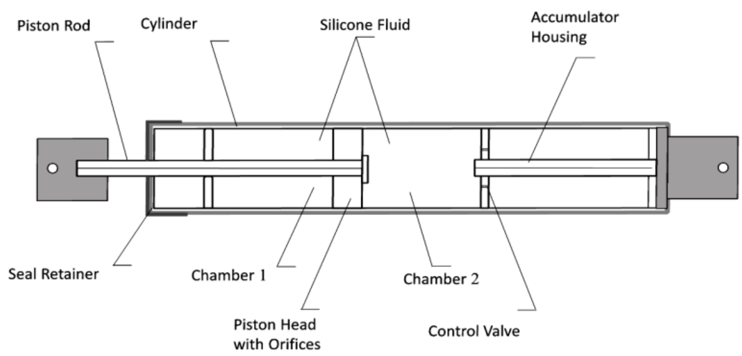

Figure 1. A typical FV damper.

FV dampers operate on the principle of fluid flow through orifices, under a dynamic excitation a highly viscous fluid is forced to flow around or through the piston (see fig. 1). A mathematical model was developed in order to describe their behavior (Constantinou & al., 1992):

$$F_D = C_D|\dot{x}|^\alpha sgn(\dot{x}) \qquad (1)$$

$sgn(.)$ is signum function, C_D is the damping coefficient i.e. viscosity, α is the damping exponent and $\dot{x}$ the relative velocity between both end of the damper. For $\alpha = 1$, the FV damper achieves a linear behavior, a solution adopted for structures subjected to small velocities, a concept that is also widely used in Tuned Mass Dampers. A nonlinear behavior for FV damper is obtained when $\alpha < 1$, which is efficient for higher velocity excitations (Soong & al., 2014). In that case a saturation of the damping force occurs for large velocities that would go beyond the design velocity. A value of $\alpha > 1$ corresponds to a "stiffening" response (see fig. 2), which is employed in lock-up devices.

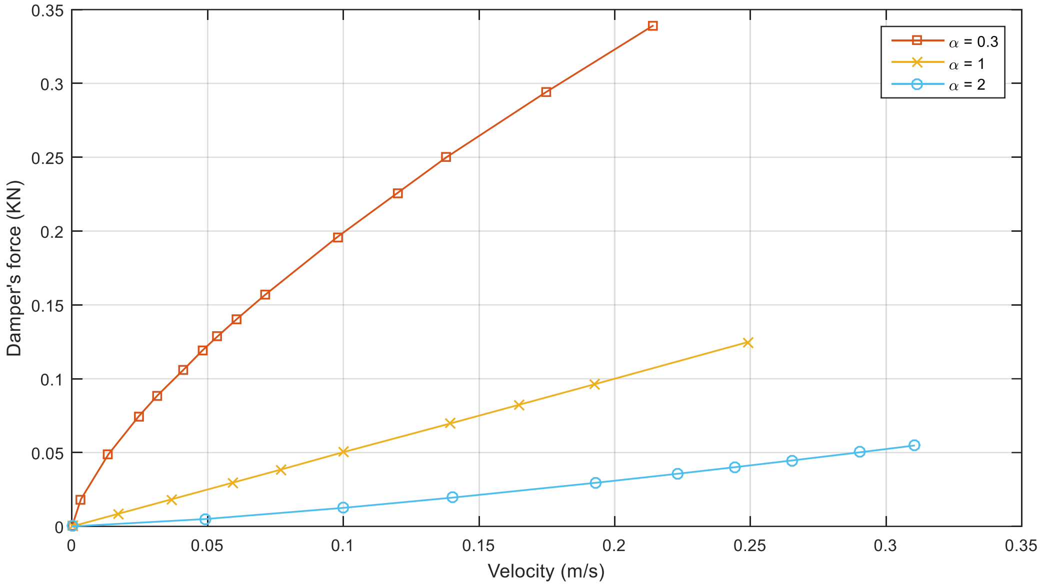

Figure 2. Force-Velocity relationship of the FV damper.

The study of structures with nonlinear FV dampers under random excitations requires a lot of attention since it is more complicated. Nonlinear time-history analysis is a powerful tool to solve the following differential equation:

$$M\ddot{x} + C_s\dot{x} + C_D|\dot{x}|^{\alpha}sgn(\dot{x}) + Kx = F \qquad (2)$$

However, it is numerically heavy and time consuming, especially in an early design stage and for large structures (Gherbi & al., 2018). Thus, frequency domain methods i.e. spectral analysis is more appreciated.

3. Stochastic methods

3.1. Spectral Analysis

The main objective of the spectral analysis is to define the spectrum of the response from the input spectrum, then integrate it in the corresponding frequency space, and thus, obtain the variance of the response (Denoël, 2015).

A stationary random process i.e. wind velocity, can be represented by its Power Spectral Density (PSD). Davenport (1961) proposed a PSD representation of wind by:

$$S_f(\omega) = \frac{\omega\,\frac{2}{3}\left(\frac{L}{U}\right)^2\sigma^2}{\left(1+\left(\frac{\omega L}{U}\right)^2\right)^{4/3}} \qquad (3)$$

with ω the frequency, L the turbulence length-scale, U is the mean wind velocity and σ^2 is the variance of the turbulent component of wind velocity. The PSD of the response $S_x(\omega)$ is obtained by a multiplication of the PSD of the loading $S_f(\omega)$ by a frequency response function i.e. Transfer function $H(\omega)$ (Newland, 2012).

$$S_x(\omega) = |H(\omega)|^2 S_f(\omega) \qquad (4)$$

Their integration over the range of frequencies yields the variance of the response.

$$\sigma_x^2 = \int_{-\infty}^{+\infty} S_x(\omega)\, d\omega = \int_{-\infty}^{+\infty} |H(\omega)|^2 S_f(\omega) d\omega \qquad (5)$$

In a similar fashion, the variance of the velocity is obtained by:

$$\sigma_{\dot{x}}^2 = \int_{-\infty}^{+\infty} S_{\dot{x}}(\omega) = \int_{-\infty}^{+\infty} \omega^2 |H(\omega)|^2 S_f(\omega) d\omega \qquad (6)$$

where the transfer function is expressed as

$$H(\omega) = \frac{1}{k^2} \frac{1}{1 - \left(\frac{\omega}{\overline{\omega}}\right)^2 + 2i\xi \frac{\omega}{\overline{\omega}}} \qquad (7)$$

with k the stifness of the structure, ξ the damping ratio, and $\overline{\omega}$ the natural frequency.

Despite the simplicity of the application of this method, it is limited to linear systems. In order to extend its domain of application to nonlinear systems, Stochastic Linearization Technique is an efficient way to achieve this extension (Roberts and Spanos, 2003).

3.2. Stochastic Linearization Technique

In 1963**,** Caughey (1963) proposed a method to solve nonlinear stochastic problems in structural dynamics. It mainly consists in the substitution of the nonlinear differential equation by a linear equivalent one.

The equation of motion of a linear deterministic system equipped with nonlinear FV dampers in equation (2) may be written as follows:

$$\ddot{x} + \eta |\dot{x}|^{\alpha} sgn(\dot{x}) + 2\,\xi^s \overline{\omega}\dot{x} + \overline{\omega}^2 x = F \qquad (8)$$

With $\eta = C_D/M$ and ξ^s is the structural damping ratio (Clough & al., 2003; Paola & al., 2007).

The SLT consists in replacing Eq (8) by:

$$\ddot{x} + 2\,\xi^e \overline{\omega}\dot{x} + \overline{\omega}^2 x = F \qquad (9)$$

The equivalent damping ξ^e is determined by minimizing an error function in a mean-square sense (Roberts & al., 2003), yielding,

$$\xi^e = \xi^s + \eta \frac{E[|\dot{x}|^{\alpha+1}]}{E[\dot{x}^2]} \qquad (10)$$

where $E[.]$ Is the expectation operator. It follows that:

$$\xi^e = \xi^s + \eta \frac{2^{1+\alpha/2}\, \Gamma(1 + \frac{\alpha}{2})}{\sqrt{2\pi}} \sigma_{\dot{x}}^{\alpha-1} \qquad (11)$$

$\Gamma(.)$ Is a gamma function, $\sigma_{\dot{x}}$ is the variance of the velocity defined in Eq (6), (Di Paola et al., 2007). It is obvious that in order to evaluate ξ^e an iterative process is required, starting from an approximation of $\sigma_{\dot{x}}$ (Canor et al., 2014)

Several methods are available to solve this iterative problem (Fixed point, Newton-Raphson). Besides, in order to validate the obtained results, the resolution of the system under the stochastic time series is realized i.e. Monte Carlo method. Using a step-by-step algorithm i.e. Newmark to solve the nonlinear differential equation.

3.3. Monte Carlo Simulations

Monte Carlo is a powerful yet complicated method. It assumes that the structural stochastic equation of motion can be interpreted as an infinite set of deterministic differential equations. It consists in the generation of samples i.e. time series of the random loading. Then, a numerical integration method is used to solve the differential equation. Thus, a larger number of simulated samples offers more accurate results, which becomes numerically heavy. However, thanks to the Ergodicity principle, statistical moments can be calculated using single sample function of the response (Roberts & al., 2003; Gherbi & al., 2020).

Based on a given PSD, the generation of equivalent time series is done by Fourier series(Shinozuka and Deodatis, 1991). Ergodicity principle states that a stationary process can be calculated from a single sample:

$$S_x(\omega) = \frac{2\pi}{T} |X(\omega, T)|^2 \qquad (12)$$

Thus by choosing:

$$X(\omega, T) = \sqrt{\frac{T}{2\pi}} \sqrt{S_x(\omega_i)}\, e^{i\varphi_j} \qquad (13)$$

With φ a random phase angle taken between 0 an 2π. The inverse Fourier transform is applied on Eq (13) to obtain the equivalent time series (Denoël, 2005).

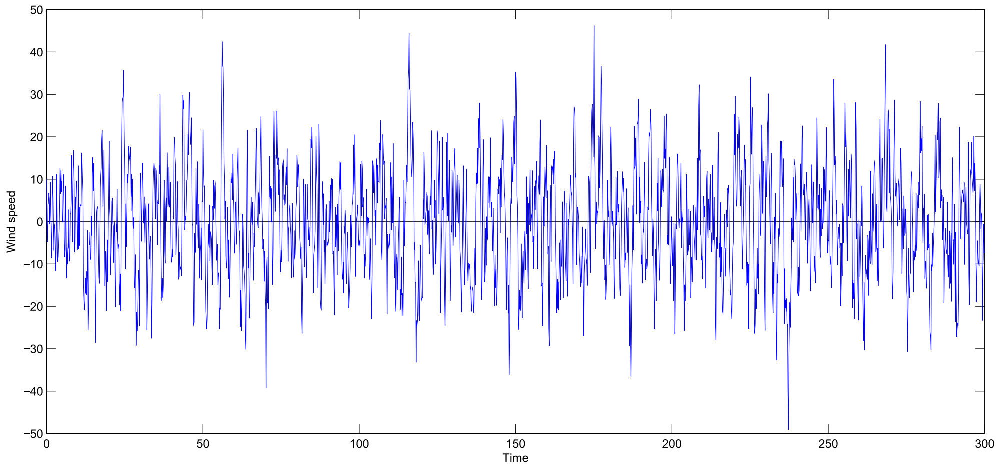

Figure 3. Generated wind velocity sample.

4. Illustration

In order to represent the simple application of the SLT and its accuracy, an SDOF system is adopted for the illustration, with a natural frequency $f = 0.9\ Hz$, and a structural damping ratio $\xi^s = 1.5\%$. Figure 4 presents Davenport's PSD described in Eq (3) along with the generated wind sample, which is transformed in the frequency domain to allow for comparison.

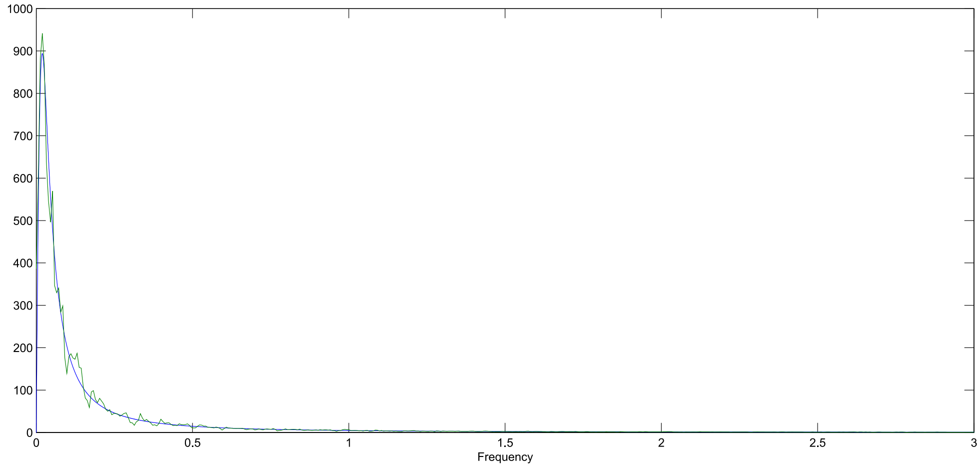

Figure 4. Frequency representation of generated sample matched with Davenport's PSD

After applying the SLT on the selected system, the response of both methods : Spectral analysis, and Monte Carlo are almost the same, since the variance of the displacement here is the area below the graph (Figure 5). The FV damper increased considerably the damping ratio $\xi^e = 9\%$ (see Eq 11).

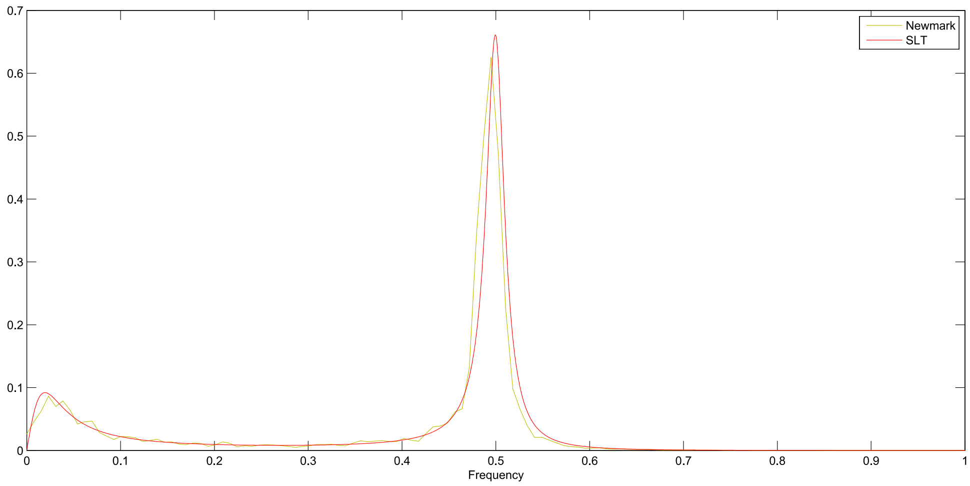

Figure 5. Response of an SDOF system presented in both domains.

Further, several analysis can be made in a small lapse of time, since the spectral analysis is a fast-forward method. it is confirmed in figure 6 that reducing the damping exponent i.e. nonlinear intensity, decreases considerably the response for a constant damping viscosity ($C_D = 50\ kN.s/m$).

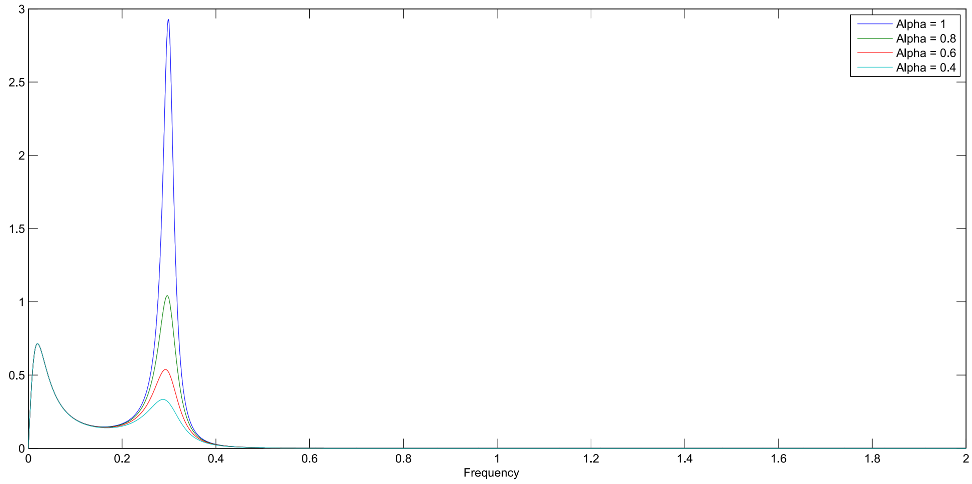

Figure 6. Response of the SDOF system for different values of α.

5. Conclusions

In this manuscript, the Stochastic Linearization Technique is applied on a system equipped with a nonlinear FV damper, which allowed to extend the applicability of the spectral analysis. This latter being known for its simplicity in application, this method offers a straightforward estimation of the response. It is thus more flexible for the designer who can readily study numerous variations of his initial project and choose suitable damping values of the FV damper (damping exponent α and viscosity C_D). An illustration of the method was presented, where a linear deterministic SDOF system was subjected to random wind load, and a nonlinear FV damper is installed in the system. The analysis was carried in two domains i.e. frequency and time, relying on the Monte Carlo simulation in order to generate wind samples equivalent with the PSD. Using Fourier transform, a comparison was possible between the two domains. The response from the SLT has a great similarity with the reference solution i.e. Monte Carlo. Further projects consist in studying larger structures equipped with several nonlinear FV dampers.

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