Graph-**B**ased **O**ptimization **M**odeling **L**anguage (GBOML)

TotalEnergies

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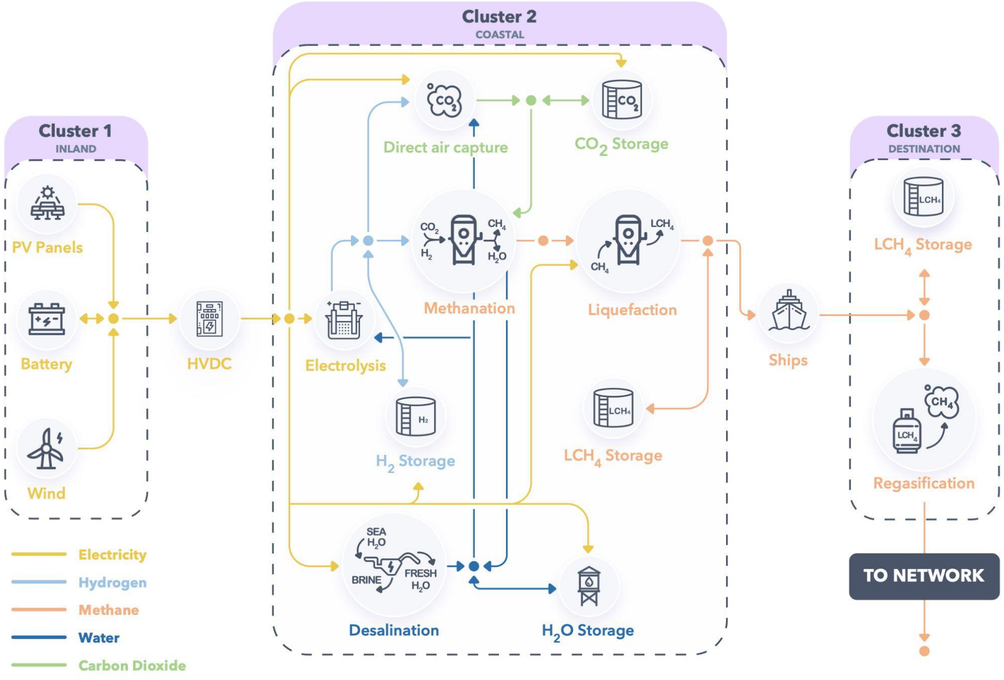
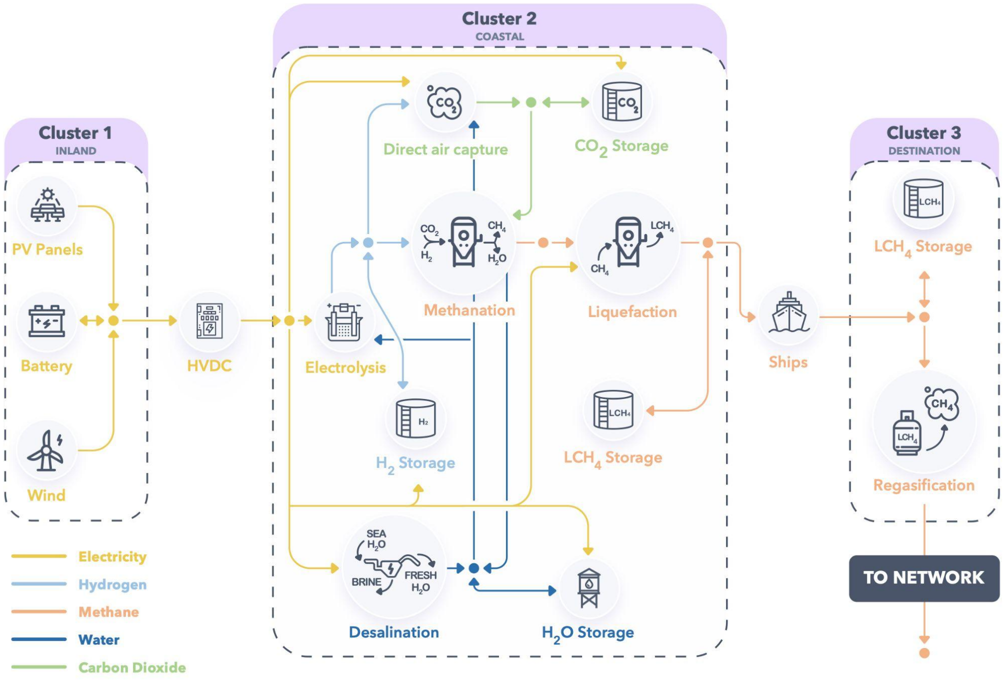
Many energy system planning and control problems can be formulated as mathematical programs



For example, we may want to *design* and *operate* a system producing synthetic fuel so as to minimise its cost

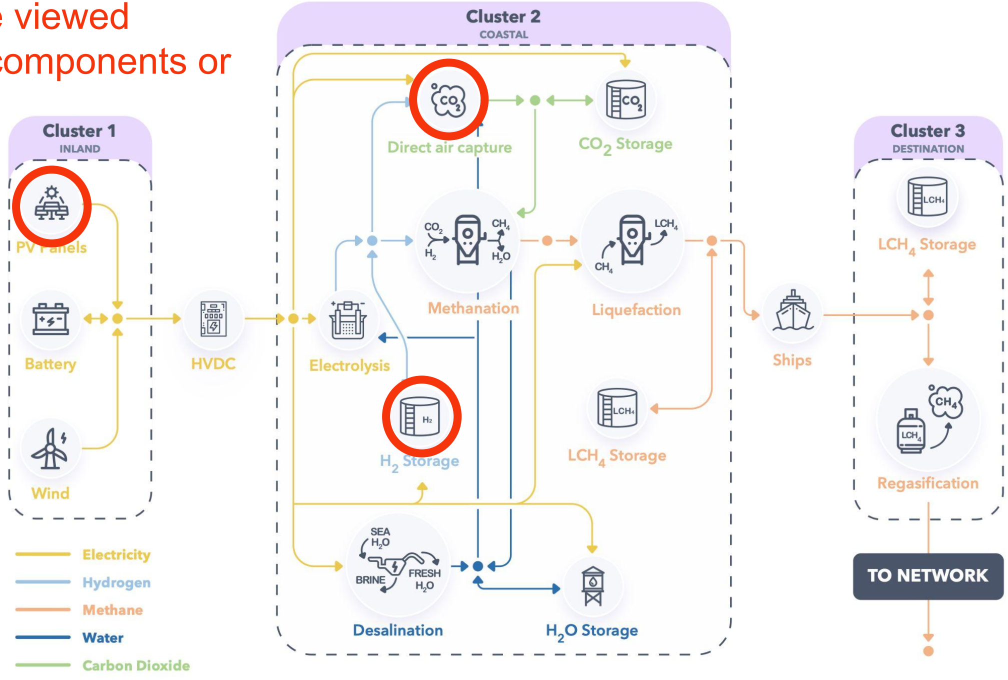
$$
\begin{aligned}
\min \quad & [\text{investment costs}] + [\text{operating costs}] \\
\text{s.t.} \quad & [\text{system design constraints}] \\
& [\text{operating constraints of subsystems}] \\
& [\text{coupling constraints between subsystems}] \\
& [\text{regulatory and environmental constraints}]
\end{aligned}
$$

Mathematical programs found in energy system planning and control applications have special structure



Notably, the underlying system is often a collection of subsystems whose operation must be optimized over time

Subsystems can be viewed as different model components or blocks



This structure can be represented via a hypergraph abstraction augmented with some concept of time-indexing

$$
\begin{aligned}
\min \quad & \textstyle\sum_{n \in \mathcal{N}} \left[f_0^n(X^n, Z^n) + \sum_{t \in \mathcal{T}} f^n(X^n, Z^n, t) \right] \\
\text{s.t.} \quad & h_k^n(X^n, Z^n, t) = 0, \ \forall t \in \mathcal{T}_k^n, \ k = 1, \dots K^n, \ \forall n \in \mathcal{N} \\
& g_k^n(X^n, Z^n, t) \le 0, \ \forall t \in \bar{\mathcal{T}}_k^n, \ k = 1, \dots \bar{K}^n, \ \forall n \in \mathcal{N} \\
& H^e(Z^e) = 0, \ \forall e \in \mathcal{E} \\
& G^e(Z^e) \le 0, \ \forall e \in \mathcal{E} \\
& X^n \in \mathcal{X}^n, Z^n \in \mathcal{Z}^n, \ \forall n \in \mathcal{N}.
\end{aligned}
$$

Sets of *nodes*, *hyperedges* and *time periods* define the hypergraph and time-indexed structure

set of nodes

set of time periods

$$
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\end{aligned}
$$

set of hyperedges

We distinguish between *internal* variables belonging to nodes and *coupling* variables used to link nodes

internal variables coupling variables

$$
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\end{aligned}
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Each node has its own objective and constraints while hyperedges implement coupling constraints

$$
\begin{aligned}
\min\quad & \sum_{n\in\mathcal{N}}\left[\overbrace{f_0^n(X^n,Z^n)+\sum_{t\in\mathcal{T}} f^n(X^n,Z^n,t)}^{\text{node objective}}\right]\\
\text{s.t.}\quad & \left.\begin{array}{l} h_k^n(X^n,Z^n,t)=0,\ \forall t\in\mathcal{T}_k^n,\ k=1,\dots K^n,\ \forall n\in\mathcal{N}\\ g_k^n(X^n,Z^n,t)\le 0,\ \forall t\in\bar{\mathcal{T}}_k^n,\ k=1,\dots \bar{K}^n,\ \forall n\in\mathcal{N}\end{array}\right\}\text{node constraints}\\
& \left.\begin{array}{l} H^e(Z^e)=0,\ \forall e\in\mathcal{E}\\ G^e(Z^e)\le 0,\ \forall e\in\mathcal{E}\end{array}\right\}\text{coupling constraints}\\
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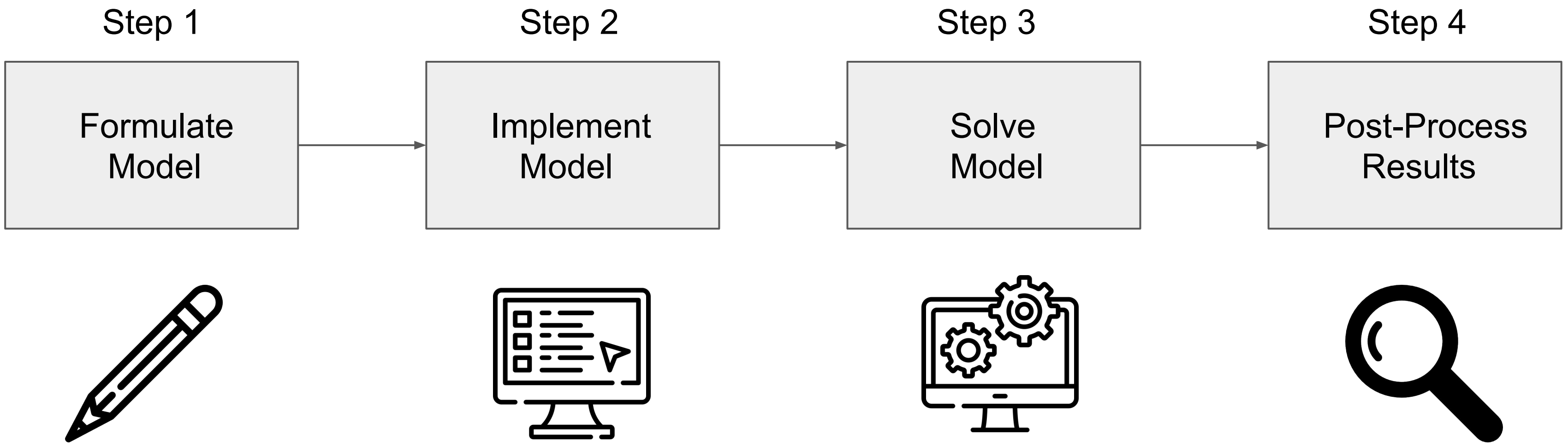
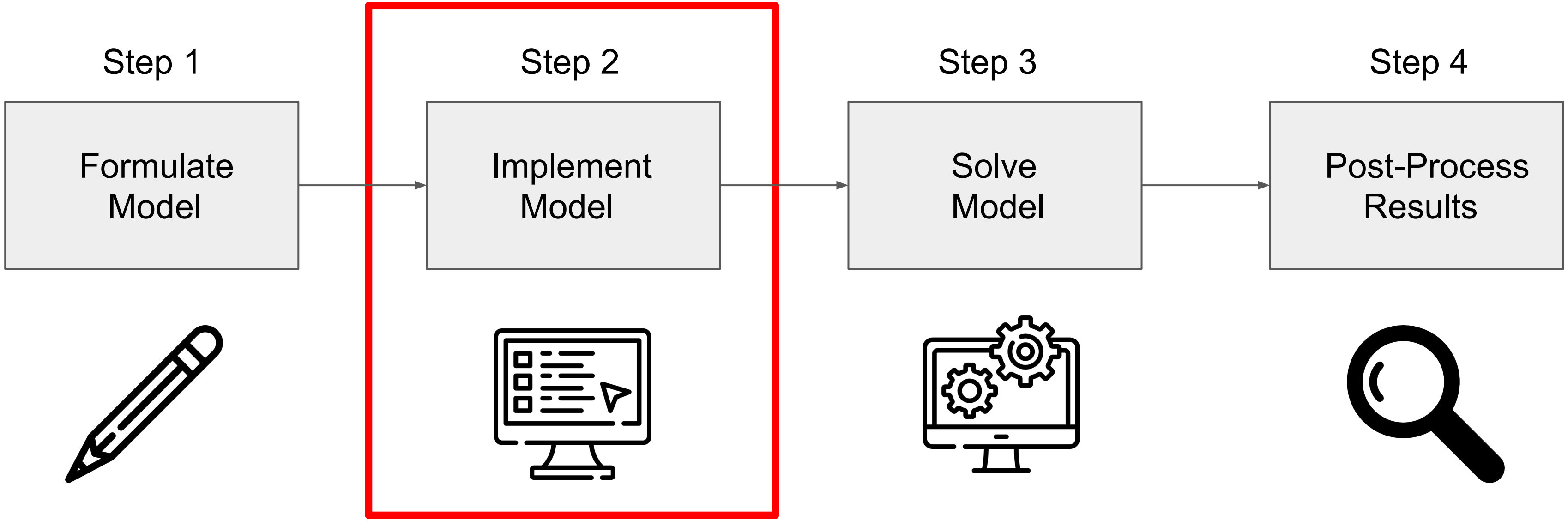
Working with optimization models involves at least four basic steps



We will focus on the second step: model encoding and implementation



Two classes of tools are available to implement models

1. Algebraic Modeling Languages (AMLs):

- **Formulation** close to **mathematical notation** (e.g., index-based notation)
- Very **expressive** (e.g., can represent any mixed-integer nonlinear program)
- Often interface with **multiple solvers**
- Sometimes **open source**
- Examples :

Two classes of tools are available to implement models

2. Object-Oriented Modeling Environments (OOMEs):

- Focus on **one** particular **application** (e.g., generation expansion planning)
- Usually make use of **predefined components** that can be "imported"
- Typically have advanced **data processing** capabilities tailored to application
- Often **open source**
- Examples :

Each approach has drawbacks

AMLs typically fail to **expose** or **exploit** block **structure**, although this may be used to:

- simplify model encoding
- enable model re-use
- speed up model generation
- facilitate the use of structure-exploiting algorithms

OOMEs, for their part:

- Lack **expressiveness**
- Often **cumbersome** to add **new components**
- Usually **rely** on **AMLs** and inherit any **shortcomings**

GBOML combines the strengths of AMLs and OOMEs

- open-source and stand-alone

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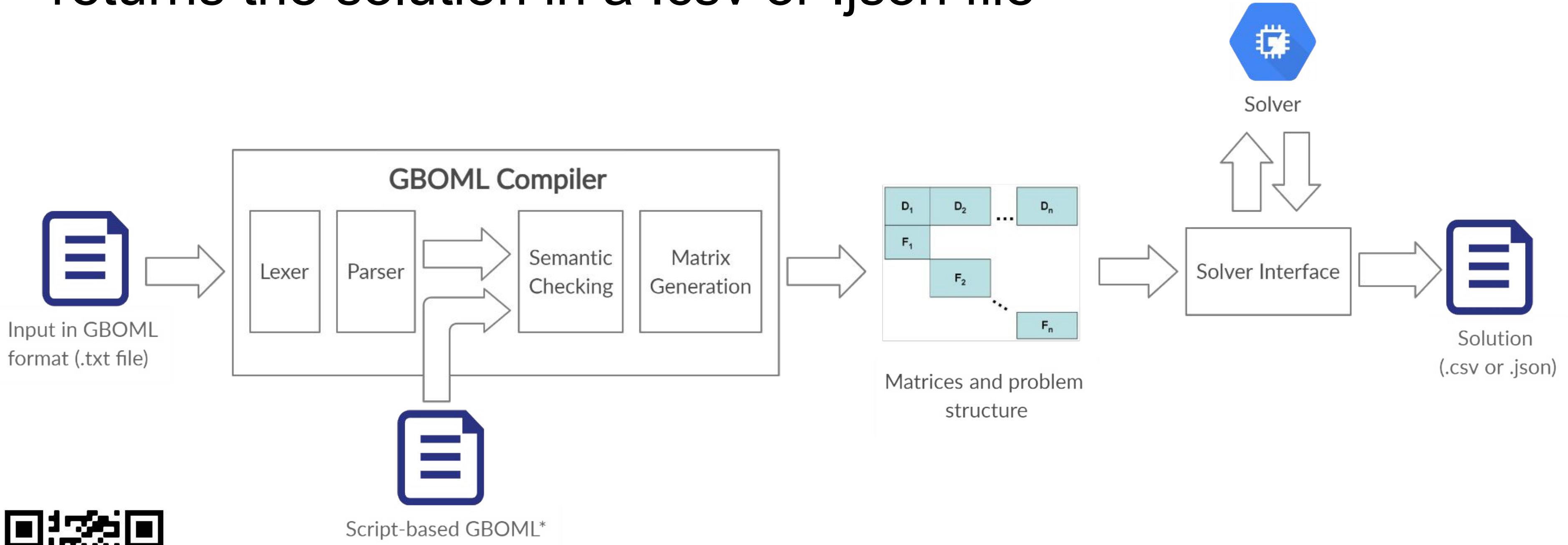
GBOML is a modeling language with its own parser

- Software developed in Python:
  - Has very **few dependencies** (PLY, NumPy, SciPy)
  - Provides two methods to **encode** models (**file** and Python **API**)
  - Interfaces with **several solvers** (Gurobi, CPLEX, Xpress, Cbc/Clp, HiGHS and DSP)
  - Produces plain **.csv** or structured **.json** outputs

- Model structure is exploited on multiple levels:
  - Model **encoding** via dedicated language constructs
  - Model **generation** via parallelism and multiprocessing
  - **Solving** via structure-exploiting solvers such as DSP

GBOML takes inputs from a file or Python script and returns the solution in a .csv or .json file



A Jupyter notebook has been prepared to illustrate how GBOML can be used

References

[1] Mathias Berger et al., (2021). “Remote Renewable Hubs for Carbon-Neutral Synthetic Fuel Production”, in Frontiers in Energy Research 9, p.200. DOI 10.3389/fenrg.2021.671279. https://www.frontiersin.org/article/10.3389/fenrg.2021.671279

[2] Miftari et al., (2022). GBOML: Graph-Based Optimization Modeling Language. Journal of Open Source Software, 7(72), 4158, https://doi.org/10.21105/joss.04158

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[4] Freepik Icons (used on slides 10-11): https://www.flaticon.com/authors/freepik