An improvement of branch-and-price algorithm for the quay crane scheduling problem

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Introduction

The increase of containers traffic has affected the competition among port container terminals. One of important indicators for container terminals to stay competitive is the vessel turnaround time, which is the average time of a vessel stays in a terminal [1]. Exposito et al. [2] state in their research that the main cause of large turnaround time is the high rate of utilization of terminal's infrastructure, thus the most effective way to reduce the terminal turnaround time is to improve the productivity of handling activities. One of the handling equipment in container terminals is quay crane (QC), which has function to load and unload the containers for berthed vessels. As one of the main bottlenecks in the container terminal, its performance largely affects terminal's throughput and efficiency [3]. Therefore, this paper explicitly focuses on quay crane scheduling problem (QCSP). According to Nguyen et al. [4], the objective of QCSP is to determine a sequence of handling operations of a vessel with a given number of QCs. From mathematical point of view, QCSP is considered as a strong mixed-integer linear programming [1] thus will result to long computational time. However, Choo et al. [5] in their research emphasized that the decision of QCSP should be obtained in a very short time, because the required inputs for solving the problem are known just before the arrival of ship.

There are two alternatives in solving QCSP, by developing either a heuristic procedure or an exact solution algorithm. Heuristic approaches outperform exact algorithms in one way because their fast computational time. On the other hand, heuristic have disadvantages since they hardly provide an optimal solution. In terms of solving QCSP, Chen et al. [1] stated that optimal solutions are more desirable since they will affect the next operations in terminal. In that sense, an exact solution algorithm is preferable to obtain the optimal solutions. One of the exact solution algorithms named branch and price (B&P) algorithm is particularly considered as the most successful tool to solve complex problem [6]. It is suitable to handle QCSP since it only considered a subset of variables at one iteration, thus it is expected to reduce the complexity of problem whilst still provide the optimal solution. However, several characteristics of B&P can result to high solutions times and reduce the efficiency of algorithm [7].

The contribution of this research is to develop an improvement for B&P algorithm in order to find optimal solutions for QCSP as fast as possible. To the best of our knowledge, there are some acceleration techniques available in the literatures; however they have not been implemented in QCSP setting so far. The main method of this paper is extending the existing model of QCSP developed by Choo et al. [5] and implementing the available improvement techniques in the literatures. The result of this research is compared to that of obtained from other algorithms presented in [5] in terms of calculation time and optimality gap. .

Literature review

Numerous studies have been conducted regarding the minimization of the turnaround time of a vessel by scheduling the quay cranes at a container terminal. Several approaches have been developed in order to obtain the solution in a fast way. Current research streams in QCSP are more opt to develop meta-heuristic or heuristic procedures to cope with the complexity of this problem, thus reducing the computational time. Some of most prominent heuristics developed are LTM algorithm to solve rich-QCSP [8], tabu search algorithm [9], simulation [10], constraint programming [11], and genetic algorithm ([12], [13], [4], [3]).

Although different mathematical models and heuristic procedures have been developed, only few studies use exact solution algorithms to solve QCSP. Those exact algorithms are branch-and-bound ([14], [15], [16]), branch-and-cut [17], and branch-and-price [5]. Those exact algorithms are not very prominent due to the high solution times. However, global optimal solution for QCSP is still more desirable thus exact algorithms are more preferable than heuristic approaches. Having said this situation, it is needed to develop an algorithm which can obtain the optimum solution for QCSP immediately. This can be done by establishing an improvement of existing exact algorithm to accelerate the computational time. To the best of our knowledge, this attempt has not been proposed so far in QCSP setting.

This research proposes several improvements to accelerate the computation of one of the exact solution algorithms, called branch-and-price (B&P) algorithm. B&P algorithm is suitable to solve a large-scale integer problem or a problem with huge number of variables such as QCSP [18]. To the best of our knowledge, the only attempt to solve exactly QCSP using B&P algorithm is the one by Choo et al. [5]. The authors reformulate the QCSP in single ship setting into set covering problem and use B&P algorithm to clarify the solution.

Model

This research considers the problem defined in [5] for QCSP in single ship setting. The objective of the model is to minimize the total make span needed to finish all the jobs. Task is defined as handling one container at one specific time. The model considers important spatial constraints such as clearance constraint between adjacent QCs and non-crossing constraint. It assumes that during planning horizon there is no other vessel berthed and the number of QCs assigned to a vessel is fixed. The output of this model is the position of each QC and the decision whether the QC handle a container at any specific time.

Solution algorithm

The main idea of B&P algorithm is to combine the dynamic column generation procedure and branch-and-bound algorithm. Using this algorithm, the problem is decomposed into smaller sub-problem. Thus, it is expected to reduce the complexity of the problem and reduce the computational time. However, B&P algorithm has drawbacks regarding its characteristics that will result to long computational time. Thereby, several improvement techniques are needed in order to maximize the utilization of B&P algorithm for solving QCSP.

One of the main shortages of B&P algorithm is the long tail convergence, which can be explained as primal degeneracy and the superfluous oscillations of dual variables [19]. This can happen since for each primal variable there are several associated dual solutions. Brunner and Stolletz [20] stated that by far the most effective way to encounter tailing-off problem is by utilizing stabilization of dual variables. The main idea is by avoiding high dual variable oscillation from one iteration to another in the column generation phase. This can be done by selecting a dual solution inside the stability center instead of using an extreme point. There are several dual stabilization techniques available in the literatures, namely interior point stabilization [21], boxstep method [22], polyhedral penalty terms [23], bundle methods [24], and convex combination of previous dual solultions [25]. In this research, the polyhedral penalty alongside with the convex combination technique are applied to improve the performance of column generation phase.

Another technique applied in this paper to improve the performance of B&P algorithm is by interfering the branch-and-bound phase. This is done by providing a better node selection procedure. There are three available branching schemes: depth-first search, best-bound search, and combination of depth-first and best-bound [20]. We applied those three procedures and compared their performances to see which one of them give the best result.

Experiments

Those approaches presented above are employed and mixed in order to find the best combination techniques thus will improve B&P algorithm for solving QCSP. The experiments are performed using Mosel modelling language and solved with Xpress IVE commercial software. Computation of basic B&P is compared to that of B&P with dual stabilization. Further, the results obtained by combining of B&P with dual stabilization and enhanced branch-and-bound scheme are also examined. Experiments are performed in several different scenarios range from small problems to large problems (realistic size) by changing the inputs such as number of bays and QCs, clearance distance, and total number of jobs. Four different measures are used for computational tests, these are: computational times, the number of column generation iterations, the number of branch-and-bound nodes, and the optimality gap. The comparison not only performed among the results of those combinations, it is also done with the result of previous research [5] in order to evaluate the improvement.

Based on the computation, the depth-first scheme is not attractive to solve the problem, because it takes a longer time than basic B&P presented in the [5]. Some of them are even not solvable. The best algorithm found in terms of computational times is the B&P algorithm with the combination of best-bound search and convex combination technique. The running times using this algorithm is on average less than 1s for small problems, which is better than heuristic procedure presented in [5]. We also tested the combination of depth-first and best-bound search, yet it did not give a better performance than best-bound search.

The implementation of dual stabilization results to fewer numbers of branch-and-bound nodes, some of instances can be solved at the root node. The implementation of dual stabilization also reduces the number of column generation iterations for about 70% in average. We conclude, based on the results of our experiments, that the enhancements (mainly dual stabilization) reduce the number of columns and consequently reduce the size of the problem. It allows faster convergence and makes an easier to find the optimal solutions. Significant reduction of the iterations proves that dual stabilization techniques have favorably tackled the tailing-off effect in the column generation procedure. Best-bound search also gives better result because in the node selection scheme, it always chooses parent node with the best value among all, thus the time to find the optimal solution is less than depth-first. Sensitivity analysis regarding the value of inputs and parameters of each improvement technique shows that the improved B&P algorithm is more robust than the basic one in terms of computational times.

Conclusion

This paper proposes the improvement of branch-and-price algorithm in order to provide the optimal solution for QCSP in single ship setting. Several enhancements are proposed, such as performing dual stabilization, performing strong branching, and combination of both. The results obtained from those improvements are compared to those from current solutions based on the algorithm developed by Choo et al. [5]. Comparisons are performed generally based on two general indicators: computational time and optimality gap. The result shows that the combination of convex dual stabilization and best-bound search provides the best result in which reducing the computational times as much as 70% over the original B&P algorithm.

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