3D Finite Element Models of Stacked Tapes Magnetic Shields

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***Abstract*—We present two 3D finite element models for magnetic shields made up of a large number of stacked tapes subjected to a transverse magnetic field. The first model consists in a stack of alternating superconducting and ferromagnetic layers, whereas in the second model, the stack is replaced by an equivalent material for which both superconducting and ferromagnetic properties are homogenized. Both models are solved with an h-ϕ-formulation and simulation results are compared with experimental measurements.**

***Keywords*—high-temperature superconductors, magnetic shielding, finite-element analysis**

I. Introduction

Stacks of flat annular HTS tapes with ferromagnetic (FM) substrates create hollow cylinders with interesting magnetic shielding properties. Under axial fields, superconducting persistent currents in the HTS prevent flux penetration inside the cylinder, while in transverse fields, the high permeability of the substrates helps channelling the flux lines away from the inside.

Modeling systems with both HTS and FM is challenging due to the strong nonlinearities of the material constitutive laws [1]. In this work, we present 3D models for the transverse field configuration based on the h-ϕ-formulation. In particular, we propose to replace the stacked materials by an homogeneous HTS-FM hybrid material having anisotropic electric and magnetic properties. This provides results in good agreement with the measurements within a limited amount of computation time.

II. Problem Definition

We consider hollow cylinders that are made up of a stack of N HTS tapes with a FM substrate, and that are maintained at a temperature of 77 K. The volume fraction of the FM substrates is $f = 0.9$. We apply a transverse field $\boldsymbol{b}_\mathrm{s}$ from 0 to 60 mT in 1 s, perpendicular to the cylinder axis. One-fourth of the geometry is modeled as represented in Fig. 1. We define the shielding factor SF as

$$\mathrm{SF} = \frac{\|\boldsymbol{b}_\mathrm{s}\|}{\|\boldsymbol{b}(P)\|}, \tag{1}$$

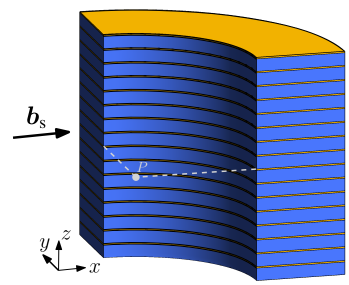

Fig. 1: Simplified detailed model of sample A: one fourth of a stack of 16 tapes (height: 24 mm, inner radius: 13 mm, outer radius: 22.5 mm). Yellow regions are HTS and blue regions are the FM substrates.

where P is the center of the stack.

In this communication, we focus on the numerical model of the system, while the measurements of the magnetic response of the materials are reported in a joint communication by S. Brialmont [2].

We model the magnetic response of the system with the magneto-quasistatic approximation. In HTS, $\mu = \mu_0$ and we assume a power law for the electrical resistivity:

$$\rho(\boldsymbol{j}) = \frac{e_\mathrm{c}}{j_\mathrm{c}} \left(\frac{\|\boldsymbol{j}\|}{j_\mathrm{c}} \right)^{n-1}, \tag{2}$$

with $e_\mathrm{c} = 10^{-4}$ V/m, $j_\mathrm{c} = 10^9$ A/m^2 and $n = 30$. In this work, j_c and n are assumed constant. The FM is assumed non-conducting and its permeability $\mu = \mu(\boldsymbol{h})$ is interpolated from the measurements at 77 K.

We consider two samples for which we have SF measurements: sample A with a height of 22 mm and $N = 294$ tapes and sample B with a height of 14.9 mm and $N = 183$. The inner and outer radii, 13 mm and 22.5 mm, respectively, are identical for both samples.

III. Finite-Element Models

Modelling the exact geometries of samples A and B is not practical because of the large values of N. Instead, we propose two approaches to simulate the magnetic response of the shields.

A. Detailed Model

In the first model, we introduce a limited number $N_1 < N$ of HTS and FM layers, with fictitious thickness so that the total height is the same as the physical sample. We then solve the magneto-quasistatic problem with an h-ϕ-formulation [1].

B. Homogeneous Model

In the second approach, we replace the detailed stack of alternating materials by an homogeneous fictitious material having both superconducting and ferromagnetic properties. To solve the problem with the h-ϕ-formulation in terms of the average field values, the material properties need to be modified accordingly. Moreover, because of the layered structure of the problem, these properties are anisotropic.

1) Anisotropic Resistivity: The FM is assumed non-conducting and does not carry any current density. In terms of the volume average value $\boldsymbol{j}$ of the current density, the current density in the HTS is therefore $\boldsymbol{j}^{\mathrm{S}} = \boldsymbol{j}/(1-f)$ with $(1-f)$ the filling factor of the HTS phase.

The current density can only flow in the x-y-plane. To prevent current from flowing in the z-direction, a large resistivity ρ_∞ is introduced. In the (x, y, z) Cartesian coordinate system represented in Fig. 1, the resistivity takes the form of a diagonal tensor:

$$\tilde{\boldsymbol{\rho}}(\boldsymbol{j}) = \operatorname{diag}\left(\rho(\boldsymbol{j}^{\mathrm{S}}),\ \rho(\boldsymbol{j}^{\mathrm{S}}),\ \rho_\infty\right). \tag{3}$$

2) Anisotropic Permeability: In the HTS, $\mu = \mu_0$ is a constant. In the FM, the permeability depends on the local magnetic field $\boldsymbol{h}^{\mathrm{F}}$, we have: $\mu = \mu(\boldsymbol{h}^{\mathrm{F}})$. From continuity considerations, the relation between the components of the average magnetic field $\boldsymbol{h}$ and those of $\boldsymbol{h}^{\mathrm{F}}$ in the FM phase read: $h_x^{\mathrm{F}} = h_x,\ \ h_y^{\mathrm{F}} = h_y$, and

$$h_z^{\mathrm{F}} = \mu_0 h_z / \left(f\mu_0 + (1-f)\mu(\boldsymbol{h}^{\mathrm{F}})\right). \tag{4}$$

Extracting $\boldsymbol{h}^{\mathrm{F}}$ from $\boldsymbol{h}$ therefore requires to solve an implicit equation.

The permeability also takes the form of a diagonal tensor:

$$\tilde{\boldsymbol{\mu}}(\boldsymbol{h}) = \operatorname{diag}\left(\bar{\mu}(\boldsymbol{h}^{\mathrm{F}}),\ \bar{\mu}(\boldsymbol{h}^{\mathrm{F}}),\ \bar{\bar{\mu}}(\boldsymbol{h}^{\mathrm{F}})\right), \tag{5}$$

with $\bar{\mu}(\boldsymbol{h}^{\mathrm{F}}) = f\mu(\boldsymbol{h}^{\mathrm{F}}) + (1-f)\mu_0$ for the (x, y)-components and $\bar{\bar{\mu}}(\boldsymbol{h}^{\mathrm{F}}) = \left(f/\mu(\boldsymbol{h}^{\mathrm{F}}) + (1-f)/\mu_0\right)^{-1}$ for the z-component.

IV. Results

The models have been implemented in GetDP. An adaptive time-stepping algorithm and a Newton-Raphson method were used during the resolution [3].

One fourth of the geometry is considered for the detailed model. For the homogeneous model, because of the additional symmetry along the z-axis, only one eighth is modeled. The parameter ρ_∞ is fixed to 0.01 Ωm.

Results for the samples A and B solved with the two approaches are shown in Fig. 2. Increasing the number of tapes in the detailed model has a very limited influence on the curves for $N_1 \gtrsim 8$. By contrast, the numerical solution of the detailed model is strongly affected by the mesh resolution inside the FM layers. When the mesh is refined, the SF curves get closer to those obtained with the homogenized model.

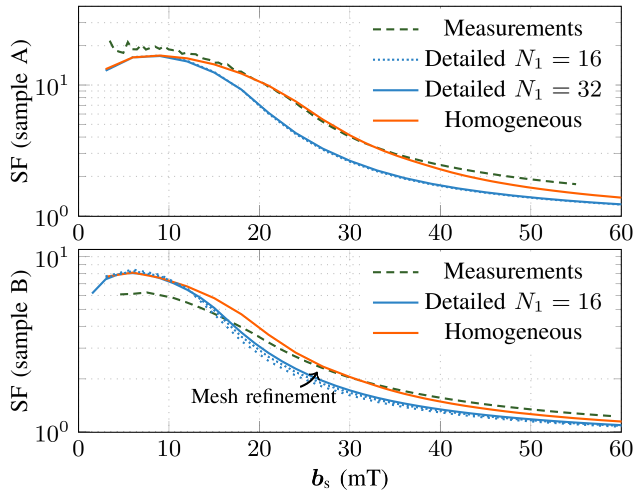

Fig. 2: Shielding factors for the two samples (up: A, down: B). Comparison of the two 3D finite element models with experimental measurements. The qualitative behavior is reproduced by both models.

The homogeneous model is less sensitive to the mesh resolution and therefore requires less computation time than the detailed approach (15 minutes compared to 25-30 minutes). In both methods however, handling the nonlinear permeability is challenging. Usually, mixed formulations involving the reluctivity $\nu = 1/\mu$ instead of the permeability demonstrate a better efficiency [1]. In this configuration however, the high surface-to-volume ratio in the detailed model or the anisotropic material properties in the homogeneous approach require a careful implementation of the mixed formulations to maintain their efficiency and accuracy. We will present the associated analysis and a comparison with the h-ϕ-formulation in the full paper, together with a more detailed verification.

V. Conclusion

We presented two methods for evaluating the magnetic shielding properties of a stack of superconducting tapes with ferromagnetic substrates. In the full paper, we will extend the study to an axial field configuration (2D-axisymmetric models). We will also assess the influence of a field-dependent critical current density on the shielding factors curves. In addition, we will evaluate the relevance of mixed formulations in this problem.

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