Dynamic identification of lightweight civil engineering structures using a portable shaker

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Abstract
This paper presents a three-step methodology relying on a portable dynamic shaker to identify the modal parameters of lightweight civil engineering structures such as footbridges or building floors. First, the shaker is used to excite the structure over the frequency band of interest using a bandlimited white noise excitation to obtain a first estimate of the natural frequencies of the vibration modes that are identifiable from the location of the shaker. The second step consists in performing stepped-sine tests in the vicinity of each resonance peak to better identify the natural frequencies and damping ratios of the modes. The last step then accurately determines the mode shapes and modal masses by exciting the structure at each of its resonance frequencies. The advantages and limitations of this methodology are demonstrated on the "La Belle Liégeoise", a footbridge crossing the Meuse river in Liège. The footbridge was not closed to the public during the measurement campaign, which makes the application of the proposed approach rather challenging.

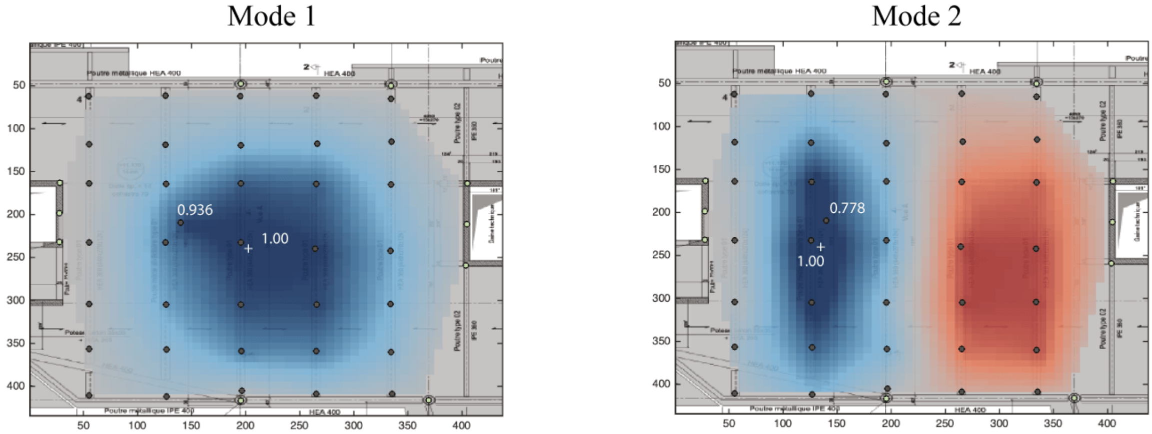
1 Introduction

Lightweight civil engineering structures are prone to vibrations induced by ambient forces such as pedestrian or wind loads. These structures can therefore be subject to human comfort issues or even suffer from structural instabilities. In some instances, they may require the use of tuned mass dampers (TMDs). As a result, identifying the modal parameters of such structures is essential to better assess their comfort and decide whether TMDs are necessary and, if they are, to determine the TMD parameters.

This article presents a dynamic identification methodology based on a portable shaker developed by the University of Liège and V2i. This methodology has already been used in several applications. For example, Figure 1 shows the results of a measurement campaign that precisely determined the natural frequencies f_j, damping ratios ξ_j and generalized masses m_j of a floor. The vibratory comfort of the floor could then be characterized knowing these modal parameters. Figure 2 shows the results of another measurement campaign carried out to characterize the efficiency of tuned-mass dampers (TMDs) installed on a footbridge. The frequency response functions (FRFs) in the undamped and damped versions have been determined using the methodology presented in this article. These FRFs made it possible to evaluate the influence of the TMDs on the dynamic behavior of the structure. This example shows that the method is sufficiently accurate to identify high damping modes and therefore to quantify the efficiency of TMDs.

These two applications, a floor and a footbridge, are just two examples of applications that have been successfully studied with this portable shaker. In usual measurement campaigns, it is possible to close or limit access to the site. This article reports a case study where access was not restricted to the structure. This test is used to discuss the validity of our methodology.

The outline of this paper is as follows. A description of each step of the methodology is given in Section 2. The shaker and the sensors used during a measurement campaign are described in Section 3. In Section 4, the methodology is demonstrated on the "La Belle Liégeoise" footbridge. The vibration modes of the footbridge which are likely to be excited by pedestrians are studied. Some guidelines regarding the use of the proposed approach are also provided.



Mode	f_j (Hz)	ξ_j (%)	m_j (t)
1	5.31	1.9	16.1
2	7.95	0.6	26.9
3	8.41	1.9	14.1

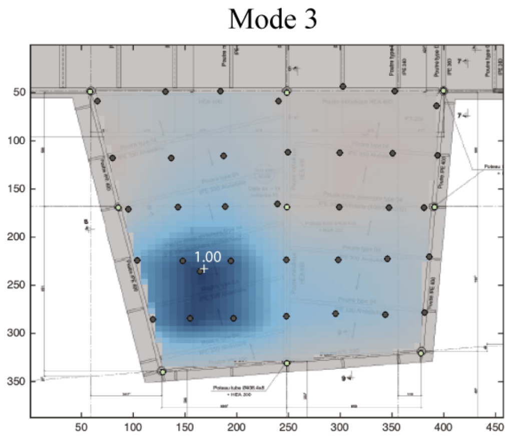


Figure 1: Summary of the identified modal properties of a building floor.

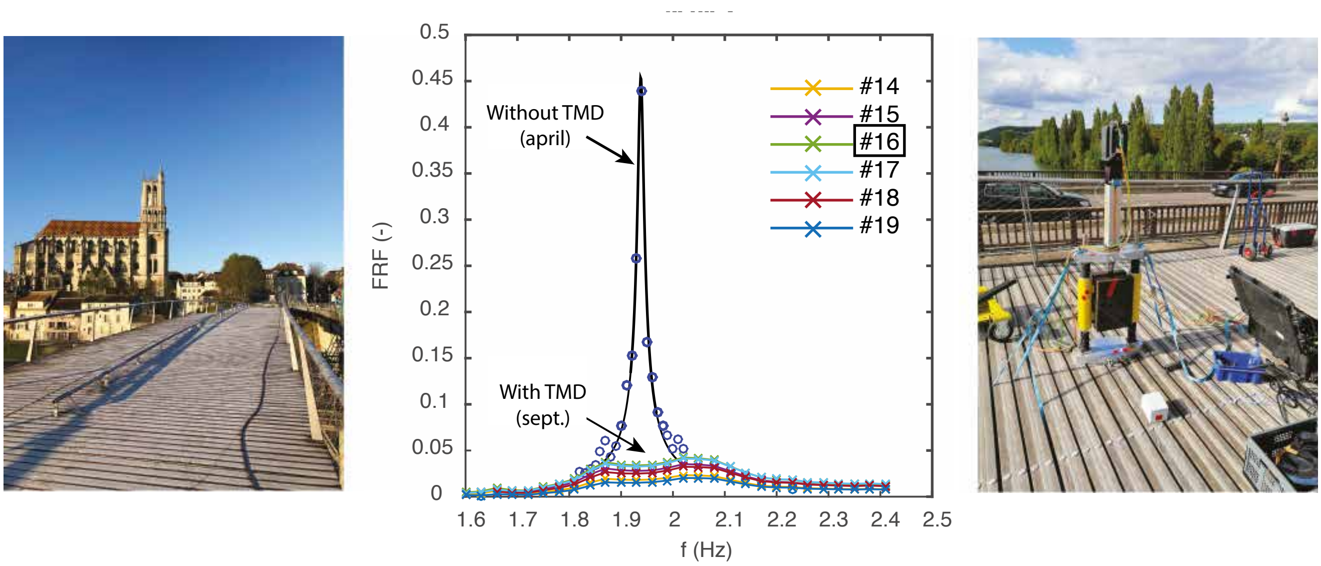


Figure 2: Measured frequency response functions in the undamped and damped versions of a footbridge.

2 Methodology

Figure 3 illustrates the main steps of the dynamic identification methodology. The overall goal is to determine the natural frequencies, damping ratios, modal masses and mode shapes of the structure. Moreover, the stepped-sine excitation provides the FRF. Each step of the methodology involves a different type of excitation and has specific objectives as described in the following.

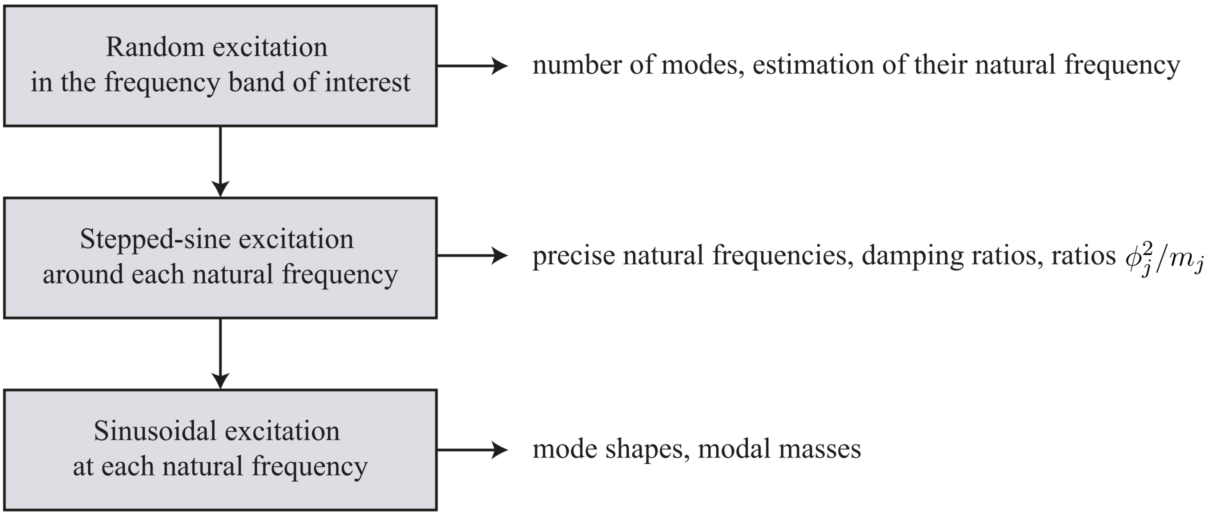


Figure 3: Overview of the main steps of the dynamic identification methodology.

2.1 Step 1 – Random excitation

First, the shaker is used to excite the structure over the frequency band of interest using a broadband random excitation at a location of supposed high modal deformations. A control accelerometer is placed on the moving mass of the shaker to know precisely the force imposed on the structure. A second accelerometer measures the response of the structure at the point of excitation. The measured frequency response provides a first estimate of the natural frequencies of the structure that are identifiable from the location of the shaker.

2.2 Step 2 – Stepped-sine excitation

The second step of the methodology consists in performing a stepped-sine excitation in the vicinity of each resonance peak identified during the random test. The frequency step is chosen small enough to correctly capture the resonance peak of each mode. The duration of each excitation frequency is chosen long enough to eliminate any transient vibration effect. These stepped-sine tests provide better estimates of the natural frequencies and damping ratios of the vibration modes as well as a combined information about the modal mass and the modal amplitude at the location of the shaker, as explained below.

Analytical expression of the frequency response curve

The equations of motion for a linear N degree-of-freedom system can be written in matrix form as [1, 2, 3]

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}(t) \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices, $\mathbf{x}$ and $\mathbf{f}$ are the vectors of displacements and external forces, respectively.

The vector of displacements $\mathbf{x}$ can be transformed to modal coordinates $\mathbf{q}$ using the modal matrix $\mathbf{\Phi}$ defined as the matrix having the mode shapes as columns:

$$\mathbf{x} = \mathbf{\Phi}\mathbf{q} \tag{2}$$

The equations of motion can be expressed in a modal coordinate system by substituting Equation (2) into Equation (1) and pre-multiplying by the transpose of the modal matrix:

$$\mathbf{\Phi}^{\mathrm{T}}\mathbf{M}\mathbf{\Phi}\ddot{\mathbf{q}} + \mathbf{\Phi}^{\mathrm{T}}\mathbf{C}\mathbf{\Phi}\dot{\mathbf{q}} + \mathbf{\Phi}^{\mathrm{T}}\mathbf{K}\mathbf{\Phi}\mathbf{q} = \mathbf{\Phi}^{\mathrm{T}}\mathbf{f} \tag{3}$$

$$\mathbf{M}_q\ddot{\mathbf{q}} + \mathbf{C}_q\dot{\mathbf{q}} + \mathbf{K}_q\mathbf{q} = \mathbf{f}_q \tag{4}$$

where the matrices $\mathbf{M}_q$, $\mathbf{C}_q$ and $\mathbf{K}_q$ are the modal mass, damping and stiffness matrices, and $\mathbf{f}_q$ is the modal external force vector.

If we impose a harmonic excitation, we have

$$\mathbf{f}_q(t) = \mathbf{F}_q\, \mathrm{e}^{i\omega t} \tag{5}$$

and the steady-state response to harmonic excitation is

$$\mathbf{q}(t) = \mathbf{Q}\, \mathrm{e}^{i\omega t} \tag{6}$$

Substituting Equations (5) and (6) into the modal equations of motion (4) and canceling the exponential term result in

$$\left[-\omega^2\mathbf{M}_q + i\omega\mathbf{C}_q + \mathbf{K}_q\right]\mathbf{Q} = \mathbf{F}_q \tag{7}$$

Assuming a proportional damping, the modal equation of motion for the jth mode can be written as

$$\left(-\omega^2 m_j + i\omega c_j + k_j\right) Q_j = F_{qj} \quad \text{for } j = 1, 2, \ldots, N \tag{8}$$

where m_j, c_j, k_j and F_{qj} are the modal mass, damping, stiffness and force for the jth mode. The complex amplitude of vibration is then

$$Q_j = F_{qj} \frac{1}{-\omega^2 m_j + i\omega c_j + k_j} \tag{9}$$

Using the jth mode damping ratio $\xi_j = c_j/(2m_j\omega_j)$ and its natural frequency $\omega_j = \sqrt{k_j/m_j}$, Equation (9) becomes

$$Q_j = \frac{F_{qj}}{m_j\omega_j^2} \frac{1}{1 - \left(\frac{\omega}{\omega_j}\right)^2 + i2\xi_j\frac{\omega}{\omega_j}} \tag{10}$$

In the case of a sinusoidal excitation imposed by the shaker at one point of the structure, the modal excitation which corresponds to the physical excitation projected in the modal basis is

$$f_{qj}(t) = \phi_j m^* \ddot{X}^*(\omega) \sin(\omega t) \tag{11}$$

where m^* is the mobile mass of the shaker, $\ddot{X}^*(\omega)$ is the amplitude of the acceleration imposed on the mobile mass at the frequency ω, and ϕ_j is the modal amplitude at the location of the shaker.

Mode-by-mode fitting

Assuming initially that the structure only responds in a single mode, the acceleration measured on the structure at the location of the excitation is equal to

$$\ddot{X}(\omega) = \phi_j\omega^2 Q_j(\omega) \tag{12}$$

Substituting Equations (10) and (11) into Equation (12) gives

$$\ddot{X}(\omega) = \phi_j \omega^2 \frac{\phi_j m^* \ddot{X}^*(\omega)}{m_j \omega_j^2} \frac{1}{1 - \left(\frac{\omega}{\omega_j}\right)^2 + i2\xi_j \frac{\omega}{\omega_j}} = \ddot{X}^*(\omega) \phi_j^2 \frac{m^*}{m_j} \frac{\left(\frac{\omega}{\omega_j}\right)^2}{1 - \left(\frac{\omega}{\omega_j}\right)^2 + i2\xi_j \frac{\omega}{\omega_j}} \tag{13}$$

Finally, the expression of the amplitude of the acceleration measured on the structure at the location of the excitation can be expressed as

$$\left|\ddot{X}(\omega)\right| = \left|\ddot{X}^*(\omega)\right| \phi_j^2 \frac{m^*}{m_j} \frac{\left(\frac{\omega}{\omega_j}\right)^2}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_j}\right)^2\right)^2 + \left(2\xi_j \frac{\omega}{\omega_j}\right)^2}} \tag{14}$$

For a given vibration mode, the experimental frequency response curve is determined frequency by frequency. The curve of Equation (14) is fitted to the measured points in the vicinity of each mode in order to determine the values of the modal properties ω_j, ξ_j and ϕ_j^2/m_j. The examination of this formula shows indeed that only these parameters can be reliably determined. There is no possibility of independently determining m_j and ϕ_j at this stage of the methodology.

Multimodal fitting

In general, the vibration modes identified mode by mode can be affected by the presence of vibration modes with close frequencies and shapes. The mode-by-mode fitting method can be improved in order to better reproduce the multimodal response. Assuming that the structure responds in M modes, the acceleration measured on the structure at the location of the excitation is equal to

$$\ddot{X}(\omega) = \sum_{j=1}^{M} \phi_j \omega^2 Q_j(\omega) \tag{15}$$

$$= \ddot{X}^*(\omega) \sum_{j=1}^{M} \phi_j^2 \frac{m^*}{m_j} \frac{\left(\frac{\omega}{\omega_j}\right)^2}{1 - \left(\frac{\omega}{\omega_j}\right)^2 + i2\xi_j \frac{\omega}{\omega_j}} \tag{16}$$

Starting from the modal properties ω_j, ξ_j and ϕ_j^2/m_j identified locally in the vicinity of each frequency, a final fitting makes it possible to refine the reproduction of the entire frequency response function by minimizing, in the least-squares sense, the difference between the magnitude of the measured acceleration $\left|\hat{\ddot{X}}(\omega_k)\right|$ and the magnitude of the acceleration predicted by the identified modal basis $\left|\ddot{X}(\omega_k)\right|$:

$$\mathcal{F}(\omega_j, \xi_j, \phi_j^2/m_j) = \sum_{k=1}^{n_{\text{freq}}} \left(\left|\hat{\ddot{X}}(\omega_k)\right| - \left|\ddot{X}(\omega_k)\right|\right)^2 \tag{17}$$

The sum in this objective function spans the frequencies used to perform the stepped-sine test.

2.3 Step 3 – Sinusoidal excitation

The last step of the methodology determines the mode shapes and modal masses by means of an appropriate forcing. The shaker is used to continuously excite the structure, one identified resonance frequency at a time. A reference accelerometer measures the vibrations at the shaker location and several wireless accelerometers are moved over the structure. The accelerations of the wireless sensors and their correlation with the reference acceleration are then used to determine the mode shapes and the modal amplitudes at the location of the shaker ϕ_j. Knowing ϕ_j, the modal masses m_j can be obtained. A lot of measurement points (about 50 or more) can be covered with a limited number of accelerometers in a short period of time (a few minutes).

3 Equipment

3.1 Portable shaker

A portable shaker has been developed by the University of Liège and V2i in order to impose different types of excitations to a structure. The shaker, shown in Figure 4, consists of a mobile mass attached to a linear actuator capable of imposing random and sinusoidal excitations of frequencies ranging from 0 to 10 Hz. The yellow conveyor weighs approximately 80 kg and can be weighted by steel plates of 3 times 50 kg maximum to boost the dynamic force generated by the device. The mobile mass is therefore equal to 230 kg at most. The available motion amplitude with the full mass goes from about 20 cm peak-to-peak at 1 Hz to 2 mm at 10 Hz.

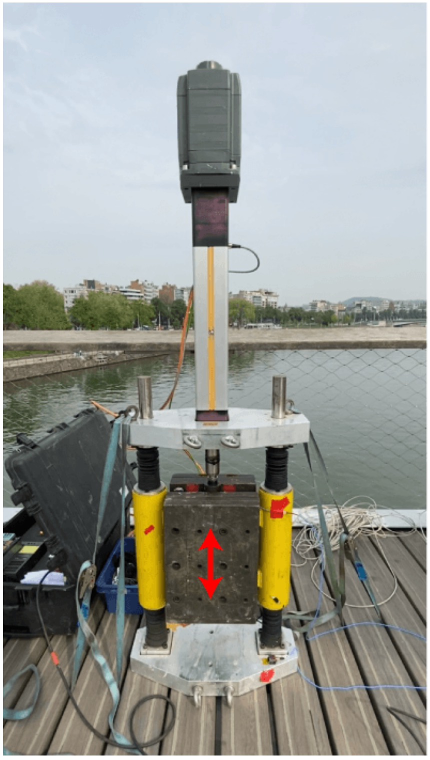


Figure 4: Photo of the shaker.

3.2 Acquisition

During the vibration tests, the moving mass of the shaker is equipped with an accelerometer to know precisely the dynamic force imposed on the structure. Another accelerometer measures the vibrations on the ground at the location of the shaker. MEMS accelerometers are used because they are suitable for measuring static and low-frequency accelerations. The measurement range of these sensors is ± 2 g with a sensitivity of 1000 mV/g.

During the mode shape identification step, the acquisition of the vibrations on the floor is done using wireless MEMS accelerometers to allow their rapid relocation to different positions on the floor. These sensors measure in the range ±2 g with a sampling frequency of up to 512 Hz which largely covers the frequencies of the study.

4 Application to the "La Belle Liégeoise" footbridge

In this section, the methodology is applied to the "La Belle Liégeoise" footbridge [4] shown in Figure 5. This hybrid bridge allows pedestrians and cyclists to cross the Meuse river from the Guillemins train station to the Boverie park in Liège (Belgium). The footbridge is 7-meter wide for a total length of 294 meters. The main span is 163-meter long. The structure is made of steel with a wooden deck. Some vibration modes are significantly damped by a set of 7 TMDs.

4.1 Configuration

In this measurement campaign, it was decided to focus on the vertical vibration modes of the central span and to consider the modes between 1.6 and 2.7 Hz, which are those likely to be excited by a crowd and a group of runners [5].

The location of the shaker is chosen by looking at the vibration modes computed with the finite element model (FEM) of Bureau Greisch, the engineering office that designed the footbridge. The vertical bending and torsion modes of the central span obtained in the frequency band of interest are given in Figure 6. Horizontal bending and local modes are not taken into account. Table 1 indicates some modal parameters. Modes 16 and 19 are each damped by a TMD.

Only one setup is chosen in this campaign. The shaker is placed at an excitation point that is presumed to provide a dynamic response of significant amplitude for all the considered vibration modes. The chosen excitation point is shown in Figure 5. The shaker is on the cantilever edge of the footbridge to better excite the torsion modes as can be seen in Figure 7. The modal amplitudes at the location of the shaker ϕ_j obtained from the FEM mode shapes are indicated in Table 1. The chosen position of the shaker can be validated during the random excitation step, which provides an overview of the excited modes.

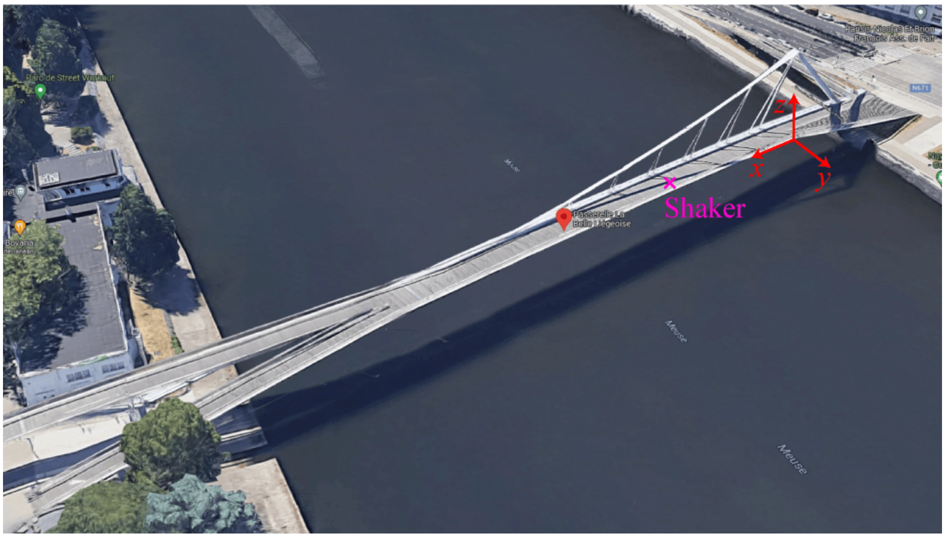


Figure 5: Photo of the "La Belle Liégeoise" footbridge. The frame of reference and the location of the shaker are also indicated. (Image from Google Maps.)

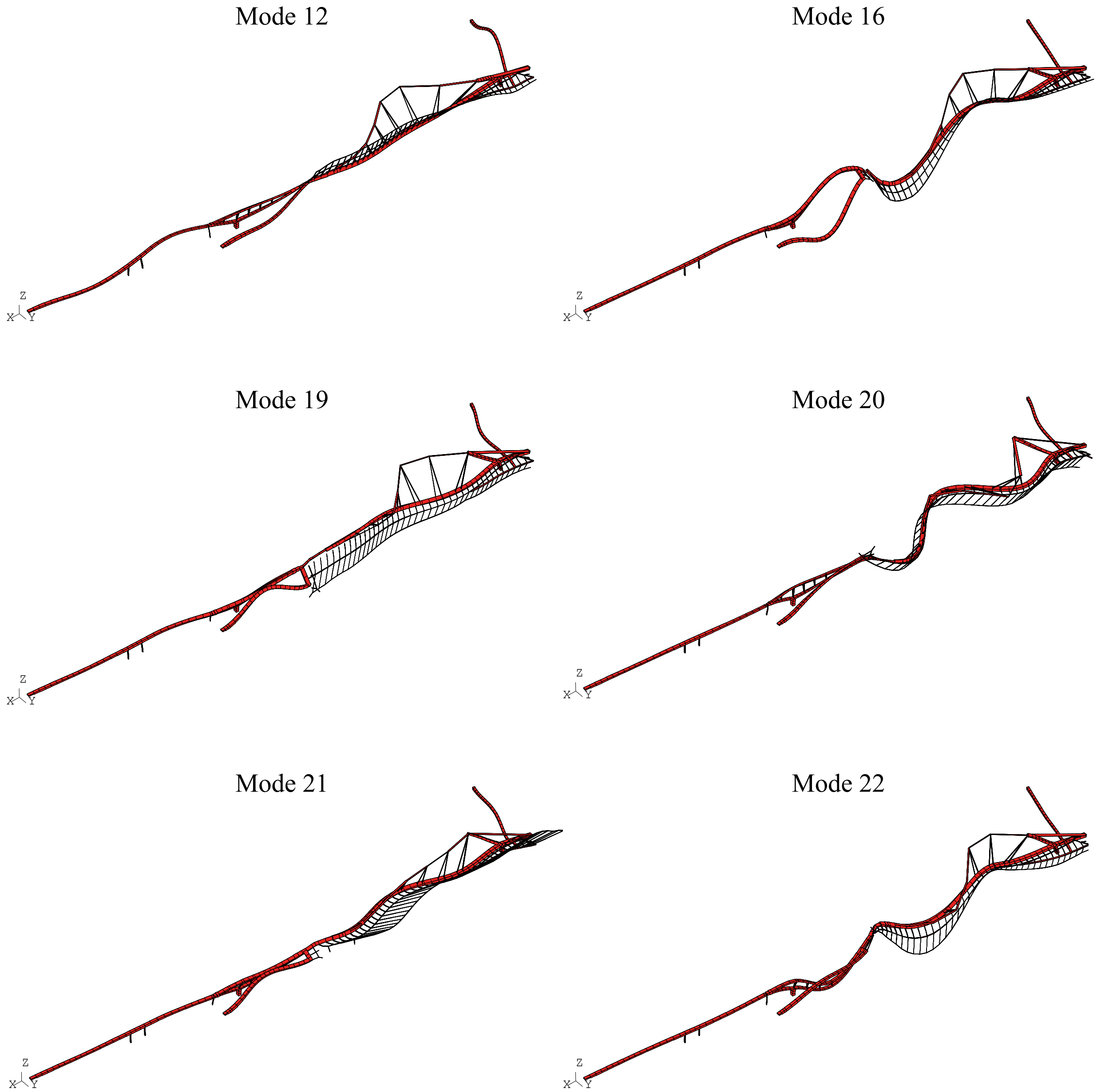


Figure 6: FEM mode shapes computed by the engineering office Bureau Greisch.

Table 1: FEM modal properties of some selected modes.

FEM mode	f_j (Hz)	M_j (t)	ϕ_j	Description
12	1.64	39	0.75	Torsion
16	1.90	91	0.84	Vertical bending and torsion (damped by a TMD)
19	2.32	31	0.38	Torsion with slight vertical bending (damped by a TMD)
20	2.39	73	0.39	Horizontal bending with slight vertical bending and torsion
21	2.45	44	0.52	Torsion
22	2.57	49	0.66	Vertical bending and torsion

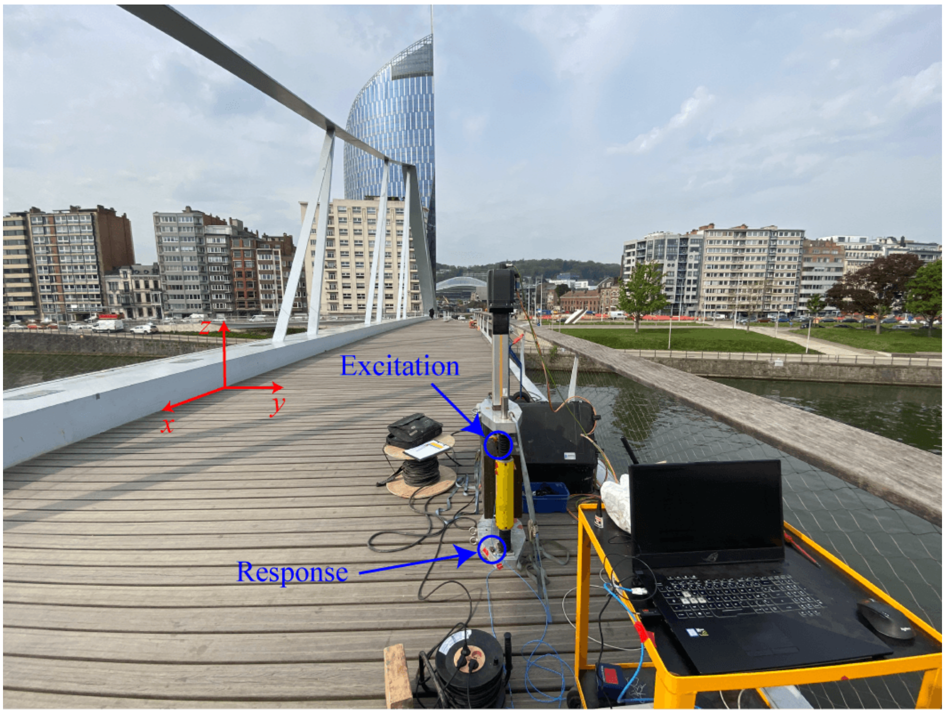


Figure 7: Photo indicating the location of the shaker on the footbridge. The accelerometer on the mobile mass of the shaker and the one on the ground at the shaker location are also shown.

4.2 Step 1 – Natural frequency determination (random excitation)

The shaker imposes a zero-mean white Gaussian noise excitation in a frequency band between 1 and 3 Hz for 15 min. The root mean square (RMS) level in terms of the displacement of the mobile mass is set at 10 mm. Figure 8 shows the vertical acceleration measured on the ground at the shaker location and the corresponding power spectral density (PSD). The footbridge was frequently crossed by pedestrians, runners and cyclists during the measurement campaign. Figure 8 also shows a measurement of the acceleration on the ground under these ambient forces for about 50 min.

In the frequency range between 1.6 and 2.1 Hz, the vibration levels measured with the shaker are similar to those obtained under ambient excitations. The vibrations of the central span in this frequency range are mainly due to the traffic. We can expect a priori that the proposed approach that relies on the dynamic force imposed by the shaker will not be applicable in this region. This difficulty is certainly common to all identification methods. The excitation imposed at this step is a white noise, we are thus in the conditions of application of a stochastic subspace identification (SSI) method [6]. This technique is applied directly to the recorded time signals to have an estimate of the natural frequencies and damping ratios (see Table 2). Two modes are observed during the random excitation in this region significantly affected by ambient forces. By comparing with the FEM modes, these modes are correlated with Modes 12 and 16. This correspondence will be confirmed at the sinusoidal excitation step by comparing the mode shapes. We can also note that several peaks are observable near Mode 16 in the frequency response because this mode is damped by a TMD.

In the region above 2.1 Hz, the modes are more weakly excited by ambient excitations. The shaker makes it possible to better excite these modes. Resonance peaks are observed during the random excitation, but the identification of the modes remains difficult. The frequency sweep will better highlight the modes in this region.

In general, the frequency response obtained at this first step provides a clear view of the number of modes identifiable from the location of the shaker. The case of this footbridge is more challenging, certainly because the ambient forces are important. In addition, the footbridge has many vibration modes with close frequencies and shapes. Some of these modes are also significantly damped by a set of TMDs.

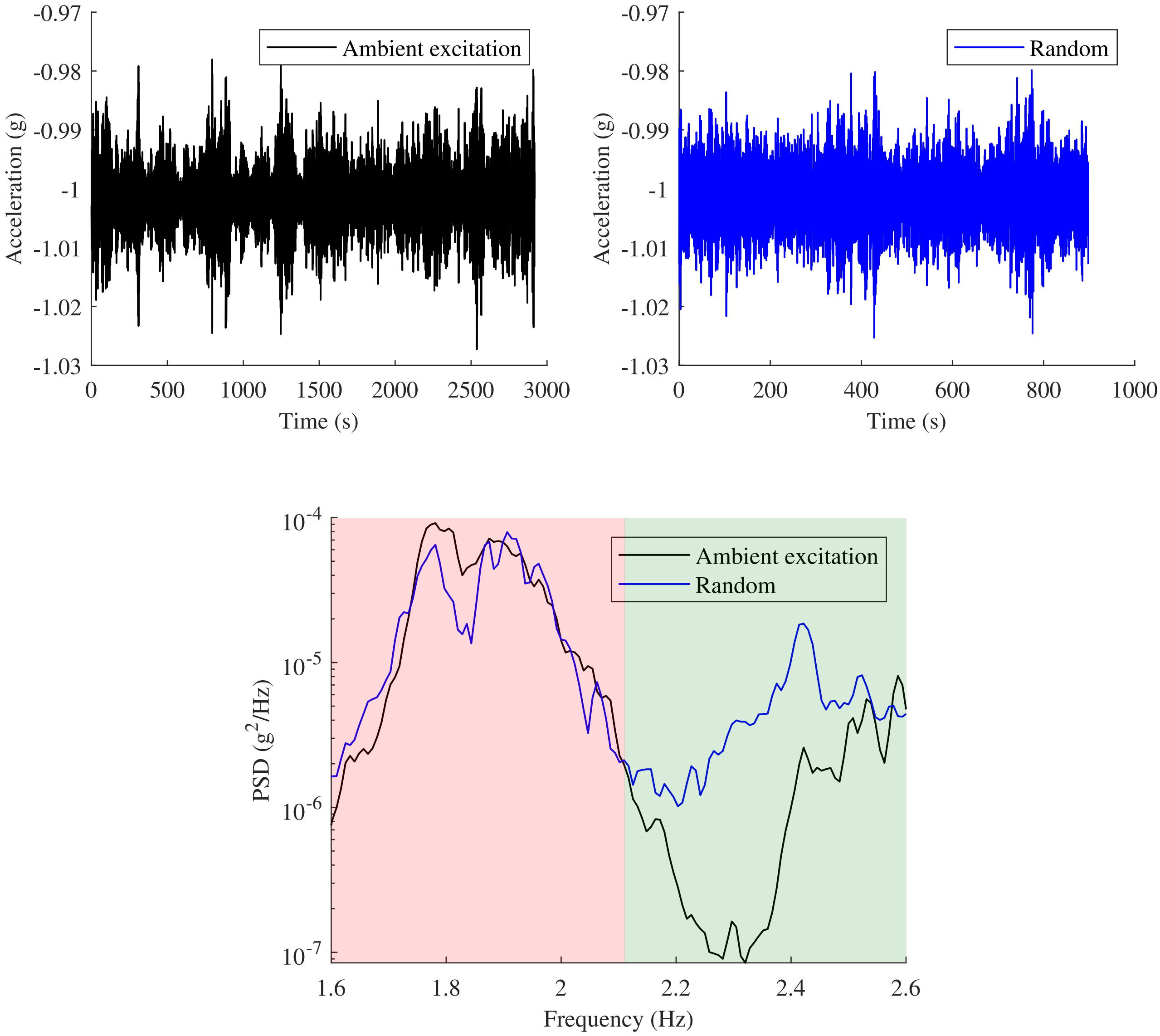


Figure 8: Vibrations measured on the ground under ambient forces and during the random excitation.

Table 2: Modal parameters identified by the SSI technique in the region between 1.6 and 2.1 Hz during the random excitation.

Experimental mode	f_j (Hz)	ξ_j (%)	Corresponding FEM mode
A	1.77	2.01	12
B	1.92	1.59	16

4.3 Step 2 – Damping identification (stepped-sine excitation)

The shaker imposes a stepped-sine excitation on the footbridge. The structure is excited one frequency at a time. The excitation is performed from 1.6 to 2.6 Hz with a step of 0.01 Hz and a duration of 30 s for each frequency. The nominal amplitude of the mobile mass is set at 10 mm. Figure 9 represents the measured frequency response function. In this graph, the amplitude of the harmonic acceleration measured on the ground is determined after applying a band-pass filter at ± 15 % of the excitation frequency. This filter eliminates the contribution of any other excitation source that is in another frequency range.

As expected, it is difficult to identify the modes in the region between 1.6 and 2.1 Hz because the operational forces are too important. The peaks observed between 2.1 and 2.27 Hz correspond to local or horizontal modes that are not studied in this campaign. We find a signature of these modes with a smaller amplitude in our measurements even though they are not excited by the shaker because of the traffic and the couplings between modes. The shaker could be placed at strategic positions to better excite the local modes. The shaker could also be put in a horizontal configuration to impose a dynamic force in the plane of the bridge floor and thus excite the horizontal modes.

Three resonance peaks are observed in the region between 2.27 and 2.55 Hz. The parameters ω_j, ξ_j and ϕ_j^2/m_j of each mode are determined using the multimodal fitting explained in Section 2. The result of the fitting is shown in Figure 9. The modal parameters will be discussed in the following section.

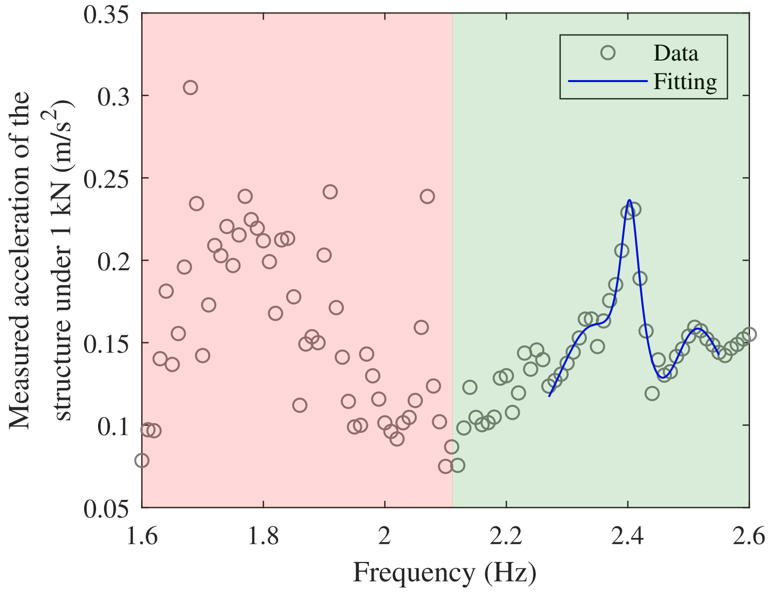


Figure 9: Frequency response curve obtained during the stepped-sine excitation. Comparison of measured data and fitted model.

4.4 Step 3 – Mode shape identification (sine excitation)

The shaker continuously excites the structure one natural frequency at a time. The mode shapes are identified using 8 wireless accelerometers. One of the sensors is placed near the shaker and serves as a reference. The other 7 sensors are each moved to 4 different positions on the central span of the bridge, resulting in a total of 28 measurement points as can be seen in Figure 10. The acceleration amplitudes obtained at the measurement points and their correlation with the reference acceleration are used to reproduce each mode shape in a short period of time.

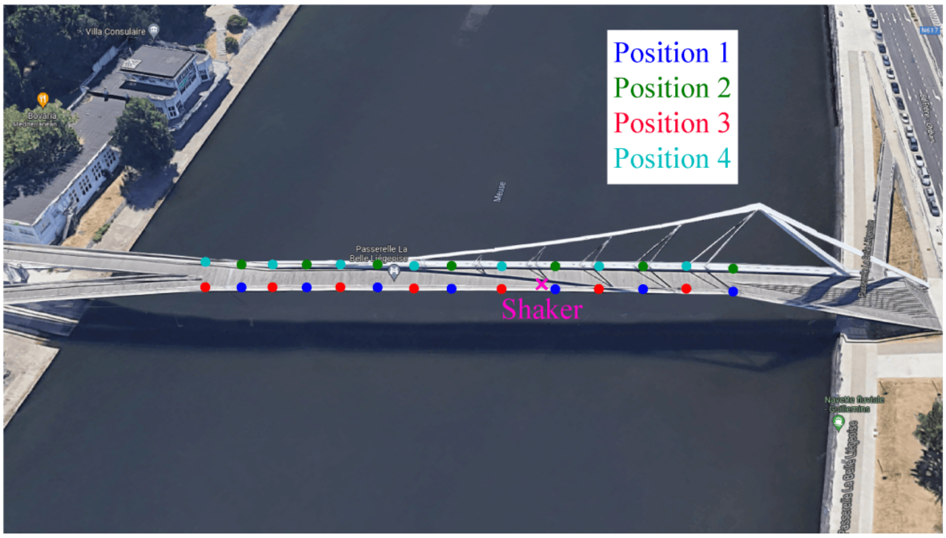


Figure 10: Representation of the measurement points.

The identified mode shapes are represented in Figure 11. The frequency, damping and mass of each mode are summarized in Table 3.

In total, 5 vertical experimental modes, labelled from A to E, have been determined in the frequency band of interest. The experimental mode D seems to correspond to a combination of the numerical modes 20 and 21. The mass of Mode D is indeed similar to the sum of the masses of Modes 20 and 21. The shape of Mode D is closer to Mode 21.

Overall, the measured mode shapes are qualitatively similar to those calculated numerically, except for Mode C which involves more bending than the corresponding numerical mode 19. The natural frequencies and the modal masses that could be identified are in agreement with the values predicted by the finite element model.

Finally, we notice that the damping of Mode C is high. This mode being damped by a TMD, it would have been necessary to carry out a stepped-sine test with a much finer frequency step to better capture the effect of the TMD as in the example of Figure 2. Furthermore, we understood that the damping of the TMD was over the typical optimal value in a regime where the two usual bumps in the FRF of a TMD-damped structure are no distinct. In this campaign, an equivalent damping is therefore identified and the effect of the TMD is replaced by an equivalent damping in a single mode.

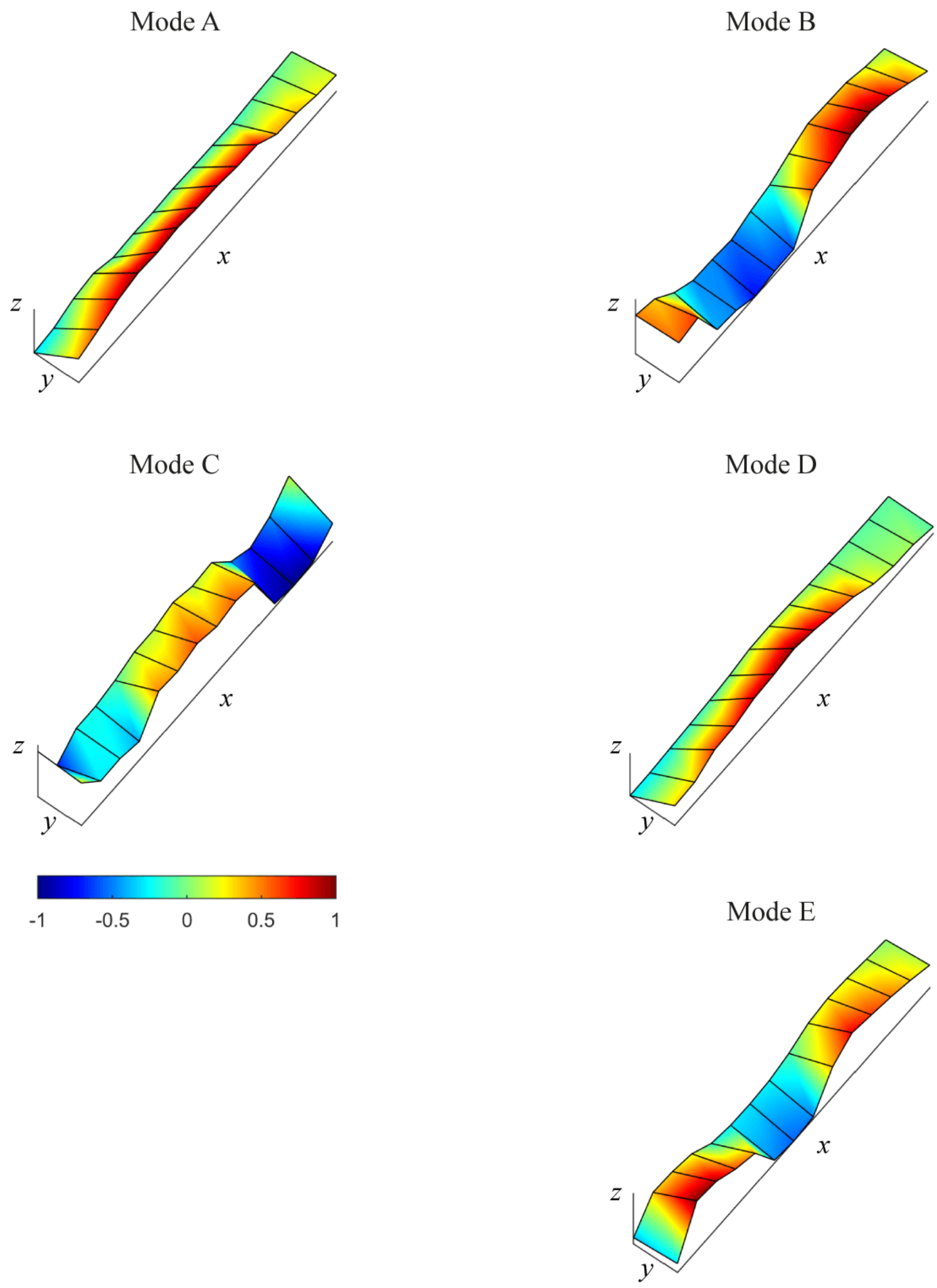


Figure 11: Mode shapes identified during the sinusoidal excitation step.

Table 3: Identified modal parameters.

Experimental mode	f_j (Hz)	ξ_j (%)	m_j (t)	ϕ_j	Corresponding FEM mode
C	2.34	3.06	37	0.52	19
D	2.40	0.79	108	0.48	20 and 21
E	2.50	2.36	49	0.50	22

5 Conclusions

A three-step dynamic identification methodology has been presented. The methodology relies on a portable shaker to impose different types of excitations and requires only a few wireless accelerometers, offering thus a practical technique for fast and reliable estimation of dynamic characteristics. The methodology has already been successfully applied to assess the comfort of building floors and footbridges before they are put into service.

In this paper, the approach was tested on a footbridge that was open to the public. This application is challenging because the ambient forces are important. It has been shown however that the proposed approach is applicable outside the frequency range significantly affected by the traffic. In these conditions, the portable shaker can excite the modes of the footbridge and the three-step methodology can be used to identify all modal parameters, including modal masses.

Acknowledgements

This work is part of the FIRST project funded by the Walloon Region. The authors would like to thank the Service Public de Wallonie (SPW) for authorizing this measurement campaign on the footbridge. Thierry Auguste from SPW also accompanied and helped us during the campaign. The authors acknowledge Yves Duchêne from Bureau Greisch for authorizing the publication of the FEM results.

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