HEC LIÈGE
Management School - Liège Université
QuantOM

Leloup Emeline
Paquay Célia
Pironet Thierry

Odysseus 2022, Tangier, Morocco
May 6, 2022

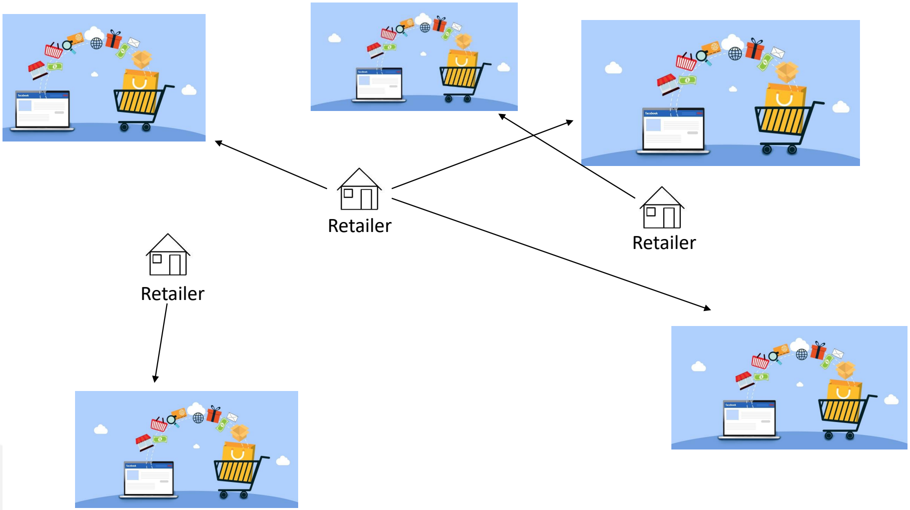
A Capacitated Vehicle Routing Problem with pickups, Time Windows and 3D packing constraints: a mathematical formulation

Outline

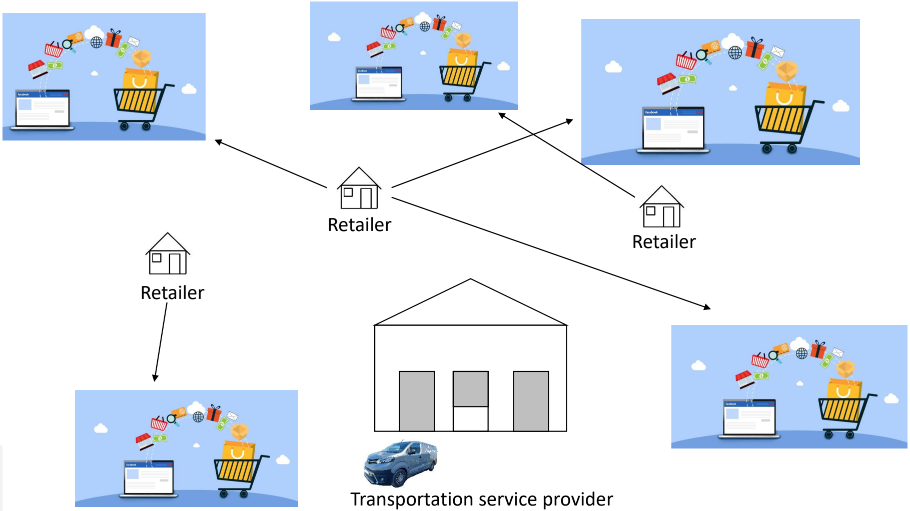
Outline

Introduction

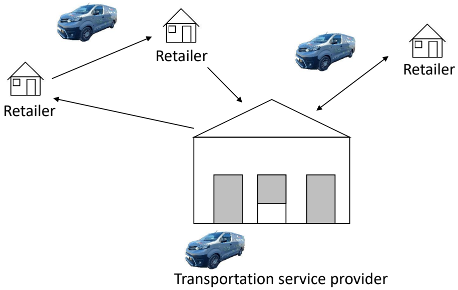
Introduction



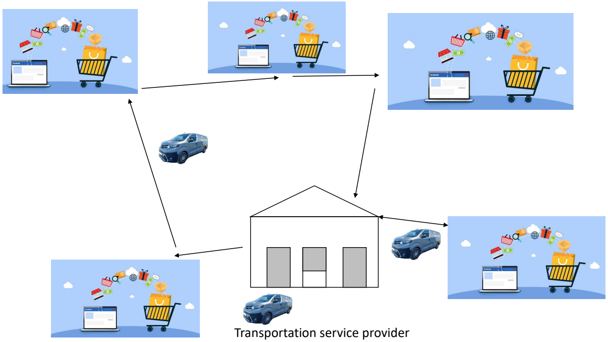
Introduction



Day 1: Pickup



Day 2: Delivery



Outline

Problem definition: practical survey

8 Belgian transportation service providers:

- unknow dimensions

Problem definition: practical survey

8 Belgian transportation service providers:

- unknow dimensions
- rectangular boxes

Problem definition: practical survey

8 Belgian transportation service providers:

- unknow dimensions
- rectangular boxes
- large time windows

Problem definition: practical survey

8 Belgian transportation service providers:

- unknow dimensions
- rectangular boxes
- large time windows
- split pickup

Problem definition: practical survey

8 Belgian transportation service providers:

- unknow dimensions
- rectangular boxes
- large time windows
- split pickup
- outsourcing (administrative burden)

Problem definition: practical survey

8 Belgian transportation service providers:

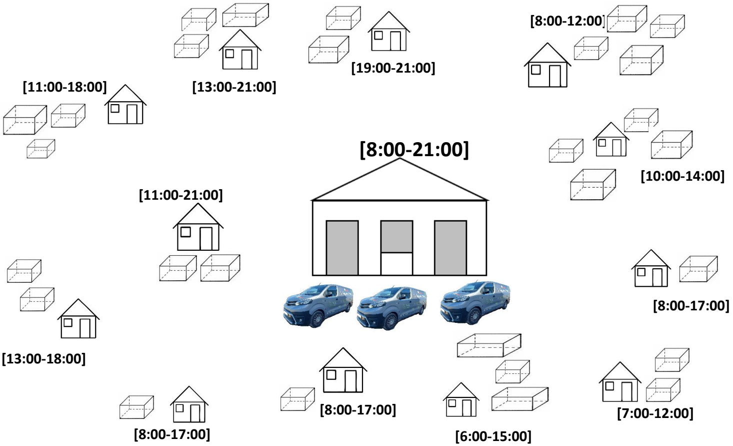
- unknow dimensions
- rectangular boxes
- large time windows
- split pickup
- outsourcing (administrative burden)

3L-CVRPTW with pickup operations, split pickups and possible outsourcing of some customers' requests

---

The problem is $\mathcal{NP}$-hard since it combines two $\mathcal{NP}$-hard problems: the Capacitated Vehicle Routing Problem and the 3D Loading Problem.

Problem definition: objective



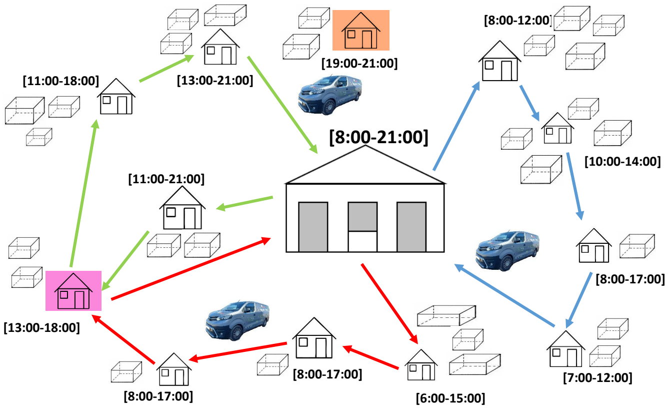
Objective function: minimise total cost while responding to all requests

Routing constraints:

- Each route starts and ends at the depot
- Each vehicle may leave the depot at most once

Time constraints:

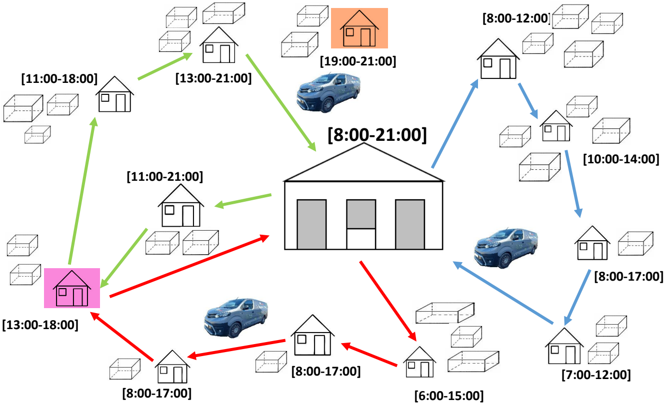
- Duration to complete a route does not exceed the maximum driver working duration
- Pickup operations must occur within the customer's time windows



Customer satisfaction: Every customer should have his boxes transported either by a vehicle of the SP or by a subcontractor.

Split pickup is allowed

Outsourced customer → all his boxes must be loaded by the subcontractor; penalty costs per outsourced customer



Problem definition: loading constraints (Bortfeldt and Wäscher (2013))

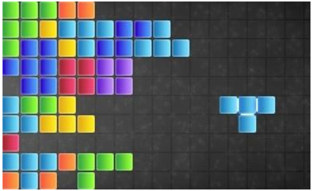
AT EACH CUSTOMER LOCATION

Weight capacity constraint

Geometric constraints

Horizontal 90°-rotation constraints
Fragility constraints

Multi-load constraints



Stability constraints → **Vertical** / **Horizontal/dynamic**

Problem definition: stability constraints

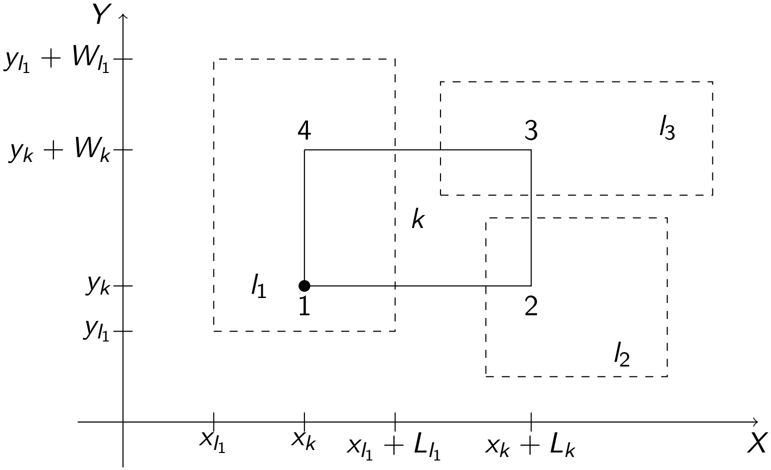


Figure: Example of four corners of a box k supported by boxes l_1, l_2 and l_3 (dashed lines)

Problem definition: stability constraints

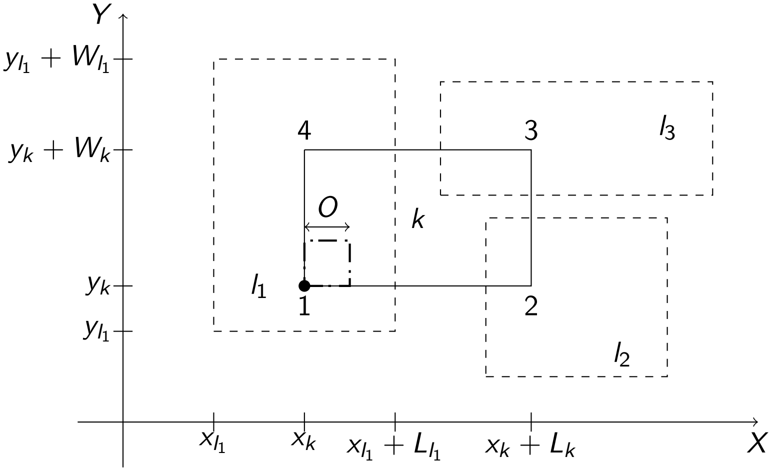


Figure: Example of four corners of a box k supported by boxes l_1, l_2 and l_3 (dashed lines)

Minimise the transportation and outsourcing costs subject to:

- customer satisfaction
- routing constraints
- time constraints
- loading constraints
  - weight capacity constraint
  - geometric constraints
  - vertical stability
  - horizontal 90°-rotation constraints
  - fragility constraints
  - multi-load constraints

Outline

Mathematical formulation: Main decisions

- Set of vehicles: $f \in \{1, ..., F\}$

Mathematical formulation: Main decisions

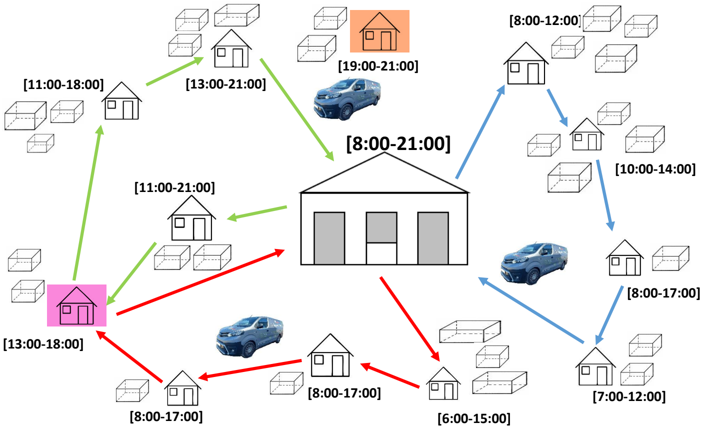
- Set of vehicles: $f \in \{1, ..., F\}$
- Depot: $i = 0$

Mathematical formulation: Main decisions

- Set of vehicles: $f \in \{1, ..., F\}$
- Depot: $i = 0$
- Set of customers: $i \in \{1, ..., N\}$

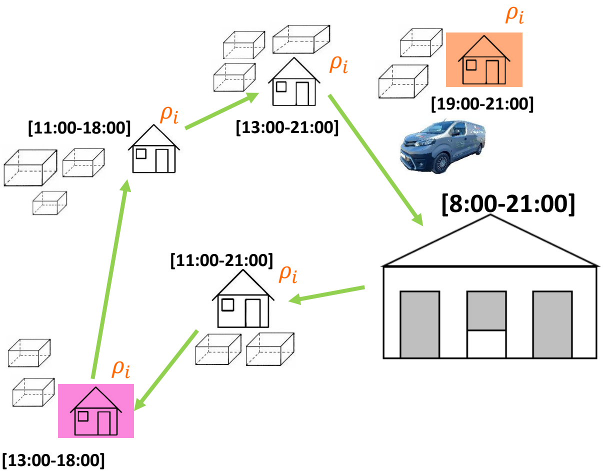
- Set of vehicles: $f \in \{1, ..., F\}$
- Depot: $i = 0$
- Set of customers: $i \in \{1, ..., N\}$
- Set of boxes per customer i: $k_i \in \{1, ..., |\mathcal{I}_i|\}$

Mathematical formulation: Main decisions



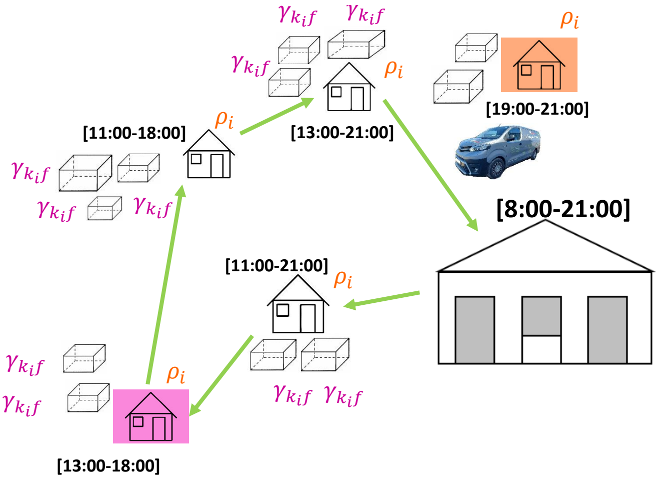
- Set of vehicles: $f \in \{1, ..., F\}$
- Depot: $i = 0$
- Set of customers: $i \in \{1, ..., N\}$
- Set of boxes per customer i: $k_i \in \{1, ..., |\mathcal{I}_i|\}$

Mathematical formulation: Main decisions



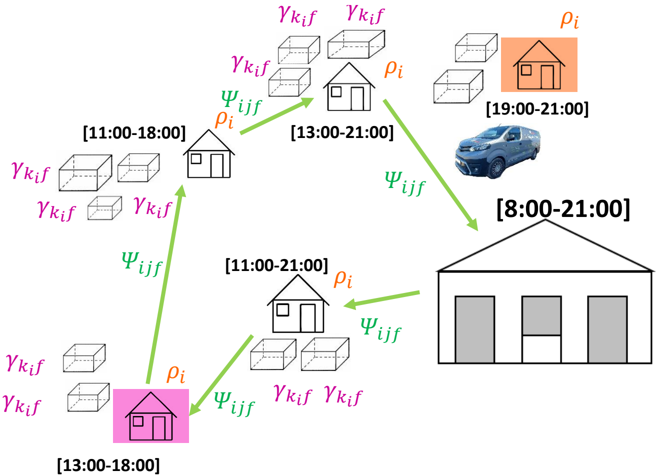
- Set of vehicles: $f \in \{1, ..., F\}$
- Depot: $i = 0$
- Set of customers: $i \in \{1, ..., N\}$
- Set of boxes per customer i: $k_i \in \{1, ..., |\mathcal{I}_i|\}$

Mathematical formulation: Main decisions



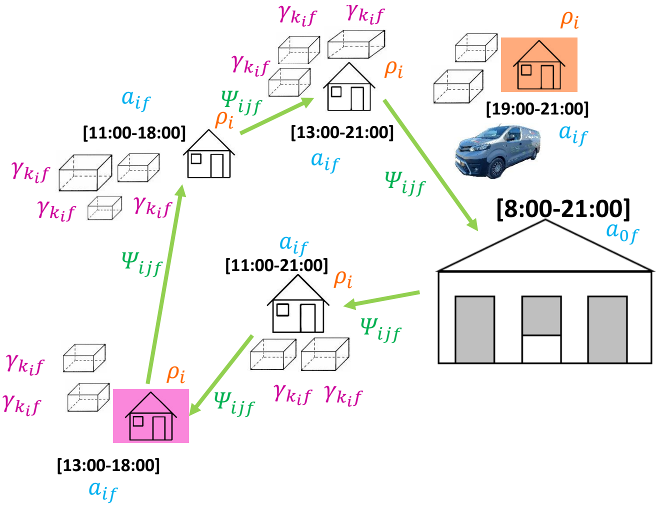
- Set of vehicles: $f \in \{1, ..., F\}$
- Depot: $i = 0$
- Set of customers: $i \in \{1, ..., N\}$
- Set of boxes per customer i: $k_i \in \{1, ..., |\mathcal{I}_i|\}$

Mathematical formulation: Main decisions



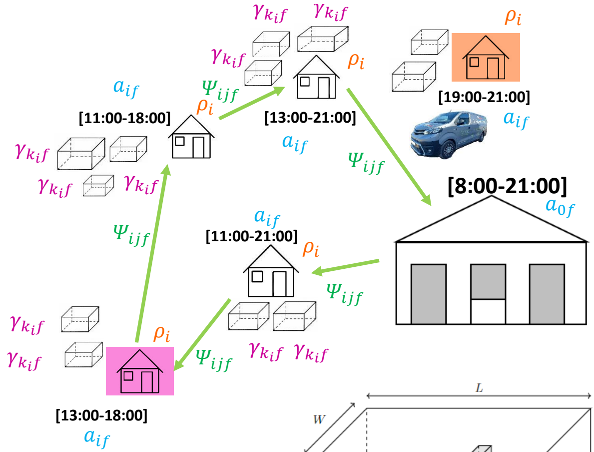
- Set of vehicles: $f \in \{1, ..., F\}$
- Depot: $i = 0$
- Set of customers: $i \in \{1, ..., N\}$
- Set of boxes per customer i: $k_i \in \{1, ..., |\mathcal{I}_i|\}$

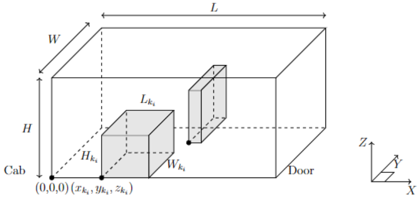
Mathematical formulation: Main decisions



- Set of vehicles: $f \in \{1, ..., F\}$
- Depot: $i = 0$
- Set of customers: $i \in \{1, ..., N\}$
- Set of boxes per customer i: $k_i \in \{1, ..., |\mathcal{I}_i|\}$

Mathematical formulation: Main decisions





- Set of vehicles: $f \in \{1, ..., F\}$
- Depot: $i = 0$
- Set of customers: $i \in \{1, ..., N\}$
- Set of boxes per customer i: $k_i \in \{1, ..., |\mathcal{I}_i|\}$

Mathematical formulation: Constraints I

Objective function: minimise the transportation and outsourcing costs of the service provider

$$\min \underbrace{\sum_{f=1}^{F}\sum_{i=0}^{N}\sum_{\substack{j=0\\ j\neq i}}^{N} C_{ij}\Psi_{ijf}}_{\text{transportation costs}} + \underbrace{\sum_{i=1}^{N} P_i\rho_i}_{\text{outsourcing costs}}$$

Customer satisfaction: $\forall$ customer i and box k_i of customer i

$$\sum_{f=1}^{F} \gamma_{k_i f} = 1 - \rho_i$$

Routing constraints:

$$\sum_{\substack{i=0\\i\neq j}}^{N} \Psi_{ijf} = \sum_{\substack{l=0\\l\neq j}}^{N} \Psi_{jlf} \qquad \forall f \in \{1, ..., F\}, j \in \mathcal{V} \qquad \text{flow conservation}$$

$$\sum_{j=1}^{N} \Psi_{0jf} \leq 1 \qquad \forall f \in \{1, ..., F\} \qquad \text{no multi-trip}$$

$$\sum_{\substack{j=0\\j\neq i}}^{N} \Psi_{ijf} \geq \gamma_{k_i f} \qquad \forall f \in \{1, ..., F\}, i \in \mathcal{V} \setminus \{0\}, k_i = 1, ..., |\mathcal{I}_i| \qquad \text{visit if loaded}$$

Mathematical formulation: Constraints III

Time constraints (I): Pickup operations must occur within the customer's time-windows

$$A_i \leq a_{if} \qquad \forall f \in \{1, \ldots, F\}, i \in \mathcal{V} \qquad \text{earliest arrival}$$

$$a_i + \sum_{\substack{j=0 \\ j \neq i}}^{N} S_i \Psi_{ijf} \leq A_i + (B_i - A_i) \sum_{\substack{j=0 \\ j \neq i}}^{N} \Psi_{ijf} \qquad \forall f \in \{1, \ldots, F\}, i \in \mathcal{V} \qquad \text{latest arrival}$$

$$a_{if} + \sum_{\substack{j=0 \\ j \neq i}}^{N} S_i \Psi_{ijf} + T_{i0} \leq B_0 \qquad \forall f \in \{1, \ldots, F\}, i \in \mathcal{V} \setminus \{0\} \qquad \text{return depot}$$

Time constraints (II):

$$a_{if} + \sum_{\substack{j=0 \\ j\neq i}}^{N} S_i \Psi_{ijf} + T_{i0} - a_{0f} \leq \Delta + (B_0 - A_0)(1 - \sum_{\substack{j=0 \\ j\neq i}}^{N} \Psi_{ijf}) \qquad \forall f \in \{1, ..., F\}, i \in \mathcal{V} \setminus \{0\}$$

maximum working duration

$$a_{if} + S_i \Psi_{ijf} + T_{ij} - a_{jf} \leq \mathbf{M'}(1 - \Psi_{ijf}) \qquad \forall f \in \{1, ..., F\}, i, j \in \mathcal{V} \setminus \{0\}, i \neq j$$

$$a_{0f} + T_{0j} - a_{jf} \leq \mathbf{M'}(1 - \Psi_{0jf}) \qquad \forall f \in \{1, ..., F\}, j \in \mathcal{V} \setminus \{0\}$$

sequencing

Weight capacity constraint:

$$\sum_{i=1}^{N} \sum_{k_i=1}^{|\mathcal{I}_i|} M_{k_i} \gamma_{k_i f} \leq M \quad \forall f \in \{1, ..., F\}$$

- Geometric constraints
- Vertical stability
- Horizontal 90°-rotation constraints
- Fragility constraints
- Multi-load constraints

Outline

Instances

- Combined the benchmark instances from Solomon (1987) and Bortfeldt and Yi (2020)

Instances

- Combined the benchmark instances from Solomon (1987) and Bortfeldt and Yi (2020)
- Generated 10 instances for 5, 10, 15, 20 customers respectively for small and large time windows

Instances

- Combined the benchmark instances from Solomon (1987) and Bortfeldt and Yi (2020)
- Generated 10 instances for 5, 10, 15, 20 customers respectively for small and large time windows
- On average 2 boxes per customer

Instances

- Combined the benchmark instances from Solomon (1987) and Bortfeldt and Yi (2020)
- Generated 10 instances for 5, 10, 15, 20 customers respectively for small and large time windows
- On average 2 boxes per customer
- On average 11%-12% of fragile boxes

Instances

- Combined the benchmark instances from Solomon (1987) and Bortfeldt and Yi (2020)
- Generated 10 instances for 5, 10, 15, 20 customers respectively for small and large time windows
- On average 2 boxes per customer
- On average 11%-12% of fragile boxes
- 3 vehicles, weight capacity 1200kg

The linear formulation is implemented in Java using IBM ILOG CPLEX 12.10 library as Branch-and-Bound (B&B) solver. Tests were performed on a workstation with a computation time limit of **one hour** for every instance run.

			Number of customers (N)			
			5	10	15	20
Small TW	Instances solved at optimality [%]		100.00	90.00	30.00	0.00
	Time [sec.]	Mean (sd.)	0.19 (0.07)	392.05 (1140.90)	533.72 (328.34)	/
	GAP [%]	Mean (sd.)	/	84.94 (0.00)	84.06 (15.75)	96.75 (2.19)
Large TW	Instances solved at optimality [%]		100.00	90.00	10.00	0.00
	Time [sec.]	Mean (sd.)	1.41 (2.51)	685.48 (1099.04)	3461.55 (0.00)	/
	GAP [%]	Mean (sd.)	/	6.47 (0.00)	94.26 (4.55)	97.81 (0.84)

Table: Evolution of the computational time and percentage of instances solved at optimality ($F = 3$)

Objective function: minimise the transportation and outsourcing costs of the service provider

$$\min \underbrace{\sum_{f=1}^{F}\sum_{i=0}^{N}\sum_{\substack{j=0\\ j\neq i}}^{N} C_{ij}\Psi_{ijf}}_{\text{transportation costs}} + \underbrace{\sum_{i=1}^{N} P_i\rho_i}_{\text{outsourcing costs}}$$

		Number of customers (N)			
		5	10	15	20
Small TW	Instances solved at optimality [%]	100.00	90.00	30.00	0.00
	Final solution outsourcing all customers [%]	0.00	0.00	10.00	80.00
Large TW	Instances solved at optimality [%]	100.00	90.00	10.00	0.00
	Final solution outsourcing all customers [%]	0.00	0.00	30.00	80.00

Table: Evolution of the percentage of outsourcing ($F = 3$)

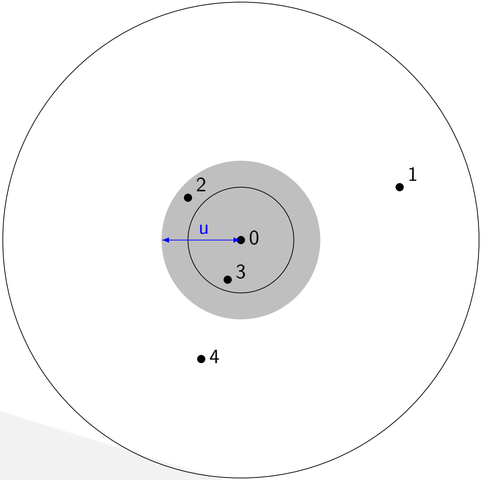
Outline

Insert-and-Fix

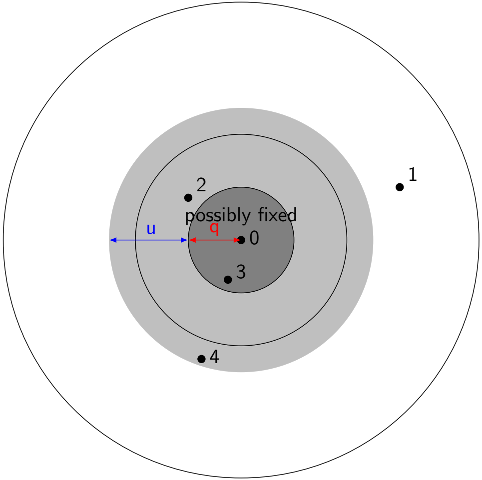
- decompose the problem into smaller subproblems
- sequential routing and packing
- adaptative to some disruptions

Constructive matheuristic: Insert-and-Fix II

Step 1:

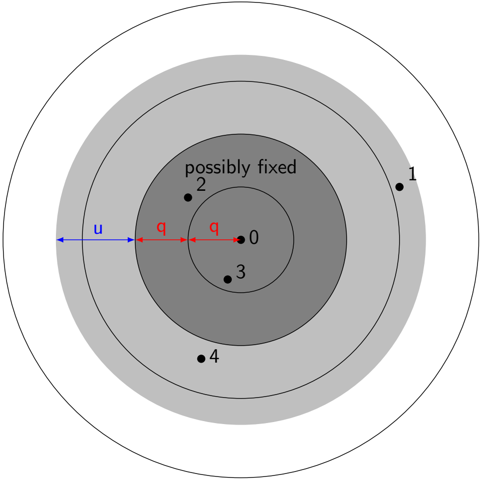


Step 2:



Constructive matheuristic: Insert-and-Fix III

Step 3:



Constructive matheuristic: sorting methods and decision policies

Sorting methods: are related to the way customers are added

Sorting method	Distance A_i	Distance Polar angle	A_i Polar angle	Distance depot Distance customers

Table: Some possible sorting methods

Decision policy: is related to the decision policy used to fix variables

Decision policy	$\gamma_{k_i f} = 1$ $(x_{k_i}, y_{k_i}, z_{k_i})$	$\rho_i = 0$	$\gamma_{k_i f} = 1$	Ψ_{jif} s.t. $\rho_i = 0$

Table: Some possible decision policies

Outline

Conclusion and future work

- Complete mathematical formulation
- Computational limitations

Perspectives:

- Constructive matheuristic: Insert-and-Fix
- Use the solution from the I&F as initial solution in CPLEX or in an improvement heuristic
- Disruptions occurring during the day


MOTOROLA
verizon
Doktor-Karl-Renner-Ring
50
JOSEFSTADT
HERNALS
WÄHRING
ALSERGRUND
Museumstraße
Rathausplatz
Rathauspark
Grete-Rehor-Park
Volksgarten
Heldenplatz
1.6km
30km/h
200
metres
11:06
1.6km


isotoner
victoria

THANK YOU

HEC LIÈGE
Management School - Liège Université
QuantOM

Odysseus 2022, Tangier, Morocco
May 6, 2022

Leloup Emeline
Paquay Célia
Pironet Thierry

A Capacitated Vehicle Routing Problem with pickups, Time Windows and 3D packing constraints: a mathematical formulation

Bortfeldt, A., & Wäscher, G. (2013). Constraints in container loading–a state-of-the-art review. European Journal of Operational Research, 229(1), 1–20.

Bortfeldt, A., & Yi, J. (2020). The split delivery vehicle routing problem with three-dimensional loading constraints. European Journal of Operational Research, 282(2), 545–558.

Solomon, M. M. (1987). Algorithms for the vehicle routing and scheduling problems with time window constraints. Operations research, 35(2), 254–265.